Multiple Selection Diversity over Exponentially Correlated Nakagami-*m* Fading Channels in the Presence of Cochannel Interference

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Abstract: - The system performances of selection combining over exponentially correlated Nakagami-*m* channels are analyzed in this paper. Selection diversity based on the signal to interference ratio (SIR) is very efficient technique that reduces fading and cochannel interference influence. Fading between diversity branches and between interferers is correlated and Nakagami-*m* distributed with exponential correlation model. Very useful closed-form expressions for the output SIR's probability density function (PDF), cumulative distribution function (CDF), and outage probability are obtained, which are the main contributions of this paper. Also, the conclusions about influence of fading severity and correlations between received desired signals and interferences on the system performances, such as outage probability, average-output SIR and amount of fading (AoF), are presented.

Key-Words: - Cochannel Interference; Signal to Interference Ratio (SIR); Exponential Correlated Nakagami-*m* fading; Selection Combinig.

1 Introduction

In wireless communication systems various techniques for reducing fading effect and influence of cochannel interference are used. Such techniques are diversity reception, dynamic channel allocation, and power control. Upgrading transmission reliability and increasing channel capacity without increasing transmission power and bandwidth is the main goal of diversity techniques.

Diversity reception, based on using multiple antennas at the receiver (space diversity), is very efficient methods used for improving system's quality of service (QoS), so it provides efficient solution for reduction of signal level fluctuations in fading channels [1, 2, 3]. Multiple received copies of signal could be combined on various ways, and most popular of them are maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC) and switch and stay combining (SSC) [4]. MRC is the optimal combining scheme, but also the most complicated. EGC provides comparable performances to MRC technique but has lower implementation complexity, so it is an intermediate solution.

With SC receiver, the processing is performed at only one of the diversity branches, which is selectively chosen, and no channel information is required. That is why SC is much simpler for practical realization. In general, selection combining, assuming that noise power is equally distributed over branches, selects the branch with the highest signal-to-noise ratio (SNR), which is the branch with the strongest signal [1-5]. In fading environments as in cellular communications systems, where the level of the cochannel interference is sufficiently high as compared to the thermal noise, SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [6]. This type of SC in which the branch with the highest SIR is selected, can be measured in real time both in base stations and in mobile stations using specific SIR estimators as well as those for both analog and digital wireless systems (e.g., GSM, IS-54) [7].

Most of recently published papers assume independent fading between diversity branches and also between cochannel interferers. However, in practice due to insufficient spacing between antennas, when diversity system is applied on small terminals with multiple antennas, correlation arises between branches [8].

While Rayleigh and Rice distributions can be indeed used to model the envelope of fading channels in many cases of interest, it has been found experimentally, that Nakagami distribution offers better fit for wider range of fading conditions in wireless communications [9]. The effect of correlated fading has been extensively analyzed on the performance metrics of wireless communication systems [10, 11]. In the paper [12] analysis of signal combining for Nakagami-*m* distribution with exponential correlation model of fading has been given, but the influence of interference was not considered. More general case is when the correlated interference is also present. Moreover to the best author's knowledge, analytical study of multibranch selection combining involving assumed exponentially correlated Nakagami-*m* fading for both desired signal and co-channel interference hasn't been reported in the literature.

2 Problem Formulation

We consider in this paper diversity system with multiple correlated Nakagami-*m* fading channels with exponential correlation model in the presence of mutually correlated interferences. We model fading and interference by Nakagami-*m* distribution with exponential correlation model, which is an adequate for the scenario of multichannel reception from equispaced diversity antennas, in which the correlation between combined signals decays as the spacing between the antennas increases [12].

In order to study the effectiveness of any modulation scheme and the type of diversity used, it is required to evaluate the system's performance over the channel conditions. In this paper, for proposed system model, expressions for probability distribution function (PDF) and cumulative distribution function (CDF) of the output SIR for selection combining diversity are derived in sections 3.1 and 3.2. Numerical results for PDF and CDF are graphically presented. Furthermore, closed-form expressions for important performance measures such as the outage probability are obtained in section 3.3. The outage probability is shown graphically for different system parameters. Numerically obtained results for average-output SIR and amount of fading (AoF) are also graphically presented. Nakagami fading (*m*-distribution) describes multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves [13]. In provides good fits to collected data in indoor and outdoor mobile-radio environments and is used in many wireless communications applications, in the frequency range from 800 MHz to 4 GHz. In this paper, wireless communication system with multiple SIR-based SC diversity is considered.

The desired signal, received by *i*-th antenna, can be written as [14]:

$$D_i(t) = R_i(t) e^{j\phi_i(t)} e^{j[2\pi f_c t + \Phi(t)]}, \quad i = 1, 2 \ldots n \quad (1)$$

where f_c is carrier frequency, $\Phi(t)$ desired information signal, $\phi_i(t)$ the random phase uniformly distributed in $[0.2\pi]$, and $R_i(t)$, Nakagami-*m* distributed random amplitude process given by [7]:

$$f_{R_i}(t) = \frac{2 t^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{t^2}{\Omega}\right); \qquad t \geq 0 \quad (2)$$

where $\Gamma(\bullet)$ is the Gamma function, $\Omega = t^2/m$, with t^2 being the average signal power, and m is the inverse normalized variance of t^2, which must satisfy

$$m \geq ½.$$

Parameter m describes the fading severity. The resultant interfering signal, received by *i*-th antenna, is:

$$C_i(t) = r_i(t) e^{j\theta_i(t)} e^{j[2\pi f_c t + \psi(t)]} \quad i = 1, 2 \ldots n \quad (3)$$

where $r_i(t)$ is Nakagami-*m* distributed random amplitude process also, $\theta_i(t)$ is the random phase, and $\psi(t)$ is the information signal. This model refers to the case of single cochannel interferer.

