Complexity Reduction by Using Triangular Matrix Multiplication in Computing Capacity for an Optimal Transmission

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Abstract: - Multiple-input-multiple-output (MIMO) technique is often employed to increase capacity in comparing to systems with single antenna. However, the computational complexity in evaluating channel capacity or transmission rate (data rate) grows proportionally to the number of employed antennas at both ends of the wireless link. Recently, the QR decomposition (QRD) based detection schemes have emerged as a low-complexity solution. After conducting QRD on a full channel matrix that results in a triangular matrix, we claim that computational complexity can be simplified by the following ways. First, to simplify channel capacity calculation, we prove that eigenvalues of the full channel matrix multiplication equals eigenvalues of the triangular channel matrix multiplication. Second, to simply evaluate optimal transmission rate constrained constellation, we propose a simplified multiplication of the resulted simple triangular matrix and a transmitted signal vector. Then, we also propose a modified mutual information calculation (MMIC) to reduce the multiplication complexity in combinational multiplication processes via the divided calculation. This divided calculation is employed in the parallel architecture for the field-programmable gate array (FPGA) implementation. That is, the number of multiplications can be reduced via increasing the number of additions in the FPGA implementation. By using the computer and FPGA implementation, simulation results show that the proposed QRD-based schemes are capable of achieving conventional performance, but at a low-complexity level.

Key-Words: Multiple-input multiple-output, channel capacity, QR decomposition, eigenvalues, mutual information, field-programmable gate array.

1 Introduction

Multiple-input-multiple-output (MIMO) technique [1]-[3] is often employed to increase capacity in comparing to systems with single antenna [4]-[5]. In wirless MIMO communications, the design of an optimal transmission is essential in order to meet demands from a large number of simultaneous data transmissions [1]-[3]. Bertrand *et al*. [6] shows the mutual information constrained constellation to achieve the maxim achievable rate at various signal-to-noise ration (SNR) conditions in MIMO system. However, with full MIMO channel matrix (**H**) [6]-[10], the computational complexity in calculating channel capacity or transmission rate (data rate) increases proportionally as the number of antennas and/or modulation order getting higher [6]-[9]. More specifically, the full MIMO channel matrix multiplication [6] leads to complicated field-programmable gate array (FPGA) implementation [10] and very-large-scale integration (VLSI) [11]. To simplify the computational complexity, the QR-decomposition (QRD) scheme [12]-[15] with restraining complexity has been devised to reduce the MIMO channel matrix multiplication due to involving a lot of zeros in a triangular matrix [13]-[15].

In this paper, we propose a simple way to realize Bertrand's scheme [6]. Towards this end, the MIMO channel matrix can be uniquely represented a triangular matrix (**R**) and a unitary matrix after QRD [13]-[15], provided that the number of receiving antennas (M) is larger than or equal to the number of transmitting antennas (N). Thus, computational complexity can be simplified by the following ways. First, to simplify channel capacity calculation, we prove that eigenvalues of the full channel matrix multiplication ($\mathbf{HH}^H$) [6] equals eigenvalues of the triangular channel matrix multiplication ($\mathbf{RR}^H$) [13]-[15]. Based on this

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equivalence, we can reduce the eigenvalue processes because $\mathbf{RR}^H$ with N-by-N has fewer dimensions than $\mathbf{HH}^H$ with M-by-M [16]-[18]. This $(\cdot)^H$ is the Hermitian transpose. Furthermore, this eigenvalue processes must establish a sin/cos look up table in the hardware implementation. Nevertheless, the memory size severely limits the precision in the hardware implementation. Therefore, for the FPGA implementation, we adopt the Coordinate Rotation Digital Computer (CORDIC) algorithm instead of the sin/cos table to achieve the memory efficiency in the eigenvalue processes [11]. Second, to design an optimal transmission rate via evaluating mutual information constrained constellation [6], we propose a simplified multiplication of the resulted simple triangular matrix and a transmitted signal vector. In addition, we propose a modified mutual information calculation (MMIC) to reduce the multiplication complexity in combinational multiplication processes via the divided calculation. This divided calculation is employed in the parallel architecture for the FPGA implementation. By using computer and FPGA implementation [19], simulation results show that the proposed QRD-based schemes are capable of achieving conventional performance (Bertrand's scheme), but at a low-complexity level [16]-[18].

This paper is organized as follows. In Section II, we give conventional MIMO equations in computing channel capacity [20]-[22]. Section III, The proposed QRD-based scheme in computing channel capacity and mutual information of constrained constellation are developed. In Section IV, we analyze the computational efficiency (CE) in the proposed QRD-based scheme. In Section V, we conduct computer simulation and FPGA implementation to confirm the effectiveness of the proposed QRD-based schemes. Finally, we conclude the paper and suggest future work in Section VI.

2 Conventional MIMO Capacity Equations

We consider a communication system with N transmitting and M receiving antennas ($M \geq N$) over a MIMO channel. The sampled basedband received signals [9] are given by

$$\mathbf{y} = \mathbf{Hx} + \mathbf{v}, \tag{1}$$

where $\mathbf{y} \in C^{M\times 1}$ is the received signal vector, $\mathbf{x} \in C^{N\times 1}$ is the transmitted signal vector and $\mathbf{H} \in C^{M\times N}$ is the MIMO channel matrix and the noise vector $\mathbf{v} \in C^{M\times 1}$ has an *i.i.d.* complex Gaussian entries and noise power is σ_v^2 . Then, the MIMO technique promises to become the technology of future wireless communication when high spectral efficiency is required. Therefore, the capacity of a random MIMO channel [6]-[9] can be expressed as follows:

