The Performance Analysis of MRC Combiner Output Signal in the Presence of Weibull Fading and Shadowing

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Abstract: - The performance of dual diversity systems in the presence of Weibull and shadowing over two uncorrelated branches will be presented in this paper. The performance measures of fading communication systems such as probability density function (PDF) of signal to noise ratio (SNR), Amount of fading (AF), and Outage probability (P_{out}) will be calculated and graphically represented for Maximal Ratio Combining. The results will be shown graphically for different signal and fading parameters values. They will be compared with respect to the kind of shadowing which is with log-normal or Gamma distribution.

Key-Words: - Weibull fading, Log-normal fading, Gamma fading, Maximal Ratio Combining, Amount of fading, Outage probabilty

1 Introduction

In wireless communications, fading causes difficulties in signal recovery. When a received signal experiences fading during transmission, its envelope and phase both fluctuate over time. The overall fading process for land mobile satelite systems is a complex combination of multipath fading and a shadowing. Multipath fading caused by the constructive and destructive combination of randomly delayed, reflected, scattered and diffracted signal components. This type of fading is relatively fast and is responsible for the short-term signal variation. In terrestrial and satellite land-mobile systems, the link quality is also affected by slow variation of the mean signal level due to the shadowing from terrain, buildings and trees.

One of the methods used to mitigate these degradation are diversity techniques, such as space diversity [1], [2]. Diversity combining has been considered as an efficient way to combat multipath fading and improve the received signal-to-noise ratio (SNR) because the combined SNR compared with the SNR of each diversity branch, is being increased. In this combining, two or more copies of the same information-bearing signal are combining to increase the overall SNR.

Multipath fading channels, or short term, fading channels has been modeled as Rayleigh, Rice, Nakagami–*m* and Weibull [1], [3]-[5]. The Weibull distribution plays an important role in several scientific fields, but it has become recently the topic of wireless communications theory [6]-[8], particularly with mobile radio systems operating in the 800/900 MHz frequency range. The Weibull model exhibits an excellent fit to experimental fading channel measurements, for both indoor [9] and outdoor [10] environments.

A composite multipath/shadowed fading environment modeled either as Rayleigh-lognormal (or Suzuki), Rician-lognormal or Nakagami-lognormal are considered in [11]-[16]. Recently, a compound analytical model was proposed to describe the shadowed fading channels [17], [18]. Up to now, composite multipath/shadowed fading environment modelled as Weibull-lognormal, has been considered only in several papers [19, 20].

2 Related Work

The use of log-normal distribution [1], [21] to model the average power, which is random variable due to shadowing, doesn't lead to a closed form solution for the probability density function (PDF) of the signal-to-noise ratio (SNR) at the receiver. This PDF can be evaluated numerically using some of software tools as Matlab and Mathematica.

A compound fading model uses a gamma distribution to account for shadowing instead of the lognormal distribution [17, 22]. This model incorporates short-term fading and shadowing and provides an analytical solution for the PDF of the SNR facilitating the analysis of wireless systems.

These channels can be described then as Rayleigh–gamma, Rice–gamma and Nakagami–*m*–gamma [14], [15], [17], [18]. These three composite models leads to a closed form solution for the probability density function of the signal-to-noise ratio at the receiver. Recently Rayleigh–gamma a composite analytical model was proposed to describe the shadowed fading channels. For this model, Rayleigh density function is used to describe amplitude of received signal and gamma density function is used to model the average power of the received signal.

In [15], a unified analytical approach to performance analyses in a gamma-shadowed Nakagami-*m* fading channel is presented. Specifically, a lognormal probability density function (PDF) of shadowing is approximated by a gamma PDF. The approximation is based on the moment matching method. The accuracy of the approximation is examined. A mathematical framework is developed for deriving key statistical parameters such as the PDFs, as well as performance metrics including outage probability and average bit-error rate of different noncoherent digital modulation schemes. These formulas are validated by specializing the general results to some particular cases whose solutions are known and by means of comparing their graphically presented results (based on the closed-form expressions) with corresponding known results evaluated by numerical techniques.

In the paper [17], wireless channels are affected by short-term fading and long-term fading (shadowing). A compound fading model was proposed for the modeling of shadowed fading channels which resulted in a closed form solution for the probability density function (PDF) of the signal-to-noise ratio (SNR). This model is applied to a case where both micro- and macro-diversity schemes are implemented to mitigate short-term fading and shadowing, respectively. Using the compound fading model, it is shown that the PDF of the signal-to-noise ratio after the implementation of maximal ratio combining (MRC) at the micro level and selection combining (SC) at the macro level can be expressed in analytical form. Even when branch correlation exists, the PDF still can be expressed in analytical form. Thus, the compound PDF model offers significant improvement over approaches which use lognormal PDF for shadowing. The performance of a coherent binary phase shift keying (BPSK) modem is evaluated using this approach. The results demonstrate the simplicity and usefulness of the compound PDF in the performance analyses of shadowed fading channels even when branch correlation exists at the base station or correlation exists between base stations [17].

