## Network Coding: An Excellent Approach for Overloaded Communication Era.

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*Abstract*— Data flow in graphs is one of the most principle challenges faced by computer scientists and mathematicians. The problem of data flow in a graph has many practical applications e.g. routing in internet and wireless networks, where major purpose is to provide quality of service guarantees improved throughput, delay reduction etc. we will be using novel technique of network coding to device strategy to achieve theoretically maximum possible data flow in a network. Network is a new paradigm which in contrast to traditional routing allows mixing of data elements. This paper will present the developed algorithms of network coding, their performance analysis, simulation results and gains in terms of quality of service. Our results will particularly focus on improving throughput and reducing number of retransmissions, better network management by using network coding approach, we also show our experimental setup and experimental results which naturally graph up the network coding in this overloaded era.

Keywords; Wired Network Coding, Routing, Gain

#### **1** Introduction

The most promising araea of todys research is "how information transfer is treated" it may the internet packet transmission or simply the signals of a digital telph network, the way information transmission is treated may not effect by the origin they belongs. They may originate from different source but they are treated in the same way. These days the routing data bits data streams error control. From about last half a century, researchers are focused on solving and improving a number of fundamental issues regarding network. The hyper growth of network traffic and its use in both wired and wireless technologies have triggered a number of controversial issues related to improving quality of service e.g. achieving the capacity of a network without sacrificing throughput, security and reliability data storage as in current networkin era all functions of networks operates on the same rule. After advent of network coding the very emerging view of "coding" arises. It describes that it is possible in any communication network wired or wireless it is possible tov allow nodes to not only

forward the incoming traffic but also performs simple mathematical operations on them. [1].

Like at the network layer intermediate bnodes can perform binary addition on independent data packets and at the physical layer of other networks the intermediate nodes may also abule to superimpose incoming signals. As data packets that are independent do not need to be carrying separate when they are transported in the network. There are many interesting ways to combine them and then extract them as they are original. Combining independent data packets offers better information flow to the network that is entirely different with conventional routing that treats information flow as commodity flow. This new idea of performing different operations on network attracted a significant interest of both electrical. verv mathematicians and computer scientist as well. In traditional networks the basic issue is to transmit the data packets from the given sources to given destinations also called sinks, by using different routing schemes like BGP, Dijkstar's, hot potato

routing etc. In traditional routing schemes the flow of data is more like commodity flow i.e. keeping data streams separate without sharing bandwidth. While Network Coding motivates the mixing of data streams and is more like information flow. The basic idea of network coding can be explained by a simple butterfly example presented by R. Ahlswede et al. Figure 1 shows a network in which one. Source s wants to transmit two packets a and b to each terminal node y and z. By using traditional routing only one of the terminals can get both packets in one transmission while the other terminal can get only one packet as shown in Figure 2. However as network coding scheme allow the intermediate nodes to mix along with forwarding and therefore both terminals can get both packets simultaneously in one transmission. As shown in Figure 3 terminal t1 will get packet a and b packet a+b and can decode packet b by simple addition over a Galois field GF (q) of both packets received i.e. a+(b+b) = b, similarly terminal y will get packet b and a+b and can decode a. One of the most promising aspects of network coding is its capability to achieve the upper bound of the capacity of the network. Capacity of a network is governed by a famous mincut max-flow theorem of the network i.e. the maximum flow of data is equal to the min-cut of that network. In Figure 1 the min-cut of network is 2 and by the mentioned theorem 2 packets can be send simultaneously which is achievable using network coding as discussed above, whereas simple forwarding cannot send 2 packets at the same time. Moreover with traditional routing, the number of transmissions required for both packets to reach both terminals is 2 while network coding makes it possible to achieve the same goal using just one transmission which means increased throughput as compared to the traditional routing. Although network coding is an attractive and fascinating approach to enhance the capacity and [1].

## 2 The Basic Frame Work of Network Coding

The basic idea of network coding can be explained by a simple butterfly example presented by R. Ahlswede et al. Figure 1 shows a network in which one source s wants to transmit two packets a and b to each terminal node y and z. By using traditional routing only one of the terminals can get both packets in one transmission while the other terminal can get only one packet as shown in Figure 2.







Figure 2. Using traditional routing: Source s transmits packet a to terminal y and b to terminal z whereas it can send either packet a or packet b on edge w,x due to which only one of the terminals can get both packets while other will get just one . [9]

Increase the throughput; it is a NP-complete probl



Figure 3. Using network coding Source s transmits packet a and packet b to terminals y and z respectively and ab to both terminals. [9]

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## **3 Applications Of Network Coding**

The practical applications developed so faro nthe base of network coding are

- Erricsonhasdevelopedanalternativetoforwarder rorcorrectionandMulti user ARQ based on network coding[18]
- Network coding is used to improve strength of network
- It provides strong ease of network Management.
- It helps in improving data rates.
- It provides better resistance against faults.
- Using network coding proficient network designing is possible.
- Increase in throughput of wireless and wired networks.

