The Role of Branch-Correlation for an MC-CDMA System Combining with Coherent Diversity over Frequency Selective Channels

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Abstract: The diversity schemes include EGC (equal gain combining) and MRC (maximal ratio combining) diversity combining with an MC-CDMA (multi-carrier coded-division multiple-access) system operating in the cases of either correlated or uncorrelated frequency selective fading channels are invested in this paper. The sum of Nakagami-*m* variates (envelope intensity) is applied for deriving the closed-form solution with arbitrarily correlated channels which is for the purpose of avoiding the difficulty of explicitly calculating the pdf (probability density function), for example, looking for the CF (characteristic function), for the SNR (signal-to-noise ratio) at the output of MRC or EGC scheme . Moreover, an alternative method utilized to obtain the jpdf (joint pdf) with arbitrary correlation coefficient in terms of the generalized Laguerre polynomial is adopted for the derivative. The analytical results of the new derived BER (bit error rate) formulas are validated by an example of the intensive Monte Carlo computer work in which consider a dual correlated branch MC-CDMA system with several assumed system parameters. It obviously shows that the well consideration and elimination the impact of correlation between fading channels for MRC or EGC diversity scheme will obtain the better system performance for an MC-CDMA system working in the environment with frequency selective fading.

Key-Words: Correlated frequency selective channels; EGC diversity; Nakagami-*m* fading; MC-CDMA system; MRC diversity

1 Introduction

The advantages of spectrum efficient and insensitivity to frequency selective channel, etc. are for evaluation whether the performance of a wireless communication system is well or not. However, all the factors mentioned above induce that multiple access system bases on DS-CDMA (direct-sequence coded-division multiple-access), which with the OFDM (orthogonal frequency division multiplexing) techniques, have drawn recent interest in the application of wireless radio systems [1-2]. Especially, MC-CDMA (multi-carrier CDMA) appears to be a considerable candidate for future mobile radio communication systems. In the wireless communication systems, the propagation channel will exhibit substantially multipath fading behavior. Whenever one desires to compensate the degradation due to multipath fading, the diversity-combining technique is one of the effective methods. It is well known that the MC-CDMA system with MRC (maximal ratio combining) receiver is an effective method for combating multipath fading over a frequency selective fading channels [3]. However, the EGC (equal gain combining) is much simpler in system configuration. The BER (bit error rate) analysis of MC-CDMA based on considering different kinds of assumptions, so far, have been dedicated in numerous previously researches [4-6]. The performance evaluation of MC-CDMA over multipath fading channels was studied in [5]. The results presented in [6] are for uplink channel using MRC with the assumed frequency offset condition in correlated fading. The performance of MC-CDMA in non-independent Rayleigh fading was studied in [7]. In [8] the method of CF (characteristic function) and residue theorem are applied to calculate the performance of downlink MC-CDMA systems. Both the effects of envelopes and phases correlation are considered in [9], in which the authors evaluate the performance of a MC-CDMA system operates in Rayleigh fading channel. In the literature of [10] assumed that the transmission channel is working over Nakagami-*m* fading channel, and the postdetection of EGC was considered illustrated the error probability of MC-CDMA systems. The research [11] analyzed the performance of the MC-DS-CDMA and considered the correlation phenomena presents in the fading of the various subcarriers. Recently, the publication [12] evaluated the system performance of MC-DS-CDMA system accompany with PBI (partial band interference) working in Nakagami-*m* fading channels. In [13, 26] claimed that is to obtain a simple approximate average BER expression for MC-CDMA systems working in correlated-Nakagami-*m* fading. On the other hand, the error probability of interleaved MC-CDMA systems with MRC receiver operating in correlated Nakagami-*m* fading channels was published in [14]. Thus, from considering the results presented in [13] and [14], one knows that is much difficult to obtain the pdf (probability density function) of SNR (signal-to-noise ratio) for the sum of correlated gamma random variables by using of the CF methods.

We aim on obtaining the much simpler expression of the generic BER performance of MC-CDMA system with MRC or EGC schemes working in both uncorrelated and correlated fading environments. The general correlation of channels with correlated Nakagami-*m* fading distribution is assumed. The closed-form of BER for MC-CDMA system was obtained via the sum of Gamma variates to avoid the difficulty of explicitly obtaining the pdf for the SNR at the MRC output. On the other hand, for the reason of simplification the joint characteristic function is applied to take over traditional methods for obtaining the jpdf of the output at the EGC output. The results show up that how the factors of channel correlation do affect the performance of MC-CDMA systems. The rest of this paper is organized as follows: section 2 gives a description of the MC-CDMA system transmitter model. In section 3 describes the receiver model of MC-CDMA system. The pdf of sum of Gamma variates for the uncorrelated and correlated Nakagami-*m* fading channel models are described in section 4. The error probability of MC-CDMA operating in uncorrelated and correlated fading cannel is carried out in section 5. There are numerically and simulation results shown in section 6. Finally, section 7 draws briefly conclusions.

