

# Performance analysis of dual selection-based macrodiversity system over channels subjected to Nakagami- $m$ fading and gamma shadowing

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*Abstract:* - This paper studies wireless communication system following microdiversity to mitigate the effects of short-term fading and macrodiversity processing to reduce shadowing effects.  $N$ -branch maximal-ratio combining (MRC) is implemented at the micro level (single base station) and selection combining (SC) with two base stations (dual diversity) is implemented at the macro level. Model in the paper assumes a Nakagami- $m$  density function for the envelope of the received signal and a gamma distribution to model the average power to account for shadowing. Analytical expressions for the probability density function (PDF), cumulative distribution function (CDF) and moments of signal after micro- and macrodiversity processing are derived. These expressions are used to study important system performance criteria such as the outage probability, average bit error probability (ABEP), average output signal value and amount of fading (AoF). Various numerical results are graphically presented to illustrate the proposed mathematical analysis and to show the effects of various system parameters to the system performance, as well as enhancement due to use of the combination of micro- and macrodiversity.

*Key-Words:* - Gamma shadowing, Macrodiversity, Microdiversity, Nakagami- $m$  fading.

## 1 Introduction

In wireless communication systems, the received signal can be exposed to both short-term fading, which is the result of multipath propagation, and long-term fading (shadowing), which is the result of large obstacles and large deviations in terrain profile between transmitter and receiver [1]. The reliability of communication over the wireless channels can be improved using diversity techniques, such as space diversity techniques [2], [3]. The most popular space diversity techniques are selection combining (SC), equal-gain combining (EGC) and maximal-ratio combining (MRC). MRC is optimal combining technique in the sense that it achieves the best performance regardless of the fading statistics on the diversity branches. However, MRC requires the knowledge of the channel fading amplitudes and phases of each diversity branch which must be continuously estimated by the receiver. These estimations require separate receiver chain for each branch of the diversity system increasing its

complexity. EGC provides performance comparable to MRC, but with simpler implementation complexity. EGC does not require the estimation of the channel fading amplitudes since it combines signals from all branches with the same weighting factor. SC is the least complicated technique. It is reposed on processing only one of the diversity branches. SC combiner chooses the branch with the highest signal value. Diversity techniques at single base station (microdiversity) reduce the effects of short-term fading. Impairments due to shadowing can be mitigated using macrodiversity techniques which employ the processing of signals from multiple base stations. The use of composite micro- and macrodiversity has received considerable interest due to the fact that it simultaneously combats both short-term fading and shadowing.

Rayleigh, Rician and Nakagami- $m$  statistical models are the most frequently used to describe fading envelope of the received signal. The Rayleigh distribution is frequently used to model

multipath fading with no direct line-of-sight (LOS) path. The Rician distribution is often used to model propagation paths consisting of one strong direct LOS component and many random weaker components. The Nakagami- $m$  distribution has gained widespread application in the modeling of physical fading radio channels. The primary justification of the use of Nakagami- $m$  fading model is its good fit to empirical fading data. It is versatile and through its parameter  $m$ , we can model signal fading conditions that range from severe to moderate, to light fading or no fading. It includes the one-sided Gaussian distribution ( $m=0.5$ ) and the Rayleigh distribution ( $m=1$ ) as special cases.

The average power, which is random variable due to shadowing, is usually modeled with lognormal distribution. A composite multipath/shadowed fading environment modeled either as Rayleigh-lognormal, Rician-lognormal or Nakagami-lognormal is considered in [4]-[7]. Unfortunately, the use of lognormal distribution to model the average power does not lead to a closed form solution for the probability density function (PDF) of the signal envelope at the receiver. This makes the analysis of system in shadowed fading environment very ponderous. Based on theoretical results and measured data, it was shown that gamma distribution does the job as well as lognormal [8], [9]. A compound fading model incorporates both short-term fading and shadowing which is modeled using gamma distribution instead of lognormal distribution [9]-[16]. Such an approach provides analytical solution for the PDF of the output signal facilitating the analysis of wireless systems.

In this paper, the system following micro- and macrodiversity reception in correlated gamma shadowed Nakagami- $m$  fading channels is analyzed. Analytical expressions for the PDF, cumulative distribution function (CDF) and moments of the output signal are derived. These analytical expressions can be used to obtain important performance measures, such as the outage probability, average bit error probability (ABEP), average signal value and amount of fading (AoF). Numerical results are graphically presented to show the effects of fading severity, shadowing severity, number of diversity branches at the micro level and correlation coefficient on system performance, as well as enhancement due to use of the combination of micro- and macrodiversity.

## 2 Problem Formulation

System model considered in the paper is shown in Fig. 1.  $N$ -branch MRC receiver is implemented at

the micro level (single base station) and SC receiver which involves the use of two geographically distributed base stations (dual diversity) is implemented at the macro level. The improvement in performance gained through the use of multiple antennas at the single base station is larger when the signals corresponding to each antenna are approximately independent, i.e. when the separation between antennas is on the order of one half of a wavelength [3]. Two base stations are treated to have nonzero correlation. It is realistic scenario because shadowing has a larger correlation distance and it is difficult to ensure that base stations operate independently, especially in microcellular systems [5].