#### **4** Contributions

Finding a solution of mixing at all intermediate nodes such as to achieve the capacity of the network is complex in terms of computations involved. Therefore research has been more focused in developing models, estimating field size required for the computations, developing heuristic, approximate and polynomial time algorithms to achieve the goal. Other important part of the network coding scheme is to device method to decode messages at the terminals because they are getting the coded/mixed packets. In Figure 3, terminals just need to do a simple addition over Galois field but in practical the specific size of some least field is required in order to decode successfully. In the given network GF (2) is required but network do exist which cannot operate on such small fields. Therefore selection of appropriate fields depending on the size of the network and number of decoding nodes is also required. However experiments have shown that a sufficiently large field under complete random settings will always work but using unnecessary high field size will complicate the problem therefore determining the least possible field size is yet another important aspect for designing a network. Network coding has find its way in many of the practical applications so far like avalanche; a digital file distributions and P2P file sharing software launched by Microsoft, wireless mesh networks, sensor network in broadcast and multi cast and unicast settings. The goal of this paper is to practical implementation of the network coding by considering its applications in wired network. The focus of this paper is to propose methods for

improving quality of service issues. The contribution of paper is three folded. Firstly, a graph theoretical model has been developed to properly calculate the parameters of concern while keeping model as close as realistic by avoiding unnecessary assumptions that hinder it from become a practical work. Secondly based on model a simple and elegant deterministic algorithm has been proposed that can work for any underlying generic network finally paper present extensive simulation results for practical channel setting for wired network and gains in terms of quality of service.

## **5** Scope

The scope of this research will be to develop algorithms for finding network codes. I shall be focusing on ways of finding algorithms or heuristics to find full rank coding matrices. In addition to that a complete set of system design parameters will be defined. The results found in this research can be applied to many of, wired internet infrastructures, mobile phone operators and network service providers to achieve maximum flow.

# 6 Problem Definition and Presented Solutions

This paper aims at developing the design and implementation methodologies required for practical implementation of network coding schemes under deterministic settings of a multi-cast network. A generic baseline network design will be used which will be implementable for wired networks. Important research point is to decrease the number of transmission so that increasing overall throughput and coding gain.

#### **Coding gain**

Coding gain is an important metric of performance defined as C=n/m (1)

Where m is number of message sent using traditional routing and n is number of messages sent using network coding. [1]

#### **Time Gain**

Gain in terms of ratio of time needed when it is required to send "h" packets from a source to each terminal using traditional routing with that of network coding is termed as time gain TG

Given multicast network (V;E; s; T) with a set of vertices V, set of edges E with weight w(e) = 18e 2 E, capacity c(e) = 18e 2 E, source s and a set of n terminals t1; t2; : : : tn 2 T with min-cut from source to

each terminal be at least h, server need to transmit "h" packets to all "n" terminals by optimizing the time required and throughput( in terms of sending more packets per communication rounds).

## **7 Network Coding Implementation**

The advent of network coding started with the seminal paper "Network Information Flow" by R. Ahlswede. Network coding is the only technique so far that guarantees the maximum possible throughput of a network to be achievable. In addition to achieving upper bound of the throughput, network coding also brings in security and reliability. The basic difference in traditional networks and the one that lies on network coding based principles is to use the nodes not just for forwarding messages but also for coding. As nothing comes without price, network coding also needs nodes with additional capability of coding along with the basic forwarding functionality.

#### 8 Network Coding in Wired networks

Wired network are most important network on which today's internet resides. Network coding in wired network like traditional internet backbones, ISPs etc can prove really helpful in solving several QoS issues. Network Coding can help in improving several network parameters in wired network. Need of such technique is very practical for this information overloaded era.

Following are the most important factors in which network coding can really help

- Increasing network utilization
- Increasing throughput
- Decreasing time response
- Load balancing on links
- Time Response
- Efficient utilization of network and its resources

And many other aspects related for improvement in the most major part of today's lives.

#### **9** Network Model

Consider a single source multi-cast network represented by a unit capacity, directed, acyclic multigraph, and let *h* denote the minimum cut between the source and any terminals  $t \in T$ .

With

One source

- A number of client nodes
- Min-cut from source to each client is "h"

And the Source want to transmit "h" packets to each terminal, and Nodes other than terminal also called as intermediate nodes with in degree more than one can mix the packets. The very important and noticeable developed point is Any node with more than one packets coming into its incoming edge is "coding node" i.e. more than one terminals need to share out coming edge of a coding node.