The performance of the multibranch SC can be carried out by considering the effect of only the strongest interferer, as in [15], assuming that the remaining interferers are combined and considered as lumped interference that is uncorrelated between antennas. Furthermore, $R_i(t)$, $r_i(t)$, $\phi_i(t)$, and $\theta_i(t)$ are assumed to be mutually independent is sufficiently high for the effect of thermal noise on system performance to be negligible (interference-limited environment) [15]. Now, due to insufficient antennae spacing, both desired and interfering signal envelopes experience correlative multivariate Nakagami-*m* fading with joint distributions.

We are considering exponential correlation Nakagmi-*m* model of distribution. Also, we are assuming arbitrary correlation coefficients between fading signals and between interferences, because correlation coefficients depend on the arrival angles of the contribution with the broadside directions of antennas, which are in general case arbitrary [16]. The exponential correlation model [17] can be obtained by setting:

$$\Sigma_{i,j} \equiv \rho^{|i-j|}$$

for $i=j$ in correlation matrix, for both desired signal and interference. Now, joint distributions of pdf for both desired and interfering signal correlated envelopes for multi-branch signal combiner could be expressed by [12]:

$$f(R_1, R_2 \ldots R_n) = \frac{R_1^{m-1} R_n^{m}}{2^{m-1}\Gamma(m)\left(1-\rho_d^2\right)^{m(n-1)}} e^{-\frac{R_1^2 + R_n^2}{2\left(1-\rho_d^2\right)} - g_1}$$

$$\times\prod_{k=1}^{n-1} R_k \left(\frac{\rho_d}{1-\rho_d^2}\right)^{-(m-1)} I_{m-1}\left(\left(\frac{\rho_d}{1-\rho_d^2}\right)R_k R_{k+1}\right), \qquad (4)$$

with:

$$g_1 = \begin{cases} 0, & n=2 \\ (\rho_d^2+1)/2(1-\rho_d^2)\sum_{k=1}^{n-1} R_k^2, & n>2 \end{cases}, \qquad (5)$$

and

$$f(r_1, r_2 \ldots r_n) = \frac{r_1^{m-1} r_n^{m}}{2^{m-1}\Gamma(m)(1-\rho_c^2)^{m(n-1)}} e^{-\frac{r_1^2+r_n^2}{2(1-\rho_c^2)}-h_1}$$

$$\times\prod_{k=1}^{n-1} r_k \left(\frac{\rho_c}{1-\rho_c^2}\right)^{-(m-1)} I_{m-1}\left(\left(\frac{\rho_c}{1-\rho_c^2}\right)r_k r_{k+1}\right), \qquad (6)$$

with:

$$h_1 = \begin{cases} 0, & n=2 \\ (\rho_c^2+1)/2(1-\rho_c^2)\sum_{k=1}^{n-1} r_k^2, & n>2 \end{cases}, \qquad (7)$$

I_{m-1} (.) is modified Bessel function of the first kind and m-1 order, ρ_d is the power correlation coefficient defined as

$$\rho_d = cov(R_i^2, R_j^2)/(var(R_i^2)var(R_j^2))^{1/2}$$

with $0 \le \rho_d \le 1$ and ρ_c is interference correlation coefficient defined as

$$\rho_c = cov(r_i^2, r_j^2)/(var(r_i^2)var(r_j^2))^{1/2}$$

with $0 \le \rho_c \le 1$.

Moreover, without loss of generality and for simplification purposes of the matrix Σ, it is assumed into (4) and (6) that

$$\Omega_{di} = 2\sigma_{di}^2, \quad \Omega_{ci} = 2\sigma_{ci}^2, \quad i=1,2\ldots n$$

with $\sigma_{di} = 1$ and $\sigma_{ci} = 1$ being the variances of desired signal and interference, respectively [12, 19].

Instantaneous values of SIR at the diversity branches input can be defined as

$$\lambda_1=R_1^2/r_1^2,\ \lambda_2=R_2^2/r_2^2 \ldots \lambda_n=R_n^2/r_n^2.$$

The selection combiner chooses and outputs the branch with the largest SIR:

$$\lambda = \max(\lambda_1, \lambda_2, \ldots, \lambda_n) \qquad (8)$$

The joint probability density function of instantaneous values of SIR in n output branches $\lambda_1, \lambda_2 \ldots \lambda_n$ is as in [6]:

$$p_{\lambda_1,\lambda_2,\ldots\lambda_n}(t_1,t_2,\ldots,t_n) = \frac{1}{2^n\sqrt{t_1 t_2\cdots t_n}}\int_0^\infty\int_0^\infty\cdots\int_0^\infty p_{R_1R_2\ldots R_n}\left(r_1\sqrt{t_1}, r_2\sqrt{t_2},\ldots,r_n\sqrt{t_n}\right)$$

$$\times p_{r_1r_2r_3}(r_1,r_2,\ldots,r_n)\, r_1 r_2\cdots r_n\, dr_1 dr_2 \cdots dr_n \qquad (9)$$