2 MC-CDMA Transmitter Model

The MC-CDMA system model is described in this section. There exists K simultaneously users with N subcarriers within a single cell is assumed in the system model. Any effect of correlation among users is going to be ignored by assuming the number of user is uniformed of distribution under the condition of uplink channel. As shown in Fig.1, a signal data symbol is replicated into N parallel copies. The signature sequence chip with a spreading code of length L is used to BPSK (binary phase shift keying) modulated each of the N subscriers of the k-th user. Where the subcarrier has frequency C/T_b Hz, and where C is an integer number [4, 5]. The technical described above is same as the performance of OFDM on a direct sequence spread-spectrum signal when assume $C=1$. The larger value of C, the more transmitting bandwidth needed. The transmitted signal, $S_k(t)$, of the k-th user for the resulting transmitted baseband signal corresponding to the M data bit size can be expressed as

$$S_k(t)=\sqrt{\frac{2P}{N}}\sum_{m'=0}^{M-1}\sum_{n=0}^{N-1}a_k[n]b_k[m']P_{T_b}(t-m'T_b)\,\mathrm{Re}[e^{j\omega_n t}] \quad (1)$$

where both $a_k[n]$ and $b_k[m']$ belong to $\{-1, 1\}$, P is the power of data bit, M denotes the number of data bit, N expresses the number of subcarriers, the sequences $a_k[0],...,a_k[N-1]$ and $b_k[0],...,b_k[M-1]$ represent the signature sequence and the data bit of the k-th user, respectively. The $P_{T_b}(t)$ is defined as an unit amplitude pulse that is non-zero in the interval of $[0, T_b]$, and $\mathrm{Re}[\cdot]$ denotes the real part of a complex number, $\omega_n = 2\pi(f_c + nC/T_b)$ is the angular frequency of the n-th subcarrier, where f_c indicates the carrier frequency, T_b is symbol duration.

A frequency-selective channel with $1/T_b \ll BW_c \ll C/T_b$ is addressed in this paper, where BW_c is the coherence bandwidth. This channel model means that each modulated subcarrier does not experience significant dispersion and with transmission bandwidth of $1/T_b$, i.e. $T_b \gg T_d$, where $1/T_d$ is the Doppler shift typically in the range of 0.3~6.1 Hz in the indoor environment [4], and the amplitude and phase remain constant over the symbol duration T_b. Besides, the channel of interest has the transfer function of the continuous-time fading channel assumed for the k-th user can be represented as

$$H_k[f_c + n\frac{C}{T_b}] = \beta_{k,n}(\cos\theta_{k,n} + j\sin\theta_{k,n}) \quad (2)$$

where $\beta_{k,n}$ and $\theta_{k,n}$ are the random amplitude and phase of the channel of the k-th user at frequency $f_c + n(C/T_b)$. In order to follow the real world case, the random amplitude, $\beta_{k,n}$ are assumed to be a set of N correlated identically distributed in one of our scenarios.

3 MC-CDMA Receiver Model

As shown in Fig. 2 with K active transmitters, the received signal $r(t)$ can be written as

$$r(t)=\sqrt{\frac{2P}{N}}\sum_{k=0}^{K-1}\sum_{m'=0}^{M-1}\sum_{n=0}^{N-1}\beta_{k,n}a_k[n]b_k[m'] \times P_{T_b}(t-m'T_b-\tau_k)cos(\omega_n t+\theta_{k,n})+n(t) \quad (3)$$

where $n(t)$ is the AWGN (additive white Gaussian noise) with a double-sided power spectral density of $N_0/2$, τ_k represents the phase difference between the transmitter and receiver. Assuming that acquisition has been accomplished for the user of interesting ($k=0$). For the reason of using coherent diversities, it is assumed that perfect phase correction can be obtained, i.e., $\hat{\theta}_{0,n}=\theta_{0,n}$. With all the assumptions for MRC or EGC combining the decision variable D_0 of the l-th data bit for the reference user is given by

$$D_0=\frac{1}{T_b}\int_{lT_b}^{(l+1)T_b} r(t)\times\sum_{n=0}^{N\text{-}1}a_0[n]\cdot d_{0,n}\,\mathrm{Re}[e^{(\omega_n t+\theta_{0,n})}]dt =U_S+I_{MAI}+\eta_0 \quad (4)$$

where $r(t)$ is the received signal shown in (3), $d_{0,n}$ is the gain factor for MRC diversity and $d_{0,n}=1$ for EGC diversity, and U_S represents the desired signal, which can be expressed as with U_S^{MRC} for MRC diversity, and with U_S^{EGC} for EGC diversity, respectively, yields as

$$U_S^{MRC}=\sqrt{\frac{P}{2N}}\sum_{n=0}^{N-1}\beta_{0,n}^2\cdot b_0[1] \quad (5)$$

$$U_S^{EGC}=\sqrt{\frac{P}{2N}}\sum_{n=0}^{N-1}\beta_{0,n}\cdot b_0[1] \quad (6)$$

, and the second term, I_{MAI}, in (4) represents the MAI (multiple access interference), which identities for whichever the MRC or EGC reception is employed, and it is contributed from all other users within the same cell, can be written as

$$I_{MAI}=\sqrt{\frac{P}{2N}}\cdot\sum_{k=1}^{K-1}\sum_{n=0}^{N-1}a_k[n]\cdot b_k[1] \times a_0[n]\cdot\beta_{k,n}\cdot\beta_{0,n}cos(\theta_k^{'}) \quad (7)$$

where $\theta_k^{'}=\theta_0-\theta_k$ and θ_k are *i. i .d.* (identically independently distributed) uniformly distributed over $[0,2\pi)$, η_0 is the AWGN term.

4 Correlated and Uncorrelated Channels

In this section there are two cases of propagating channels, that is, uncorrelated and correlated Nakagami-m channels considered for comparison reason, though we are interested in diversity reception for the MC-CDMA system. The identity fading severities are considered for all of the channels, namely, $m_i=m_j=m$, for $i\neq j$, $i,j=0,\ \ldots,\ N-1$. The pdf of fading amplitude for the k-th user with n-th channel, $\beta_{k,n}$, are assumed as an r.v. (random variable) with Nakagami-m distributed and given as [15]

$$f(\beta)=\frac{2\beta^{2m-1}}{\Gamma(m)}\cdot(\frac{m}{\Omega})^m\cdot exp(-\frac{m\beta^2}{\Omega}),\quad \beta\geq 0 \quad (8)$$

where $\Gamma(\cdot)$ is the gamma function defined by $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$, $\Omega=E[\beta^2]$ denotes the average power of the fading signal, the fading figure (parameter), m, of the amplitude distribution characterizes the severity of the fading, and it is defined as $m=\Omega^2/E[(\beta^2-\Omega)^2]\geq 0.5$.