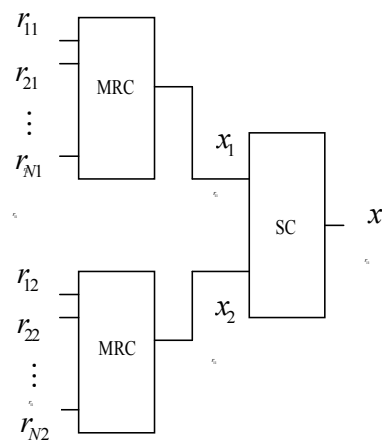


Fig. 1 System model

The PDF of the signal received by the  $i$ th antenna at the  $j$ th base station in the presence of Nakagami- $m$  fading is

$$p_{r_{ij}}(r_{ij}) = \frac{2m^m r_{ij}^{2m-1}}{\Gamma(m)\Omega_j^m} \exp\left(-\frac{m}{\Omega_j} r_{ij}^2\right), \quad i = \overline{1, N}, j = 1, 2 \quad (1)$$

where  $\Gamma(\cdot)$  is gamma function,  $\Omega_j$  is the average power of the signal at the  $j$ th base station and  $m$  is Nakagami fading parameter which describes fading severity ( $m \geq 0.5$ ). As parameter  $m$  increases, the fading severity decreases. After transformation  $x_{ij} = r_{ij}^2$ , (1) becomes

$$p_{x_{ij}}(x_{ij}) = \frac{m^m x_{ij}^{m-1}}{\Gamma(m)\Omega_j^m} \exp\left(-\frac{m}{\Omega_j} x_{ij}\right), \quad i = \overline{1, N}, j = 1, 2. \quad (2)$$

The result signal at the MRC output of the  $j$ th base station is the sum of squared envelopes of Nakagami- $m$  faded signals,  $x_j = \sum_{i=1}^N r_{ij}^2$ , or equivalently,  $x_j = \sum_{i=1}^N x_{ij}$  with PDF given by [17, (71)]

$$p_{x_j}(x_j|y_j) = \frac{x_j^{M-1} M^M}{\Gamma(M) y_j^M} \exp\left(-\frac{M}{y_j} x_j\right), \quad j=1,2 \quad (3)$$

where  $y_j$  is the total input power ( $y_j=N\Omega_j$ ) and  $M=Nm$ .

The conditional nature of the PDF in (3) reflects the existence of shadowing with  $y_j$  being random variable. The joint PDF of  $y_1$  and  $y_2$  follows the correlated gamma distribution [18], [19]

$$p_{y_1 y_2}(y_1, y_2) = \frac{\rho^{\frac{c-1}{2}} (y_1 y_2)^{\frac{c-1}{2}}}{\Gamma(c)(1-\rho) y_0^{c+1}} \exp\left(-\frac{y_1 + y_2}{(1-\rho) y_0}\right) \times I_{c-1}\left(\frac{2\sqrt{\rho y_1 y_2}}{(1-\rho) y_0}\right) \quad (4)$$

where  $\rho$  is the correlation between  $y_1$  and  $y_2$ ,  $c$  is the order of gamma distribution,  $y_0$  is related to the average power of  $y_1$  and  $y_2$  and  $I_n(\cdot)$  is the first kind and  $n$ th order modified Bessel function. The severity of gamma shadowing is measured in terms of  $c$ . The lower value of  $c$  means the higher shadowing while the value of  $c = \infty$  corresponds to a pure short-term fading channel. The relationship between the parameter  $c$  and standard deviation  $\sigma$  of shadowing in dB in the lognormal shadowing exists through [9,(6)],  $\sigma(dB) = 4.3429\sqrt{\psi'(c)}$ , where  $\psi'(\cdot)$  is the trigamma function. The typical values of  $\sigma$  are between 2-12 dB.

Selection diversity is applied at the macrolevel. Namely, the base station with the larger average power is selected to provide service to the user. Using the concepts of probability, the PDF of the signal after diversity combining at the micro- and macrolevel can be derived as

$$p_x(x) = \int_0^\infty dy_1 \int_0^{y_1} p_{x_1}(x|y_1) p_{y_1 y_2}(y_1, y_2) dy_2 + \int_0^\infty dy_2 \int_0^{y_2} p_{x_2}(x|y_2) p_{y_1 y_2}(y_1, y_2) dy_1 = 2 \int_0^\infty p_{x_1}(x|y_1) dy_1 \int_0^{y_1} p_{y_1 y_2}(y_1, y_2) dy_2. \quad (5)$$

which, by substituting (3) and (4) and using [20, (8.445), (3.381/2) and (3.471/9)], yields [15]

$$p_x(x) = \frac{1}{\Gamma(M)\Gamma(c)} \sum_{n,k=0}^{\infty} \frac{\rho^n M^{\frac{n+c+k+M}{2}}}{2^{n+c+\frac{k-M}{2}-2} (1-\rho)^{\frac{n+k+M}{2}}} \times \frac{x^{\frac{n+c+k+M}{2}-1}}{n! \Gamma(n+c) y_0^{\frac{n+c+k+M}{2}} \prod_{l=0}^k (n+c+l)} \times K_{2(n+c)+k-M} \left( 2 \sqrt{\frac{2Mx}{y_0(1-\rho)}} \right). \quad (6)$$

The CDF of signal at the  $j$ th base station output is

$$F_{x_j}(x_j|y_j) = \int_0^{x_j} p_{x_j}(t|y_j) dt, \quad j=1,2. \quad (7)$$

Substituting (3) in (7) and using [20, (3.381/2)], the CDF of  $x_j$  is

$$F_{x_j}(x_j|y_j) = \frac{1}{\Gamma(M)} \exp\left(-\frac{Mx_j}{y_j}\right) \sum_{k=0}^{\infty} \frac{1}{\prod_{l=0}^k (M+l)} \left(\frac{Mx_j}{y_j}\right)^{M+k}. \quad (8)$$