Substituting (4) and (6) in (9), and after using famous relation:

$$I_n(x) = \sum_{i=0}^{\infty}\frac{x^{2i+n}}{2^{2i+n}\, i!\,\Gamma(i+n+1)}, \qquad (10)$$

$p_{\lambda 1,\lambda 2,\ldots,\lambda n}(t_1,t_2,\ldots,t_n)$ can be written as:

$$p_{\lambda_1,\lambda_2,\lambda_n}(t_1,t_2\ldots t_n) = \sum_{i_1,\ldots i_{n-1}=0}^{\infty}\sum_{l_1,\ldots,l_{n-1}}^{\infty}\Big[\frac{\lambda_1^{m+i_1-1}\lambda_2^{m+i_1+i_2-1}\cdots\lambda_n^{m+i_n-1}}{\Gamma(m)^2\, 2^{n(2m-1)}(1-\rho_d^2)^{2(n-1)m}(1-\rho_c^2)^{2(n-1)m}}$$

$$\times\frac{\left(\frac{\rho_d}{1-\rho_d}\right)^{i_1+\ldots+i_{n-1}}\left(\frac{\rho_c}{1-\rho_c}\right)^{l_1+\ldots+l_{n-1}}}{\prod_{j=1,\,p=1}^{n-1} 4^{i_j}4^{l_p}(i_j!)(l_p!)\Gamma(m+i_j)\Gamma(m+l_p)}$$

$$\times\left(\frac{(1-\rho_d^2)(1-\rho_c^2)}{t_1(1-\rho_c^2)+(1-\rho_d^2)}\right)^{2m+i_1+l_1}$$
$$\times\left(\frac{(1-\rho_d^2)(1-\rho_c^2)}{t_2(1-\rho_c^2)(1+\rho_d^2)+(1-\rho_d^2)}\right)^{2m+i_1+i_2+l_1+l_2}$$

$$\times\left(\frac{2(1-\rho_d^2)(1-\rho_c^2)}{t_{n-1}(1-\rho_c^2)(1+\rho_d^2)+(1-\rho_d^2)}\right)^{2m+i_{n-2}+i_{n-1}+l_{n-2}+l_{n-1}}$$
$$\times\left(\frac{2(1-\rho_d^2)(1-\rho_c^2)}{t_n(1-\rho_c^2)+(1-\rho_d^2)}\right)^{2m+i_{n-1}+l_{n-1}}$$

$$\times\Gamma(2m+i_1+l_1)\Gamma(2m+i_1+i_2+l_1+l_2)\cdots\Gamma(2m+i_{n-1}+l_{n-1})\ \Big] \qquad (11)$$

This expression is equivalent to the expression:

$$p_{\lambda_1,\lambda_2,\lambda_n}(t_1,t_2\ldots t_n) = \sum_{i_1,\ldots i_{n-1}=0}^{\infty}\sum_{l_1,\ldots,l_{n-1}}^{\infty}\ [\ G_1\times$$

$$\times \frac{t_1^{m+i_1-1}\cdots t_n^{m+i_n-1}}{\left(t_1\left(1-\rho_c^2\right)+\left(1-\rho_d^2\right)\right)^{i_1+l_1+2m}\ldots+\left(t_n\left(1-\rho_c^2\right)+\left(1-\rho_d^2\right)\right)^{i_{n-1}+l_{n-1}+2m}}\ \Bigg] \tag{12}$$

where G_1 is:

$$G_1=\frac{1}{\Gamma(m)^2}\frac{\left(1-\rho_d^2\right)^{m(n+1)+2(l_1+\ldots l_{n-1})}\left(1-\rho_c^2\right)^{m(n+1)+2(i_1+\ldots+i_{n-1})}\rho_d^{2(i_1+\ldots+i_{n-1})}\rho_c^{2(l_1+\ldots+l_{n-1})}}{\prod_{\substack{j=1\\p=1}}^{n-1} i_j!\,l_p!\,\Gamma(m+i_j)\Gamma(m+l_p)}$$

$$\times\Gamma(2m+i_1+l_1)\Gamma(2m+i_1+i_2+l_1+l_2)\cdots\Gamma(2m+i_{n-1}+l_{n-1}) \tag{13}$$

3 Problem Solution

3.1 Cumulative distribution function

For this case cumulative distribution function can be written as in [6]:

$$F_\lambda(t)=\underbrace{\int_0^t\cdots\int_0^t}_{n} p\left(x_1 x_2,\ldots,x_n\right)\underbrace{dx_1\cdots dx_n}_{n} \tag{14}$$

Substituting expression (12) in (14), and after multiple integration joint cumulative distribution function becomes:

$$F_\lambda(t)=\sum_{i_1,\ldots i_{n-1}=0}^{\infty}\ \sum_{l_1,\ldots l_{n-1}=0}^{\infty}\Bigg[\ G_2\ \frac{\left(\dfrac{t}{\dfrac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)^{i_1+m}}{(i_1+m)}$$

$$\times\frac{\left(\dfrac{t}{\dfrac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}+t}\right)^{i_1+i_2+m}}{(i_1+i_2+m)}\times\cdots\times\frac{\left(\dfrac{t}{\dfrac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)^{i_{n-1}+m}}{i_{n-1}+m}$$

$$\times\,{}_2F_1\left(i_1+m,1-l_1-m;1+i_1+m;\frac{t}{\dfrac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)$$