First, if the propagation channels are assumed as *i. i. d*, then by use of the variable changing, the variable γ is assigned as the fading power of the channel. The pdf of γ is given follows as a gamma distribution, can readily be obtained by the processing of random stochastic and expressed as

$$P_\gamma(\gamma)=\frac{\gamma^{m-1}e^{-\gamma/\Omega}}{\Gamma(m)\Omega^m} \quad (9)$$

where γ is considered as the instantaneous power of the fading amplitude, that is, $\gamma=\beta^2$ is assumed. The power at the output of the MRC is a function of the sum of the squares of signal strengths, and is given as $R=\sum_{i=0}^{N-1}\gamma_i$, which is a r.v. represents the summation of the MRC output power. Hence applying the pdf results shown in [17] extended by Alouini, and Abdi from the reference [16] proposed by Kavehthe. The pdf of the power at the output of a MRC receiver can be expressed as [17]

$$f_R(\gamma)=\prod_{i=0}^{N-1}(\frac{\lambda_1}{\lambda_i})^m\sum_{v=0}^{\infty}\frac{\mu_v\gamma^{mN+v-1}e^{-\gamma/\lambda_1}}{\lambda_1^{mN+v}\Gamma(mN+v)} \quad (10)$$

where the coefficients μ_v can be obtained recursively by the following formula given as

$$\begin{cases} \mu_0 = 1 \\ \mu_{v+1} = \dfrac{m}{v+1} \displaystyle\sum_{i=1}^{v+1} \left[\sum_{j=1}^{N} \left(1 - \frac{\lambda_1}{\lambda_j} \right)^i \right] \mu_{v+1-i} \end{cases} \quad (11)$$

, $v = 0,\ 1,\ 2, \ldots.$

where $\lambda_1 = min\{\lambda_i\}$, and $\lambda_i,\ i = 0, \ldots, N-1$ are the eigenvalues of the matrix $\underline{Z} = \underline{X}\underline{Y}$, where $\underline{X}$ is a $N \times N$ diagonal matrix with the entries of average power $\Omega_i,\ i = 0, \ldots, N-1$, where the subcarrier paths are considered correlated each other, the entries of Ω_i can be obtained by taking the minimum value of $\Omega_i = \gamma_i / m_i$. The matrix $\underline{Y}$ is a $N \times N$ positive definite matrix defined by

$$\underline{Y} = \begin{bmatrix} 1 & \rho_{12}{}^{1/2} & \cdots & \rho_{1N}{}^{1/2} \\ \rho_{21}{}^{1/2} & 1 & \cdots & \rho_{2N}{}^{1/2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1}{}^{1/2} & \cdots & \cdots & 1 \end{bmatrix}_{N \times N} \quad (12)$$

where ρ_{ij} denotes the correlation coefficient between γ_i and γ_j, $i \neq j$, $i, j = 1,\ 2, \ldots,\ N$, and ρ_{ij} can be expressed as

$$\rho_{ij} = \frac{Cov(\gamma_i,\ \gamma_j)}{[Var(\gamma_i) \cdot Var(\gamma_j)]^{1/2}},\ 0 \le \rho_{ij} \le 1 \quad (13)$$

where $Var(\cdot)$ and $Cov(\cdot,\cdot)$ are the variance as well as the covariance operators, respectively. The plot of pdf shown in (10) versus different correlation coefficient values, with $\rho = 0.1,\ 0.4$, and 0.8, is shown in Fig. 3. We apply the numerical analysis for the circular correlation case with $N = 5$, $m = 2.5$ and the average power is assumed as unit, which were ever considered in [16]. From observing the results shown in Fig. 3 it is obviously known that the much concentrate for the pdf distribution, the little values of correlation coefficient value are.

On the other hand, let $[\gamma_i],\ i = 0, \ldots, N-1$ be a set of N correlated identically distributed, and all the figure parameters and the average power are assumed equivalently, that is, $m_i = m_j = m$, and $\Omega_i = \Omega_j = \Omega$, where $i \neq j$, for $i, j = 0, \ldots, N-1$. The BER analysis for MC-CDMA system with the two types of diversity techniques will be evaluated as follows. It is known that only the phase shift induced by the fading channel without amplitude weighting will be modified in EGC (equal gain diversity) technique. Each subcarrier will modify its phase shift by a factor of the combining coefficient, $G_{k,n} \triangleq H^*_{k,n} / |H_{k,n}|$, where $H_{k,n}$ is the channel impulse response of the *n*-th subcarrier of the *k*-th user. Hence it is not only the phase shift, but the weights the received signal with the attenuation of the channel fading also be modified in MRC diversity technique. The modified factor of the combining coefficient will be, $G_{k,n} \triangleq H^*_{k,n}$. Thereafter, the pdf of the sum of the instantaneous signal amplitudes, $\beta_{k,n}$, $n = 0, \ldots, N-1$, at the output of EGC need to be calculated. Hence by means of the results shown in [18], the generalized jpdf (joint pdf) of $B = \sum_{n=1}^{N} \beta_{k,n}$ with branch correlation can be written as [26]