The CDF of signal at the output of a dual-port selection based macrodiversity can be obtained as

$$F_x(x) = \int_0^\infty dy_1 \int_0^{y_1} F_{x_1}(x|y_1) p_{y_1 y_2}(y_1, y_2) dy_2 + \int_0^\infty dy_2 \int_0^{y_2} F_{x_2}(x|y_2) p_{y_1 y_2}(y_1, y_2) dy_1 = 2 \int_0^\infty F_{x_1}(x|y_1) dy_1 \int_0^{y_1} p_{y_1 y_2}(y_1, y_2) dy_2 \quad (9)$$

which, by substituting (4) and (8) and using [20, (3.381/2) and (3.471/9)] to solve integrals, becomes [15]

$$F_x(x) = \frac{1}{\Gamma(M)\Gamma(c)} \sum_{n,i,k=0}^{\infty} \frac{\rho^n}{2^{n+c+\frac{i-k-M}{2}-2} (1-\rho)^{\frac{i+k+M}{2}}} \times \frac{(Mx)^{n+c+\frac{i+k+M}{2}}}{n! \Gamma(n+c) y_0^{\frac{n+c+i+k+M}{2}} \prod_{j=0}^i (n+c+j) \prod_{l=0}^k (M+l)} \times K_{2(n+c)+i-k-M} \left( 2 \sqrt{\frac{2Mx}{y_0(1-\rho)}} \right). \quad (10)$$

The  $L$ th order moment of the received signal can be derived as [21, eq. (5-38)]

$$x_L = \varepsilon \langle x^L \rangle = \int_0^\infty x^L p_x(x) dx. \quad (11)$$

Substituting (6) in (11) and using [20, eq. (6.561/16)],  $L$ th moment can be written as

$$x_L = \frac{1}{\Gamma(M)\Gamma(c)} \times \sum_{n,k=0}^{\infty} \frac{\rho^n (1-\rho)^{L+c} y_0^L \Gamma(L+2(n+c)+k) \Gamma(L+M)}{2^{L+2(n+c)+k-1} n! M^L \Gamma(n+c) \prod_{l=0}^k (n+l+c)}. \quad (12)$$

### 3 Performance analysis

#### 3.1 Outage probability

The outage probability,  $P_{out}$ , is one of the widely accepted performance measure for diversity systems operating in fading environment. It is defined as the probability that the output signal value falls bellow a

given outage threshold  $\lambda_{th}$ . The outage threshold is a protection value above which the quality of service (QoS) is satisfactory. The outage probability can be obtained by replacing  $x$  with  $\lambda_{th}$  in (10), i.e.

$$P_{out} = F_x(\lambda_{th}). \tag{13}$$

Expression for the outage probability is in the form of the nested infinite multiple sums. But, it converge rapidly, and therefore, it can be used efficiently. As an indicative example, the convergence rate of expression for the outage probability is examined in Table 1. Table 1 summarizes the number of terms that need to be summed to achieve accuracy at the 4<sup>th</sup> significant digit. The number of the required terms depends strongly on  $\rho$  and  $\lambda_{th}$ . It is obvious that the number of terms increases as  $\rho$  and/or  $\lambda_{th}$  increases.

TABLE 1 The number of terms needed to be summed in (13) to achieve accuracy at the 4<sup>th</sup> significant digit ( $m=1.8, \sigma=3\text{dB}, y_0=5\text{ dB}$  and  $N=2$ )

$\lambda_{th}$ (dB)	$\rho = 0.2$	$\rho = 0.6$
-5	7	12
5	11	15

In Fig. 2, the outage probability versus the normalized outage threshold,  $\lambda_{th}/y_0$ , for different fading and shadowing severity is plotted. As expected, system performance improves as severity of both fading and shadowing decreases (i.e.  $m$  increases and  $\sigma$  decreases). In addition, fading severity has a larger influence on the outage probability of system under a lighter shadowing.

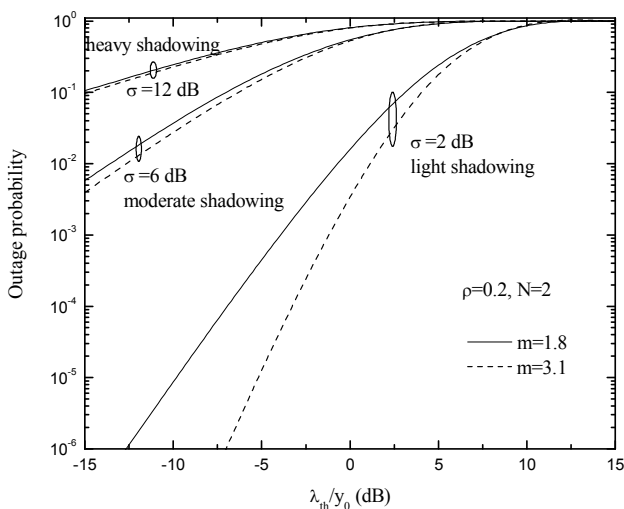


Fig. 2 Outage probability versus normalized outage threshold for different fading and shadowing severity

The outage probability versus the normalized outage threshold for different number of diversity branches and correlation coefficient is plotted in Fig. 3. The system performance is better in the case of larger number of diversity branches at the base stations and for lower values of the correlation coefficient, i.e. for larger spatial separation between base stations. If the correlation is too high, then deep fades in the macrodiversity branches will occur simultaneously resulting in low improvement degree of considered space diversity system.