$$C = E\left\{ \max_{p(\mathbf{x}):tr(\mathbf{R}_{xx})=N} I(\mathbf{x};\mathbf{y}) \right\}, \tag{2}$$

where $\mathbf{R}_{xx} := E\{\mathbf{xx}^H\}$ is the covariance matrix of the transmitted symbol vector $\mathbf{x}$ and $p(\mathbf{x})$ denotes all possible transmitter statistical distribution. By using a MIMO channel, the mutual information [6]-[9] between $\mathbf{x}$ and $\mathbf{y}$ which can be given as

$$I(x;y) = E\left\{ \log_2\left[\det\left(\mathbf{I}_M + \frac{P_T}{\sigma_v^2 N}\mathbf{HR}_{xx}(\mathbf{H})^H \right)\right]\right\}. \tag{3}$$

where total power P_T is limited, irrespective of the number of transmitting antennas. Substituting (3) into (2), we have

$$C = \max_{tr(\mathbf{R}_{xx})\leq N} E\left\{ \log_2\left[\det\left(\mathbf{I}_M + \frac{P_T}{\sigma_v^2 N}\mathbf{HR}_{xx}(\mathbf{H})^H \right)\right]\right\}. \tag{4}$$

In this work, assuming the channel coefficient is unknown to the transmitter and hence the uniform power distribution is considered in transmitter. The covariance matrix of $\mathbf{x}$ is then given by $\mathbf{R}_{xx} = \mathbf{I}_M$, which implies that the transmitted symbol x is an *i.i.d.* random variable with zero-mean and unit-variance. As a result, the ergodic capacity for a spatially white MIMO channel [6]-[9] can be given as

$$\begin{aligned} C &= E\left\{ \log_2\left[\det\left(\mathbf{I}_M + \frac{P_T}{\sigma_v^2 N}\mathbf{H}(\mathbf{H})^H \right)\right]\right\} \\ &= E\left\{ \log_2\left[\det\left(\mathbf{I}_M + \frac{\rho}{N}\mathbf{H}(\mathbf{H})^H \right)\right]\right\}, \end{aligned} \tag{5}$$

where $\rho := \frac{P_T}{\sigma_v^2}$ is the average SNR at each receiver branch. By using the eigenvalue decomposition (EVD) [16]-[17], we can get

$$\mathbf{HH}^H = \mathbf{E\Lambda_H E}^H, \tag{6}$$

where $\mathbf{E}$ is an $M \times M$ matrix which $\mathbf{EE}^H = \mathbf{E}^H\mathbf{E} = \mathbf{I}_M$ and $\mathbf{\Lambda_H}$ = diag$\{\Lambda_{\mathbf{H},1}, \Lambda_{\mathbf{H},2}, \ldots \Lambda_{\mathbf{H},M}\}$ is a diagonal matrix with $\Lambda_{\mathbf{H},i}$ 0 employed in full MIMO channel matrix. Assuming $\Lambda_{\mathbf{H},i}$'s are ordered so that $\Lambda_{\mathbf{H},i}$ $\Lambda_{\mathbf{H},i+1,}$ then we have $\Lambda_{\mathbf{H},i} = 0$ if $d+1$ i M, where d is given as

$$d = \text{rank}(\mathbf{H}) \quad N, \tag{7}$$

therefore, the capacity of a MIMO channel [6]-[9] can be rewritten as

$$C = \sum_{i=1}^{d} E\left\{\log_2(1+\frac{\rho}{N})\Lambda_{\mathbf{H},i}\right\}. \tag{8}$$

In outage analysis, we consider the q% outage capacity $C_{out,q}$ as the information rate that is guaranteed for (100 - q)% of the channel realization as

$$Prob(C \le C_{out,q}) = q\%, \tag{9}$$

in upper bound, the outage capacity is larger than the channel capacity when a finite probability q is considered. Furthermore, when the channel knowledge is known at the transmitter, the capacity of a MIMO channel is the sum of the capacities associated with the parallel SISO channels [7]-[9] given by

$$C = \sum_{i=1}^{r} E\left\{\log_2\left(1+P_i\frac{\rho}{N}\Lambda_{\mathbf{H},i}\right)\right\}, \tag{10}$$

where transmitting power in the i^{th} sub-channel is $P_i := E\{|x_i|^2\}$ for $i = 1, 2, \ldots, r$ and total power satisfies $P_1 + P_2 + \ldots + P_r = N$. Therefore, the transmitter can access the spatial sub-channels, it can allocate variable power across the sub-channels to maximize the channel capacity. The channel capacity with power loaded optimization is

$$C = \max_{P_T = N} \sum_{i=1}^{r} E\left\{\log_2\left(1+P_i\frac{\rho}{N}\Lambda_{\mathbf{H},i}\right)\right\}. \tag{11}$$

Hence, the solution can be obtained by using the Lagrangian methods [7]-[9]. The optimal power allocation of the i^{th} sub-channel is denoted as

$$P_i^o = \left(\mu - \frac{N}{\rho\Lambda_{\mathbf{H},i}}\right)_+, \tag{12}$$

where μ is a constant chosen to satisfy the power constraint of (12) and $(x)_+$ denotes

$$(x)_+ = \begin{cases} x & x \ge 0 \\ 0 & x < 0 \end{cases}. \tag{13}$$

The optimal power allocation in (13) is found iteratively through the "waterfilling" algorithm [7]-[9].

3 Reduced-complexity for MIMO Capacity Equations

In this section, the QRD-based scheme is proposed to reduce matrix multiplication in computing A) channel capacity and B) mutual information of constrained constellation as follows.

