Study on Power and Rate Control Algorithm for Cognitive Wireless Networks

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Abstract:-Rate control and power control are two important issues in cognitive radio networks. In this paper, three rate control strategies are first introduced, namely Dirty Paper Coding (DPC) strategy, Noise strategy, and Opportunistic Interference Cancellation (OIC) strategy. The achievable rates of the three strategies are also compared. Then, optimal power control algorithms are proposed, each corresponding to one rate control strategy. The algorithms achieve the maximum transmit rate for the cognitive user by appropriately controlling the transmit power on each subchannel under the interference temperature constraint of the primary user. Simulation results show that the proposed algorithms can dramatically improve the transmit rate of cognitive user.

Key-Words:-Cognitive radio, Interference temperature, Power control, Rate control

1 INTRODUCTION

Wireless spectrum is quite a scarce resource, and as the rapid growth of the wireless standards and the population of wireless users, the scarcity of radio spectrum resource is becoming a serious issue, which imposes increasingly severe restrictions on the development of wireless communication industry. In recent years, a new wireless communication technology called Cognitive Radio (CR), which enables multi-radio system to share spectrum opportunistically, is emerging as a promising technique to deal with this increasingly tense situation. This technology improves the efficiency of spectrum utilization, and to some extent, solves the problem of the scarcity of radio spectrum resource [1]-[3].

Power control and rate control are two key technologies in cognitive radio system. In cognitive wireless networks, the primary user (authorized user) has priority in accessing the licensed spectrum; cognitive user detects the usage of spectrum and opportunistically access the unoccupied channel. In this paper, primary user shares spectrum with cognitive user, and in order to provide a protection for primary user's communication, the transmit power of cognitive user is strictly controlled. On the other hand, cognitive user adjusts its transmit rate based on channel condition and the interference from primary transmitter, in order to achieve the optimal utilization of spectrum resource.

There are several literatures concerning the above problems from the perspective of information theory. *On the problem of cognitive wireless channel capacity*: Devroye, et al [4] regard cognitive channel as a special interference channel with the entire knowledge of interference, and obtain the capacity limit of cognitive user by using Gel'fand-Pinsker's coding. Jovicic, et al [5] further point out that, in order to guarantee the transmit rate of primary user, cognitive user should divide its transmit power into two parts: one part relays primary user's information, and the other part transmits its own information. Also, they assume that the primary user's information is known to the cognitive user, so the cognitive transmitter can employ Dirty Paper Coding (DPC) to cancel the interference caused by the primary user at the cognitive receiver. In [6] and [7], the authors discuss the achievable rate region of cognitive multi-access channel and the methods to achieve the capacity. *On the problem of power control*: Ghasemi, et al [8] study the capacity limit of cognitive user and the optimal strategies of power allocation and rate control under the constraint of the interference temperature. Literature [9] proposes an opportunistic power control strategy in fading wireless channel, which can maximize the ergodic capacity of cognitive user under the premise of maintaining the primary user's outage probability. *On the problem of rate control*: Popovski, et al [10] propose the Opportunistic Interference Cancellation (OIC) strategy where the transmitter employs superposition coding while the receiver applies successive decoding. This strategy can significantly improve the transmit rate of cognitive user.

In this paper, we jointly consider the technologies of rate control and power control in cognitive radio networks. First, three commonly used rate control strategies are reviewed and compared: *Dirty Paper Coding (DPC) strategy, Noise strategy, Opportunistic Interference Cancellation (OIC) strategy*. Then, we focus on OFDM based multi-channel cognitive wireless system. Corresponding to the above rate control strategies, three optimal power control strategies are proposed. Aiming at maximizing the transmit rate of cognitive user, these strategies determine the optimal transmit power on each subchannels under the constraint of average interference power at the primary user. Simulation results show that, the proposed power control strategies can significantly improve the transmit rate of cognitive user.