In Fig. 4, the influence of number of diversity branches at the micro level is in focus. It is obvious that system performance enhancement due to increase of number of diversity branches reduces with increment of that number, i.e. the gap among the adjacent curves decreases as number of branches increases.

In Fig. 5 the influence of average power  $y_0$  is examined. As expected, the outage probability decreases with increase of  $y_0$ .

In Fig. 6, we wanted to see and establish improvement obtained through macrodiversity. For example, for  $m=1.8\text{ dB}, N=2$  and  $\rho=0.2$ , at an  $P_{out}$  of 0.1, the macrodiversity gain for  $\sigma=2\text{ dB}$  is about 1.3 dB and for  $\sigma=6\text{ dB}, 5.4\text{ dB}$ . The macrodiversity gain here is defined as the increase in the normalized outage threshold compared to the case of having no macrodiversity. Therefore, the simultaneous use of both micro- and macrodiversity is more than reasonable, especially in environment under higher shadowing severity.

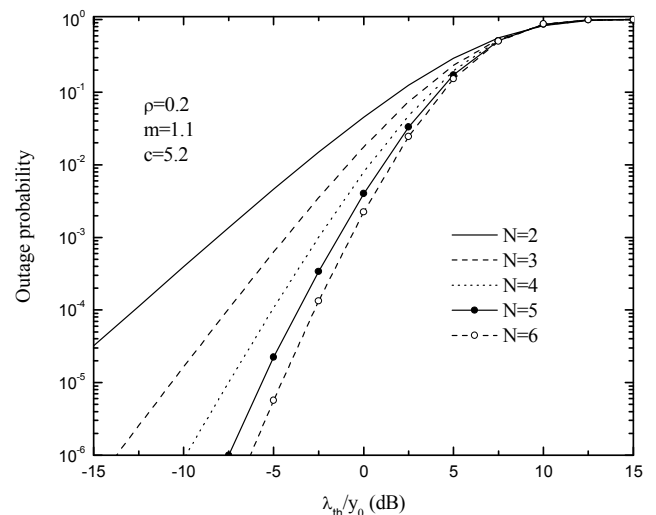


Fig. 3 Outage probability versus normalized outage threshold for different number of branches and correlation

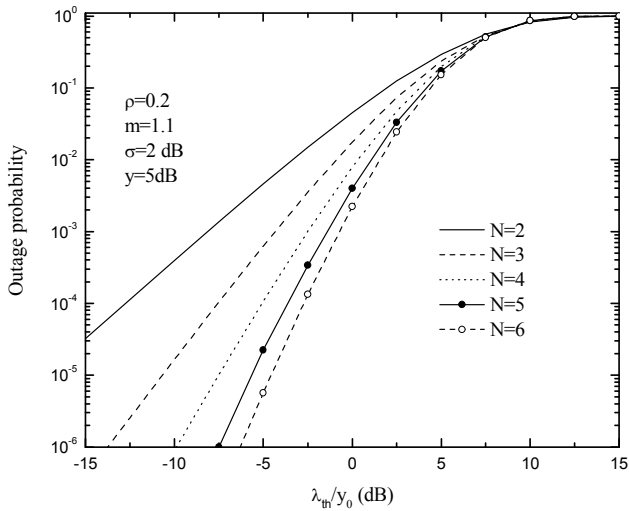


Fig. 4 Outage probability versus normalized outage threshold for different number of branches

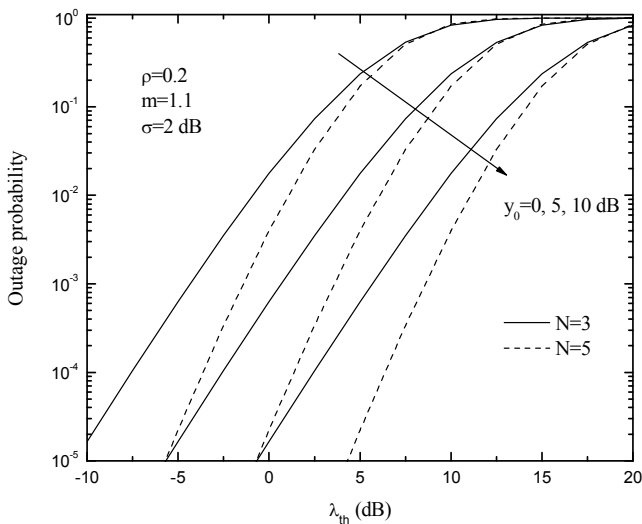


Fig. 5 Outage probability versus outage threshold for different values of average power  $y_0$  and number of branches

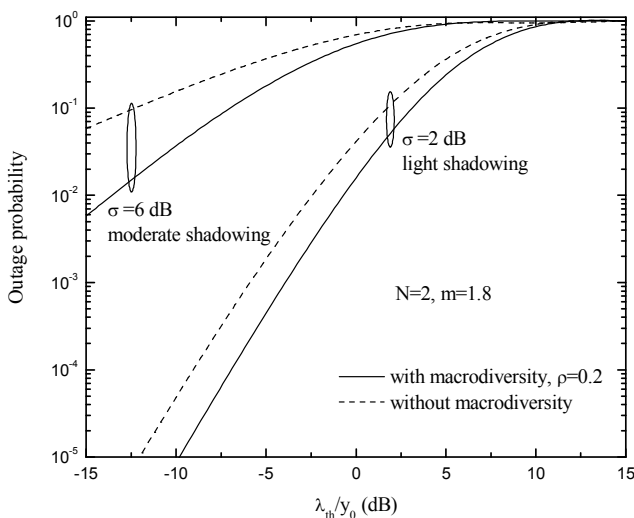


Fig. 6 Outage probability versus normalized outage threshold for systems with and without macrodiversity for different shadowing severity

