Capacity of MIMO System in the Presence of Directional Interferences

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Abstract: - Capacity of Multiple Input Multiple Output (MIMO) system degrades significantly in the presence of interferences, especially in the presence of strong directional interferences in military communication scenario. In this paper, the capacity of a MIMO system with directional interferences is investigated. The focus is on the case where the channel state information (CSI) is known to the receiver but unknown to the transmitter. Further, attention is restricted to narrowband MIMO system. The capacity of a MIMO system with directional interferences is derived respectively for deterministic and random fading MIMO channels. And the asymptotic lower limit in high interference-to-noise ratio (INR) is also analyzed. It is indicated that the relationship between the numbers of receive antennas and interferences is critical to the capacity in high INR. The number of receive antennas should be greater than that of interferences in order to ensure the system work normally in high INR, and the corresponding capacity limit approaches the capacity of another MIMO system using same number of transmit antennas and fewer receive antennas without interference. Moreover, the impacts of the correlation and directions of the interferences on capacity are discussed respectively. It is indicated that higher correlation and closer directions of the interferences both yields higher capacity. Monte Carlo simulation results are given and well coincide with our theoretical analysis results.

Key-Words: - MIMO, capacity, directional interference.

1 Introduction

In recent years, Multiple Input Multiple Output (MIMO) technique using multiple antennas at both transmitter and receiver has attracted much attention owing to its promise of dramatic system performance and capacity gains [1~3]. It is widely considered as a promising technique in the next generation communication system and has been widely adopted in various wireless communication standards, such as the 3GPP cellular standard [4], WiMAX (IEEE 802.16) [5], and IEEE 802.11n [6]. Besides in wireless commercial communication realm, MIMO technique is also attractive for military communication systems to provide communication reliability as possible.

There has been a great deal of studies on the capacity of MIMO system. For single user communications over the Rayleigh fading channel, Telatar [2], Foschini [3], etc. proved that the capacity can be multiplied several times by using MIMO technique under independence assumptions for the fades and noises. By investigating MIMO capacity under various propagation conditions, such as Ricean channels in [7]; correlated fading channels in [8]; and time varying channels in [9], this significant result has been extended. In addition, the measurement, evaluation and optimization of MIMO channel capacity are also investigated. In [10], capacity measurements for wideband MIMO channels in indoor office environment were performed for different propagation conditions. The genetic algorithm technique was applied for optimization process of channel capacity for indoor MIMO system in [11]. And in [12], the vector parabolic equation method was used to investigate the capacity of MIMO systems in indoor corridors.

In a practical scenario, unfortunately, there are many kinds of interferences such as co-channel interference, directional interference (including jamming interferences in a military application), which degrade the capacity of MIMO system significantly or even make MIMO system fail to transmit useful information.

The capacity limits of MIMO system with co-channel interference has been investigated in many references [13~15]. It is concluded that MIMO system provides higher capacity with fewer co-channel interferences each with higher power than with more ones each with lower power in [13]. And in [14], a rigorous explanation of this conclusion is provided. The optimum signaling scheme for capacity of narrowband MIMO system with co-channel interference was considered in [15], where there is no channel state information (CSI) at the transmitter, but CSI is assumed at the receiver.

However, even though directional interference exists widely, literatures focusing on the capacity of MIMO system with directional interference are limited in recent work. In this paper, the capacity of a MIMO system with directional interferences is investigated. The MIMO system is restricted to narrowband system. With the assumption that CSI is known to the receiver but unknown to the transmitter, the capacity of the MIMO system with directional interferences is derived for both deterministic and random fading channels in detail. And we also analyze the asymptotic lower limit on capacity in high interference-to-noise ratio (INR) since the power of directional interference is often much higher than that of noise. For a MIMO system using N_T transmit antennas and N_R receive antennas with N_J directional interferences in high INR, the analysis in this paper indicates that the relationship between the numbers of receive antennas and interferences is critical to the capacity in high INR. When the number of receive antennas is less than or equal to that of interferences, i.e., $N_R \le N_J$, the capacity limit in high INR approaches zero. And when the number of receive antennas is greater than that of interferences, i.e., $N_R > N_J$, the capacity limit in high INR approaches the capacity of another MIMO system using N_T transmit and $\left(N_R - N_J\right)$ receive antennas without interference. Therefore, the condition $N_R > N_J$ should be satisfied in order to ensure the MIMO system with directional interferences work normally in high INR. Moreover, the impacts of the correlation and directions of the interferences on capacity are also discussed respectively. It is indicated that higher correlation and closer directions of the interferences both yields higher capacity. Monte Carlo simulation results are given and accord well with the theoretical analysis results.

The remainder of this paper is organized as follows: In section 2, the system model for a MIMO system with directional interferences is introduced briefly. In section 3, the capacity of a MIMO system with directional interferences is derived in detail for both deterministic and random fading channels. Then, the asymptotic lower limit on capacity in high INR, the impacts of the correlation and directions of the interferences on capacity are further analyzed respectively in section 4 and 5. Monte Carlo simulation results are given in section 6. Finally, We conclude the paper in section 7.

2 System Overview

Consider a narrowband $N_T \times N_R$ MIMO system with N_J directional interferences as shown in Fig 1. Denoting the transmitted signal vector at time periods t by $\mathbf{s} = \left[s_1, s_2, \cdots s_{N_T} \right]^T$. Usually, it is assumed that the entries of $\mathbf{s}$ have zero mean. In order to constrain the total average energy transmitted over a symbol period, the covariance matrix of the transmitted signal vector $\mathbf{s}$, $\mathbf{R_{ss}} = E\left(\mathbf{ss}^H\right)$, should satisfy $Tr\left(\mathbf{R_{ss}}\right) = N_T$ where $Tr\left(\bullet\right)$ denotes the trace of a matrix.