## **10 Random Network Coding Approach**

This is a simple algorithm that uses random coding coefficients over a Galois field of size **F**. Each encoding node chooses random coding coefficient uniformly from the selected Galois field size. For example consider an encoding node v that receives two packets a and b from the incoming links then node v will generate a packet c on outgoing link as

$$c = \alpha a + \beta b$$
 (2)

Where  $\alpha$  and  $\beta$  is randomly chosen from symbols of field  ${\it F}$ 

## **11 Developed Algorithms**

There are 2 phases of our developed algorithm

#### Phase 1:

We Find "h" edge disjoint paths from source to each destination

#### Phase 2:

After visiting all edges in topological order and then find linear network coding employed for that edge. Pack the information of coding coefficients with packets sent. After transmission of packets we have developed some basic steps for better encoding of packed information that is

- Encoding of an edge is done after encoding all edges that leads to that edge
- Encoding is done in a way all destination nodes will receive "h" linearly independent packets.

## 12 Minimum Cut, Maximum Flow Theorem [2][5]

Network denoted as a graph G

• G=(V, E)

- E Set of Edges (line joining pairs of vertices)
- V Set of vertices (nodes)

#### Rij Capacity of an edge (i,j)

- S Source
- T1, ..., TL Set of sinks/destinations [2][5]

*Cut*: A partition of vertex set into 2 classes, S containing source and S' containing the sink.[2][5]

*Value of the cut:*  $\sum C(e)$  (Sum of the capacities of the edges leaving the cut), where C(e) is the rate constraint/capacity of each edge [2][5]

 $\mathbf{F} = [F_{ij}, (i, j) \in E]$  is a flow in G from S to Tl

- for all  $(i, j) \in E \longrightarrow 0 \le Fij \le Rij$
- Such that for all  $i \in V$  except for S and Tl

---> i.e. the total flow into node i is equal to the total flow out of node i.

• Fij the value of F (flow) in the edge (i, j).

#### The value of F is

**F** is a max-flow from S to  $T_i$  in G if **F** is a flow from S to  $T_i$  whose value is greater than or equal to any other flow from S to  $T_i[2][5]$ 

For one source and one sink, the value of a maxflow is called the capacity of the graph.

For multiple sinks the max-flow is

$$F_{multicast} = \min_{i \in (t_1, \dots, t_L)} F(i)$$

The value of the Max-Flow is equal to the value of the Min-Cut. [2][5]

## 13 Implementation of Developed Algorithm

After successful development of algorithm, we have implemented these algorithms on a basic model for performance evaluation.

• **Phase 1**: min-cut for the given network is 2 so find 2 disjoint paths from source to terminal "Y"(Green paths) and terminal "Z"(Red paths) • **Phase 2**: Edges are ordered in topological order.

Our algorithm consists of two stages. In the first stage, a set of h edge-disjoint paths from the source s to each terminal  $t \in T$ . The second stage is a greedy algorithm that visits each edge topologically and compute the linear encoding coefficients for that edge. The problem is to choose such a linear combination that guarantees h linearly independent combinations for each destination. It can be done by maintaining an  $h^*h$ matrix called Global Encoding Matrix for each destination that keeps the coding coefficients for each packet sent along each disjoint path.. Whereas h linearly independent combinations for each destination can be ensured by keeping that Global Encoding Matrix to be invertible and of minimum rank. Destination nodes can decode the packets easily by multiplication of received vector with the inverse of Global Encoding Matrix.

Consider an network shown in Fig 4 with h=2. The execution of algorithm is carried out in two phases. During first phase find two disjoint paths from source to terminals *Y*(pink paths) and terminal *Z*(Red paths). During second phase initially edges are ordered in topological order as shown in Fig 4. After that source *s* send 2 packets on its 2 outgoing links with Global Encoding matrix  $B_v$  and  $B_z$ .

• Source sends 2 packets on its 2 outgoing links with Global Encoding matrix By and Bz. Where :

$B_z =$	1	0	<i>R</i> =	1	0
	1	1	$D_z$ –	0	1

When packets comes to encoding node W (Pink node) then new Global Encoding Matrix will be

$$B_{y} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad \qquad B_{z} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

While traversing edges in topological order when outgoing edges of node W (Pink node) then new Global Encoding Matrix will be Node Y and Z can receives two linearly independent packets and can decode the received.

## **14 Computing Encoding Coefficients**

A Global Encoding Matrix M is associated with each terminal of size hxh. Column of Global Encoding Matrix represents the h disjoint paths edges at the cutunder consideration and rows represents the h packets. Each entry in a column of Global Encoding Matrix contains encoding coefficients for each packet send on that disjoint path. Global Encoding Matrix is sent along with each packets from source. Packets are mixed up at encoding nodes while ensuring that new packet is linearly independent. This is done by checking the Global Encoding Matrix to be of minimum rank and invertible. packets.



Figure 4. The network model [9]

Node Y and Z can receives 2 linearly independent packet.