2 SYSTEM MODEL

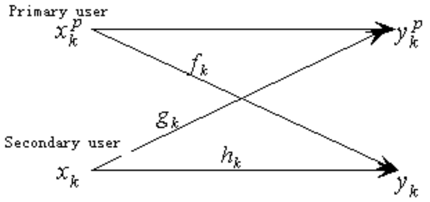

Fig.1 Channel model of cognitive radio networks

Channel model of cognitive radio networks is shown in Figure 1. The primary receiver communicates with the primary transmitter using the licensed spectrum. The cognitive user communicates using the same spectrum. The primary user and cognitive user both employ the OFDM modulation technique to transmit information. Assume that there are K subchannels in the system, and for the k-th subchannel, g_k denotes the channel gain between cognitive transmitter and primary receiver, h_k denotes the channel gain between cognitive transmitter and cognitive receiver, f_k denotes the channel gain between primary transmitter and cognitive receiver. In addition, we also assume that the cognitive system obtains the channel gain information through certain methods, and adjust the transmit power and rate on each subchannel based on the channel information.

The received signal on the k-th subchannel of the cognitive receiver can be described as:

$$y_k = h_k \sqrt{p_k} x_k + f_k \sqrt{p_k^p} x_k^p + z_k \qquad (1)$$

where p_k, p_k^p denote the transmit power of cognitive user and primary user on the k-th subchannel respectively, x_k, x_k^p denote the transmit symbols, and z_k denotes the Gaussian noise with power σ_k^2.

In cognitive radio networks, the primary user ignores the presence of cognitive user. In other words, primary user's transmit power p_k^p and transmit rate R_k^p are determined by only primary user and are uncorrelated with g_k, h_k and f_k. So, in this paper, we regard p_k^p and R_k^p as known parameters.

On the other hand, in order to avoid the interference at the primary user caused by the cognitive user, the average interference power at primary receiver is strictly constrained. Let $\overline{Q}$ be the maximum average power that the primary user can tolerate at its receiver, thus

$$\sum_{k=1}^{K} g_k^2 p_k \le \overline{Q}. \qquad (2)$$

Note that, $\overline{Q}$ in the above inequality is determined by the interference temperature of the primary receiver.

3 RATE CONTROL

The problem of cognitive user rate control can be described as: given h_k, p_k, f_k, p_k^p, R_k^p and z_k, what is the maximum achievable rate of cognitive user on the *k*-th subchannel? From (1), we can see that the received signal at the cognitive receiver contains the interference signal caused by the primary user, so the key problem to deal with is how we can cancel the primary user's interference at the cognitive receiver. At present, there are three popular strategies on rate control of cognitive user.

A. Dirty Paper Coding (DPC) Strategy

In [4] and [5], the authors employ this strategy to cancel the interference caused by the primary user. If the cognitive transmitter has the knowledge of x_k^p and f_k, the receiver can completely cancel the primary user interference by using Dirty Paper Coding. In this case, the transmit rate of cognitive user is:

$$R_k^{DPC} = C(\gamma_k) \qquad (3)$$

where $C(x) = \log_2(1+x)$, $\gamma_k = \dfrac{h_k^2 p_k}{\sigma_k^2}$.

In fact, the Dirty Paper Coding strategy requires that the cognitive transmitter should have prior knowledge of the primary user's interference, which is merely not operational in actual systems.

B. Noise Strategy

Noise strategy refers to the simple but widely used strategy of rate control [8], [9]. Regarding the interference as noise, the transmit rate of cognitive user can be described as:

$$R_k^{Noise} = C\left(\frac{\gamma_k}{\gamma_k^p + 1}\right) \qquad (4)$$

where $\gamma_k^p = \dfrac{f_k^2 p_k^p}{\sigma_k^2}$.