3.1 Channel Capacity

In this subsection, we consider the triangular matrix multiplication to achieve low computational complexity via the QRD-based scheme. Thus, the full MIMO channel matrix [13]-[15] can be expressed as

$$\mathbf{H} = [\mathbf{Q}_1, \mathbf{Q}_2]\begin{bmatrix}\mathbf{R} \\ \mathbf{0}\end{bmatrix} = \mathbf{QR}, \tag{14}$$

where $\mathbf{Q} = [\mathbf{Q}_1 \in C^{N\times N}, \mathbf{Q}_2 \in C^{N\times(M-N)}]$ is an $M \times N$

unitary matrix so $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$ and $\mathbf{R}$ is an $N \times N$ upper triangular matrix [13]-[15] as

$$\mathbf{R} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N} \\ 0 & r_{2,2} & \cdots & r_{2,N} \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & r_{N,N} \end{bmatrix} \in C^{N \times N}, \qquad (15)$$

where $\mathbf{R}$ involves a lot of zeros and hence matrix multiplication can be reduced. Based on (10), considering low-complexity in the matrix multiplication, we thus prove that the eigenvalues of full channel matrix multiplication are equivalent to the eigenvalues of triangular channel matrix multiplication as follows.

Theorem 1: Assuming $\mathbf{H} \in C^{M \times N}$, let $\mathbf{Q}$ and $\mathbf{R}$ be the **QR** decomposition of $\mathbf{H}$ (i.e. $\mathbf{H} = \mathbf{QR}$). Then, we have $\boldsymbol{\Lambda}(\mathbf{HH}^H) = \boldsymbol{\Lambda}(\mathbf{RR}^H)$ where $\Lambda(\cdot)$ is the set of eigenvalues.

Proof: We first consider the "$\subseteq$" case, let $\boldsymbol{\Lambda}_\mathbf{H} \in \boldsymbol{\Lambda}(\mathbf{HH}^H)$. There is a $\mathbf{a} \neq \mathbf{0}$, such that

$$\mathbf{HH}^H\mathbf{a} = \boldsymbol{\Lambda}_\mathbf{H}\mathbf{a}. \qquad (16)$$

So $\mathbf{H} = \mathbf{QR}$ implies

$$\mathbf{Q}^H(\mathbf{QRR}^H\mathbf{Q}^H\mathbf{a}) = \mathbf{Q}^H(\boldsymbol{\Lambda}_\mathbf{H}\mathbf{a}) = \boldsymbol{\Lambda}_\mathbf{H}\mathbf{Q}^H\mathbf{a}. \qquad (17)$$

Since $\mathbf{Q}^H\mathbf{a} \neq \mathbf{0}$, we have $\boldsymbol{\Lambda}_\mathbf{H} \in \boldsymbol{\Lambda}(\mathbf{RR}^H)$ and hence proof is done. Next, for the "$\supseteq$" case, let $\boldsymbol{\Lambda}_\mathbf{R} \in \boldsymbol{\Lambda}(\mathbf{RR}^H)$. There is a $\mathbf{b} \neq \mathbf{0}$. So $\mathbf{a} = \mathbf{Qb}$ implies

$$\mathbf{Q}^H\mathbf{a} = \mathbf{b}, \quad \forall\, \mathbf{a} \neq \mathbf{0}. \qquad (18)$$

So $\mathbf{RR}^H\mathbf{b} = \boldsymbol{\Lambda}_\mathbf{R}\mathbf{b}$ implies

$$\mathbf{RR}^H\mathbf{Q}^H\mathbf{a} = \boldsymbol{\Lambda}_\mathbf{R}\mathbf{Q}^H\mathbf{a}. \qquad (19)$$

By multiplying $\mathbf{Q}$ on left of (14), we have

$$\mathbf{QRR}^H\mathbf{Q}^H\mathbf{a} = \boldsymbol{\Lambda}_\mathbf{R}\mathbf{QQ}^H\mathbf{a} = \boldsymbol{\Lambda}_\mathbf{R}\mathbf{a}. \qquad (20)$$

Notice that because of (16), we have $\mathbf{QRR}^H\mathbf{Q}^H\mathbf{a} = \mathbf{HH}^H\mathbf{a} = \boldsymbol{\Lambda}_\mathbf{R}\mathbf{a}$. Thus, proof is done.

(S.1) Symmetric QR algorithm [16 p.421] is applied to

$$\mathbf{RR}^H = \mathbf{P}^H\mathbf{TP} = \mathbf{P}^H \begin{bmatrix} a_1 & b_1 & & \\ b_1 & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & b_n & a_n & b_{n-1} \end{bmatrix} \mathbf{P} \quad ; \text{//}\mathbf{P} \text{ is the Givens processes}$$

(D.1) **procedure** *DC*(**T**, **Q**, Λ, n : input size) // *DC*: divide-and-conquer
(D.2) **begin**
(D.2) **if** $n <= n_0$ **then**
(D.3) Solve problem without sub-division;
(D.4) **else**
(D.5) sub-division: $\mathbf{T} = \begin{bmatrix} \mathbf{T}_1 & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{T}_k \end{bmatrix} + b_m\mathbf{vv}^H$; // $1 \le m \le n$
(D.6) Recursive: call *DC*($\mathbf{T}_i$, $\mathbf{Q}_i$, Λ_i, n/k) *for* $i = 1$ *to* k; // k: positive integer;
(D.7) *Dcompose* $\mathbf{D}_i + \rho_i\mathbf{uu}^H = \mathbf{V}_i\Lambda_i\mathbf{V}_i^H$; // $\mathbf{v}^H = [\mathbf{0}\ 1\ 1\ \mathbf{0}]$ as $k = 2$
(D.8) Update the eigenvector: $\mathbf{Q}_i = \mathbf{Q}_i \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$; // $\mathbf{u} = \begin{bmatrix} \mathbf{Q}_1^H & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{Q}_k^H \end{bmatrix} \mathbf{v}$
(D.9) Return **Q** and Λ;
(D.10) **endif**
(D.11) **end** *DC*.