$$\times\,{}_2F_1\left(i_1+i_2+m,1-l_1-l_2-m;1+i_1+i_2+m;\frac{t}{\dfrac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1+\rho_d^2\right)\left(1-\rho_c^2\right)}+t}\right)$$

$$\times\cdots\times\,{}_2F_1\left(i_{n-1}+m,1-l_{n-1}-m;1+i_{n-1}+m;\frac{t}{\dfrac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)\Bigg] \tag{15}$$

with:

$$G_2=\frac{1}{\Gamma(m)^2}\frac{\left(1-\rho_1^2\right)^m\left(1-\rho_2^2\right)^m\rho_1^{2(i_1+\ldots+i_{n-1})}\rho_2^{2(l_1+\ldots+l_{n-1})}}{\left(1+\rho_1^2\right)^{(n-2)m+(i_1+\ldots+i_{n-1})}\left(1+\rho_2^2\right)^{(n-2)m+(l_1+\ldots+l_{n-1})}}$$

$$\times\frac{\prod\limits_{\substack{s=0\\ i_0=i_n=0\\ l_0=l_n=0}}^{n-1}\Gamma\left(i_s+i_{s+1}+l_s+l_{s+1}+2m\right)}{\prod\limits_{\substack{j=1\\p=1}}^{n-1} i_j!\,l_p!\,\Gamma\left(m+i_j\right)\Gamma\left(m+l_p\right)} \tag{16}$$

and ${}_2F_1\,(u_1,u_2;u_3;x)$, being the Gaussian hypergeometric function [18, (9.100)].

3.2 Probability distribution function

Probability density function (PDF) of the output SIR can be obtained easily from previous presented expression (15):

$$p_\lambda(t)=\frac{d}{dt}F_\lambda(t)=\sum_{i_1,\ldots i_{n-1}=0}^{\infty}\ \sum_{l_1,\ldots l_{n-1}=0}^{\infty}\Bigg[\ G_2 t^{2i_1+\ldots+2i_{n-1}+(n-1)m-1}$$

$$\times\left\{\frac{\left(\dfrac{1-\rho_d^2}{1-\rho_c^2}\right)^{m+l_1}}{\left(\dfrac{1-\rho_d^2}{1-\rho_c^2}+t\right)^{3m+i_1+l_1+i_{n-1}}\left(\dfrac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t\right)^{(n-2)m+i_1+2i_2+\ldots+i_{n-1}}(i_1+i_2+m)\cdots(i_{n-1}+m)}\right.$$

$$\times\,{}_2F_1\left(i_1+i_2+m,1-l_1-l_2-m;i_1+i_2+m+1;\frac{t}{\dfrac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t}\right)$$

$$\times\cdots\times\,{}_2F_1\left(i_{n-1}+m,1-l_{n-1}-m;i_{n-1}+m+1;\frac{t}{\dfrac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)\Bigg]$$

$$+\left[\frac{\left(\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}\right)^{m+l_1+l_2}}{\left(\frac{1-\rho_d^2}{1-\rho_c^2}+t\right)^{i_1+i_{n-1}+2m}\left(\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t\right)^{(n-1)m+i_1+l_1+l_2+2i_2+\ldots+i_{n-1}}(i_1+m)\cdots(i_{n-1}+m)}\right.$$

$$\times {}_2F_1\left(i_1+m,1-l_1-m;i_1+m+1;\frac{t}{\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)$$

$$\left.\left.\times\cdots\times {}_2F_1\left(i_{n-1}+m,1-l_{n-1}-m;i_{n-1}+m+1;\frac{t}{\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)\right]\right]$$

$$\times\left\{\left[\frac{\left(\frac{1-\rho_d^2}{1-\rho_c^2}\right)^{m+l_{n-1}}}{\left(\frac{1-\rho_d^2}{1-\rho_c^2}+t\right)^{3m+i_1+l_{n-1}+i_{n-1}}\left(\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t\right)^{(n-2)m+i_1+2i_2+\ldots+i_{n-1}}(i_{n-1}+m)\cdots(i_1+i_2+m)}\right.\right.$$

$$\times {}_2F_1\left(i_1+m,1-l_1-m;i_1+m+1;\frac{t}{\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)$$

$$\times\cdots\times {}_2F_1\left(i_{n-2}+i_{n-1}+m,1-l_{n-2}-l_{n-1}-m;i_{n-2}+i_{n-1}+m+1;\frac{t}{\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t}\right) \quad (17)$$

Following these expressions for triple branch case we obtain the CDF and PDF of the output SIR in the form of:

$$F_\lambda(t)=\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{l_1=0}^{\infty}\sum_{l_2=0}^{\infty}\left[G_3\times\frac{\left(\frac{t}{\left[\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t\right]}\right)^{i_1+m}\left(\frac{t}{\left[\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t\right]}\right)^{i_1+m+l_1}\left(\frac{t}{\left[\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t\right]}\right)^{l_1+m}}{(i_1+m)(i_1+m+l_1)(m+l_1)}\right.$$

$$\times {}_2F_1\left(i_1+m,1-l_1-m;i_1+m+1;\frac{t}{\left[\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t\right]}\right)$$

$$\times {}_2F_1\left(i_1+i_2+m,1-l_1-l_2-m;1+i_1+i_2+m;\frac{t}{\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1+\rho_d^2\right)\left(1-\rho_c^2\right)}+t}\right)$$