The MIMO channels between the transmitter and receiver can be described by a matrix $\mathbf{H}$ of dimension $N_R \times N_T$. Here, we restrict the MIMO channels to quasi-static, frequency-flat Rayleigh fading channels. The entries of $\mathbf{H}$, $\left[\mathbf{H}\right]_{n_R, n_T}$, represents the channel coefficient between the $n_T th$ transmit antenna and the $n_R th$ receive antenna for $n_T = 1,\ 2,\ ...,\ N_T$ and $n_R = 1,\ 2,\ ...,\ N_R$.

Denoting the transmitted interference vector at time periods t by $\mathbf{z} = \left[z_1, z_2, \cdots, z_{N_J} \right]^T$, where z_j , $j = 1, 2, \cdots, N_J$, represents the interference transmitted from the jth directional interference source, and is assumed to be zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance.

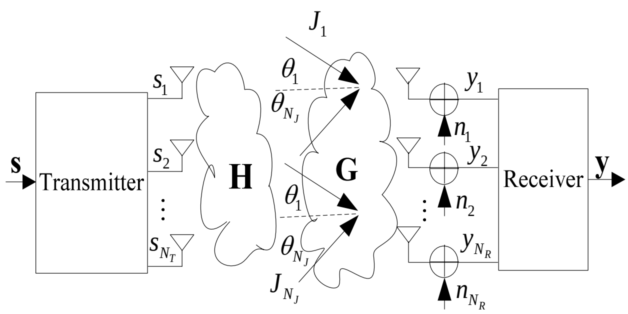

Fig.1 A $N_T \times N_R$ MIMO system with N_J directional interferences

The interferences may suffer scattering since there are often rich scatterers around the receiver. Assumed that the direct component P_{Dj} of the jth directional interference arrives at the receiver directly from θ_j, and the rest scattering component P_{Sj} suffers Rayleigh fading and arrives at the receiver in scattered paths for $j = 1, 2, \cdots, N_J$. Then, the transmission vector of the jth directional interference, $\mathbf{g}_j$, can be modelled as follows

$$\mathbf{g}_j = \sqrt{\frac{K_j}{K_j + 1}} \cdot \mathbf{a}(\theta_j) + \sqrt{\frac{1}{K_j + 1}} \cdot \tilde{\mathbf{g}}_j \quad , \quad j = 1, 2, \cdots N_J \tag{1}$$

where $\mathbf{a}(\theta_j)$ is the directional vector of the jth interference, $\tilde{\mathbf{g}}_j$ represents the scattered paths of the jth interference which has independent and identically distributed (i.i.d) ZMCSCG entries with unit variance, and K_j is the Ricean factor of the jth directional interference and is given by the ratio of the power in direct component P_{Dj} to the power in scattering component P_{Sj}, i.e.,

$$K_j = \frac{P_{Dj}}{P_{Sj}}, \quad j = 1, 2, \cdots, N_J .$$

Assume that pulse shaping, match-filtering and sampling are all optimal, therefore, the $N_R \times 1$ vector of received complex base band samples after matched filtering becomes

$$\begin{aligned} \mathbf{y} &= \sqrt{\frac{\rho}{N_T}} \mathbf{Hs} + \mathbf{G\Phi z} + \mathbf{n} \\ &= \sqrt{\frac{\rho}{N_T}} \mathbf{Hs} + \mathbf{j} + \mathbf{n} \end{aligned} \tag{2}$$

where ρ is the signal-to-noise ratio (SNR), $\mathbf{n}$ is the noise vector of dimension $N_R \times 1$. Usually, it is assumed that the channel matrix $\mathbf{H}$ and the noise vector $\mathbf{n}$ have i.i.d ZMCSCG entries with unit variance and are themselves mutually independent. And the term $\mathbf{j} = \mathbf{G\Phi z}$ represents the received interference vector. The matrix $\mathbf{\Phi}$ is a $N_J \times N_J$ diagonal matrix, where $[\mathbf{\Phi}]_{j,j} = \sqrt{\eta_j}$ and η_j denotes the INR of the jth directional interference for $j = 1, 2, ..., N_J$. The matrix $\mathbf{G}$ of dimension $N_R \times N_J$ represents the transmission matrix of the directional interferences, where $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \cdots \mathbf{g}_{N_J}]$. And the entries of the vector $\mathbf{z}$ are assumed to be independent from those of the channel matrix $\mathbf{H}$ and the noise vector $\mathbf{n}$. The covariance matrix of the received interference vector $\mathbf{j}$ is given by

$$\mathbf{R_{jj}} = E(\mathbf{jj}^H) = \mathbf{G\Phi R_{zz} \Phi G}^H , \tag{3}$$

where $\mathbf{R_{zz}} = E(\mathbf{zz}^H)$ is the covariance matrix of the interference sources. And the entries of $\mathbf{R_{zz}}$, $[\mathbf{R_{zz}}]_{j,q}$, represents the correlation coefficient between the jth interference source and the qth interference source for $j, q = 1, 2, ..., N_J$.

3 Capacity of MIMO System with Directional Interferences

In this section, the capacity of a MIMO system with directional interferences is derived in detail for both deterministic and random fading channels with the assumption that the channel matrix $\mathbf{H}$ and the transmission matrix $\mathbf{G}$ are known to the receiver but unknown to the transmitter.

First, assume that the channel matrix $\mathbf{H}$ and the transmission matrix $\mathbf{G}$ are both deterministic. The capacity of MIMO system is defined as the following expression [2~3, 16]

$$C = \max_{f(\mathbf{s})} I(\mathbf{s}, \mathbf{y}) , \tag{4}$$

where $f(\mathbf{s})$ is the probability distribution of the transmitted signal vector $\mathbf{s}$, and $I(\mathbf{s}, \mathbf{y})$ is the mutual information between the vectors $\mathbf{s}$ and $\mathbf{y}$. Note that

$$I(\mathbf{s}, \mathbf{y}) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{s}), \tag{5}$$

where $H(\mathbf{y})$ is the differential entropy of the received signal vector $\mathbf{y}$, while $H(\mathbf{y}|\mathbf{s})$ is the conditional differential entropy of the received signal vector $\mathbf{y}$, given knowledge of the transmitted signal vector $\mathbf{s}$. Usually, the received interference vector $\mathbf{j}$ and the noise vector $\mathbf{n}$ are both independent from the transmitted signal vector $\mathbf{s}$, then we have $H(\mathbf{y}|\mathbf{s}) = H(\mathbf{j} + \mathbf{n})$. Therefore, (5) simplifies to

$$I(\mathbf{s}, \mathbf{y}) = H(\mathbf{y}) - H(\mathbf{j} + \mathbf{n}). \tag{6}$$