Fig. 1. Pseudo code for EVD in computer implementation.

Based on the Theorem 1, to reduce the eigenvalue processes, the capacity of a MIMO channel of (8) can be rewritten as

$$C = \sum_{i=1}^{d} E\left\{\log_2(1 + \frac{\rho}{N})\Lambda_{\mathbf{R},i}\right\}, \qquad (21)$$

where eigenvalues $\boldsymbol{\Lambda}_\mathbf{R}$ are obtained by $\mathbf{RR}^H = \mathbf{U}\boldsymbol{\Lambda}_\mathbf{R}\mathbf{U}^H$ and $\boldsymbol{\Lambda}_\mathbf{R} = \boldsymbol{\Lambda}_\mathbf{H}$, depicted in Theorem 1. Similarly, the channel capacity with power loaded optimization of (11) can be rewritten as

$$C = \max_{P_T = N} \sum_{i=1}^{r} E\left\{\log_2\left(1 + P_i \frac{\rho}{N} \Lambda_{\mathbf{R},i}\right)\right\}. \qquad (22)$$

Thus, by using $\mathbf{RR}^H$, we can reduce the eigenvalue processes as the capacity value of (21) and (22) are equivalent to the capacity value of (5) and (8). In this paper, channel capacity calculation will be developed in computer and FPGA implementation as follows. For computer implementation, the symmetric QR and divide-and-conquer algorithm [16 p.421-444] are proposed to process the symmetric eigenvalue problem in Fig. 1. For FPGA implementation, we consider the Jacobi method to realize symmetric eigenvalue problem [16] as follows

$$\mathbf{U}^H\mathbf{A}\mathbf{U} = \mathbf{U}_n^H\mathbf{U}_{n-1}^H \ldots \mathbf{U}_1^H\mathbf{A}\mathbf{U}_1 \ldots \mathbf{U}_{n-1}\mathbf{U}_n = \Lambda, \quad (23)$$

where **U,** the jacobi matrix, performs a sequence of orthogonal two sided plan rotations to the symmetric matrix **A** ($\mathbf{A} = \mathbf{R}\mathbf{R}^H$). The Jacobi rotation matrices $\mathbf{U}(p, q, \theta)$ is formed as

$$\mathbf{U}(p, q, \theta) = \begin{bmatrix} 1 & & & & 0 \\ & \ddots & \vdots & \cdot^{\cdot^{\cdot}} & \\ & \cos\theta\cdots & 0\cdots & \sin\theta & \\ & \cdots 0 & \cdots 1 \cdots & 0 & \\ & -\sin\theta\cdots & 0\cdots & \cos\theta & \\ & \cdot^{\cdot^{\cdot}} & \vdots & \ddots & \\ 0 & & & & 1 \end{bmatrix}. \quad (24)$$

Then, assuming $\mathbf{U}_i$ ($1 \le i \le n$) is an orthonormal plane rotation via an angle θ, we thus have $U_{pp} = \cos\theta$, $U_{pq} = \sin\theta$, $U_{qp} = -\sin\theta$, and $U_{qq} = \cos\theta$. The 2-by-2 EVD algorithm [16] is presented as follows

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^H \begin{bmatrix} a_{pp} & a_{pq} \\ a_{qp} & a_{qq} \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} b_{pp} & b_{pq} \\ b_{qp} & b_{qq} \end{bmatrix}, \quad (25)$$

where $a_{pp}, a_{qq}, a_{pq}, a_{qp} \in \mathbf{A}_i$ is i^{th} ($1 \le i \le n$) iteration in processing EVD of **A**. To compute θ in (24), the algorithm gives symmetric **A**, p and q to satisfy $b_{pq} = b_{qp} = 0$, these processes can be depicted in Fig. 2. To achieve the memory efficiency in the channel capacity evaluation, we use the coordinate rotation digital computer (CORDIC) algorithm to realize the sin/cos generator instead of lookup table in the FPGA implementation [11]. In Fig. 3, the three main blocks of the EVD process are depicted as follows: 1) The CORDIC blocks work in vector rotation and arctangent process. In Fig. 3, we use two CORDICs to achieve the speed efficiency of the system computation. In these two CORDICs, one works in the vector rotations and the other works in the arctangent calculations to solve (23)-(25). 2) The memory block stores the eigenvalue and eigenvector temporarily when data bus is busy. 3) The random access memory (RAM) stores the eigenvectors and eigenvalues via the CORDIC processes as well as the variable data with various iterations. Especially, in this paper, the EVD process can be calculated easily due to invoking symmetric characteristic [16]-[18].

```
if A(p, q) ≠ 0
  τ = (A(q, q) − A(p, p))/2A(p, q);
    if τ ≥ 0
       θ = tan^{-1}( 1 / (τ + sqrt(1 + τ^2)) ); // tan^{-1}(·) is an arctangent operation
    else
       θ = tan^{-1}( −1 / (−τ + sqrt(1 + τ^2)) );
    endif
else
    θ = 0;
endif.
```

Fig. 2. Pseudo code for achieving the rotation angle in FPGA implementation.

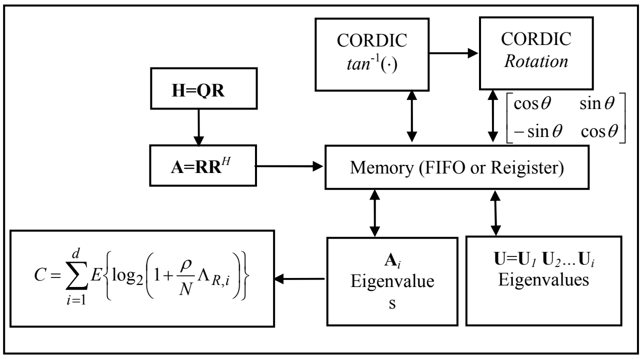


Fig. 3. Channel capacity calculation for FPGA implementation.