$$f_B(\beta_0, \beta_1, \cdots, \beta_{N-1}) = \prod_{n=0}^{N-1} f_\beta(\beta_n) \cdot \sum_{h=0}^{\infty} \frac{(m/2)_h}{h!} \left\{ \sum_{i<j} V_{i,j} \left[\frac{\mathsf{L}(\beta_i, m)}{m} \cdot \frac{\mathsf{L}(\beta_j, m)}{m} \right] + \cdots + V_{01\cdots(N-1)} \left[\frac{\mathsf{L}(\beta_0, m)}{m} \cdots \frac{\mathsf{L}(\beta_{(N-1)}, m)}{m} \right] \right\}^h \quad (14)$$

where the term $V_{0,1,\cdots,(N-1)}$ is used to measure the correlation among the diversity branches , and is the determinant of the covariance matrix C_X, usually written as

$$V_{0,1,\cdots,(N-1)} = (-1)^N \cdot \det \begin{bmatrix} 0 & \rho_{01} & \cdots & \rho_{0(N-1)} \\ \rho_{10} & 0 & \cdots & \rho_{1(N-1)} \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{(N-1)0} & \rho_{(N-1)1} & \cdots & 0 \end{bmatrix}_{N \times N} \quad (15)$$

where the h-th power of the Laguerre polynomial divided by *m* in (14) can be written as

$$\left[\frac{\mathsf{L}(x, m)}{m} \right]^h = \frac{\mathsf{L}_h(x, m)}{(m)_h} \quad (16)$$

where $\mathsf{L}_h(x, y)$ is the generalized Laguerre polynomial of degree h ,and denotes the Pochhammer symbol.

5 Error Probability Analysis

Once the statistical formulas are determined in the previous section, following the normal procedures of calculating system performance, then the generalized average BER for the *k*-th user using coherent BPSK (binary phase shift keying) modulation scheme is able to be derived in this section. For coherent demodulation in the presence of AWGN channel, the probability of error conditioned on the instantaneous *SNR* can be expressed as [19]

$$P_e(S)=\frac{\Gamma(\frac{1}{2},\sigma_0 S)}{2\sqrt{\pi}}=0.5Q\left(\sqrt{2\sigma_0 S}\right) \quad (17)$$

where the incomplete gamma function is defined as $\Gamma(z,t)=\int_t^{\infty}x^{(z-1)}e^{-x}dx$, and the symbols $\sigma_0\ S$ is viewed as the SNR at the output of the combiner, where $\sigma_0 S$ will be replaced with $\sigma_0^{MRC}S^{MRC}$ and $\sigma_0^{EGC}S^{EGC}$ for distinguishing between MRC and EGC diversity cases. As the condition is that *S* is a deterministic variable, the previous equation can be defined as the conditional error probability.

5.1 *BER Analysis for MC-CDMA System with MRC Diversity*

de In order to determine the received instantaneously *SNR*, which conditioned on $\gamma_{0,n}=\beta_{0,n}^2$, at output of the MRC receiver, by adopting the square of (5) and putting the results into *SNR* formula, then it can be calculated as

$$\frac{(U_S^{MRC})^2}{\sigma_T^2}=\frac{\frac{P}{2N}\sum_{n=0}^{N-1}\beta_{0,n}^2}{\sigma_{I_{MAI}}^2+\sigma_\eta^2} \quad (18)$$

where $\sigma_{I_{MAI}}^2$ is the variance of I_{MAI}, which is shown in (7). In the limiting case of large *N* values and by the methods of CLT (central limit theorem), the MAI term in (8) can be approximated by a Gaussian r.v. with zero mean and the variance, $\sigma_{I_{MAI}}^2$, can be determined as [10]

$$\begin{aligned}\sigma_{I_{MAI}}^2&=E[I_{MAI}^2]=\frac{P}{2}(K-1)\cdot E[\beta_{k,n}^2]\cdot E[cos^2\overline{\theta}_{k,n}]\\&=\frac{P}{4}(K-1)\Omega_{k,n}\end{aligned} \quad (19)$$

where $\Omega_{k,n}=E[\beta_{k,n}^2]$, $E[cos^2\overline{\theta}_{k,n}]=1/2$. On the other hand, the background noise term η_0 is a random variable with zero mean and the variance can be calculated as

$$\sigma_{\eta_0}^2=E[{\eta_0}^2]=\frac{NN_0}{4T_b}\beta_{0,n}^2 \quad (20)$$

The received SNR at the output of the receiver can be obtained by substituting (19) and (20) into (18) and can be expressed as (see the Appendix A)

$$\frac{(U_S^{MRC})^2}{\sigma_T^2}=\frac{\sigma_0^{MRC}\cdot S^{MRC}}{2N} \quad (21)$$

where $S^{MRC}=\sum_{n=0}^{N-1}\beta_{0,n}^2/\Omega_{k,n}$, and

$$\begin{aligned}\sigma_0^{MRC}&=\left(\frac{NN_0}{4PT_b\Omega_{k,n}}+\frac{K-1}{4}\right)^{-1}\\&=\left(\frac{N}{4\gamma_0}+\frac{K-1}{4}\right)^{-1}\end{aligned} \quad (22)$$

where $\gamma_0=PT_b\Omega_{k,n}/N_0=E_b\Omega_{k,n}/N_0$ is the *SNR* of each received bit, and $E_b=PT_b$ denotes the bit energy, where P is the power of each bit same as that of shown in (1). It is known that the decision variable in (4) has a Gaussian distribution conditioned on the uncorrelated and correlated channel power $\beta_{0,\mathrm{n}}^2$. The AWGN, η_0, and the MAI, η_{MAI}, are mutually independent. Therefore, the calculation of probability of error for an MC-CDMA system with MRC diversity and by means of BPSK modulation scheme conditioned on the instantaneously SNR as given in (18), can be evaluated with two different situations as follows.