### 3.2 Average signal value and amount of fading

A composite multipath/shadowed fading environment can be also described by the average signal value and AoF. The analytical expression for the moments is very useful since it can be used to directly obtain these important performance measures. The average signal value can be obtained by setting  $L=1$  in (12), i.e.

$$\bar{x} = \frac{1}{\Gamma(M)\Gamma(c)}$$

$$\times \sum_{n,k=0}^{\infty} \frac{\rho^n (1-\rho)^{1+c} y_0 \Gamma(1+2(n+c)+k) \Gamma(1+M)}{2^{2(n+c)+k} n! M \Gamma(n+c) \prod_{l=0}^k (n+l+c)} \quad (14)$$

The AoF can be expressed in terms of first and second order moments as

$$AoF = \frac{x_2}{x_1^2} - 1 \quad (15)$$

Typically, this performance measure is independent of the average power. The higher order moments ( $L > 2$ ) are also useful in signal processing algorithms for signal detection, classification and estimation and they play a fundamental role in studying the performance of wideband communication systems.

TABLE 2 The number of terms needed to be summed in (14) to achieve accuracy at the 4<sup>th</sup> significant digit ( $m = 1.1, y_0 = 20$  dB and  $N = 2$ )

$\sigma$ (dB)	$\rho=0.2$	$\rho=0.4$	$\rho=0.6$
12	14	16	20
5	14	15	18
2	22	25	34

TABLE 3 The number of terms needed to be summed in (15) to achieve accuracy at the 4<sup>th</sup> significant digit ( $m = 1.1$  and  $\sigma = 5$  dB)

$N$	$\rho \rightarrow 0$	$\rho = 0.5$	$\rho \rightarrow 1$
2	13	18	151
3	18	18	150
4	18	22	150
5	18	23	151

Analytical expressions for the average signal value and AoF are in the form of the nested infinite multiple sums. These expressions also converge rapidly, and therefore, they can be used efficiently. Tables 2 and 3 summarize the number of terms that need to be summed in expressions for the average signal value and AoF, respectively, to achieve

accuracy at the 4<sup>th</sup> significant digit. The number of terms depends strongly on the correlation coefficient. In the case of strong correlation, the number of terms is large.

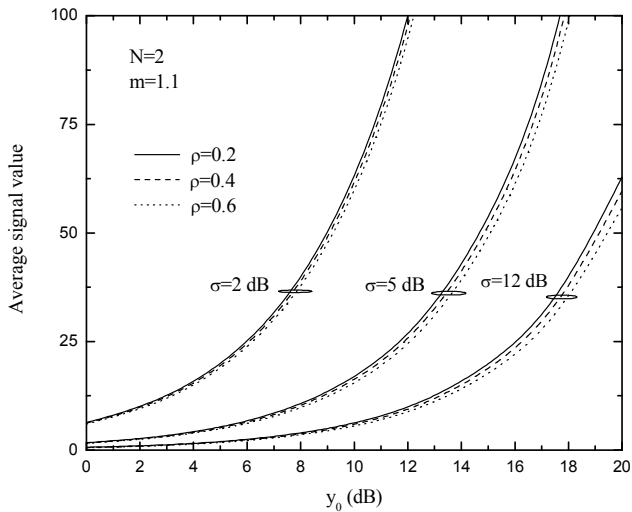


Fig. 7 Average signal value versus  $y_0$  for different shadowing severity and correlation coefficient

In Fig. 7, the average signal value is plotted as a function of  $y_0$ . As it was expected, system performance improves with a decrease of correlation coefficient and shadowing severity. The influence of correlation coefficient is larger in the case of higher shadowing severity.

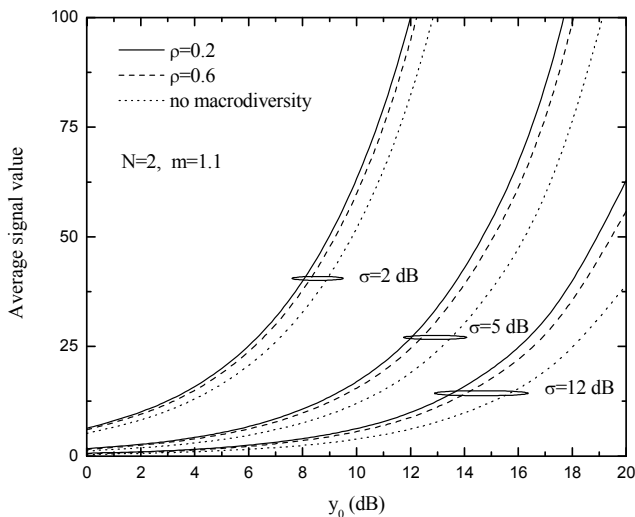


Fig. 8 Average signal value versus  $y_0$  for systems without and with macrodiversity for different shadowing severity and correlation coefficient

Improvement in average signal value obtained through macrodiversity is explored in Fig. 8. For example, for  $N=2$  and  $m=1.1$ , at the  $\bar{x}$  of 25, the macrodiversity gain for  $\sigma=2$  dB and  $\rho=0.2$  is about 0.9 dB, for  $\rho=0.6$ , 0.7 dB; for  $\sigma=12$  dB and  $\rho=0.2$  is about 2 dB while for  $\rho=0.6$  is 1.5 dB. The

macrodiversity gain here is defined as the reduction in the  $y_0$  compared to the case of having no macrodiversity. Therefore, similar to the outage probability, it can be deduced clear conclusion that the combination of micro- and macrodiversity provides significant improvement in average signal value, especially in environment under higher shadowing severity. Also, improvement is greater in the case when separation between the base stations at the micro level is larger.