3.2 Mutual Information of Constrained Constellation

In this section, to design an optimal transmission rate, the mutual information of constrained constellation is essentially evaluated. To achieve the effort of various constellations on achieving maximum rate in (1), the mutual information between received signals and transmitted signals [6] can be denoted as

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{x}), \quad (26)$$

where $H(\mathbf{y}|\mathbf{x}) = N\cdot\log(2\pi\sigma_v^2 e)$ is a standard division assuming Gaussian channel for any symbol constellation. $H(\mathbf{y})$ is entropy function as

$$H(\mathbf{y}) = -E\log p(\mathbf{y}) = -E\log\sum_{\mathbf{x}} p(\mathbf{y}\,|\,\mathbf{x})p(\mathbf{x}), \quad (27)$$

where probability of event $\mathbf{x}$ is given as $p(\mathbf{x}) = 2^{-mN}$ and m is an modulation order. The condition probability $p(\mathbf{y}|\mathbf{x})$ [6] can be given as

$$p(\mathbf{y}\,|\,\mathbf{x}) = \frac{1}{(2\pi\sigma_v^2)^N}\exp\left(-\frac{1}{2\sigma_v^2}\|\mathbf{Y}-\mathbf{Hx}\|^2\right), \quad (28)$$

therefore, mutual information of (26) can be written as

$$I(\mathbf{x},\mathbf{y}) = -E\log\left(\frac{1}{2^{mN}(2\pi\sigma_v^2)^N}\sum_{\mathbf{x}\in C^N}\exp\left[-\frac{1}{2\sigma_v^2}\|\mathbf{Y}-\mathbf{Hx}\|^2\right]\right) - N\log 2\pi\sigma_v^2 e. \quad (29)$$

In this paper, we will simplify the multiplication ($\mathbf{Hx}$) of the resulted a full channel matrix and transmitting signal vector in (29). By using QRD, a simplified multiplication of the resulted a simple triangular matrix and transmitted signal vector [13]-[15] can be given as

$$\begin{aligned}\|\mathbf{y}-\mathbf{Hx}\|^2 &= \mathbf{x}^H\mathbf{H}^H\mathbf{Hx} - 2\mathbf{y}^H\mathbf{Hx} + \mathbf{y}^H\mathbf{y}\\ &= \mathbf{x}^H\mathbf{R}^H\mathbf{Rx} - 2\mathbf{z}^H\mathbf{Rx} + \mathbf{z}^H\mathbf{z}\\ &= \|\mathbf{z}-\mathbf{Rx}\|^2. \end{aligned} \quad (30)$$

By applying this simplified multiplication ($\mathbf{Rx}$) of (30), the mutual information of (29) can be rewritten as

$$I(\mathbf{x},\mathbf{y}) = -E\log\left(\frac{1}{2^{mN}(2\pi\sigma_v^2)^N}\sum_{\mathbf{x}\in C^N}\exp\left[-\frac{1}{2\sigma_v^2}\|\mathbf{z}-\mathbf{Rx}\|^2\right]\right) - N\log 2\pi\sigma_v^2 e. \quad (31)$$

Therefore, we can simplify multiplication as the mutual information value of (31) because it is equivalent to the mutual information value of (29). In practical, calculating all possible symbol vectors in (31) are high complex when number of antennas is large. Especially, in the FPGA implementation, the number of multiplications is limited when the parallel architecture is considered. Therefore, to reduce the number of multiplications via increasing the number of additions in the FPGA implementation, the terms calculation inside the norm in (31) will be divided two parts as

$$I(\mathbf{x},\mathbf{y}) = -E\log\left(\frac{1}{2^{mN}(2\pi\sigma_v^2)^N}\sum_{\mathbf{x}\in C^N}\exp\left[-\frac{1}{2\sigma_v^2}\left\|\mathbf{z}-\left(\overline{\mathbf{R}}\overline{\mathbf{x}}+\mathbf{R}_1x_{1,i}\right)\right\|^2\right]\right) - N\log 2\pi\sigma_v^2 e \quad (32)$$

where $\overline{\mathbf{R}} \in C^{N-1\times N}$, $\mathbf{R}_1 \in C^N$ and $\overline{\mathbf{x}} = [x_2, x_3,\ldots, x_N]^T \in C^{N-1}$. Specifically, to reduce m^N (m is a modulation order) iterations to m^{N-1} iterations in (31), the terms calculation inside the norm of (32) is divided into brackets and the equivalent received signals $\mathbf{z} = \mathbf{Q}^H\mathbf{y}$. That is, in this brackets ($\mathbf{Rx} = \overline{\mathbf{R}}\overline{\mathbf{x}} + \mathbf{R}_1x_1$), the size of $\overline{\mathbf{x}}$ is less than the size of $\mathbf{x}$. Thus, the number of iterations in $\overline{\mathbf{R}}\overline{\mathbf{x}}$ has only m^{N-1} iterations for calculating all possible symbol vectors. To depict this multiplication reduction, we give an example by using BPSK (m = 2) and $N = M$ = 3 as follows

$$\mathbf{Rx}_i = \begin{bmatrix} r_{1,1}\\ 0\\ 0\end{bmatrix}x_{1,i} + \begin{bmatrix} r_{1,2} & r_{1,3}\\ r_{2,2} & r_{2,3}\\ 0 & r_{3,3}\end{bmatrix}\begin{bmatrix} x_{2,i}\\ x_{3,i}\end{bmatrix} = \mathbf{R}_1x_{1,i} + \overline{\mathbf{R}}\overline{\mathbf{x}}_i, \quad (33)$$