In [23] wireless communication systems are subject to short- and long-term fading of the channel. Instead of the commonly used Nakagami–lognormal model to account for the conditions existing in these shadowed fading channels, a compound probability density function (PDF) model is used to evaluate the performance of wireless systems. While the Nakagami–lognormal lacks a closed-form solution to the PDF of the received power in shadowed fading channels, the compound PDF has an analytical expression for the PDF of the received signal power. The synergy between these two models for the analysis of wireless systems is explored by calculating the bit error rate in a DPSK modem as well as the outage probability in a wireless system in a shadowed fading channel. This is followed by the computation of the outage probability in the general case where both the desired and cochannels are subject to shadowing and fading. The analyses were carried out for both fixed number of cochannels and random number of cochannels. Results demonstrate the usefulness of the compound PDF model for the performance analyses of wireless systems in shadowed fading channels.

In [24] the compound PDF which incorporates the short-term fading (Rayleigh fading) and long-term fading (gamma shadowing) is used to study the effectiveness of macro- and micro-diversity techniques to mitigate the degradation problems in shadowed fading channels. That paper considers macrodiversity system which has two microdiversities with maximal ratio combining (MRC) at the micro level and selection combining (SC) at the macro level.

This paper presents Maximal-Ratio Combining procedure for communication system where the diversity combining is applied over two uncorrelated branches (*ρ=0*), which are given as channels with Weibull and log-normal fading.

In this environment the receiver does not average the envelope fading due to multipath but rather, reacts to the instantaneous composite multipath/shadowed signal. This is often the scenario in congested downtown areas with slow-moving pedestrians and vehicles. This type of composite fading is also observed in land mobile satellite systems that are subjected to vegetative or urban shadowing

Maximal-Ratio Combining (MRC) is one of the most widely used diversity combining schemes whose SNR is the sum of the SNR's of each individual diversity branch. MRC is the optimal combining scheme, but its price and complexity are high, since MRC requires cognition of all fading parameters of the channel.

Combining techniques like maximal ratio combining (MRC) and equal gain combining (EGC) require all or some of the amount of the channel state information of received signal. Second, MRC and EGC require separate receiver chain for each branch of the diversity system, which increase its complexity of the system.

3 System and Channel Models

A dual-branch diversity system over two uncorrelated channels in the presence of Weibull and log-normal fading is being considered. Under these conditions, instantaneous SNR, $p(\gamma)$ is obtained by averaging the instantenous Weibull fading average power over the conditional pdf (probability density function) of the log-normal shadowing, which results with combination of with combination of Weibull distribution (for fast multipath feding) and log-normal distribution (for shadowing) [1], [25]:

$$p(\gamma/\Omega)=\frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\Omega}\right)^{c/2}\cdot\gamma^{c/2-1}\cdot$$

$$\cdot\exp\left[-\left(\frac{\gamma}{\Omega}\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}\right],\qquad \gamma\geq 0 \tag{1}$$

$$p_{\Omega}(\Omega)=\frac{\xi}{\sqrt{2\pi\sigma}\Omega}\cdot\exp\left(-\frac{(10\log_{10}\Omega-\mu)^2}{2\sigma^2}\right) \tag{2}$$

$$p(\gamma)=\int_0^{\infty}p_{\gamma}(\gamma/\Omega)p_{\Omega}(\Omega)d\Omega \tag{3}$$

Substituting Eq. (1) and Eq. (2) in Eq. (3), $p(\gamma)$ can be written as:

$$p(\gamma)=\int_0^{\infty}\frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\Omega}\right)^{c/2}\cdot\gamma^{c/2-1}\cdot$$

$$\cdot\exp\left[-\left(\frac{\gamma}{\Omega}\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}\right]\cdot$$

$$\cdot\left\{\frac{\xi}{\sqrt{2\pi}\sigma\Omega}\exp\left[-\frac{(10\log_{10}\Omega-\mu)^2}{2\sigma^2}\right]\right\}d\Omega$$

$$\gamma\geq 0 \tag{4}$$

where

$$\xi=10/\ln 10=4.3429,$$

μ_i (db) is mean of $10\log_{10}\gamma$, σ_i (db) is standard deviation of $10\log_{10}\gamma$, and c is the Weibull shape parameter.

Instead of the lognormal distribution, for shadowing, it can be used gamma distribution [17]:

$$p_{\Omega}(\Omega)=\frac{\Omega^{c_g-1}}{\Gamma(c_g)}\mu_g^{\ -c_g}\exp\left(-\frac{\Omega}{\mu_g}\right) \tag{5}$$

where c_g is the order of the Gamma distribution and μ_g is average SNR.
μ can be done in terms of μ_g and c_g as [27, 28]:

$$\mu(db)=\xi\left[\ln(\mu_g)+\psi(c_g)\right] \tag{6}$$

The relationship between c_g and σ is [27, 28]:

$$\sigma(db)=\xi\sqrt{\psi'(c_g)} \tag{7}$$

where ψ and ψ' are digamma and trigamma functions.