C. Opportunistic Interference Cancellation (OIC) strategy

Opportunistic Interference Cancellation (OIC) strategy is proposed in [10]. In this strategy, the cognitive channel is regarded as a multi-access channel (MAC), which contains the primary user's interference signal and the cognitive user signal, as shown in figure 2. In the figure, the decoding order at L_p is: first decode cognitive user signal, and then decode primary user's interference signal. The rate pair is:

$$\left(R_k, R_k^p\right) = \left(C\left(\frac{\gamma_k}{\gamma_k^p + 1}\right), C\left(\gamma_k^p\right)\right) \qquad (5)$$

Decoding order at L_c is just the opposite: first decode primary user's interference signal, and then decode the cognitive user signal. The rate pair is:

$$\left(R_k, R_k^p\right) = \left(C(\gamma_k), C\left(\frac{\gamma_k^p}{\gamma_k + 1}\right)\right) \qquad (6)$$

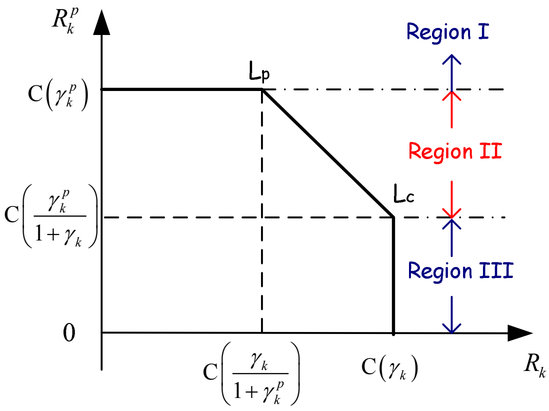

Fig.2 The capacity region of MAC

In OIC strategy, the transmit rate of cognitive user is relevant to the transmit rate of cognitive user R_k^p. Given R_k^p in different regions, the maximum achievable rates of the cognitive user have different expressions as described below:

1. $R_k^p > C(\gamma_k^p)$

This case corresponds to Region I in figure 2. The interference signal of the primary user can not be decoded correctly in this region. So we have to treat the interference signal as noise, and then the achievable rate can be described as:

$$R_k^{OIC} = C\left(\frac{\gamma_k}{\gamma_k^p + 1}\right) \tag{7}$$

2. $C\left(\frac{\gamma_k^p}{\gamma_k + 1}\right) < R_k^p \le C(\gamma_k^p)$

This case corresponds to Region II in figure 2. The cognitive transmitter divides the transmit signal into two parts by using superposition coding [10]:

$$x_k = (1-a_k)x_k^{(1)} + a_k x_k^{(2)} \tag{8}$$

This strategy resembles the Rate Splitting proposed in [11] and [12]. The decoding order at the cognitive receiver is: first decode $(1-a_k)x_k^{(1)}$ at rate $R_k^{(1)} = C\left(\frac{(1-a_k)\gamma_k}{\gamma_k^p + a_k\gamma_k + 1}\right)$, and then decode the interference signal at rate $C\left(\frac{\gamma_k^p}{a_k\gamma_k + 1}\right)$. If a_k satisfies the equation:

$$C\left(\frac{\gamma_k^p}{a_k\gamma_k + 1}\right) = R_k^p, \tag{9}$$

the interference signal of the primary user can be decoded correctly. At last, decode $a_k x_k^{(2)}$ at rate $R_k^{(2)} = C(a_k\gamma_k)$.

According to (9), we can get $a_k = \frac{\gamma_k^p / \beta_k^p - 1}{\gamma_k}$, where β_k^p satisfies $R_k^p = C(\beta_k^p)$. So, the transmit rate of the primary user is:

$$\begin{aligned} R_k^{OIC} = R_k^{(1)} + R_k^{(2)} &= \log_2\left(\frac{1+\gamma_k^p}{1+\beta_k^p}\right) + C\left(\frac{\gamma_k}{\gamma_k^p + 1}\right) \\ &= M_k + C\left(\frac{\gamma_k}{\gamma_k^p + 1}\right) \end{aligned} \tag{10}$$