where i ($1 \le i \le m$) is a constellation index and $x_{1,i}$, $x_{2,i}$, $x_{3,i}$ $\in$ {−1, 1}. Then, the processes for calculating all possible combinational signal vectors are given as

$$\begin{aligned}\mathbf{Rx}_1 &= \mathbf{R}_1x_{1,1} + \overline{\mathbf{R}}\overline{\mathbf{x}}_1,\\ \mathbf{Rx}_2 &= \mathbf{R}_1x_{1,1} + \overline{\mathbf{R}}\overline{\mathbf{x}}_2,\\ &\vdots\\ \mathbf{Rx}_4 &= \mathbf{R}_1x_{1,1} + \overline{\mathbf{R}}\overline{\mathbf{x}}_4,\\ \mathbf{Rx}_5 &= \mathbf{R}_1x_{1,2} + \overline{\mathbf{R}}\overline{\mathbf{x}}_1,\\ &\vdots\\ \mathbf{Rx}_8 &= \mathbf{R}_1x_{1,2} + \overline{\mathbf{R}}\overline{\mathbf{x}}_4. \end{aligned} \quad (34)$$

Based on (32)-(34), the block diagram of the modified mutual information calculation (MMIC) is proposed in Fig. 4.

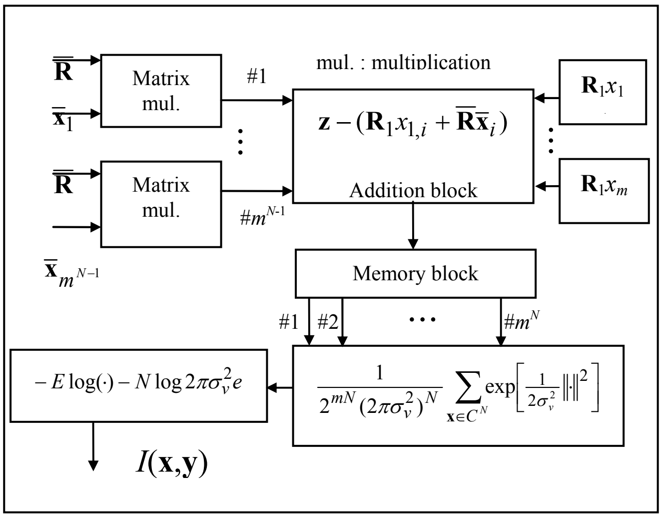


Fig. 4. The block diagram of the mutual information calculation.

4 Computational Complexity

In this subsection, we analyze the computational efficiency (CE) in evaluating channel capacity and mutual information of constrained constellation as follows. For simplicity, the computational complexity is only in terms of the number of complex multiplications. To evaluate channel capacity, the symmetric QR and divide-and-conquer algorithms [16 p.421-444] are proposed to process the symmetric eigenvalue problem, where algorithms require about $4n^3/3$ and $O(n^2)$ [17]-[18], respectively. Considering the multiplication of the full channel matrix ($\mathbf{HH}^H \in C^{MxM}$) [6]-[9], the computational complexity of (8) [16]-[17] is given as

$$Compl_{\mathbf{H}} \cong \frac{4}{3}M^3 + M^2 + NM^2. \qquad (35)$$

To reduce the eigenvalue processes in computing channel capacity, the QR-based method is employed where QR-decomposition complexity is about $O(2MN^2 - 2N^3/3)$ [16]. Considering the multiplication of triangular channel matrix ($\mathbf{RR}^H \in C^{NxN}$), the computational complexity of (21) [16]-[17] is given as

$$Compl_{\mathbf{R}} \cong \frac{4}{3}N^3 + N^2 + \sum_{i=1}^{N} i^2 + MN^2 - \frac{N^3}{3}. \qquad (36)$$

Based on (32) and (35), the computational complexity requirements of (8) and (21) for evaluating channel capacity are compared by the CE ratio:

$$CE_c := \frac{Complex_{\mathbf{R}}(\text{capacity})}{Complex_{\mathbf{H}}(\text{capacity})} \cong \frac{\frac{4}{3}N^3 + N^2 + \sum_{i=1}^{N} i^2 + MN^2 - \frac{N^3}{3}}{\frac{4}{3}M^3 + M^2 + NM^2}. \qquad (37)$$

To compute mutual information under various constellations, by using a full MIMO channel matrix multiplication [6], the computational complexity of mutual information of (29) is given as

$$Complex_{\mathbf{H}}(\text{mutual}) \cong 2^{mN} \cdot M \cdot N + 2^{mN}(M+1), \qquad (38)$$

where m is an modulation order. By using the triangular channel multiplication, the computational complexity of mutual information of (31) [13]-[15] is given as

$$Complex_{\mathbf{R}}(\text{mutual}) \cong 2^{m(N-1)} \cdot \left(\frac{N^2+N}{2}\right) + 2^m \cdot N + 2^{mN}(M+1) + 2MN^2 - 2N^3/3. \qquad (39)$$

Based on (36) and (39), assuming m and/or N is large enough, the computational complexity requirements of (28) and (31) for computing mutual information is compared by the CE ratio:

$$CE_m := \frac{Complex_{\mathbf{R}}(\text{mutual})}{Complex_{\mathbf{H}}(\text{mutual})} \cong \frac{\frac{N^2+N}{2} + M + 1}{2^m NM + M + 1} \le 1, \quad \forall M \ge N. \qquad (40)$$

Clearly, by using (40), the proposal QRD-based scheme in (31) is less computational complexity than the conventional scheme in (29) [6] because of the triangular matrix multiplication. Beside, analyzing channel capacity evaluation and mutual information evaluation in FPGA implementations, the complexity of the proposed QR-based schemes are analyzed by using the number of logic elements (LEs) in next.