According to the nature of the Gaussian distribution [17], $H(\mathbf{y})$ and $H(\mathbf{j} + \mathbf{n})$ are given by:

$$H(\mathbf{y}) = \log_2 \det(\pi e \mathbf{R_{yy}}) \quad \text{bps/Hz} , \tag{7}$$

$$H(\mathbf{j} + \mathbf{n}) = \log_2 \det(\pi e \mathbf{R_{jn}}) \quad \text{bps/Hz} , \tag{8}$$

where $\mathbf{R}_{\mathbf{yy}}$ is the covariance matrix of the received signal vector $\mathbf{y}$, and $\mathbf{R}_{\mathbf{jn}}$ is the covariance matrix of the combined the received interference vector $\mathbf{j}$ plus the noise vector $\mathbf{n}$. The matrices $\mathbf{R}_{\mathbf{yy}}$ and $\mathbf{R}_{\mathbf{jn}}$ are respectively given by

$$\mathbf{R}_{\mathbf{yy}} = E\left(\mathbf{y}\mathbf{y}^H\right) = \frac{\rho}{N_T}\mathbf{H}\mathbf{R}_{\mathbf{ss}}\mathbf{H}^H + \mathbf{R}_{\mathbf{jn}}, \qquad (9)$$

$$\mathbf{R}_{\mathbf{jn}} = E\left\{(\mathbf{j}+\mathbf{n})(\mathbf{j}+\mathbf{n})^H\right\} = \mathbf{G\Phi R}_{\mathbf{zz}}\mathbf{\Phi G}^H + \mathbf{I}_{N_R}. \qquad (10)$$

Note that the covariance matrix $\mathbf{R}_{\mathbf{jn}}$ is positive definite matrix. Then, $I(\mathbf{s},\mathbf{y})$ in (6) reduces to

$$I(\mathbf{s},\mathbf{y}) = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\mathbf{H}\mathbf{R}_{\mathbf{ss}}\mathbf{H}^H\left(\mathbf{R}_{\mathbf{jn}}\right)^{-1}\right)\text{bps/Hz}. \qquad (11)$$

Therefore, it follows from (4) that the capacity of the $N_T \times N_R$ MIMO system with N_J directional interferences is given by:

$$C = \max_{Tr(\mathbf{R}_{\mathbf{ss}})=N_T} \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\mathbf{H}\mathbf{R}_{\mathbf{ss}}\mathbf{H}^H\left(\mathbf{R}_{\mathbf{jn}}\right)^{-1}\right)\text{bps/Hz}. \qquad (12)$$

Note that $\mathbf{\Phi} \to \mathbf{0}$ as the INR of each interference approaches zero, i.e., $\eta_j \to 0$ for $j = 1,2,\cdots,N_J$. Hence, according to (10), the matrix $\left(\mathbf{R}_{\mathbf{jn}}\right)^{-1}$ approaches

$$\left(\mathbf{R}_{\mathbf{jn}}\right)^{-1} = \left(\mathbf{G\Phi R}_{\mathbf{zz}}\mathbf{\Phi G}^H + \mathbf{I}_{N_R}\right)^{-1} \to \mathbf{I}_{N_R}$$
$$\text{as } \eta_j \to 0 \quad \text{for } j = 1,2,\cdots,N_J \;,. \qquad (13)$$

And the capacity in (12) approaches

$$C \to \max_{Tr(\mathbf{R}_{\mathbf{ss}})=N_T} \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\mathbf{H}\mathbf{R}_{\mathbf{ss}}\mathbf{H}^H\right)\text{bps/Hz}$$
$$\text{as } \eta_j \to 0 \quad \text{for } j = 1,2,\cdots,N_J, \qquad (14)$$

which is the capacity of a $N_T \times N_R$ MIMO system without interference for AWGN channels given in [2~3]. This conclusion is consistent with intuitive inference.

3.1 Capacity of Deterministic Channels

When the channel matrix $\mathbf{H}$ and the transmission matrix $\mathbf{G}$ are both deterministic and completely unknown to the transmitter, the transmitted signal vector $\mathbf{s}$ may be chosen to be statistically non-preferential, i.e.,

$$\mathbf{R}_{\mathbf{ss}} = \mathbf{I}_{N_T}. \qquad (15)$$

This implies that the signals are independent and allocated equal power at the transmit antennas.

Substituting (15) into (12), the capacity of the MIMO system with directional interferences for deterministic channels in the absence of CSI at the transmitter is given by

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\mathbf{H}\mathbf{H}^H\left(\mathbf{R}_{\mathbf{jn}}\right)^{-1}\right)\text{bps/Hz}. \qquad (16)$$

Note that the eigen decomposition of the covariance matrix $\mathbf{R}_{\mathbf{jn}}$ [18] is in the form of

$$\mathbf{R}_{\mathbf{jn}} = \mathbf{U}\mathbf{D}\mathbf{U}^H, \qquad (17)$$

where $\mathbf{D}$ is a $N_R \times N_R$ diagonal matrix, which is given by

$$\mathbf{D} = diag\left(\delta_1, \delta_2, \cdots \delta_{N_R}\right). \qquad (18)$$

And $\delta_1 \ge \delta_2 \ge \cdots \ge \delta_{N_R} \ge 1$ are the eigen values of the covariance matrix $\mathbf{R}_{\mathbf{jn}}$, and the matrix $\mathbf{U}$ is a $N_R \times N_R$ unitary matrix consisting of the eigen vectors of $\mathbf{R}_{\mathbf{jn}}$. It is not difficult to obtain the form of $\left(\mathbf{R}_{\mathbf{jn}}\right)^{-1}$ from (17)

$$\left(\mathbf{R}_{\mathbf{jn}}\right)^{-1} = \mathbf{U}\mathbf{D}^{-1}\mathbf{U}^H. \qquad (19)$$