In the case of gamma distribution for shadowing, $p(\gamma)$ can be expressed as:

$$p(\gamma)=\int_0^\infty \frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\Omega}\right)^{c/2}\cdot\gamma^{c/2-1}\cdot$$

$$\cdot\exp\left[-\left(\frac{\gamma}{\Omega}\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}\right]\cdot$$

$$\cdot\frac{\Omega^{c_g-1}}{\Gamma(c_g)}\mu_g^{-c_g}\exp\left(-\frac{\Omega}{\mu_g}\right)d\Omega$$

$$\gamma\ge 0 \tag{8}$$

In Fig 1, 2. and 3. the PDF curves $p(\gamma)$ are shown for various values of the factors μ, σ and c. For the case where $c=2$, an overall fading consists of Rayleigh and log-normal (gamma).

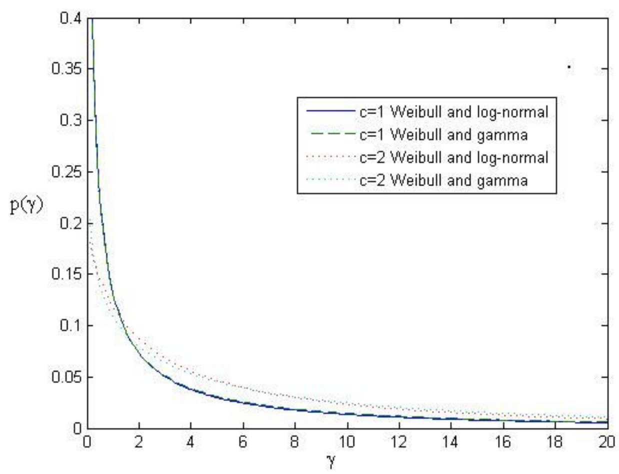

Fig 1. The PDF $p(\gamma)$ for $\mu=10db$, $\sigma=5db$, $c=1;2$

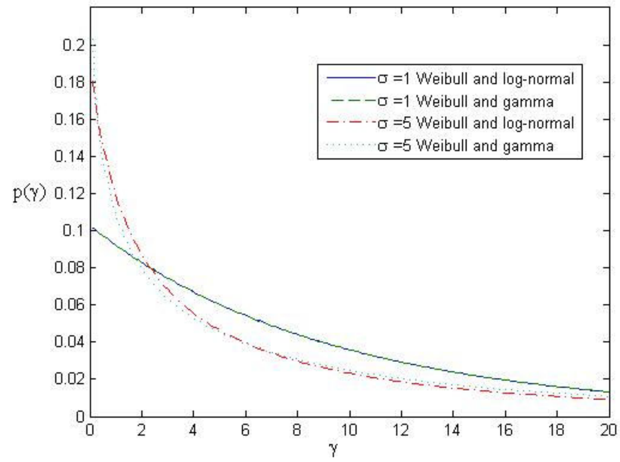

Fig.2. The PDF $p(\gamma)$ for $\mu=10db$, $\sigma=1;5db$, $c=2$

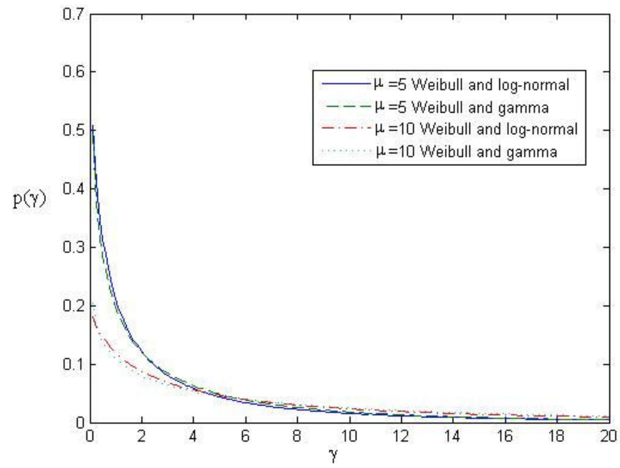

Fig. 3. The PDF $p(\gamma)$ for $\mu=5;10db$, $\sigma=5db$, $c=2$

For further evaluations it is important to find the moments of combined SNR.

N-th moment of the output SNR is given by [21]:

$$E[\gamma^n]=\int_0^\infty \gamma^n p_\gamma(\gamma)d\gamma \tag{9}$$

N-th moment for distribution in Eq. (4) can be written as

$$E[\gamma^k]=\frac{\Gamma\left(1+\frac{2k}{c}\right)}{\Gamma\left(1+\frac{2}{c}\right)^k}\exp\left[\frac{k}{\xi}\mu+\frac{1}{2}\left(\frac{k}{\xi}\right)^2\sigma^2\right] \tag{10}$$

and for (5) as:

$$E[\gamma^k] = \frac{\Gamma\left(1+\frac{2k}{c}\right)}{\Gamma\left(1+\frac{2}{c}\right)^k} \frac{\Gamma(c_g + k)}{\Gamma(c_g)} \mu_g^{\ k} \tag{11}$$

Amount of fading (AF) is a unified measure of the severity of fading for particular channel model and is typically independent of the average fading power, but it is dependent of the instantaneous SNR.