5 Simulation Result

In this section, first, computer simulation results are presented to characterize the proposed QRD-based scheme [13]-[15] described in capacity. Second, we also demonstrate the proposed QRD-based method which can reduce computational complexity in achieving mutual information under various constellations. In this MIMO system, the channel coefficient of (1) is obtained from the transmitting antenna n (n = 1, 2,..., N) to the receiving antenna m (m = 1, 2,..., M) [13]-[15] as

$$h_{m,n} = h_{\mathrm{R}} + jh_{\mathrm{I}}\,, \tag{41}$$

where h_{R} and h_{I} are complex Gaussian random variables with a zero mean that denote the real part and the image part, respectively. Each point on the curves was obtained with averaging over 10^5 trials in the Monte-Carlo simulation. Assuming channel is perfectly known at receiver, the proposed QRD-based schemes evaluate the effects of A) channel capacity, optimal power, B) the mutual information of constrained various constellations and C) reduced-complexity analysis.

5.1 Channel capacity and optimal power

Based on (10) and (21), Fig. 5 and Fig. 6 respectively show the ergodic capacity and 10% outage capacity for different SNR conditions and numbers of antennas. As expected, the ergodic capacity and outage capacity increases when SNR or the number of antennas was increased, respectively. For optimal power in (11) and (22), Fig. 7 shows the complementary cumulative distribution function (ccdf) versus capacity for various antenna configurations at SNR = 10 dB. As observed, assuming the same antenna configuration, the figure shows that the capacity is larger with channel knowledge known at the transmitter than without channel knowledge at the transmitter. That is, this capacity gains because the transmitter involves power allocation when the channel knowledge is given at transmitter. In Fig. 7, K and U denote that the channel knowledge is known and unknown at transmitter. Thus, Fig. 5, Fig. 6 and Fig. 7 demonstrate that the proposed triangular matrix (**R**) in (21) and (22) can be employed to realize the full MIMO channel matrix (**H**) in (10) and (11) [6]-[9], where it involves less computational complexity by using the triangular matrix multiplication.

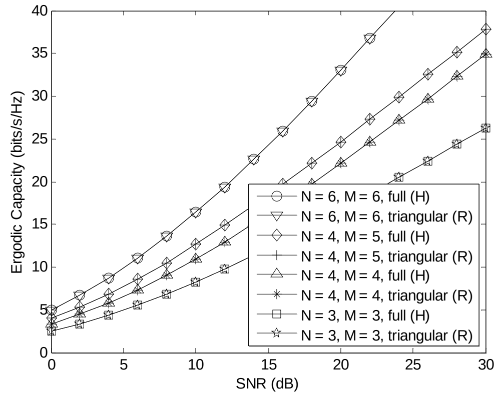


Fig. 5. Ergodic capacity versus SNR performance for various antenna configurations (s **denotes second).**

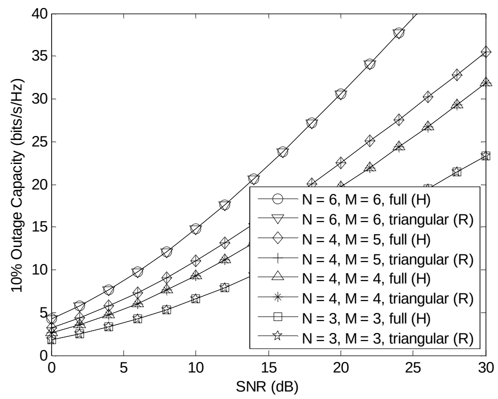


Fig. 6. Outage capacity versus SNR performance for various antenna configurations.

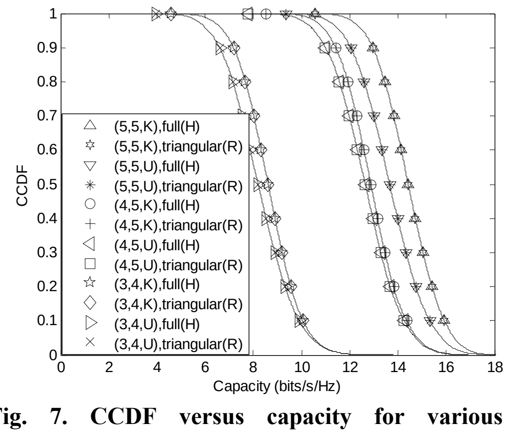
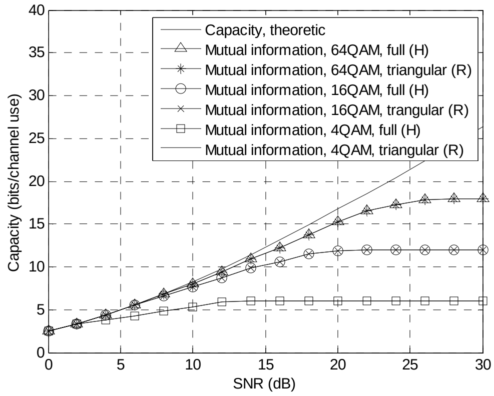
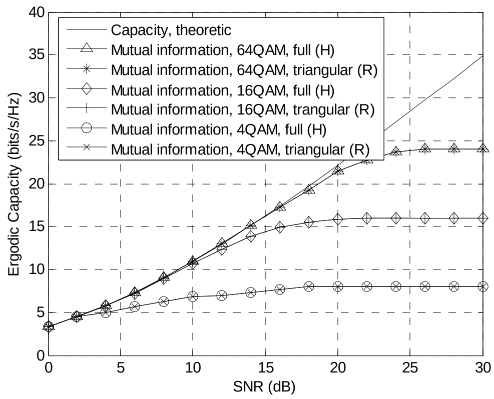


Fig. 7. CCDF versus capacity for various antenna configurations at SNR = 10 dB.