Using (19) and the following identity

$$\det\left(\mathbf{I}_m + \mathbf{AB}\right) = \det\left(\mathbf{I}_n + \mathbf{BA}\right), \qquad (20)$$

for matrices $\mathbf{A}(m \times n)$ and matrix $\mathbf{B}(n \times m)$, the capacity of the MIMO system with directional interferences for deterministic channels in the absence of CSI at the transmitter in (16) is equivalent to

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\mathbf{D}^{-1}\right)\text{bps/Hz} \quad (21)$$

where $\tilde{\mathbf{H}}$ is defined as

$$\tilde{\mathbf{H}} = \mathbf{U}^H\mathbf{H}. \qquad (22)$$

Note that the column vectors of the matrix $\mathbf{U}$ is the eigen vectors of the covariance matrix $\mathbf{R}_{\mathbf{jn}}$, $\tilde{\mathbf{H}}$ can be viewed as the modified version of the channel matrix $\mathbf{H}$ incorporating interferences and noise. According to the nature of Gaussian distribution [17], the entries of the matrices $\tilde{\mathbf{H}}$ and $\mathbf{H}$ have same statistical characteristics since the matrix $\mathbf{U}$ is a unitary matrix. Therefore, the entries of the matrix $\tilde{\mathbf{H}}$, $\left[\tilde{\mathbf{H}}\right]_{n_R,n_T}$, are i.i.d ZMCSCG random variables with unit variance for $n_T = 1,\ 2,\ ...,\ N_T$ and $n_R = 1,\ 2,\ ...,\ N_R$.

3.2 Capacity of Random Channels

There are two commonly used statistics to describe the capacity of random channels. One statistic is the ergodic capacity [2~3,16], the other statistic is the outage capacity [2,16]. The ergodic capacity of a MIMO channel is the average of the ensemble average of the information rate over the distribution of the channel matrix $\mathbf{H}$, and the outage capacity is used to describe the information rate that is guaranteed with a certain level of reliability. In this paper, the former statistic, i.e., ergodic capacity, is used in the discussion of the capacity of MIMO system with directional interferences for random channels.

When the channel matrix $\mathbf{H}$ and the transmission matrix $\mathbf{G}$ are both random and ergodic, according to the definition of the ergodic capacity given in [2~3], the ergodic capacity of the MIMO system with directional interferences for random channels is the ensemble average of (12) over the distribution of the entries of the matrices $\mathbf{H}$ and $\mathbf{G}$.

Similar to the case that the channel matrix $\mathbf{H}$ and the transmission matrix $\mathbf{G}$ are both deterministic, the covariance matrix of the transmitted signal vector $\mathbf{s}$, $\mathbf{R}_{\mathbf{ss}}$, is chosen to be the identity matrix when the matrices $\mathbf{H}$ and $\mathbf{G}$ are unknown to the transmitter. Therefore, the corresponding ergodic capacity of the MIMO system with directional interferences for random channels in the absence of CSI at the transmitter is given by the ensemble average of (16) or (21) over the distribution of the entries of the matrices $\mathbf{H}$ and $\mathbf{G}$, i.e.,

$$C_{ERG} = E(C) = E\left(\log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\mathbf{H}\mathbf{H}^H\left(\mathbf{R}_{\mathbf{jn}}\right)^{-1}\right)\right) \text{bps/Hz}, \quad (23)$$

or

$$C_{ERG} = E(C) = E\left(\log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\mathbf{D}^{-1}\right)\right) \text{bps/Hz}. \quad (24)$$

where $E(\bullet)$ denotes expectation operation.

Since the capacity of deterministic channels in (16) or (21) can be viewed as a special case of the ergodic capacity of random channels in (23) or (24), we will focus on the ergodic capacity of random channels in the following discussions.

4 Asymptotic Lower Limit on Capacity in High INR

Usually, in military application, the power of directional interferences is much stronger than that of noise, i.e., $\eta_j \to +\infty$, for $j=1,2,\cdots,N_J$. Therefore, it is necessary to analyze the capacity limit in high INR. The interference sources are assumed to be uncorrelated, i.e., $\mathbf{R}_{\mathbf{zz}} = \mathbf{I}_{N_J}$.

It is not difficult to obtain the form of $\mathbf{D}^{-1}$ from (18)

$$\mathbf{D}^{-1} = diag\left(\delta_1^{-1}, \delta_2^{-1}, \cdots \delta_{N_R}^{-1}\right). \quad (25)$$

When the number of receive antennas is less than or equal to that of interferences, i.e., $N_R \le N_J$, it is observed from the definition of $\mathbf{U}$ as an unitary matrix that any increase in the INR of the *kth* interference η_k will result in an increase of the eigen values δ_k of the covariance matrix $\mathbf{R}_{\mathbf{jn}}$ for $k=1,2,\cdots,N_R$. Therefore, according to (25), as the INR of each interference approaches infinite, i.e., $\eta_k \to +\infty$ for $k=1,2,\cdots,N_R$, we have

$$\delta_k^{-1} \to 0$$

$$\mathbf{D}^{-1} = diag\left(\delta_1^{-1}, \delta_2^{-1}, \cdots \delta_{N_R}^{-1}\right) \to \mathbf{0}, \quad (26)$$

and the ergodic capacity in (24) decreases gradually and approaches

$$C_{ERG} = E\left(\log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\mathbf{D}^{-1}\right)\right) \to 0 \text{bps/Hz}. \quad (27)$$

And when the number of receive antennas is greater than that of interferences, i.e., $N_R > N_J$, the matrix $\mathbf{D}$ is in the form of

$$\mathbf{D} = diag\left(\delta_1, \delta_2, \cdots, \delta_{N_J}, 1, 1, \cdots, 1\right). \quad (28)$$

Thus, $\mathbf{D}^{-1}$ is given by

$$\mathbf{D}^{-1} = diag\left(\delta_1^{-1}, \delta_2^{-1}, \cdots, \delta_{N_J}^{-1}, 1, 1, \cdots, 1\right). \quad (29)$$

Similarly, increasing the INR of the *kth* interference η_k will increase the eigen values δ_k of the covariance matrix $\mathbf{R}_{\mathbf{jn}}$ for $k=1,2,\cdots,N_J$, but for $k=N_J+1,\cdots,N_R$, δ_k will remain fixed at unity.