3. $R_k^p \le C\left(\frac{\gamma_k^p}{\gamma_k + 1}\right)$

This case corresponds to Region III in figure 2. The cognitive receiver can correctly decode interference signal caused by the primary user first, and then decode its own signal. In this case, the cognitive user can achieve a rate of:

$$R_k^{OIC} = C(\gamma_k). \tag{11}$$

In conclusion, the maximum achievable rate of the cognitive user in OIC strategy can be expressed as follows:

$$R_k^{OIC} = \begin{cases} C\left(\frac{\gamma_k}{\gamma_k^p + 1}\right), & R_k^p > C(\gamma_k^p) \\ \log_2\left(\frac{1+\gamma_k^p}{1+\beta_k^p}\right) + C\left(\frac{\gamma_k}{\gamma_k^p + 1}\right), & C\left(\frac{\gamma_k^p}{\gamma_k + 1}\right) < R_k^p \le C(\gamma_k^p) \\ C(\gamma_k), & R_k^p \le C\left(\frac{\gamma_k^p}{\gamma_k + 1}\right) \end{cases} \tag{12}$$

This strategy is *opportunistic* because the maximum achievable rate of the cognitive user depends on the primary user's transmit rate, as well as the instantaneous channel side information between primary transmitter and cognitive receiver. When the transmit rate of the primary user is high (case 1), the cognitive receiver cannot correctly decode the primary user's interference signal, thus has to treat the interference signal as noise. In this case, the OIC is identified with Noise strategy. When the transmit rate of the primary user is quite low (case 3), the primary user's interference signal can be decoded correctly. So the interference can be cancelled completely. In this case, the OIC strategy is identified with DPC strategy. Note that, in case 2 and 3, the inflection point is at $\gamma_k^* = \gamma_k^p / \beta_k^p - 1$.

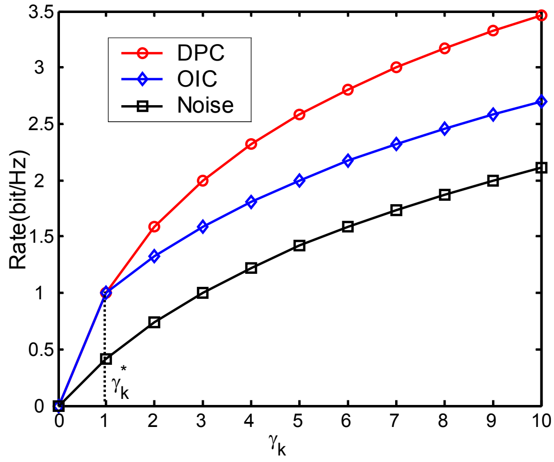

Fig.3 Transmit rate vs. γ_k

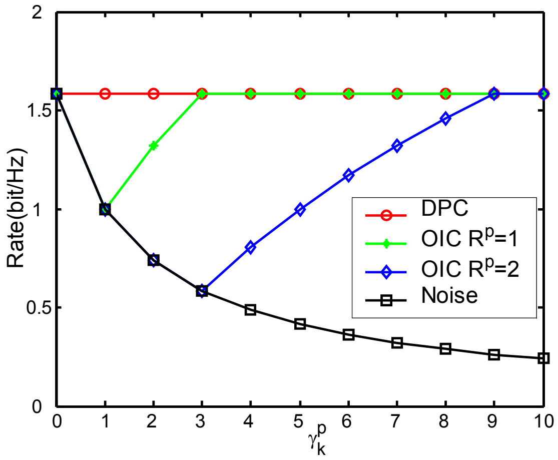

Fig.4 Transmit rate vs. γ_k^p

Figure 3 gives the numerical illustration of relations between transmit rates and γ_k in different strategies, where R_k^p =1bit/Hz, γ_k^p =2. Because $R_k^p \le C(\gamma_k^p)$, the transmit rate of cognitive user is determined by equations (10) and (11). Observed from the figure, there exists an inflection point on the curve of R_k^{OIC}, before which the OIC curve behaves like the DPC curve (Equ.11), and after which the OIC curve performs as an upper shift M_k of the curve in Noise strategy. The inflection point is at $\gamma_k^* = \gamma_k^p / \beta_k^p - 1 = 1$.