5.1.1 *BER Analysis over* Uncorrelated Channel

Based on the assumption of that the BPSK modulation scheme is applied to the MC-CDMA system operating over uncorrelated fading channel. The event of involving Gaussian *Q*-function is necessary for calculating the BER over uncorrelated channel. However, we adopt an alternative expression for the Gaussian Q-function, which has been proposed in [20], and expressed as

$$Q(t)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}e^{(-\frac{t^2}{2\sin^2\varphi})}d\varphi,\ \mathrm{t}\geq 0 \quad (23)$$

and the random variables $\{\gamma_i,\ i=0,1,\ldots,N-1\}$ are assumed to be independent in this uncorrelated channels case. Such that the equation (17) can be expressed as [22]

$$P_e^{MRC}(s)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{n=1}^{N}J_{0,n}(\Omega_{0,n},\varphi)d\varphi \quad (24)$$

where the term $J_{0,n}(\Omega_{0,n},\varphi)$ stands for an integral function, can be determined as (see the Appendix B)

$$\begin{aligned}J_{0,n}(\Omega_{0,n},\varphi)&=\int_0^{\infty}e^{(-\frac{\gamma_{0,n}}{\sin^2\varphi})}P_\gamma(\gamma_{0,n})d\gamma_{0,n}\\&=\left(\frac{\mathrm{m}}{\Omega_{0,n}}\right)^m\left(\frac{1}{\sin^2\varphi}+\frac{\mathrm{m}}{\Omega_{0,n}}\right)^{-m}\end{aligned} \quad (25)$$

where $\Omega_{0,n}$ indicates the average value of SNR $\gamma_{0,n}$. With the assumption of Nakagami-*m* fading channels, it is known that the $\gamma_{0,n}$ follows the gamma distribution as the expression shown in (9). All *N* subcarriers are assumed *i. i. d*, and the average bit error probability, $P_{(uncor)}^{MRC}$, of uncorrelated-branch case, can be calculated by using of the simple form of a single integral with finite limits and obtained as

$$P_{(uncor)}^{MRC} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left[J_{0,n}(\Omega_{0,n}, \varphi) \right]^N d\varphi \tag{26}$$

5.1.2 *BER Analysis over Correlated Channel*

The condition of correlated channels are considered in this section, and the average bit error probability, $P_{(cor)}^{MRC}$, for correlated channel case can be calculated by averaging (10) and (17) after the substitution of (21) is completed, and yield as

$$\begin{aligned} P_{(cor)}^{MRC} &= \int_0^\infty f_R(r)\cdot P_e(S)dS \\ &= 0.5\prod_{n=1}^{N}\left(\frac{\lambda_1}{\lambda_n}\right)^m \sum_{v=0}^{\infty}\frac{\mu_v}{\lambda_1^{mN+v}\Gamma(mN+v)} \\ &\times \frac{\Gamma(mN+2)}{2\sqrt{\pi}(2mN+2v-1)\left(\sqrt{2\sigma_0^{MRC}}\Big/2\right)^{2mN+v-1}} \\ &\times {}_2\mathrm{F}_1\left(mN+v-\frac{1}{2}, mN+v; mN+v+\frac{1}{2}; \frac{-1}{2\lambda_1\sigma_0^{MRC}}\right) \end{aligned} \tag{27}$$

where μ_v is shown in (11), λ_1 and σ_0^{MRC} are shown in (10) and (22), respectively, and the formula [22]

$$\begin{aligned} &\int_0^\infty [1-\Phi(\omega x)]\cdot e^{\mu^2 x^2}\cdot x^{\nu-1}dx \\ &= \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi}\upsilon\omega^\nu}\times {}_2F_1\left(\frac{\nu}{2},\frac{\nu+1}{2};\frac{\nu}{2}+1;\frac{\mu^2}{\omega^2}\right) \end{aligned} \tag{28}$$

has been applied in the calculation, $\Phi(x) = erf(x)$ denotes the error function, $\Gamma(x)$ is the gamma function, and ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ denotes the confluent hyper-geometric function [22].

5.2 *BER Analysis for MC-CDMA System with EGC Diversity*

Since space reason, only the evaluation of BER for MC-CDMA system with EGC over correlated branch case will be studied in this subsection. The same procedures of section 5.1.2 will be followed as that is for the MRC diversity, the *SNR* can be obtained first as

$$\frac{(U_S^{EGC})^2}{\sigma_T^2} = \frac{\sigma_0^{EGC}\cdot S^{EGC}}{2N} \tag{29}$$

where

$$S^{EGC} = \left(\sum_{n=0}^{N-1}\beta_{0,n}\right)^2 \Bigg/ \Omega_{k,n} \tag{30}$$

, and

$$\begin{aligned} \sigma_0^{EGC} &= \left(\frac{K-1}{4} + \frac{NN_0}{4E_b\Omega_{k,n}}\right)^{-1} \\ &= \left(\frac{K-1}{4} + \frac{N}{4\gamma_0}\right)^{-1} \end{aligned} \tag{31}$$

where $\gamma_0 = PT_b\Omega_{k,n}/N_0 = E_b\Omega_{k,n}/N_0$ is the *SNR* of each bit, and $E_b = PT_b$ denotes the bit energy, where P is the power of each bit the same as shown in (1).

It is known that the decision variable in (4) has a Gaussian distribution conditioned on the uncorrelated and correlated channel power $\beta_{0,n}^2$, respectively, and the AWGN, η_0, the MAI, and the η_{MAI} are mutually independent. Therefore, the probability of error by means of BPSK modulation conditioned on the instantaneously *SNR* has been given in (17).