Therefore, according to (29), as the INR of the *kth* interference $\eta_k \to +\infty$ for $k=1,2,\cdots,N_J$, we see that

$$\delta_k^{-1} \to 0,$$

$$\left(\mathbf{D}^{-1}\right)_k \to \mathbf{0}, \quad (30)$$

where $\left(\mathbf{D}^{-1}\right)_k$ denotes the *kth* column of $\mathbf{D}^{-1}$, and the ergodic capacity in (24) decreases gradually and approaches

$$C_{ERG} = E\left(\log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\mathbf{D}^{-1}\right)\right) \to E\left(\log_2 \det\left(\mathbf{I}_{N_R} + \frac{\rho}{N_T}\mathbf{H}_P\mathbf{H}_P^H\right)\right) \text{bps/Hz}, \quad (31)$$

where $\mathbf{H}_P$ is a $(N_R - N_J)\times N_T$ matrix, which is given by

$$\mathbf{H}_P = \mathbf{U}_\mathbf{n}^H \mathbf{H}. \quad (32)$$

And $\mathbf{U}_\mathbf{n}$ is a $N_R \times (N_R - N_J)$ matrix consisting of the basis vectors of noise subspace, which can be obtained by partitioning the unitary matrix $\mathbf{U}$ in (19) into two sub-matrices as follows

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_\mathbf{j} & \mathbf{U}_\mathbf{n} \end{bmatrix}, \quad (33)$$

where $\mathbf{U}_\mathbf{j}$ is a $N_R \times N_J$ matrix consisting of the basis vectors of interferences subspace .

Obviously, the entries of the matrix $\mathbf{H}_P$ are i.i.d ZMCSCG random variables with unit variance, which can be used to describe the channel matrix of another $N_T \times (N_R - N_J)$ MIMO system without interference under Rayleigh fading propagation condition. Therefore, under Rayleigh fading propagation condition, the capacity limit of the $N_T \times N_R$ MIMO system with N_J directional interferences in high INR given by (31) approaches the capacity of another $N_T \times (N_R - N_J)$ MIMO system without interference when $N_R > N_J$.

According to the above analysis, we get the following conclusions. Under Rayleigh fading propagation condition, the relationship between the numbers of receive antennas and interferences is critical to the capacity of a $N_T \times N_R$ MIMO system with N_J directional interferences in high INR. When the number of receive antennas is less than or equal to that of interferences, i.e., $N_R \le N_J$, the ergodic capacity limit in high INR approaches zero and dose not be improved as SNR increases. In this case, the information can not be transmitted reliably. And when the number of receive antennas is greater than that of interferences, i.e., $N_R > N_J$, the system will be able to support some capacity, and the corresponding capacity limit in high INR approaches the capacity of another $N_T \times (N_R - N_J)$ MIMO system without interference. Therefore, the condition $N_R > N_J$ should be satisfied in order to ensure the MIMO system work normally in high INR.

5 Impacts of the Correlation and Directions of the interferences

In this section, the impacts of the correlation and directions of interferences on capacity are analyzed, respectively. In the following, it is also assumed that the matrices $\mathbf{H}$ and $\mathbf{G}$ are known perfectly to the receiver and unknown to the transmitter, and that the number of receive antennas is greater than that of interferences.

5.1 Impact of the Correlation of the Interferences

The correlation of the interferences indicates the correlation of the interference sources in this paper. When the interferences are correlated, the matrix $\mathbf{R}_\mathbf{zz}$ is not the identity matrix, and the matrix $\mathbf{R}_\mathbf{jn}$ will be affected.

According to (23), in high SNR, the corresponding ergodic capacity of the MIMO system with directional interferences for random channels in the absence of CSI at the transmitter can be written as

$$C_{ERG} \approx E\left(\log_2 \det\left(\frac{\rho}{N_T}\mathbf{H}\mathbf{H}^H\right)\right) - E\left(\log_2 \det\left(\mathbf{R}_\mathbf{jn}\right)\right) \text{ bps/Hz}. \quad (34)$$

Let

$$Q = E\left(\log_2 \det\left(\mathbf{R}_\mathbf{jn}\right)\right). \quad (35)$$

As we know that the covariance matrix $\mathbf{R}_\mathbf{jn}$ is positive definite matrix. Hence, we have $Q > 0$. Substituting (10) into (35), we have

$$Q = E\left(\log_2 \det\left(\mathbf{I}_{N_R} + \mathbf{G}\mathbf{\Phi}\mathbf{R}_\mathbf{zz}\mathbf{\Phi}\mathbf{G}^H\right)\right). \quad (36)$$

From (36), it is clear that when the transmission matrix $\mathbf{G}$ is unknown to the transmitter, the value of Q is maximized by choosing $\mathbf{R}_\mathbf{zz} = \mathbf{I}_{N_J}$, so that the ergodic capacity in (34) is minimized. Therefore, we get the conclusion that the correlation of the interferences has positive impact on capacity, i.e., higher correlation of the interferences yields higher capacity.

5.2 Impact of the Directions of the Interferences

We focus on the case that the transmission matrix $\mathbf{G}$ consists of the direct component only, i.e., the Rican factor $K_j \to +\infty$, for $j = 1, 2, \cdots, N_J$. It is assumed that the directional interferences are uncorrelated.