Figure 4 shows the relations between the transmit rate of cognitive user and γ_k^p, where γ_k=2. From the figure, we can see that, as to Noise strategy, the transmit rate decreases with γ_k^p. As to OIC strategy, when γ_k^p is low, $R_k^p > C(\gamma_k^p)$, the primary user's interference signal cannot be decoded. So the achievable rate is the same as the rate in Noise strategy. With the increase of γ_k^p, the primary user's interference signal can be decoded and cancelled gradually, and the achievable rate increases gradually. When $\gamma_k^p \ge (1+\gamma_k)\beta_k^p$, the primary user's interference signal can be cancelled completely, and the achievable rate curve coincides with the curve in DPC strategy.

4 POWER CONTROL

In Section 3, we studied on the rate control strategies for determining p_k, and the main focus is how to cancel the interference caused by the primary user. In this section, we will research on power control strategies of cognitive user. The objective is to maximize the total achievable rate of cognitive users, under the constraint of interference temperature at primary receiver.

First, let $l_k = \frac{g_k^2 \sigma_k^2}{h_k^2}$. Recall that $\gamma_k = \frac{h_k^2 p_k}{\sigma_k^2}$, so the constraint inequality (2) can be transformed as: $\sum_{k=l}^{K} l_k \gamma_k \le \overline{Q}$. Then, consider that the achievable rate is an increasing function of γ_k, so the constraint inequality can be further simplified: $\sum_{k=l}^{K} l_k \gamma_k = \overline{Q}$. Consequently, the optimization problem can be described as:

$$\max_{p_k} \sum_{k=1}^{K} R_k(\gamma_k)$$

$$\text{s.t.} \ \sum_{k=l}^{K} l_k \gamma_k = \overline{Q}.$$

A. Dirty Paper Coding (DPC) Strategy

For this strategy, Lagrange's method of multipliers is applied. To find the optimal solution of the power allocation, we form the Lagragian:

$$J = \sum_{k=1}^{K} \log_2(1+\gamma_k) - \lambda \sum_{k=1}^{K} l_k \gamma_k \tag{13}$$

Let $\frac{\partial J}{\partial \gamma_k} = 0, \forall k$. Considering the nonnegativity of γ_k, we derive:

$$\gamma_k = \left(\frac{1}{\lambda l_k} - 1 \right)^{+} \tag{14}$$

where $(x)^{+} = \begin{cases} x, & x > 0 \\ 0, & x \le 0 \end{cases}$. The water level is determined by the following equation:

$$\sum_{k=1}^{K} \left(\frac{1}{\lambda} - l_k \right)^{+} = \overline{Q} \tag{15}$$

B. Noise Strategy

For this strategy, we can get the optimal solution of the power allocation using the same approach:

$$\gamma_k = \left(\frac{1}{\lambda l_k} - 1 - \gamma_k^p \right)^{+} \tag{16}$$

Similarly, λ satisfies $\sum_{k=1}^{K} \left(\frac{1}{\lambda} - l_k - l_k \gamma_k^p \right)^{+} = \overline{Q}$.

C. Opportunistic Interference Cancellation (OIC) strategy

For OIC strategy, when $R_k^p \le C\left(\gamma_k^p\right)$, rate control function $R_k(\gamma_k)$ is a piecewise analytic function of γ_k, which is nondifferentiable at the inflexion point γ_k^*. So Lagrange's method of multipliers can not be applied, and we have to transform the optimization problem. Let γ_k^{opt} be the optimal solution of the power allocation. According to γ_k^{opt}, the subchannels can be classified into four sets:

$I_1 = \left\{ k \mid R_k^p > C\left(\gamma_k^p\right) \right\}$,

$I_2 = \left\{ k \mid R_k^p \le C\left(\gamma_k^p\right), \gamma_k^{opt} > \gamma_k^* \right\}$,

$I_3 = \left\{ k \mid R_k^p \le C\left(\gamma_k^p\right), \gamma_k^{opt} = \gamma_k^* \right\}$,