If the condition of correlated channels are considered in this section, and the average bit error probability for the correlated branch case can be calculated by averaging (14) and (17) after substituting (29) into (17), and yield as

$$\begin{aligned} P_{(cor)}^{EGC} &= \int_0^\infty P_B(\beta_0,\beta_1,\cdots,\beta_{L-1})P_e(S)dS \\ &= \overbrace{\int_0^\infty\cdots\int_0^\infty}^{N-folds} \frac{0.5}{\pi}\int_0^{\frac{\pi}{2}} d\varphi e^{-\frac{1}{2\sin^2\varphi}\frac{S\sigma_0}{N}} \\ &\times f_{\beta_{0,0},\cdots,\beta_{0,n}}(\beta_{0,0},\cdots,\beta_{0,n})\cdot d\beta_{0,0},\cdots,d\beta_{0,n} \end{aligned} \tag{32}$$

Hereafter last formula involves *N*-folds integration and may be computed with the method given by Alouini and Goldsmith in [14]. Now, for the purpose of validating the accuracy a correlated dual-branch case will be illustrated as an example, where the average BER, $P_{(cor)}^{EGC}$, of an MC-CDMA system with EGC can be calculated as

$$P_{(cor)}^{EGC} = \int_0^{\frac{\pi}{2}} \frac{1}{\pi} \sum_{u=0}^{\infty} \frac{\left(\frac{m}{2}\right)_u}{u!(m)_u} \frac{\Gamma(u+m+1)}{\Gamma(m+1)} \times (V_{01})^k \left(1+\frac{\sigma_0^{EGC}/N}{2\sin^2\varphi}\right)^{-2m} \times \left[{}_2F_1\left(-u, m; 1+m; \left(1+\frac{\sigma_0^{EGC}/N}{2\sin^2\varphi}\right)^{-1}\right)\right]^2 \tag{33}$$

where σ_0^{EGC} is shown in (31), and $(m)_u$ denotes the Pochhammer symbol.

6 Numerical Results

The results of SNR (in dB) versus BER and user capacity, *K*, versus BER for MC-CDMA system works over uncorrelated fading channels are illustrated in Fig. 4 and Fig. 5, respectively. In additions, both of the results presented in Fig. 6 and Fig. 7 are corresponding to the BER performance for MC-CDMA system work in correlated channels environments. In Fig. 4 the results show the different BER performance plots figured out adopts different fading parameters, $m=2$ and $m=4$ and different subcarrier numbers, $N=8,\ 16,\ 32$. The results presented that the much more subcarrier number the better BER performance. The user number, *K*, versus BER for MC-CDMA system operate in uncorrelated fading channels is illustrated in Fig. 5, in which the *SNR=5dB* is assumed. The capacity of user number is limited both of the SNR and the subcarrier number. The results shown in Fig. 4 and Fig. 5 can be validated by the previously works as shown in [16], and the simulation results also presented. Simulation results (shown with little circle) are in excellent agreement with the theoretical curves (shown with solid lines).

The different correlation coefficients between the correlated channels are assumed to be $\rho = 0.1$, 0.4, and 0.8, which are adopted from [16] (the values of $\rho = 0.01$, 0.16, and 0.64 are applied in [16], but the square root is the effective values for putting into the covariance matrix (12), the subcarrier number and the user number are assumed corresponding to 16 and 8 in Fig. 6. To calculate the BER, the mean power of the desired signal is assumed equal to the mean power of each interfering mobile unit. Base on the results shown in Fig. 6, we should know that the performance of BER will become degraded gradually by the increasing of the correlation coefficient, ρ, between the correlated channels. It is reasonable that the system performance BER becomes much better when the fading parameter is increasing. The performance of BER versus user number is shown in Fig. 7, in which the correlation coefficients are assumed as $\rho =$ 0.4, 0.6, and 0.8, the average bit SNR is set as 5dB [16]. From these figures it should be noted that the impact of channel correlation definitely affects the performance of MC-CDMA system.

Next the numerical results from (32) of the BER for MC-CDMA system with EGC schemes are presented. The Gaussian correlation model which was discussed in [23] of an equally spaced linear array with an arbitrary correlation coefficient is applied in this paper. It is of interest to note that the correlation matrix followed by the linear array has a Toeplitz form and is revealed as

$$\lambda_{ij} = \exp[-0.5\eta(i-j)^2(\frac{d}{\lambda})^2],\ i,\ j = 0,\cdots,N-1 \tag{34}$$

where d denotes the separation between the transmitter and the receiver, $\eta \cong 21.4$ is a coefficient chosen from setting this correlation model equal to Bessel correlation model [24] with -3 dB point, and d/λ is the normalized distance between two neighboring branches, with λ being the wavelength of the carrier frequency. The parameter d/λ is applied to determine the threshold level of correlation. In this numerical analysis, the assigned values of d/λ are $0,\ 0.1,\ 0.3$, and ∞, in which $d/\lambda = 0$ and $d/\lambda \to \infty$ represent two extreme conditions, i.e., fully correlated and uncorrelated branches, respectively.

The effect of fading parameters for MC-CDMA system is presented in Fig. 8. As the expectation that the BER results of MC-CDMA system becomes much better when the fading parameter is increasing. The BER versus E_b/N_0 in dB with different d/λ values $0,\ 0.1,\ 0.3$ are shown in Fig. 9, the fading parameter is set as $m=4$, and the subcarrier numbers *N* are with the values of 8, 16, and 32, respectively. It is reasonable to note that the much subcarrier numbers the better system performance is. The impact of the branch correlation is also presented in Fig. 9, that is, the larger the correlation coefficient, the lower the average BER, and that the performance of MC-CDMA system is going to be degraded when the correlation coefficient approaches the fully correlated condition, $d/\lambda = 0$.