When there is only one directional interference from θ_1, the covariance matrix $\mathbf{R}_\mathbf{jn}$ in (10) becomes

$$\mathbf{R}_\mathbf{jn} = \eta_1 \mathbf{a}(\theta_1)\mathbf{a}(\theta_1)^H + \mathbf{I}_{N_R}. \quad (37)$$

According to (28) and (33), the corresponding unitary matrix $\mathbf{U}$ and the diagonal matrix $\mathbf{D}$ are respectively in the form of

$$\mathbf{D} = diag\left(\delta_1, 1, \cdots 1\right),$$
$$\mathbf{U} = \left[\mathbf{u}_1 \quad \mathbf{U}_\mathbf{n}\right]. \tag{38}$$

where $\mathbf{u}_1$ is the eigen vector of $\mathbf{R}_\mathbf{jn}$, corresponding to the maximal eigen value δ_1. Note that

$$\mathbf{R}_\mathbf{jn} \cdot \frac{\mathbf{a}(\theta_1)}{\left\|\mathbf{a}(\theta_1)\right\|} = \left(\eta_1 \left\|\mathbf{a}(\theta_1)\right\|^2 + 1\right) \cdot \frac{\mathbf{a}(\theta_1)}{\left\|\mathbf{a}(\theta_1)\right\|}. \tag{39}$$

Obviously, $\left(\eta_1 \left\|\mathbf{a}(\theta_1)\right\|^2 + 1\right)$ is the maximal egien value of $\mathbf{R}_\mathbf{jn}$, and $\frac{\mathbf{a}(\theta_1)}{\left\|\mathbf{a}(\theta_1)\right\|}$ is the corresponding egien vector of $\mathbf{R}_\mathbf{jn}$. Hence, we have

$$\delta_1 = \eta_1 \left\|\mathbf{a}(\theta_1)\right\|^2 + 1.$$
$$\mathbf{u}_1 = \frac{\mathbf{a}(\theta_1)}{\left\|\mathbf{a}(\theta_1)\right\|}. \tag{40}$$

Then, from (38) and (40), $\mathbf{D}^{-1}$ becomes

$$\mathbf{D}^{-1} = diag\left(\left(\eta_1 \left\|\mathbf{a}(\theta_1)\right\|^2 + 1\right)^{-1}, 1, \cdots, 1\right). \tag{41}$$

It is indicated from (41) that the value of θ_1 dose not affect the matrix $\mathbf{D}^{-1}$. Hence, according to (24), the ergodic capacity of the MIMO system with only one directional interference from θ_1 for random channels is independent from θ_1, i.e., the corresponding ergodic capacity is not affected by the value of θ_1.

When there are more than one directional interference, i.e., $N_J > 1$, the covariance matrix $\mathbf{R}_\mathbf{jn}$ in (10) becomes

$$\mathbf{R}_\mathbf{jn} = \left(\sum_{j}^{N_J} \eta_j \mathbf{a}(\theta_j) \mathbf{a}(\theta_j)^H\right) + \mathbf{I}_{N_R}. \tag{42}$$

Since the directional vectors of different interferences are correlated, the corresponding unitary matrix $\mathbf{U}$ and the diagonal matrix $\mathbf{D}$ are both affected by the values of θ_j, $j = 1, 2, \cdots, N_J$. Intuitively, the N_J interferences will appear to be just one, higher power, interference by simply sum from the receiver's point of view as the directions of these interferences are close enough. Therefore, it is plain that the ergodic capacity of a MIMO system with N_J ($N_J > 1$) directional interferences for random channels increases as the directions of the interferences get closer.

6 Simulation Results

In this section, Monte Carlo simulations are carried out to verify the above theoretical derivation and analysis results. In the following simulations, a 4×3 MIMO system is adopted. The MIMO channels are assumed to be Rayleigh fading channels. And we used 10000 instantiations of the channel matrix $\mathbf{H}$ and the transmission matrix $\mathbf{G}$. It is assumed that the matrices $\mathbf{H}$ and $\mathbf{G}$ are known to the receiver but unknown to the transmitter.

Firstly, we focus on the impact of directional interference on capacity. The ergodic capacity curves for cases of only one interference where $\theta_1 = -20\text{Deg}$ and two interferences where $\theta_1 = -20\text{Deg}$ and $\theta_2 = 40\text{Deg}$ are shown in Fig.2. The INR of all the interferences are the same and set as 0dB, 10dB, and 20dB in order. The Ricean factor of all the interferences is fixed at 6. The corresponding cumulative distribution (CDF) curves of information rate when $\rho = 25\text{dB}$ are shown in Fig.3. It can be seen from Fig.2 and Fig.3, the capacity of a MIMO system with interferences decreases obviously compared with that of a MIMO system without interference. Moreover, the capacity decreases with increasing INR and also with increasing the number of interferences.

Secondly, we turn to testing the asymptotic lower limit on capacity in high INR. There are four cases. The first case is that there is only one interference from $\theta_1 = -20\text{Deg}$. The second case is that there are two interferences from $\theta_1 = -20\text{Deg}$ and $\theta_2 = 40\text{Deg}$. The third case is that there are three interferences from $\theta_1 = -20\text{Deg}$, $\theta_2 = 40\text{Deg}$ and $\theta_3 = 70\text{Deg}$. And the last case is that there are four interferences from $\theta_1 = -20\text{Deg}$, $\theta_2 = 40\text{Deg}$, $\theta_3 = 70\text{Deg}$, and $\theta_4 = -50\text{Deg}$. The INR and Ricean factor of all the interferences are set as 40dB and 6 respectively.