Amount of fading is defined by [3]:

$$AF = \frac{\mathrm{var}(\alpha^2)}{(E[\alpha^2])^2} = \frac{E[(\alpha^2-\Omega)^2]}{\Omega^2} \tag{12}$$

$$AF = \frac{E[\gamma^2]-(E[\gamma])^2}{(E[\gamma])^2} = \frac{E[\gamma^2]}{(E[\gamma])^2} - 1 \tag{13}$$

where α is fading amplitude, Ω is average fading power, $E[]$ denotes statistical average and *var( )* denotes variance.

The joint probability density function in the case of two uncorrelated fading channels is given by [26]:

$$p_{\gamma_1,\gamma_2}(\gamma_1,\gamma_2) = p_{\gamma_1}(\gamma_1)\cdot p_{\gamma_2}(\gamma_2) \tag{14}$$

4 Maximal Ratio Combining

The total SNR at the output of the MRC combiner is given by:

$$\gamma_{MRC} = \sum_{l=1}^{L} \gamma_l \tag{15}$$

where L is number of branches.

The average combined SNR at the MRC (maximal ratio combining) output with two branches is given by:

$$\gamma_{MRC} = \gamma_1 + \gamma_2 \tag{16}$$

Probability density function of the sum of the first and second branch $\gamma_{MRC} = \gamma_1 + \gamma_2$ can be written as

$$p_{\gamma_{MRC}}(\gamma_{MRC}) = \int_0^{\gamma_{MRC}} p_{\gamma 2}(\gamma_{MRC} - \gamma_1) p_{\gamma 1}(\gamma_1) d\gamma_1 \tag{17}$$

Substituting Eq. (4) in Eq. (16), $p_{\gamma_{MRC}}(\gamma_{MRC})$ for Weibull-lognormal distribution can be obtained as

$$p_{\gamma_{MRC}}(\gamma_{MRC}) = \int_0^{\gamma_{MRC}} \left[\int_0^{\infty} \frac{c_2}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_2}\right)}{\Omega} \right)^{c_2/2} \cdot \right.$$

$$\cdot (\gamma_{MRC} - \gamma_1)^{c_2/2-1} \cdot \exp\left[-\left(\frac{\gamma_{MRC}-\gamma_1}{\Omega_2} \Gamma\left(1+\frac{2}{c_2}\right)\right)^{c_2/2} \right] \cdot$$

$$\cdot \left\{ \frac{\xi}{\sqrt{2\pi}\sigma_2\Omega_2} \exp\left[\frac{10\log_{10}\Omega_2 - \mu_2)^2}{2\sigma_2^{\ 2}} \right] \right\} d\Omega_2 \cdot$$

$$\cdot \int_0^{\infty} \frac{c_1}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_1}\right)}{\Omega_1} \right)^{c_1/2} \cdot \gamma_1^{c_1/2-1} \cdot$$

$$\cdot \exp\left[-\left(\frac{\gamma_1}{\Omega_1} \Gamma\left(1+\frac{2}{c_1}\right)\right)^{c_1/2} \right] \cdot$$

$$\left. \cdot \left\{ \frac{\xi}{\sqrt{2\pi}\sigma_1\Omega_1} \exp\left[\frac{10\log_{10}\Omega_1 - \mu_1)^2}{2\sigma_1^{\ 2}} \right] \right\} d\Omega_1 \right] d\gamma_1 \tag{18}$$

Analogous to (18) in (19) is obtained expression for Weibull-Gamma distribution:

$$p_{\gamma_{MRC}}(\gamma_{MRC}) = \int_0^{\gamma_{MRC}} \left[\int_0^{\infty} \frac{c_2}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_2}\right)}{\Omega} \right)^{c_2/2} \cdot \right.$$

$$\cdot (\gamma_{MRC} - \gamma_1)^{c_2/2-1} \cdot \exp\left[-\left(\frac{\gamma_{MRC}-\gamma_1}{\Omega_2} \Gamma\left(1+\frac{2}{c_2}\right)\right)^{c_2/2} \right] \cdot$$

$$\cdot \left\{ \frac{\Omega_2^{\ c_{g2}-1}}{\Gamma(c_{g_2})} \mu_{g_2}^{\ -c_{g2}} \exp\left(-\frac{\Omega_2}{\mu_{g_2}} \right) \right\} d\Omega_2 \cdot$$

$$\cdot \int_0^{\infty} \frac{c_1}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_1}\right)}{\Omega_1} \right)^{c_1/2} \cdot \gamma_1^{c_1/2-1} \cdot$$

$$\cdot \exp\left[-\left(\frac{\gamma_1}{\Omega_1}\Gamma\left(1+\frac{2}{c_1}\right)\right)^{c_1/2}\right]\cdot$$

$$\cdot\left\{\frac{\Omega_1^{c_{g1}-1}}{\Gamma(c_{g_1})}\mu_{g_1}^{-c_{g1}}\exp\left(-\frac{\Omega_1}{\mu_{g_1}}\right)\right\}d\Omega_1\Bigg]d\gamma_1 \qquad (19)$$

Relatively simple closed form expressions to represent $p_{\gamma_{MRC}}(\gamma_{MRC})$ can not be derived, because Eq. (18) is too complex for tractable communication system analyses. This pdf can be evaluated numerically using some of software tools (Matlab, Mathematica).