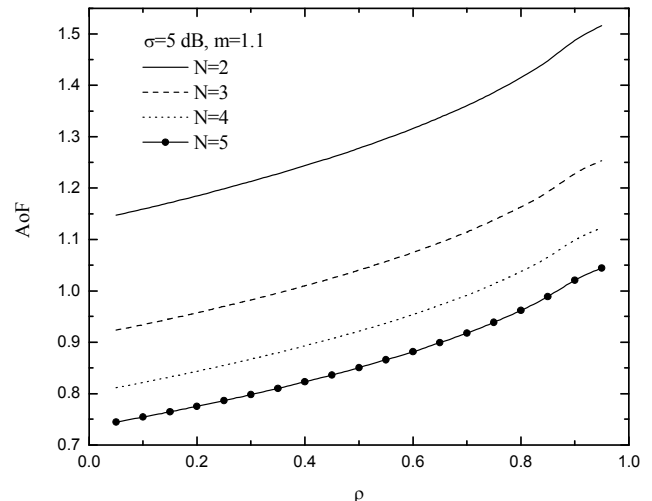


Fig. 9 AoF versus correlation coefficient for different diversity branches number

Fig. 9 presents the AoF versus correlation coefficient for different number of diversity branches. It is observed that AoF decreases with an increase of  $N$  while when  $\rho$  increases, AoF also increases resulting in performance degradation.

### 3.3 Average bit error probability

ABEP is another useful performance criterion characteristic of wireless communication systems. Conditional bit error probability (BEP) is a nonlinear function of  $x$  and the nature of the nonlinearity is a function of the modulation/detection scheme employed by the system. The conditional BEP is

$$P_e(x) = \frac{1}{2} e^{-gx} \tag{16}$$

where  $g$  denotes modulation constant, i.e.  $g = 1$  for binary differential phase shift keying (BDPSK) and  $g = 1/2$  for binary frequency shift keying (BFSK). ABEP can be evaluated directly by averaging the conditional BEP over the PDF of  $x$ .

$$P_e = \int_0^{\infty} p_x(x) P_e(x) dx \tag{17}$$

In Fig. 10, ABEP of BDPSK versus  $y_0$  for several values of correlation coefficient and number

of diversity branches is presented. Similar as in previous figures, conclusion that system performance improves as number of diversity branches and separation between base stations increase can be extracted. It is very interesting to observe that for lower values of  $y_0$  performance of system with five branches coincides with performance of system with two branches but with larger spatial separation between base stations. In that case, it is more economic to ensure large separation between base stations than to increase number of antennas (diversity branches) and to hold small separation between base stations. Fig. 11 shows that the gap among the adjacent curves due to increase of  $N$  reduces. This implies that ABEP improvement due to increase of diversity branches number reduces with increment of that number.

Fig. 12 and Fig. 13 present ABEP of BFSK. The similar findings as from Fig. 10 and Fig. 11 can be extracted. Fig. 14 shows that system performance is better for BDPSK modulation.

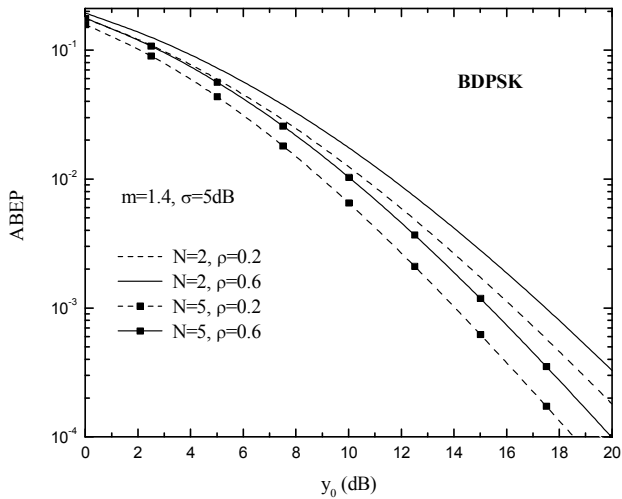


Fig. 10 ABEP of BDPSK versus  $y_0$  for different values of correlation coefficient and diversity branches number