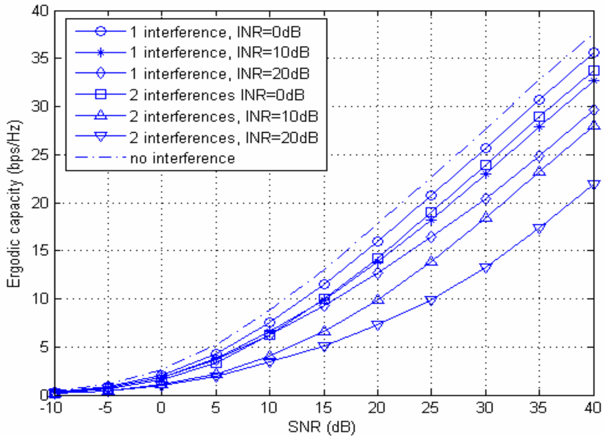

Fig.2 Capacity of a 4×3 MIMO system with directional interferences

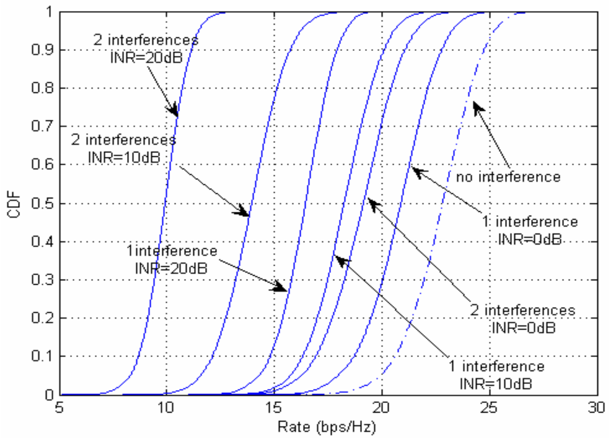

Fig.3 CDF of information rate of a 4×3 MIMO system with directional interferences

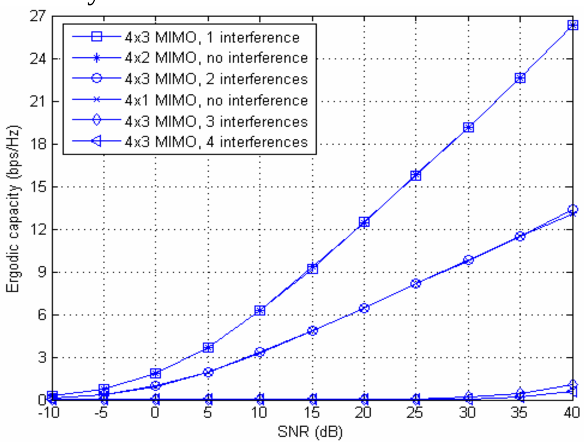

Fig.4 Asymptotic lower limit on capacity in high INR

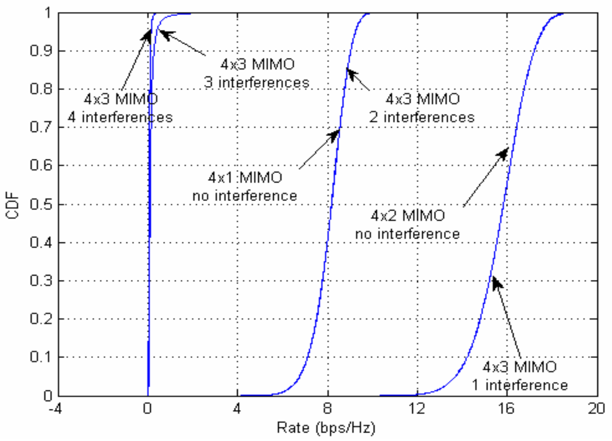

Fig.5 CDF of information rate in high INR

The curves of the capacity limit in high INR are shown in Fig.4. And the capacity curves of a 4×2 MIMO system and a 4×1 MIMO system without interference are also given as reference. From Fig.4, it is obviously that the curves of the capacity limit of the 4×3 MIMO system in the cases of only one interference and two interferences in high INR respectively coincide with the capacity curves of the 4×2 MIMO system and the 4×1 MIMO system without interference. It is consistent with the conclusion drawn in section 4 that in Rayleigh MIMO channels environment, the capacity limit of a $N_T \times N_R$ MIMO system with N_J directional interferences in high INR approaches the capacity of another $N_T \times (N_R - N_J)$ MIMO system without interference when $N_R > N_J$. Moreover, the capacity limit of the 4×3 MIMO system in high INR in the cases of three interferences and four interferences are both very close to zero and can not be improved as SNR increases. It is also consistent with the conclusion drawn in section 4 that the capacity of a $N_T \times N_R$ MIMO system with N_J directional interferences in high INR approaches zero when $N_R < N_J$. Same conclusions also can be drawn by observing the CDF curves of information rate with a SNR of 25dB as shown in Fig.5. Therefore, in order to ensure the MIMO system work normally in high INR the condition that $N_R > N_J$ should be satisfied as the conclusion drawn in section 4.

Thirdly, we turn to testing the impact of the correlation of the interferences on capacity. In the simulation, there are two interferences where $\theta_1 = -20\text{Deg}$, $\theta_2 = 40\text{Deg}$, $K_1 = K_2 = 6$ and $\eta_1 = \eta_2 = 20\text{dB}$, the ergodic capacity is simulated for the cases as follows:

Case A: two Interferences are independent to each other, i.e., $\mathbf{R}_{zz} = \mathbf{I}_{N_J}$;