7 Conclusion

The SNR and user number versus probability error rate for MC-CDMA system with MRC and EGC receipt works in uncorrelated and correlated

Nakagami-*m* fading channels has been evaluated with numerical and simulation results in this paper. The results explicitly show that the phenomena of channel correlation do degrade the performance of MC-CDMA communication systems with whichever kinds of the combining techniques are adopted. Therefore the consideration of correlation coefficient for channel fading should pay much attention while designing the MC-CDMA systems for wireless radio systems.

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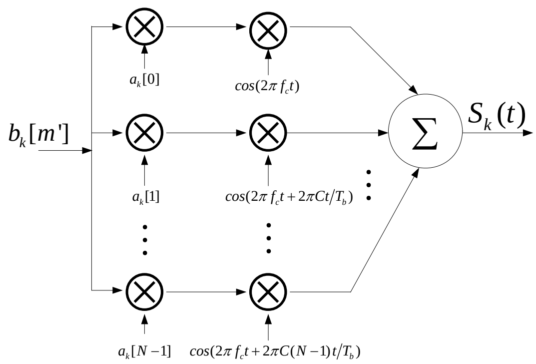

Fig. 1. The transmitter model of an MC-CDMA system.

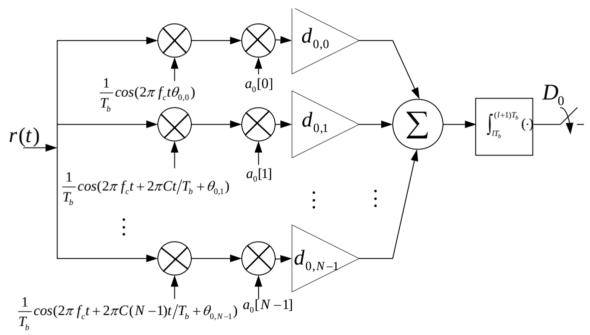

Fig. 2. The receiver model of an MC-CDMA system

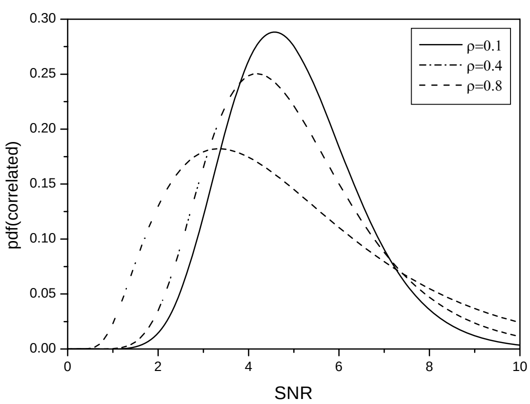

Fig. 3. The plot of comparison between different correlation coefficients for correlated Nakagami-*m* distributed.

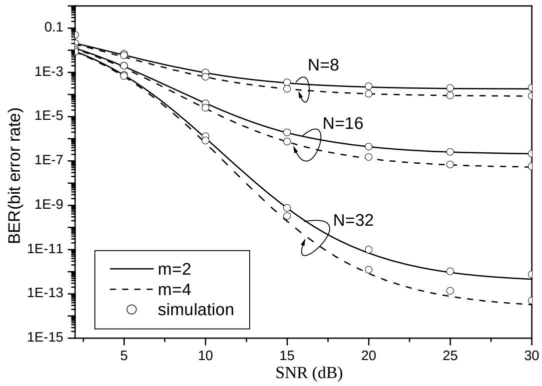

Fig. 4. The SNR vs. BER for MC-CDMA system with MRC for different subcarrier number and fading parameters over uncorrelated channels with $K = 50$

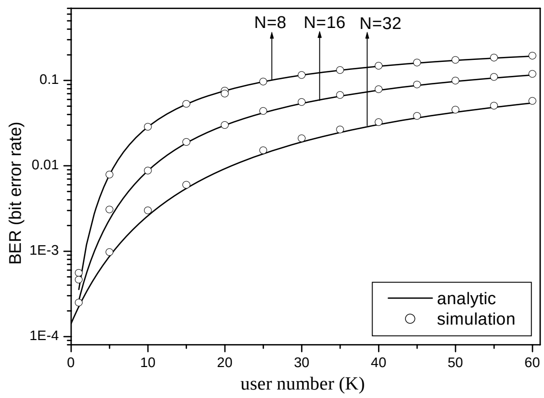

Fig. 5. The user number vs. BER for MC-CDMA system with MRC in different subcarrier number over uncorrelated channels with $E_b / N_0 = 5dB$.

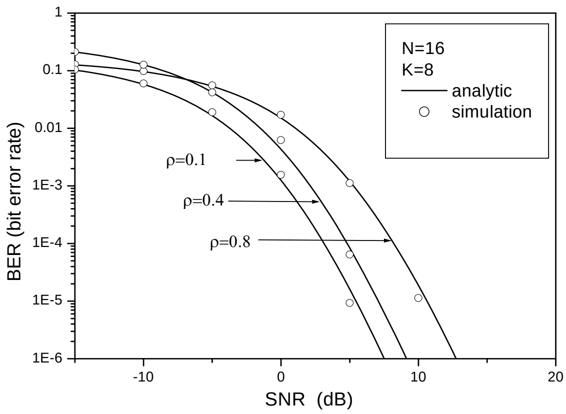

Fig. 6. The SNR vs. BER for MC-CDMA system with MRC in different correlation coefficient with $m = 2$

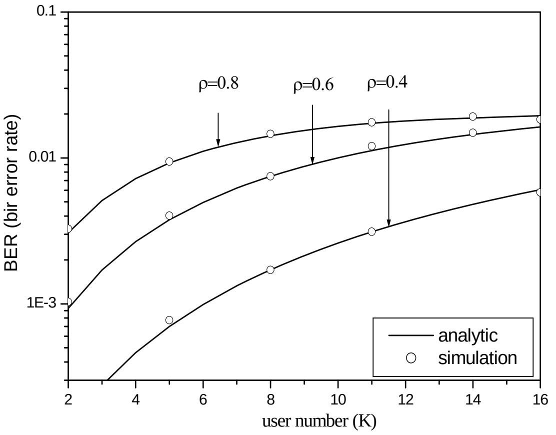

Fig. 7. The user number vs. BER for MC-CDMA system with MRC in different correlation coefficients with $E_b / N_0 = 5dB$, $m = 2$, $N = 32$.