## **15 Decoding at Terminals**

Receiver can decode the packet by a simple matrix multiplication of input received. packet with the inverse of global encoding matrix [12].

Consider the the graph shown in Fig 2, min-cut for the given network is 2. During Phase1 we first find 2 disjoint paths from source to terminal T1 (Red paths) and terminal T2 (Pink paths) as shown in Fig 2 and Fig 4 respectively. During Phase2 vertices are ordered in topological order as shown . While traversing nodes the only mixing is done at Green vertex where network coding is done. The coding matrix when source S send 2 packets on its 2 outgoing links with Global Encoding

matrix BT1 and BT2. Terminals T1 and T2 receives 2 linearly independent packet and therefore can decode.

## **16** Perforance Evalauation Measures

As discussed above that Network Coding can help in improving several QOS parameters in wired network. Need of such technique is very practical for this information overloaded era. In previous chapters network coding has shown to improve the following important QoS factors:

- Increasing network utilization
- Increasing throughput
- Decreasing time response
- Load balancing on links

A source has to transmit "h" packets to a number of clients. Using the existing network routing technique the routers can simply forward the packets whereas network

coding calls for mixing of packets at intermediate nodes. But the Problem is to keep all "h" packets for each client independent at every step while try to share links between different terminals.

- We have defined coding gain to be the gain in terms of throughput of network over traditional routing
- We have defined Time gain to be the % decrease in response time over traditional routing.

## **17 Experiments**

For experimental setup we have drawn A random graph is formed where nodes and their connections

(edges) are all random. A source and number of terminals are selected at random using uniform distribution.

• Find min-cut of the network

- Identify network coding nodes
- Apply simple "XOR" over incoming packets and forward it on outgoing links.
- Calculate time and throughput using both network coding and traditional routing.
- Calculation of coding gain in terms of number of clients.
- Calculation of average percentage decrease time response with reference to conventional routing approaches

We have conducted a set of experiment for 2 3 and 4 terminal nodes and have generated random topologies check how network coding works as compared to the traditional approach. I have been focusing on two most important factors i.e.

For throughput it has been observed that with increase in number of clients the average coding gain decreases as shown in Fig 5 whereas for Fig 6 i.e. Time

Response, average time response decreases with the increase in number of terminals in a multicast network...

Time taken by all terminals to decode and throughput of the network. the results are shown in Fig 5 for Throughput and Fig 6 for Time response.

## **18 Simulation Results**

Simulation results shows a network of 100 nodes and showing the average coding gain for 2, 3 and 4 number of clients, the observation shows that with increase in number of clients the average coding gain decreases. The second simulation shows the results of average percentage decrease time response for 2,3 and 4 number of clients, the both simulation results shows the network coding effectiveness as compared to traditional routing approach. Which never ever achieves maximum flow in any network, but the network does it very effectively and very simply. These results makes network coding an excellent approach for maximum data flow with minimum number of transmissions. [1]



Figure 5. A Network of 100 nodes. Average coding gain for 2, 3 and 4 number of clients is shown in top middle and bottom figure respectively.

Observation: With increase in number of clients the average coding gain decreases



Figure 6. A Network of 100 nodes. Average %decrease in response time for 2,3 and 4 number of clients is shown in top middle and bottom figure respectively.

## **19** Conclusion

Network coding is proved as an emerging area in coding theory it also render a new view on multicasting in a wired scenario where maximum flow is most promising aspect. Network coding also offers additional benefits compared to traditional routing techniques, and using network coding fewer network resources consumed, it also offered ease of Network management provides excellent robustness against link failures. In this paper, we have presented a network coding algorithm and simulation results of algorithm for wired multicast scenario that definitely achieves the max-flow of the information transmission rate. A significant feature of our algorithm is its simpleness which definitely makes encoding and decoding very simple and easy to implement in practical networks. Our network coding algorithm works for all types of wired networks it may be a simple internet backbone or ISP, These types networks are excellent member for network coding, and, in fact, the possible use of network coding is for achieving maximum throughput over wired channels. Our presented algorithms addressed the main problem of computing maximum flow using minimum path and achieves optimal throughput, our algorithm have a positive force on the design of new wired networks it also allows enhanced extend the information flows in networks. In any

Situation, it can make straightforward shortest path for maximum data flow.

Finally our experimental results shows how network Coding is capable to facilitate the design of efficient networks as this elementary difficulty was before viewed as Very hard and rigid. We also showed the most major gain of network coding is not to achieve maximum throughput, but to make networks practical to attain upper bound rate of any network.

So, our paper makes the following contributions. First, we define the network model

Secondly, we developed an optimal algorithm in which the network coding can be performed over a larger networks in addition, we proved the how coding gain effects with the number of clients, and how to achieve

Upper bounds of coding gain and for the more practical wired networks. Simulation results also recommend the coding gain and average percentage decrease in response time with reference to number of clients.

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