Case B: two Interferences are correlated, where

$$\mathbf{R}_{zz} = \begin{bmatrix} 1 & 0.7 \\ 0.7 & 1 \end{bmatrix}$$

Case C: two Interferences are correlated, where

$$\mathbf{R}_{zz} = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$$

Case D: two Interferences are correlated, where

$$\mathbf{R}_{zz} = \begin{bmatrix} 1 & 0.99 \\ 0.99 & 1 \end{bmatrix}$$

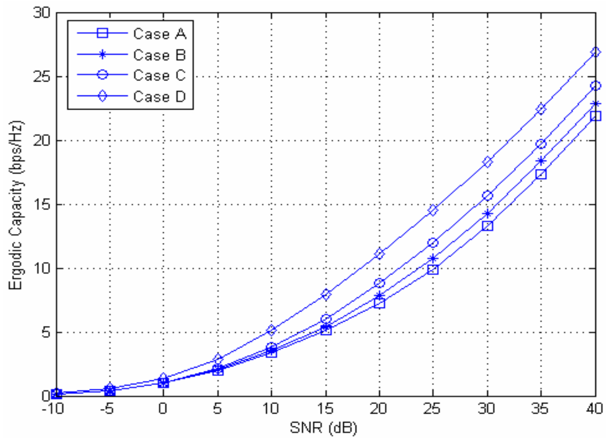

Fig.6 Impact of the correlation of the interference on capacity

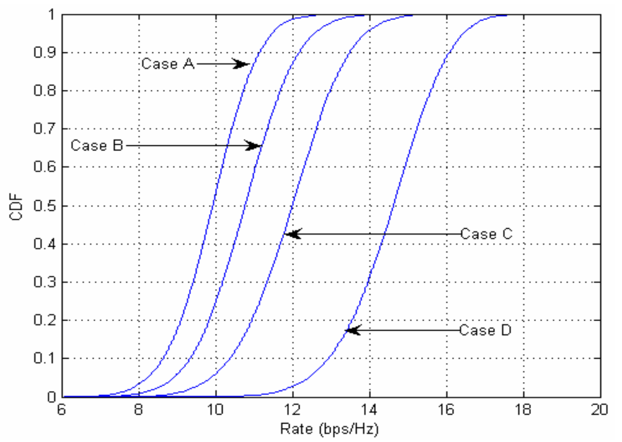

Fig.7 Impact of the correlation of the interference on CDF of information rate

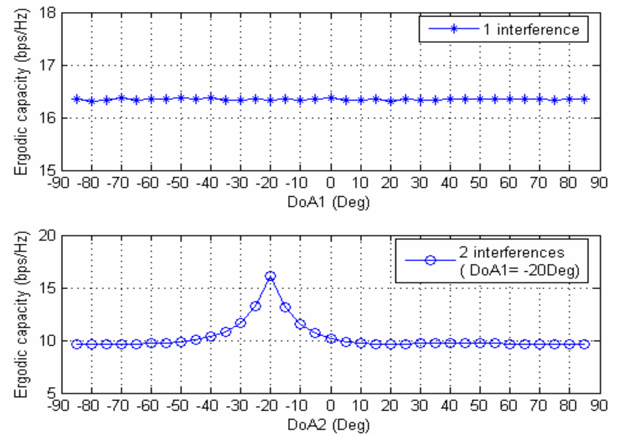

Fig.8 Impact of the directions of the interferences on capacity

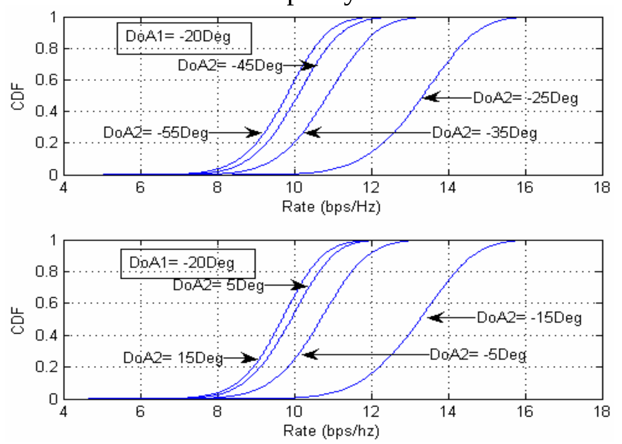

Fig.9 Impact of the directions of the interferences on CDF of information rate

The corresponding results are shown in Fig.6. And Fig.7 shows the CDF curves of information rate with a SNR of 25dB for the above four cases. It can be seen from Fig.6 and Fig.7 that higher correlation of the interference yields higher capacity, which is consistent with the conclusion drawn in section 5 that the correlation of the interferences has positive impact on capacity.

Finally, the impact of the directions of the interferences on capacity is explored. In the simulation, $\rho = 25\text{dB}$. It is assumed that the transmission matrix of the interferences, $\mathbf{G}$, consists of the direct component only. Fig.8 shows how the ergodic capacity varies with the directions of the interferences. There are two cases. One is that there is only one interference where $\eta_1 = 20\text{dB}$ and θ_1 changes from -85Deg to 85Deg. The other is that there are two interferences where $\eta_1 = \eta_2 = 20\text{dB}$, θ_1 is fixed at -20Deg while θ_2 changes from -85Deg to 85Deg. And the CDF curves of information rate in the case of two interferences are given in Fig.9. It can be seen from Fig.8 that the ergodic capacity in the case of only one directional interference from θ_1 is not affected by the value of θ_1, which is consistent with the conclusion drawn in section 5 that the ergodic capacity of a MIMO system with only one directional interference for random channels is independent from the direction of the interference. Moreover, we can see from Fig.8 and Fig.9 that the ergodic capacity increases as θ_2 approaches θ_1 in the case of two interferences. This also well accords with the conclusion drawn in section 5 that the ergodic capacity of the MIMO system with N_J ($N_J > 1$) directional interferences increases as the directions of the interferences get closer.

7 Conclusion

In directional interference scenario, the capacity of a MIMO system for deterministic and random fading MIMO channels is given through theoretical derivation in this paper, focusing on the case where the CSI is known to the receiver but unknown to the transmitter. The asymptotic lower limit on capacity in high INR is also analyzed. It is indicated that the relationship between the numbers of receive antennas and interferences is critical to the capacity in high INR. In order to ensure MIMO system with directional interferences works normally in high INR, the condition that the number of receive antennas is greater than that of interferences should be satisfied. Otherwise, the directional interferences with high INR will cause the capacity approaches zero. When the condition is satisfied, the corresponding capacity limit of the system approaches to the capacity of another MIMO system using same number of transmit antennas and fewer receive antennas without interference. In addition, the impacts of the correlation and directions of the interferences on capacity are further analyzed. It is seen that capacity increases as the correlation gets higher and the directions of the interferences get closer. Monte Carlo simulation results are given and well coincide with the theoretical analysis conclusions.

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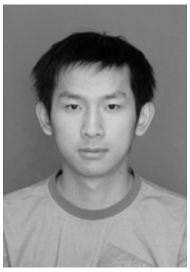

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