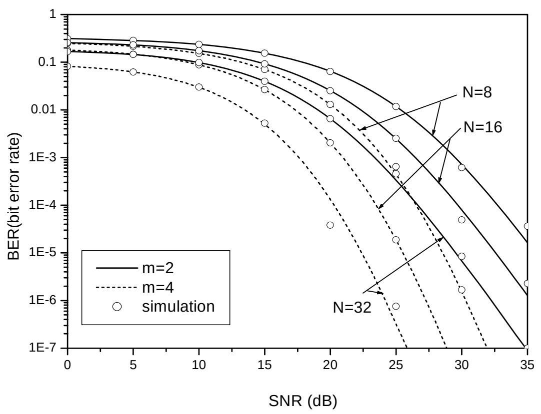

Fig. 8. The SNR vs. BER for MC-CDMA system with EGC in different subcarrier number and fading parameters with $d / \lambda = 0.1$.

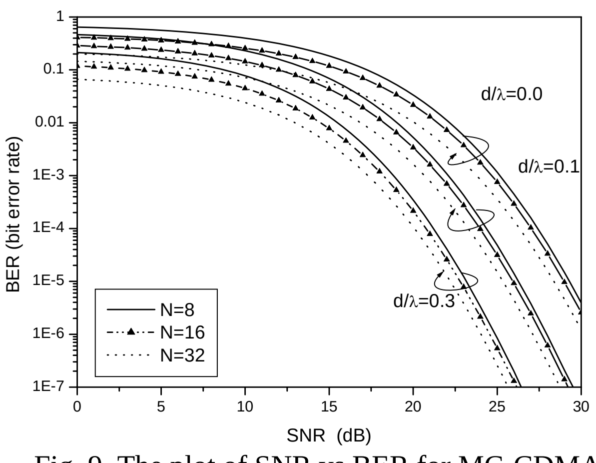

Fig. 9. The plot of SNR vs BER for MC-CDMA system with EGC in different subcarrier number over uncorrelated channel.

Appendix A:

The instantaneously *SNR* expressed in (21) is obtained by substituting (5), (19) and (20) into (18), thus yields as

$$\frac{\left(U_S^{MRC}\right)^2}{\sigma_T{}^2} = \frac{\dfrac{P}{2N}\displaystyle\sum_{n=0}^{N-1}\beta_{0,n}{}^2}{\dfrac{NN_0}{4T_b}\beta_{0,n}^2 + \dfrac{P}{4}(k-1)\Omega_{k,n}}$$

$$= \frac{P\left(\dfrac{1}{2N}\displaystyle\sum_{n=0}^{N-1}\beta_{0,n}{}^2\right)}{P\left(\dfrac{NN_0}{4PT_b}\beta_{0,n}^2 + \dfrac{(k-1)}{4}\Omega_{k,n}\right)}$$

$$= \frac{\dfrac{1}{2N}\displaystyle\sum_{n=0}^{N-1}\beta_{0,n}{}^2}{\Omega_{k,n}\left(\dfrac{NN_0}{4PT_b} + \dfrac{(k-1)}{4}\right)} = \frac{\sigma_0^{MRC} S^{MRC}}{2N}$$

where S^{MRC}, and σ_0^{MRC} are defined in (22), and $E_b = PT_b$ and $\Upsilon_0 = \dfrac{\mathrm{PT_b\Omega_{k,n}}}{\mathrm{N_0}} = \dfrac{\mathrm{E_b\Omega_{k,n}}}{\mathrm{N_0}}$.

Appendix B:

In this appendix the (25) will be derived. By substituting (9) into the first equivalent of (25), then the results can be obtained as

$$J_{0,n}(\Omega_{0,n}, \varphi)$$

$$= \int_0^\infty e^{(-\frac{\gamma_{0,n}}{\sin^2\varphi})} P_\gamma(\gamma_{0,n}) d\gamma_{0,n}$$

$$= \int_0^\infty e^{(-\frac{\gamma_{0,n}}{\sin^2\varphi})} \frac{\gamma_{0,n}{}^{m-1} e^{-\gamma_{0,n}/\Omega_{0,n}}}{\Gamma(m)\Omega_{0,n}^m} d\gamma_{0,n} \qquad \text{(B.1)}$$

$$= \int_0^\infty \frac{1}{\Gamma(m)\Omega^m} \exp\left[-\gamma_{0,n}\left(\frac{1}{\sin^2\varphi} + \frac{1}{\Omega_{0,n}^m}\right)\right]$$

$$= \left(\frac{m}{\Omega_{0,n}}\right)^m \left(\frac{1}{\sin^2\varphi} + \frac{m}{\Omega_{0,n}}\right)^{-m}$$

where the formula [22]

$$\int_0^\infty x^{\upsilon-1} \cdot e^{-\mu x} dx = \frac{\Gamma(\upsilon)}{\mu^\upsilon} \qquad \text{(B.2)}$$

has been adopted in this calculation.