$I_4 = \left\{ k \mid R_k^p \le C\left(\gamma_k^p\right), \gamma_k^{opt} < \gamma_k^* \right\}$, corresponding to the subchannels in Region I, Region II, Inflexion Point γ_k^* and Region III respectively. Then the objective function $\sum_{k=1}^{K} R_k(p_k)$ can be transformed as:

$$\sum_{k \in I_1} C\left(\frac{\gamma_k}{\gamma_k^p + 1} \right) + \sum_{k \in I_2} \left(M_k + C\left(\frac{\gamma_k}{\gamma_k^p + 1} \right) \right) + \sum_{k \in I_3} C\left(\gamma_k^*\right) + \sum_{k \in I_4} C\left(\gamma_k\right).$$

Employ Lagrange's method of multipliers to derive γ_k^{opt}:

$$\gamma_k^{opt}=\begin{cases}\left(\frac{1}{\lambda l_k}-1-\gamma_k^p\right)^+, & k\in I_1\cup I_2\\ \gamma_k^*, & k\in I_3\\ \left(\frac{1}{\lambda l_k}-1\right)^+, & k\in I_4\end{cases} \quad (17)$$

where λ satisfies $\sum_{k=1}^{K} l_k\gamma_k^{opt}=\overline{Q}$. Then the original optimization problem transforms to a new problem: to find the optimal value of λ and the division of sets I_2, I_3, I_4. Note that I_2, I_3, I_4 are relevant to γ_k^{opt}, but I_1 is not. So we propose the following iterative algorithm to solve the problem:

Step1: Initialize $I_1=\{k \mid R_k^p > C(\gamma_k^p)\}$, $I_2=\{k \mid R_k^p \le C(\gamma_k^p)\},\ I_3=I_4=\Phi$.

Step2: Find λ which satisfies

$$\sum_{k\in I_1\cup I_2}\left(\frac{1}{\lambda}-l_k-l_k\gamma_k^p\right)^+ +\sum_{k\in I_3} l_k\gamma_k^* +\sum_{k\in I_4}\left(\frac{1}{\lambda}-l_k\right)^+ .$$
$$=\overline{Q}$$

Step3: If $\forall k\in I_2$, the inequality $\frac{1}{\lambda l_k}-1-\gamma_k^p\ge\gamma_k^*$ holds, break from the loop; otherwise, update:

$$I_4=I_4+\left\{k\left|\frac{1}{\lambda l_k}-1<\gamma_k^*, k\in I_2\cup I_3\right.\right\},$$

$$I_3=I_3+\left\{k\left|\frac{1}{\lambda l_k}-1\ge\gamma_k^*, k\in I_2\right.\right\}.$$

Step4: Repeat step 2.

The principle of this iterative algorithm is as follows: first assume that the transmit power on each subchannel is high, and the subchannels belong to set I_1 or I_2. Based on this assumption, find the water level λ^{-1}. If the inequality $\frac{1}{\lambda l_k}-1-\gamma_k^p\ge\gamma_k^*$ is satisfied by all subcarriers belonging to set I_2, whose power are below the water level, the initial division of the sets is correct, and the algorithm converges; otherwise, adjust the division based on the result of power control, and search for the water level until the algorithm converges. After each iteration, the water level λ^{-1} declines, so part of subchannels in I_2, I_3 transfer to I_3 or I_4. When the algorithm converges, γ_k and I_2, I_3, I_4 will satisfy: 1)For the optimal solution γ_k, the division of the sets I_2, I_3, I_4 is determined; 2)The optimal solution γ_k satisfies equation (17) under the division of the sets I_2, I_3, I_4. Therefore, the algorithm converges to the optimum solution of power control. Also, the complexity of the proposed algorithm is not quite high. In the worst case, the algorithm iterates *K* times, and only one subchannel transfers to other sets each time.