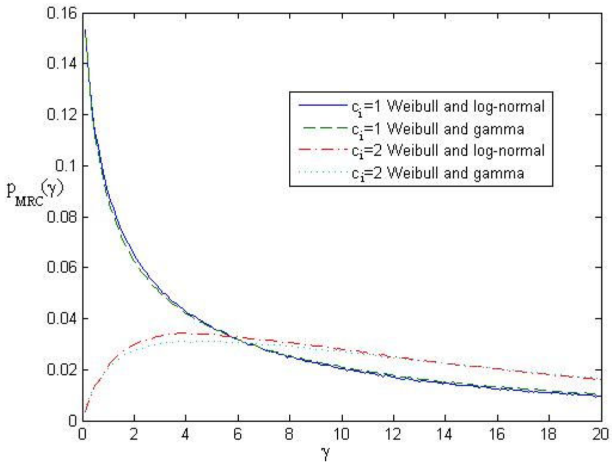

Fig 4. The PDF $p_{\gamma_{MRC}}(\gamma_{MRC})$ for $\mu_i = 10db$, $\sigma_i = 5db$, $c_i = 1; 2$

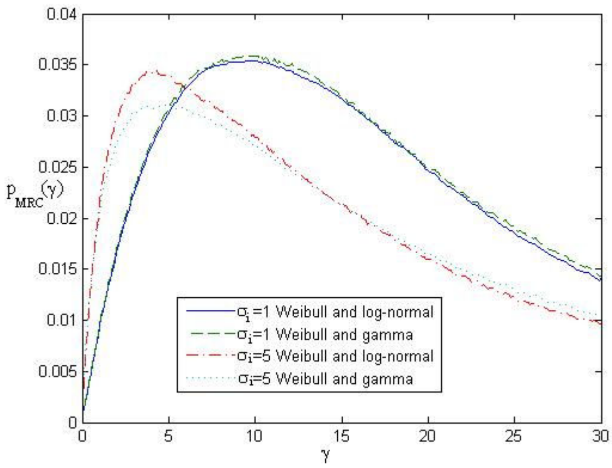

Fig 5. The PDF $p_{\gamma_{MRC}}(\gamma_{MRC})$ for $\mu_i = 10db$, $\sigma_i = 1;\ 5db$, $c_i = 2$

Very often it is assumed for the performance analysis of communication systems, that channel coefficients are uncorrelated and identically distributed.

Figs 4. to 6. depict PDF of γ_{MRC} at the output of MRC combiner for two uncorrelated identically distributed channels with identical parameters and for different values of the parameters μ, σ and *c*.

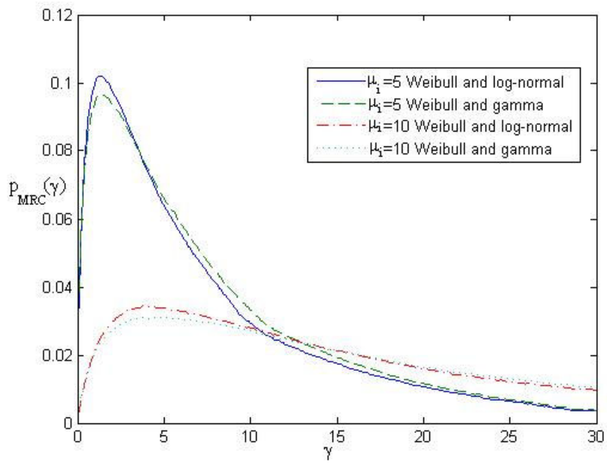

Fig 6. The PDF $p_{\gamma_{MRC}}(\gamma_{MRC})$ for $\mu_i = 5{,}10db$, $\sigma_i = 5db$, $c_i = 2$

N-th moment of γ_{MRC} can be expressed as [1]

$$E\left[\gamma^n{}_{MRC}\right] = E\left[(\gamma_1+\gamma_2)^n\right] \qquad (20)$$

Using the binomial expansion, (20) can be written as

$$E\left[\gamma^n{}_{MRC}\right] = E\left[\sum_{k=0}^{n}\binom{n}{k}\gamma_1{}^k\gamma_2{}^{n-k}\right] = $$

$$= \sum_{k=0}^{n}\binom{n}{k}E\left[\gamma_1{}^k\gamma_2{}^{n-k}\right] \qquad (21)$$

The average combined SNR $\bar{\gamma}_{MRC}$ at the MRC output can be written as

$$\bar{\gamma}_{MRC} = E\left[\gamma^1{}_{MRC}\right] = \bar{\gamma}_1 + \bar{\gamma}_2 \qquad (22)$$

The second moment of γ_{MRC} is given by

$$E\left[\gamma^2{}_{MRC}\right] = E\left(\gamma_1{}^2\right) + 2\bar{\gamma}_1\bar{\gamma}_2 + E\left(\gamma_2{}^2\right) \tag{23}$$