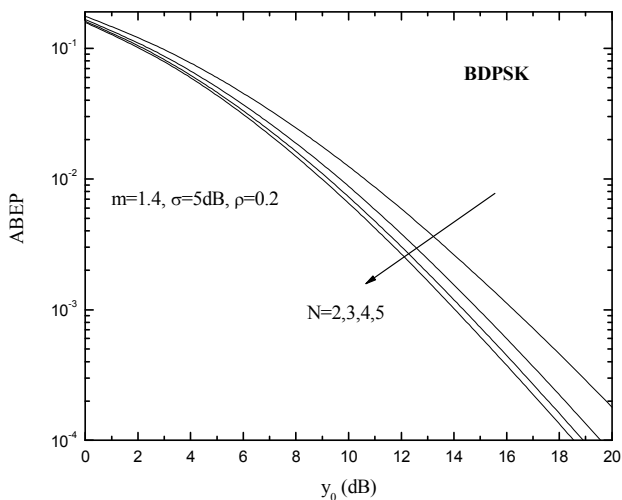


Fig. 11 ABEP of BDPSK versus  $y_0$  for different number of diversity branches

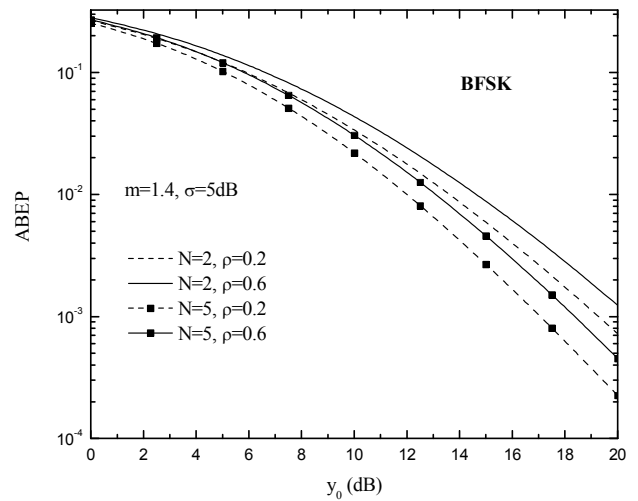


Fig. 12 ABEP of BFSK versus  $y_0$  for different values of correlation coefficient and diversity branches number

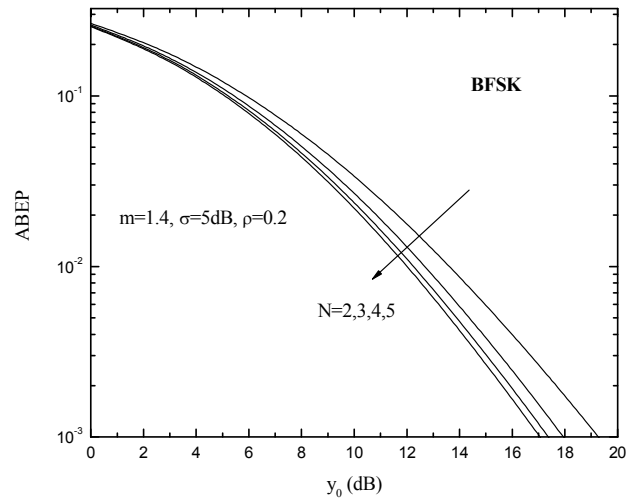


Fig. 13 ABEP of BFSK versus  $y_0$  for different number of diversity branches

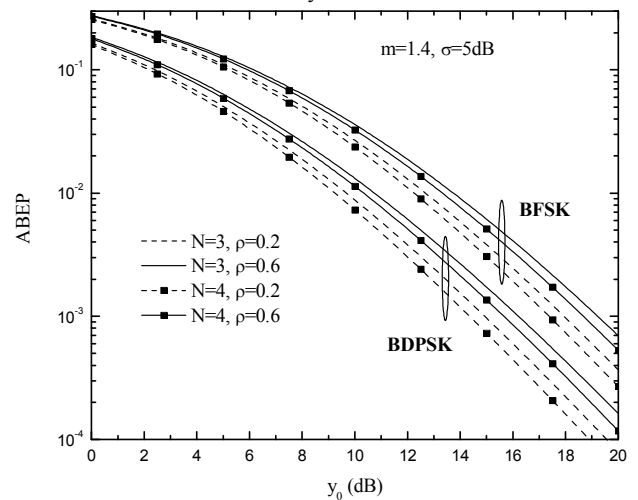


Fig. 14 ABEP of BDPSK and BFSK versus  $y_0$  for different values of correlation coefficient and number of diversity branches

In Fig. 15 and Fig. 16, ABEP of BDPSK and BFSK, respectively, for systems without and with macrodiversity versus  $y_0$  for several values of

correlation coefficient and number of diversity branches is presented. It is interesting to observe that in the case of higher shadowing, system with two branches at the micro level and with larger separation between base stations (smaller correlation coefficient) shows better performance than system with five branches at the micro level and smaller separation between base stations. Fig. 17 shows that system with BDPSK signalling shows better performance than system with BFSK signalling. For lower values of  $y_0$  and lower values of shadowing severity, system with BDPSK signalling without macrodiversity shows better performance than system with BFSK signalling with macrodiversity.

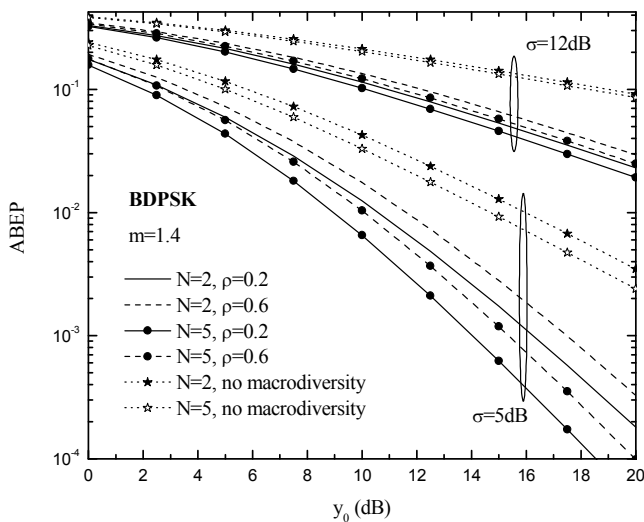


Fig. 15 ABEP of BDPSK versus  $y_0$  for systems without and with macrodiversity for different values of correlation coefficient and number of diversity branches