$$\left.\times {}_2F_1\left(i_2+m,1-l_2-m;i_2+m+1;\frac{t}{\left[\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t\right]}\right)\right]; \quad (18)$$

with:

$$G_3=\frac{1}{\Gamma(m)^2}\frac{\left(1-\rho_d^2\right)^m\cdot\left(1-\rho_c^2\right)^m\cdot\rho_d^{2i+2l}\cdot\rho_c^{2j+2n}}{i!\cdot j!\cdot l!\cdot n!\cdot\left(1+\rho_d^2\right)^{i+l+m}\cdot\left(1+\rho_c^2\right)^{j+n+m}}$$

$$\times\frac{\Gamma(i+j+2m)\cdot\Gamma(i+j+l+n+2m)\cdot\Gamma(l+n+2m)}{\Gamma(i+m)\Gamma(j+m)\Gamma(l+m)\Gamma(n+m)} \quad (19)$$

Now we have:

$$p_\lambda(t)=\frac{d}{dt}F_\lambda(t)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\left[G_3t^{2i+2l+3m-1}\right.$$

$$\times\left\{\left[\frac{\left(\frac{1-\rho_d^2}{1-\rho_c^2}\right)^{i_1+m}}{\left(\frac{1-\rho_d^2}{1-\rho_c^2}+t\right)^{i_1+l_1+i_2+3m}\left(\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t\right)^{i_1+i_2+m}(i_1+i_2+m)(i_1+m)}\right.\right.$$

$$\times {}_2F_1\left(i_1+i_2+m,1-l_1-l_2-m;i_1+i_2+m+1;\frac{t}{\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t}\right)$$

$$\left.\left.\times {}_2F_1\left(i_2+m,1-l_2-m;i_2+m+1;\frac{t}{\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)\right]\right]$$

$$+\left[\frac{\left(\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}\right)^{m+l_1+l_2}}{\left(\frac{1-\rho_d^2}{1-\rho_c^2}+t\right)^{i_1+i_{n-1}+2m}\left(\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t\right)^{(n-1)m+i_1+l_1+l_2+2i_2+\ldots+i_{n-1}}(i_1+m)\cdots(i_{n-1}+m)}\right.$$

$$\times {}_2F_1\left(i_1+m,1-l_1-m;i_1+m+1;\frac{t}{\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t}\right)$$

$$\times {}_2F_1\left(i_2+m, 1-l_2-m; i_2+m+1; \frac{t}{\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t} \right) \Bigg]\Bigg]$$

$$\times \left\{ \frac{\left(\frac{1-\rho_d^2}{1-\rho_c^2}\right)^{m+l_{n-1}}}{\left(\frac{1-\rho_d^2}{1-\rho_c^2}+t\right)^{3m+i_1+l_{n-1}+i_{n-1}} \left(\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t\right)^{(n-2)m+i_1+2i_2+..+i_{n-1}} (i_{n-1}+m)\cdots(i_1+i_2+m)} \right.$$

$$\times {}_2F_1\left(i+l+m, 1-j-n-m; i+l+m+1; \frac{t}{\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t} \right)$$

$$\times {}_2F_1\left(i+m, 1-j-m; i+m+1; \frac{t}{\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t} \right) \Bigg]\Bigg]\Bigg\} \quad] \qquad (20)$$

3.3 Outage probability

One of the accepted performance measure for diversity systems operating in fading environments is the outage probability P_{out}. This performance measure is very useful in wireless communication systems design especially for the cases when cochannel interference is present.

In the interference limited environment, P_{out} is defined as the probability that the output SIR of the SC falls below the given outage threshold γ also known as the protection ratio:

$$P_{out} = P_R\left(\xi < \gamma\right) = \int_0^{\gamma} p_{\xi}\left(t\right)dt = F_{\xi}\left(\gamma\right) \quad . \qquad (21)$$

Protection ratio depends on modulation technique and expected QoS demands.

3.4 Average-output SIR

Average-output SIR is another important performance criterion for SIR-based wireless communications systems operating in the cochannel interference environment. The average SIR at the SC output can be derived by averaging the instantaneous output SIR over the pdf [8]. Hence, taking into account (17), it can be written as

$$\bar{S} = \int_0^{\infty} t p_{\lambda}\left(t\right) dt \qquad (22)$$

3.5 Amount of fading (AoF)

By definition, the moments of the output SIR are given as:

$$E(t^n) = \int_0^{\infty} t^n p_{\lambda}(t) dt \qquad (23)$$

Amount of Fading (AoF) is a unified measure of fading severity. This performance criterion is independent of the average fading power. The AoF of the SC's output is defined as the ratio of the variance to the square mean SC output SIR and using (23) can be evaluated as:

$$AoF = \frac{var\left(t\right)}{\bar{S}^2} = \frac{E\left(t^2\right)}{\bar{S}^2} - 1 \qquad (24)$$

3.6 Numerical results

Probability density functions of the output signal to interference ratio, versus SIR at the branches inputs and for various values of the correlation coefficient are shown at Fig.1 for the case of triple branch combiner.