The Amount of fading can be calculated using Eq. (10), Eq. (22) and Eq. (23) in Eq. (13) for log-normal shadowing as

$$AF_{MRC} = \left[\frac{\Gamma\left(1+\frac{4}{c_1}\right)}{\Gamma\left(1+\frac{2}{c_1}\right)^2} \exp\left[\frac{2}{\xi}\mu_1 + \frac{1}{2}\left(\frac{2}{\xi}\right)^2 \sigma_1{}^2\right] + \right.$$

$$+ \frac{\Gamma\left(1+\frac{4}{c_2}\right)}{\Gamma\left(1+\frac{2}{c_2}\right)^k} \exp\left[\frac{k}{\xi}\mu_2 + \frac{1}{2}\left(\frac{2}{\xi}\right)^2 \sigma_2{}^2\right] +$$

$$+ 2\exp\left[\frac{1}{\xi}\mu_1 + \frac{1}{2}\left(\frac{1}{\xi}\right)^2 \sigma_1{}^2\right] \cdot$$

$$\left. \cdot \exp\left[\frac{1}{\xi}\mu_2 + \frac{1}{2}\left(\frac{1}{\xi}\right)^2 \sigma_2{}^2\right]\right] \cdot$$

$$\cdot \left(\exp\left[\frac{1}{\xi}\mu_1 + \frac{1}{2}\left(\frac{1}{\xi}\right)^2 \sigma_1{}^2\right] + \right.$$

$$\left. + \exp\left[\frac{k}{\xi}\mu_2 + \frac{1}{2}\left(\frac{k}{\xi}\right)^2 \sigma_2{}^2\right]\right)^{-2} - 1 \tag{24}$$

and for Gamma shadowing as:

$$AF_{MRC} = \left[\frac{\Gamma\left(1+\frac{4}{c_1}\right)}{\Gamma\left(1+\frac{2}{c_1}\right)^2} \frac{\Gamma\left(c_{g_1}+2\right)}{\Gamma\left(c_{g_1}\right)} \mu_{g_1}^2 + \right.$$

$$+ \frac{\Gamma\left(1+\frac{4}{c_2}\right)}{\Gamma\left(1+\frac{2}{c_2}\right)^k} \frac{\Gamma\left(c_{g_2}+2\right)}{\Gamma\left(c_{g_2}\right)} \mu_{g_2}^2 +$$

$$\left. + 2\frac{\Gamma\left(c_{g_1}+1\right)}{\Gamma\left(c_{g_1}\right)} \mu_{g_1} \frac{\Gamma\left(c_{g_2}+1\right)}{\Gamma\left(c_{g_2}\right)} \mu_{g_2} \right] \cdot$$

$$\cdot \left(\frac{\Gamma\left(c_{g_1}+1\right)}{\Gamma\left(c_{g_1}\right)} \mu_{g_1} + \frac{\Gamma\left(c_{g_2}+1\right)}{\Gamma\left(c_{g_2}\right)} \mu_{g_2} \right)^{-2} - 1 \tag{25}$$

The amount of fading for MRC has less values then single channel receiver for identically distributed channels that is shown in Fig 7. to 9.

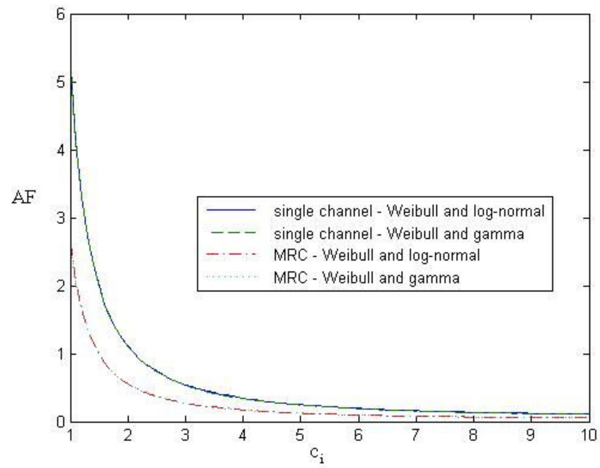

Fig 7. Amount of fading for single channel and MRC receiver versus c_i for $\mu_i = 10db$, $\sigma_i = 1db$

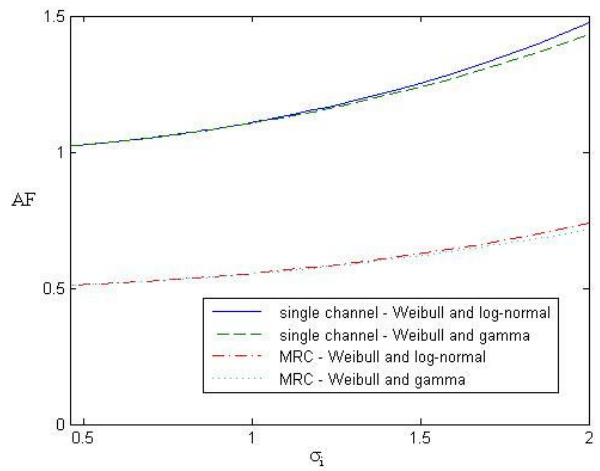

Fig 8. Amount of fading for single channel and MRC receiver versus σ_i for $\mu_i = 10db$, $c_i = 2$

From Fig. 8 we can see that only for single channel a little disagreement between lognormal and gamma shadowing beside Weibull fading exist only for $\sigma_i > 1,5db$.