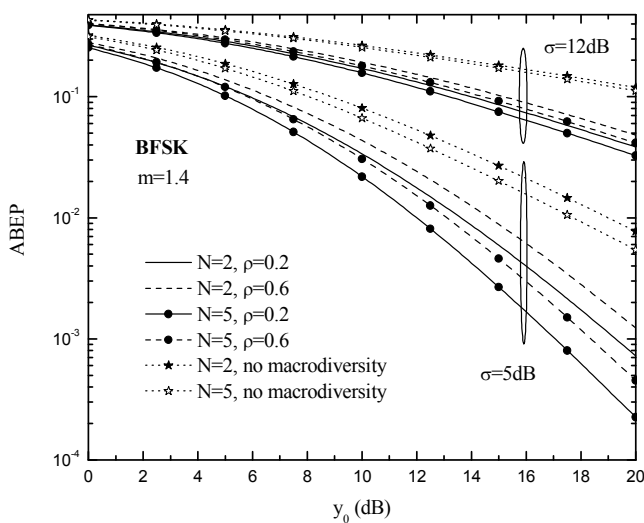


Fig. 16 ABEP of BFSK versus  $y_0$  for systems without and with macrodiversity for different values of correlation coefficient and number of diversity branches

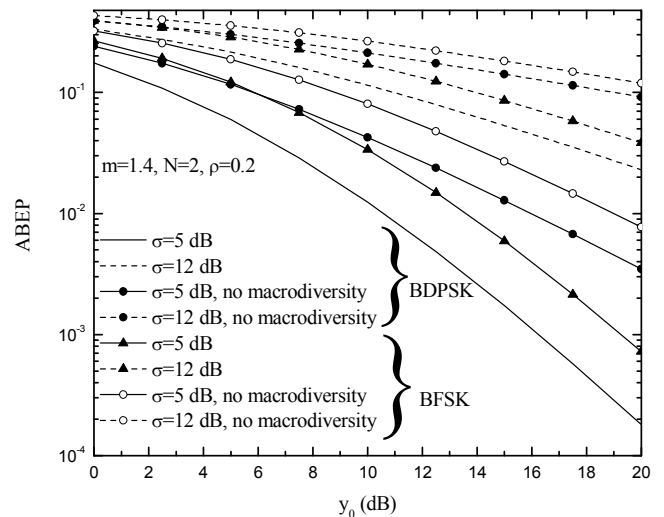


Fig. 17 ABEP of BDPSK and BFSK versus  $y_0$  for systems without and with macrodiversity for different shadowing severity

### 4 Conclusion

In this paper, system with micro- and macrodiversity reception was considered. The received signal envelope has a Nakagami- $m$  distribution and it also suffers gamma shadowing. Microdiversity scheme is based on MRC and macrodiversity scheme is based on SC. Expressions for the PDF, CDF and moments of the output signal are derived. These expressions are used to study important system performance criteria such as the outage probability, ABEP, average signal value and AoF. The presented infinite-series representations for the mentioned performance measures converge for any value of the parameters and accordingly, they enable great accuracy of the evaluated and graphically presented results. They show that the system performance improves with an increase of the Nakagami- $m$  factor, number of diversity branches at the micro level and order of gamma distribution while an increase of the correlation coefficient leads to deterioration of the system performance. Improvement achieved through macrodiversity is also established. Expressions in the paper enable the designers of wireless communication systems to simulate different fading and shadowing conditions and readjust systems operating parameters in order to meet the QoS demands.

### 5. Appendix: The case of no macrodiversity

The PDF of the signal at the output of single base station (the case of no macrodiversity) in shadowed Nakagami- $m$  fading channels is



$$p_x(x) = \int_0^{\infty} p_x(x|y)p_y(y)dy \quad (18)$$

where  $p_y(y)$  is the gamma PDF of the average power given by [19]

$$p_y(y) = \frac{y^{c-1} \exp(-y/y_0)}{\Gamma(c)y_0^c}. \quad (19)$$

Substituting (3) and (19) in (18) and after some straightforward manipulations, integral can be solved with the use of [20, (3.471/9)], resulting in analytical expression for the PDF of  $x$

$$p_x(x) = \frac{2M^{\frac{c+M}{2}} x^{\frac{c+M}{2}-1}}{\Gamma(M)\Gamma(c)y_0^{\frac{c+M}{2}}} K_{c-M} \left( 2\sqrt{\frac{Mx}{y_0}} \right). \quad (20)$$

The CDF of  $x$  can be obtained using

$$F_x(x) = \int_0^{\infty} F_x(x|y)p_y(y)dy. \quad (21)$$

Substituting (8) and (19) in (21) and after some straightforward manipulations, integral can be solved with the use of [20, (3.471/9)], resulting in analytical expression for the CDF of  $x$

$$F_x(x) = \frac{2}{\Gamma(M)\Gamma(c)} \sum_{k=0}^{\infty} \frac{(Mx)^{\frac{c+M+k}{2}}}{y_0^{\frac{c+M+k}{2}} \prod_{l=0}^k (M+l)} \quad (22)$$

$$\times K_{c-M-k} \left( 2\sqrt{\frac{Mx}{y_0}} \right).$$

Substituting (20) in (11) and using [20, (6.561/16)],  $L$ th moment for the case of no macrodiversity can be written as

$$x_L = \frac{y_0^L \Gamma(c+L)\Gamma(M+L)}{\Gamma(M)\Gamma(c)M^L}. \quad (23)$$

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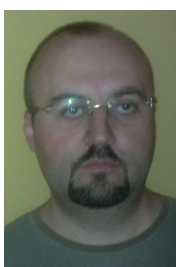
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