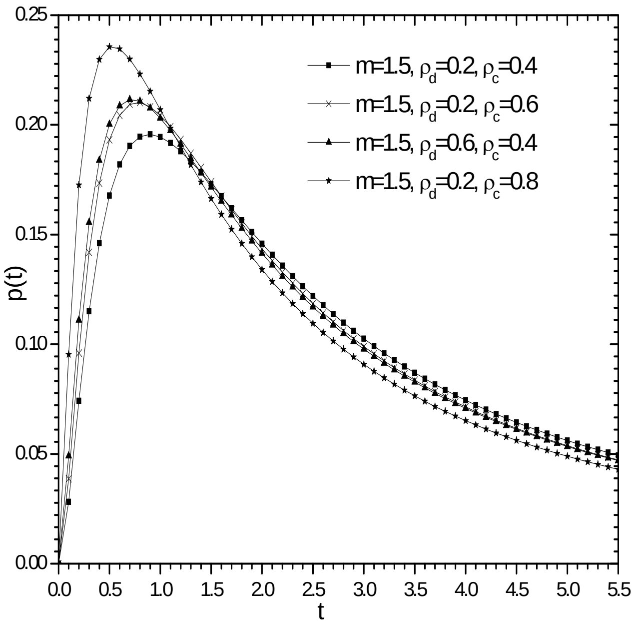

Fig.1 Probability density functions of output SIR for various values of correlation coefficients and triple branch case

Fig.2 shows the outage probability curves versus the power correlation coefficient ρ_d for desired signal and several values of m and for triple branch case. It is evident that for strong interference the outage probability increases slowly as the correlation coefficient increases, while a small increase of m does not have significant effect on the outage probability. For higher values of normalized SIR the influence of m and ρ_d becomes stronger.

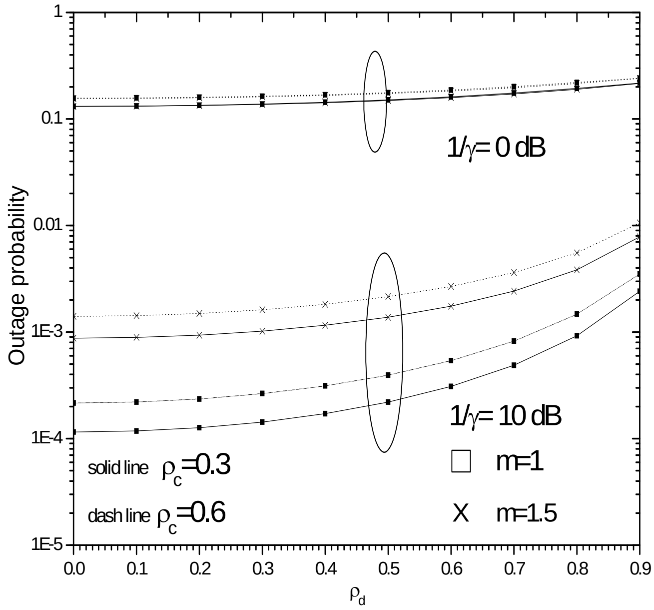

Fig.2 The outage probability curves versus ρ_d for triple branch case

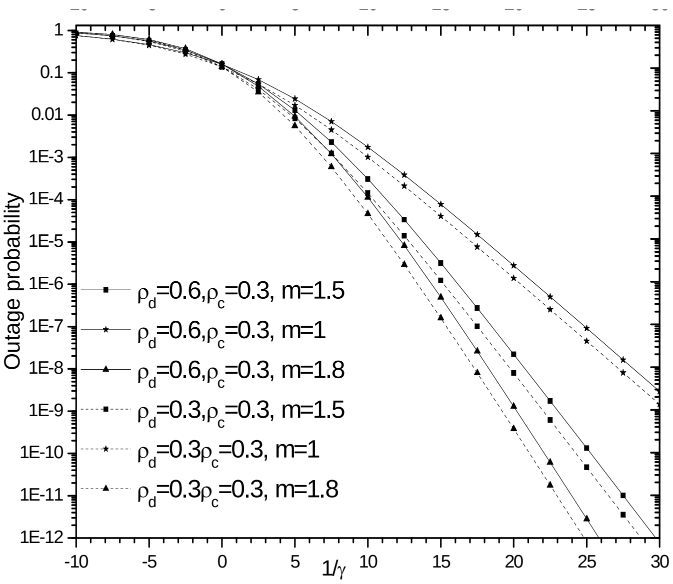

Fig. 3 The outage probability curves versus average SIR normalized by protection ratio for various values of correlation coefficients and fading severity for triple branch case

Fig.3 shows the outage probability versus average SIR normalized by the protection ratio for various values of the power correlation coefficient ρ_d for desired signal and several values of m and for triple branch case. Average SIR's at the i-th branch input of the triple-branch selection combiner is defined as $S_i = \Omega_{di} / \Omega_{ci}$.

Without loss of generality, it is assumed that $\Omega_{di}=\Omega_{ci}=1$. It is very interesting to observe that for lower values of $1/\gamma$ (<3dB) the outage probability deteriorates when the fading severity decreases due to interference domination. For higher values of $1/\gamma$ (when desired signal dominates), interference fading severity increase leads to the outage probability decrease. Also, it is shown that as the fading severity index m increases and normalized outage threshold decreases, P_{out} slightly decreases as well. However, as the signal correlation coefficient, ρ_d increases and normalized outage threshold decreases, P_{out} increases.

Fig.4 shows the outage probability versus the power correlation coefficient ρ_d for desired signal and several values of m and for the case of four branches combining.