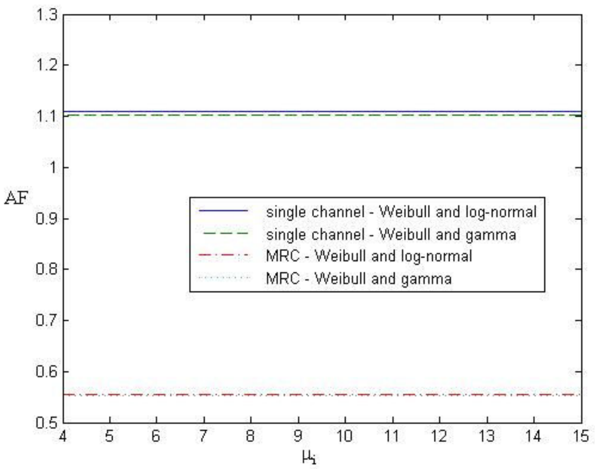

Fig 9. Amount of fading for single channel and MRC receiver versus μ_i for $\sigma_i = 1db$, $c_i = 2$

The outage probabilty is standard performance criterion of diversity systems opereting over fading chanels and it is defined as the probability that the instantaneous error rate exceeds a specified value, or equivalently, that combined SNR of MRC falls below a predetermined threshold γ_{th}.

$$P_{out}^{MRC}(\gamma_{th}) = P\left[\gamma_{MRC} = \gamma_1 + \gamma_2 \le \gamma_{th}\right] \tag{26}$$

$P_{out}^{MRC}(\gamma_{th})$ is defined in the form of integral by

$$P_{out}^{MRC}(\gamma_{th}) = \int_0^{\gamma_{th}} p_{\gamma MRC}(\gamma_{MRC})\, d\gamma_{MRC} \tag{27}$$

Substituting Eq. (4) in Eq. (27) $P_{out}^{MRC}(\gamma_{th})$ can be written as

$$P_{OUT}^{MRC}(\gamma_{th}) = \int_0^{\gamma_{th}} \int_0^{\gamma_{MRC}} \left[\int_0^{\infty} \frac{c_2}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_2}\right)}{\Omega} \right)^{c_2/2} \right.\cdot$$

$$\cdot\left(\gamma_{MRC}-\gamma_1\right)^{c_2/2-1} \exp\left[-\left(\frac{\gamma_{MRC}-\gamma_1}{\Omega_2}\Gamma\left(1+\frac{2}{c_2}\right)\right)^{c_2/2}\right]\cdot$$

$$\cdot\left\{\frac{\xi}{\sqrt{2\pi}\sigma_2\Omega_2}\exp\left[\frac{10\log_{10}\Omega_2-\mu_2)^2}{2\sigma_2{}^2}\right]\right\}d\Omega_2\cdot$$

$$\cdot\int_0^{\infty}\frac{c_1}{2}\left(\frac{\Gamma\left(1+\frac{2}{c_1}\right)}{\Omega_1}\right)^{c_1/2}\gamma_1{}^{c_1/2-1}\cdot$$

$$\cdot\exp\left[-\left(\frac{\gamma_1}{\Omega_1}\Gamma\left(1+\frac{2}{c_1}\right)\right)^{c_1/2}\right]\cdot$$

$$\left.\cdot\left\{\frac{\xi}{\sqrt{2\pi}\sigma_1\Omega_1}\exp\left[\frac{10\log_{10}\Omega_1-\mu_1)^2}{2\sigma_1{}^2}\right]\right\}d\Omega_1\right]d\gamma_1 d\gamma_{MRC} \tag{28}$$

Similarly, substituting Eq. (5) in Eq. (27) $P_{out}^{MRC}(\gamma_{th})$ can be written as

$$P_{OUT}^{MRC}(\gamma_{th}) = \int_0^{\gamma_{th}} \int_0^{\gamma_{MRC}} \left[\int_0^{\infty} \frac{c_2}{2} \left(\frac{\Gamma\left(1+\frac{2}{c_2}\right)}{\Omega} \right)^{c_2/2} \right.\cdot$$

$$\cdot\left(\gamma_{MRC}-\gamma_1\right)^{c_2/2-1} \exp\left[-\left(\frac{\gamma_{MRC}-\gamma_1}{\Omega_2}\Gamma\left(1+\frac{2}{c_2}\right)\right)^{c_2/2}\right]\cdot$$

$$\cdot\left\{\frac{\Omega_2{}^{c_{g2}-1}}{\Gamma(c_{g_2})}\mu_{g_2}{}^{-c_{g2}}\cdot\exp\left(-\frac{\Omega_2}{\mu_{g_2}}\right)\right\}d\Omega_2\cdot$$

$$\cdot\int_0^{\infty}\frac{c_1}{2}\left(\frac{\Gamma\left(1+\frac{2}{c_1}\right)}{\Omega_1}\right)^{c_1/2}\gamma_1{}^{c_1/2-1}\cdot$$

$$\cdot \exp\left[-\left(\frac{\gamma_1}{\Omega_1}\Gamma\left(1+\frac{2}{c_1}\right)\right)^{c_1/2}\right]$$

$$\cdot\left\{\frac{\Omega_1^{c_{g_1}-1}}{\Gamma(c_{g_1})}\mu_{g_1}^{-c_{g_1}}\exp\left(-\frac{\Omega_1}{\mu_{g_1}}\right)\right\}d\Omega_1\Bigg]d\gamma_1 d\gamma_{MRC} \tag{29}$$

Figs. 10. to 12. show the outage probability versus instantaneous SNR for the same factors as in Figs 7. to 9.