5 SIMULATION RESULTS

Consider a cognitive system in the following scenario. The system bandwidth *B* is 5MHz and the number of subchannels *K*=64. The noise power on each cognitive sunchannel is equal and normalized to 1, i.e., $\sigma_k^2=1,\forall k$. The transmit power on each primary subchannel is equal to 10*dB*, i.e., p_k^p=10dB, $\forall k$, and the transmit rate on each primary subchannel is equal to R_k^p=2bit/Hz. Furthermore, g_k, h_k and f_k are subject to Rayleigh distribution with parameters $\sigma_g^2,\sigma_h^2,\sigma_f^2$ respectively. In the simulation, for simplicity, we fix $\sigma_g^2=0$ *dB,* σ_h^2=10*dB*. We name the algorithm proposed in Section 4 as the Optimal Power Control (OPC) strategy. In order to evaluate the performance of OPC, we also propose a simple power control algorithm, namely Equal Power Control (EPC), which assumes that the interference power is equal on each subchannel, i.e., $p_k = \overline{Q}/(Kg_k)$, to compare with it.

Fig.5 shows the performance of different joint power and rate control algorithms when $\sigma_f^2 = 0\,dB$. As shown in the figure, the achievable rate of DPC is maximum, while the achievable rate of Noise strategy is minimum, and the achievable rate of OIC falls somewhere in between. On the other hand, OPC has a better performance than EPC. However, with the increase of $\overline{Q}$, the difference between the two strategies becomes narrower. When $\overline{Q}$=20dB, there is no significant difference. The result indicates that, EPC approximates to OPC when high interference temperature is tolerable.

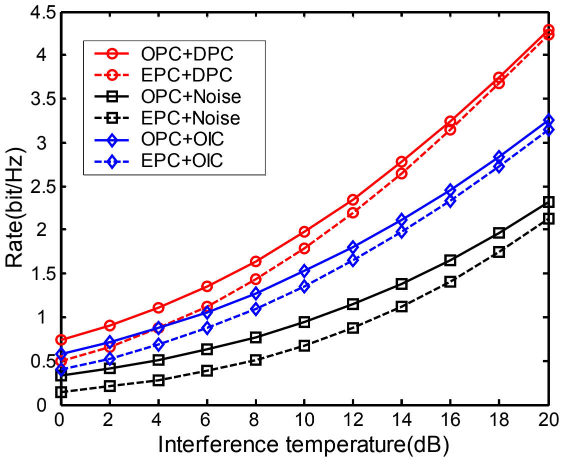

Fig.5 Transmit rate vs. interference temperature

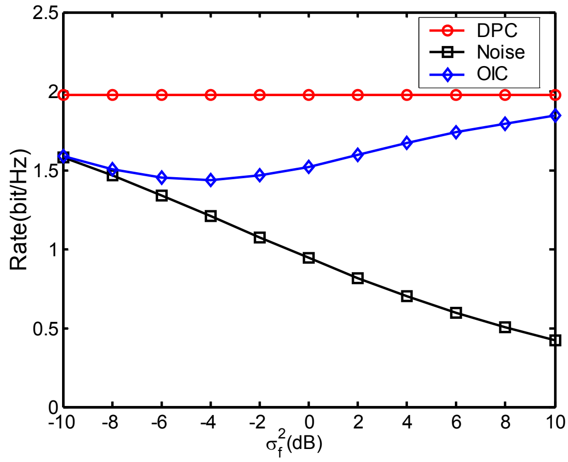

Fig.6 Transmit rate vs. σ_f^2

Figure 6 plots the curve of achievable rates of different algorithms versus σ_f^2. Note that $\overline{Q}$=10dB and OPC is adopted in the computation. From the figure, we can see that the performance of DPC is independent of σ_f^2. However, the achievable rate of Noise strategy decreases sharply with σ_f^2, because the interference of primary user increases with σ_f^2. For OIC, the achievable rate first decreases but then increases with σ_f^2 increases, and approximates to OPC gradually, for the same reason as in Fig.4 .

6 CONCLUSION

In this paper, we first introduced three rate control strategies in cognitive radio network: Dirty Paper Coding strategy, Noise strategy, and Opportunistic Interference Cancellation strategy. Corresponding to these rate control strategies, we proposed three optimal power control algorithms for cognitive user with multi- subchannels. Simulation results showed that, the power control strategies we proposed can significantly improve the transmit rate of cognitive user.

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