5.2 Mutual information of constrained various constellations



(a)



(b)

Fig. 8. Ergodic capacity versus SNR performance for various modulation orders employing (a) N = 3, M = 3, (b) N = 4 and M = 4.

In Fig. 8(a), with $N = M = 3$, the theoretic channel capacity of (5) is represented by the highest curve. Based on (29) and (32), the remaining curves indicate mutual information of constrained various constellations when various SNR conditions are employed. That is, the curves show maximum transmiision rate variability achieved with 64-QAM, 16-QAM and 4-QAM. As a result, the design of a suitable transmitted data rate (C) can be ensured by channel code rate (R), modulation order (m) and the number of transmitting antennas (N). For example, considering an error-free transmission rate possibly for $R{\cdot}N{\cdot}m \leq C$ in Fig. 8(a), we can be sure R = 1 at SNR ≥ 24 dB when C = 18 (bits/channel use), N = 3 and m = 6 are employed. Beside, we can ensure R = 0.5 at about SNR = 10 dB when C = 9 (bits/channel use), N = 3 and m = 6 are employed. Similarly, with increasing the number of antennas, Fig. 8(b) shows the capacity increase when modulation order and/or SNR is increased. As expected, Fig. 8(a) and Fig. 8(b) demonstrate that the proposed triangular matrix (**R**) in (32) can be employed to realize the full MIMO channel matrix (**H**) in (29) [6], but at a low complexity level.

5.3 Reduced-complexity analysis in the proposed QRD-based scheme

In this subsection, we investigate 1) channel capacity evaluation and 2) mutual information evaluation in both computer simulation and FPGA implementation. First, regarding channel capacity evaluation in computer simulation, considering M is fixed, Table I demonstrates that the CE ratio of (37) decreases when N is decreased in practical. Specifically, in Table I, the proposed QRD-based scheme can reduce the computational complexity by about 91% over [6]-[9], when N = 10 and M = 30. That is, we can reduce the eigenvalue processes because $\mathbf{RR}^H$ with N-by-N has fewer dimensions than $\mathbf{HH}^H$ with M-by-M. In Fig. 3, channel capacity evaluation has been designed in Verilog and implemented with FPGA via Xilinx Virtex-5 (XC5VSX240T) [19]. Table II demonstrates that the CE ratio of (37) decreases when N is decreased. Second, regarding mutual information evaluation in computer simulation, in practical, Table III demonstrates that the proposed QRD-based scheme in (32) has less computational complexity than conventional scheme in (29). For FPGA implementation in Fig. 4, Table IV shows that the proposed QRD-based scheme can reduce the computational complexity than conventional scheme [6]. Thus, the CE ratio of (37) and (40) are claimed via Table 1 - IV in computer and FPGA implementation.

TABLE I. EVALUATING CHANNEL CAPACITY COMPLEXITY WITH COMPUTATIONAL IMPELMENTATION

Empolying in (34)	CE_c (Mul./ Mul.) with QPSK
M=30, N=30	81131/81153=0.9997
M=30, N=26	56811/76581=0.7418
M=30, N=22	37398/72009=0.5193
M=30, N=18	22841/67437=0.3387
M=30, N=14	12491/62856=0.1987
M=30, N=10	5695/45900=0.0977

TABLE II. EVALUATING CHANNEL CAPACITY COMPLEXITY WITH FPGA IMPELMENTATION

Empolying in (34)	CE_c (LEs/LEs) with QPSK
M=20, N=20	23279/24473=0.9512
M=20, N=19	21476/23707=0.9
M=20, N=18	18726/23199=0.8072
M=20, N=17	16214/22691=0.7146
M=20, N=16	13929/22183= 0.6279
M=20, N=15	7421/12237= 0.5473

TABLE III. EVALUATING MUTUAL INFORMATION COMPLEXITY WITH COMPUTATIONAL IMPELMENTATION

Empolying in (37)	CE_m (Mul./ (Mul.)
M=10, N=10 (QPSK)	25954107/116391936=0.223
M=10, N=9 (QPSK)	5833874/26476544=0.2203
M=10, N=8 (QPSK)	1311726/5963776=0.1992
M=5, N=5 (16QAM)	8729701/39007121=0.2238
M=5, N=4 (16QAM)	2044726/521235= 0.2549

TABLE IV. EVALUATING MUTUAL INFORMATION COMPLEXITY WITH FPGA IMPELMENTATION

Empolying in (37)	CE_m (LEs/LEs) with QPSK
M=6, N=6 (QPSK)	54488/187128=0.292
M=6, N=5 (QPSK)	38283/132457=0.289
M=6, N=4 (QPSK)	2587/8967=0.2887
M=3, N=3 (16QAM)	74547/25206=0.3381
M=3, N=2 (16QAM)	1571/3584=0.4385

6 Conclusion

In this paper, considering low-complexity, the triangular matrix multiplication is proposed to achieve channel capacity, optimal power and mutual information of constrained constellation for MIMO communications. Beside, we also propose a modified mutual information calculation (MMIC) to reduce the multiplication complexity via the divided calculation in the FPGA implementation. To evaluate low-complexity in the computer and FPGA implementation, the MIMO channel capacity and mutual information was analyzed by using the proposed computational efficiency (CE) in simulation results. Our future work will investigate MIMO techniques in accordance with precoder and STBC for mobile broadband wireless access applications.

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