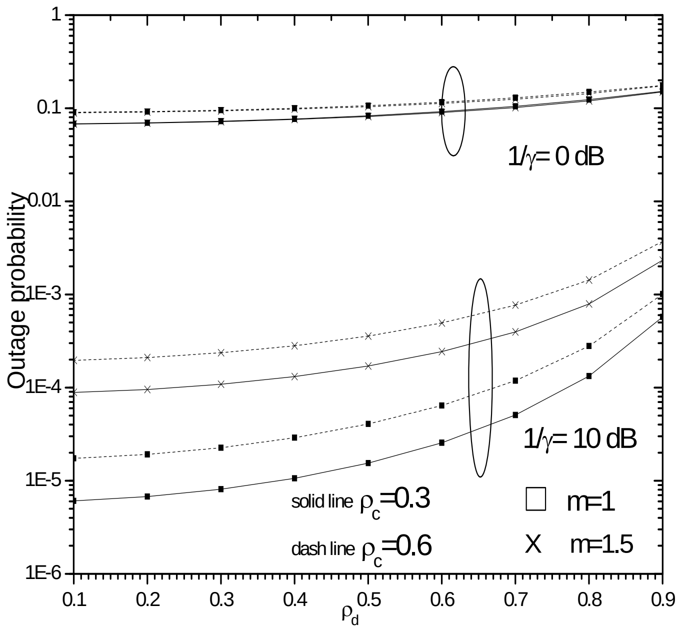

Fig. 4 The outage probability versus ρ_d for the four branches combining

Fig.5 shows the outage probability versus average SIR normalized by the protection ratio for various values of the power correlation coefficient ρ_d for desired signal and several values of m, for the case of four branches combining.

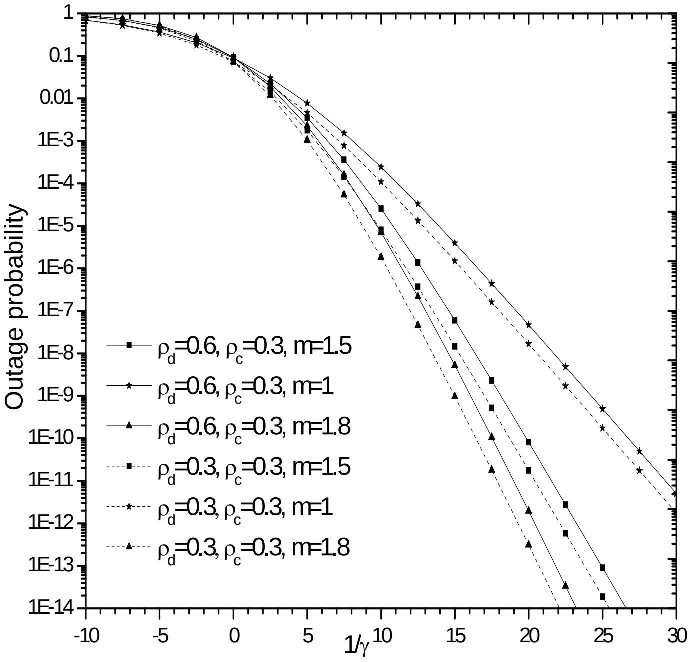

Fig. 5 The outage probability versus average SIR normalized by the protection ratio for various values of the correlation coefficients and fading severity for the four branches combining

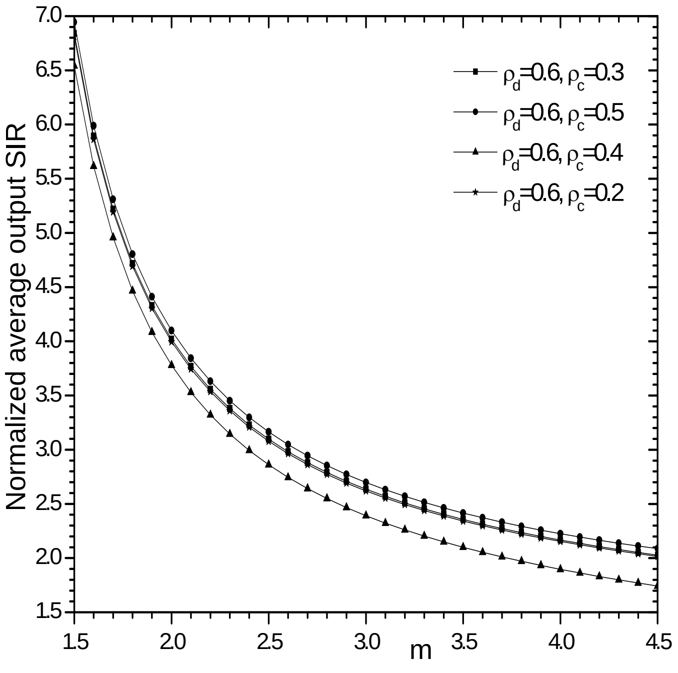

Fig. 6 Normalized average-output SIR for various values of fading severity m for the triple branch combining

By comparing results from figures which show the outage probability for different order of diversity selection used, we derive conclusion that the outage probability behavior improves as the diversity order (number of branches) increases. Also we can observe that differences in the outage probabilities between low and high correlation coefficients increase as diversity order grows.

Fig.6 shows normalized average-output SIR for various values of fading severity m for the case of triple branch combining.