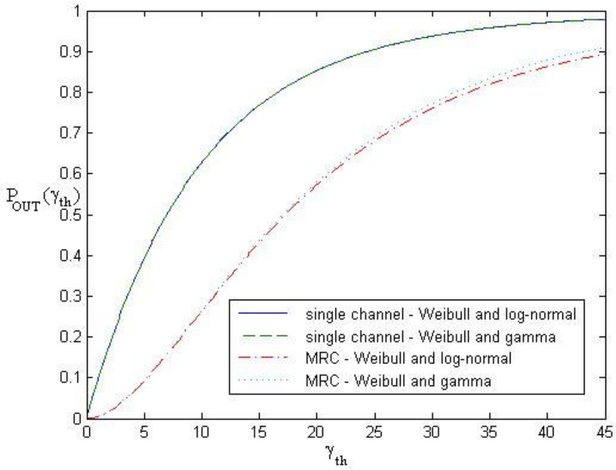

Fig 10. The outage probability for single channel and MRC receiver for $\mu_i = 10db$, $\sigma_i = 1db$, $c_i = 2$

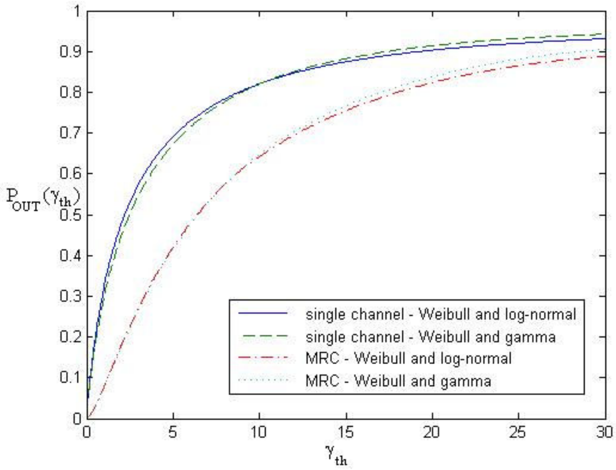

Fig 11. The outage probability for single channel and MRC receiver for $\mu_i = 5db$, $\sigma_i = 5db$, $c_i = 2$

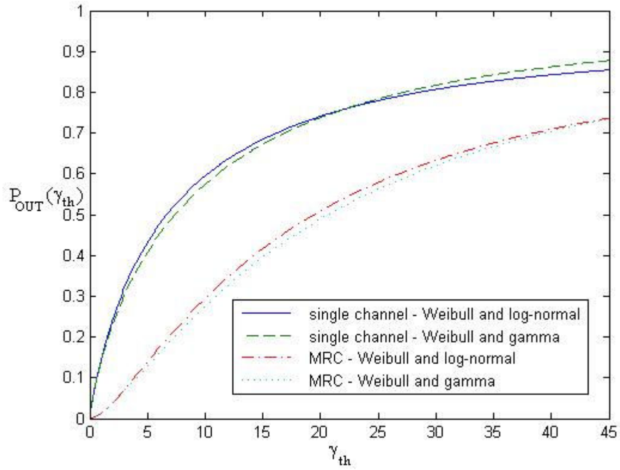

Fig 12. The outage probability for single channel and MRC receiver for $\mu_i = 10db$, $\sigma_i = 5db$, $c_i = 2$

It can be seen from these figures that values for probability density function of SNR, the amount of fading and the outage probability are very closed especially for small values of parameter σ, whatever which functionis is used for shadowing, lognormal or gamma.

From these figures we can see that MRC combiner has better performances then single channel receiver as the theory says.

5 Conclusion

In this paper a unified performance analysis for dual diversity MRC over uncorrelated Weibull fading and shadowing channels is presented. The shadowing was given by log-normal and Gamma distribution. The composite multipath/shadowed fading environment modelled as Weibull and shadowing has not been considered detail up to now except in several papers where Weibull /log-normal fading was investigated.

The probability density function of SNR, the amount of fading and the outage probability are derived in the form of multiple integrals. That can not be obtained in relatively simple closed-form expression for evaluation of these parameters, because the system structure is too complex and it was performed numerical calculation of them. As ilustration of this aproach, the characteristics of the receiver are shown for MRC dual diversity case to point out the effect of the overall fading. The calculated curves are shown graphically for different signal and fading parameters values.

It is shown in this paper that MRC receiver has better performances then single channel receiver. Also, it can be seen that values for probability density function of SNR, the amount of fading and the outage probability are very closed especially for small values of parameter σ, whatever which functionis is used for shadowing, lognormal or gamma.

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