It is evident from Fig. 6, where the normalized output SIR versus the cochannel fading severity parameter m is depicted, that a significant improvement at the diversity gain is observed for lower values of m ($m<2$). From the mathematical point of view, this behavior is due to the widespread of the output SIR pdf for small values of m. From the physical point of view, lower values of m mean deep fading behavior for the cochannel interferers that lead to an increase of the average SIR at the output of the SC. The fading severity of the interference plays the main role to output SIR performance.

Fig.7 shows normalized average-output SIR for various values of the power correlation coefficient ρ_d for the case of triple branch combining.

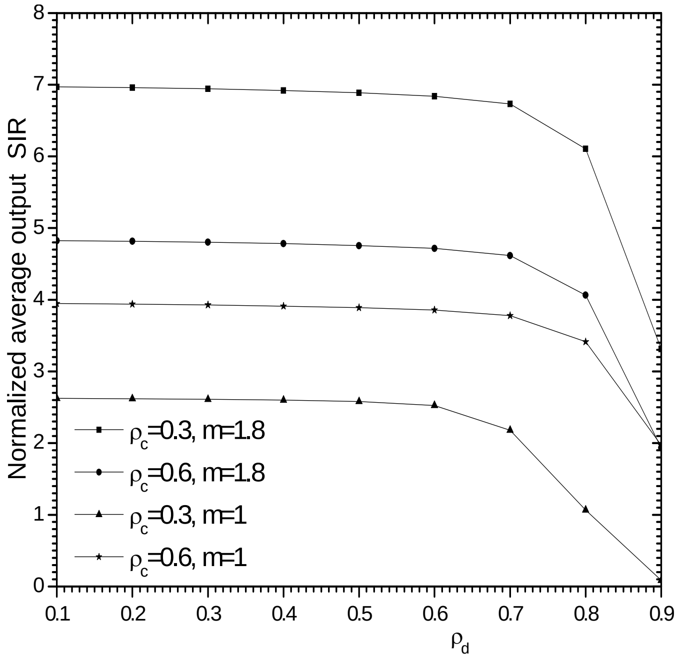

Fig. 7 Normalized average-output SIR for various values of the power correlation coefficient ρ_d for the triple branch combining

It is observed here that, with constant m, the effect of ρ_c to the diversity gain performance is small. Moreover, diversity gain decreases with an increase of the correlation, as expected. Finally, the output SIR decreases as the parameter m increases. Similar behavior is also observed in [12], where the average output SNR of a dual SC was studied. It is interesting to note here that diversity gain changes rapidly with a small change of *m*.

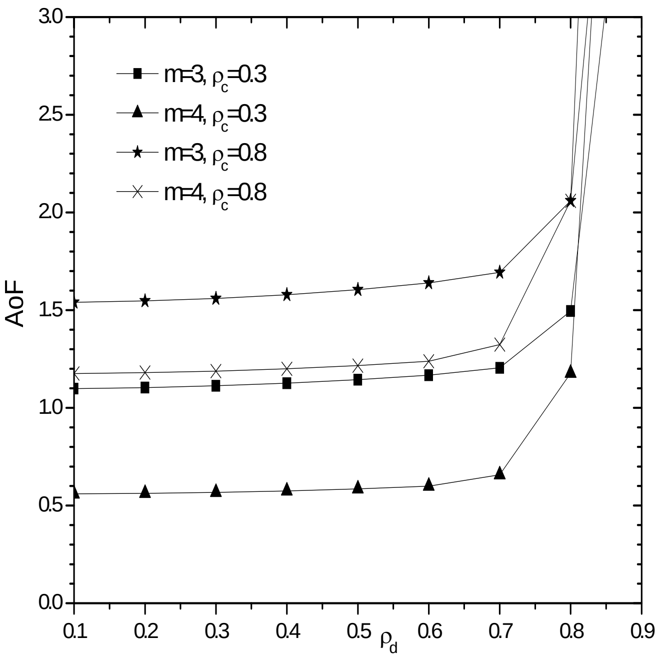

Fig. 8 Amount of Fading (AoF) for various values of fading severity *m* and interference correlation coefficient ρ_c for the triple branch combining

Fig.8 shows numerical results obtained for amount of fading for various values of the power correlation coefficient ρ_d for the case of triple branch combining.

It is evident from Fig.8 that an increase of ρ_d leads to corresponding increase of AoF resulting in performance degradation. Also, proposed analysis confirms that the AoF changes rapidly with a small change of *m*. All of these facts are shown in Fig. 8. It is because of deep fading behavior of the cochannel interferers (lower values of *m*) which leads to an increase of the average SIR at the output of the SC. Similarly, an increase of ρ_c leads to increase of AoF resulting in performance degradation.

4 Conclusion

The performance of the system with selection combining over exponentially correlated Nakagami-*m* channels in the presence of interference was studied in this paper. The fading was modeled as exponentially correlated Nakagami-*m* process. This flexible model is providing very good fit to experimental fading channel measurements for both, indoor and outdoor environments. The channel interference is also modeled as an exponentially correlated Nakagami-*m* process. The complete statistics for the SC output SIR is given in the closed form, i.e., PDF and CDF are given. Using these new formulae, the outage probability was efficiently evaluated. Also, the results for another important performance measures, such as average output SIR and amount of fading, are numerically evaluated. As an illustration of the mathematical formalism, numerical results of these performance criteria are graphically presented, describing their dependence on correlation coefficient and fading severity.

The main contribution of this analysis of multibranch signal combiner is that it has been done for the first time considering general case when correlated cochanel interference is present.

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