# An Algorithm Reliability of Wireless Networks for Sum of Disjoint Poroduct and Topological Structure

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*Abstract:* - A new formula for computing ST reliability of wireless networks from Source to Terminal t was presented. The formula is a combination of IE and SDP, and the Satyaranayanna's formula is its special case. The new formula contains terms which correspond one by one to a class Special subnetworks , For a given networks, the terms of the new formula are fewer than those corresponding Satyaranayanna's formula. An algorithm for computing ST reliability was presented, it computed ST reliability or produced a ST reliability expression by enumerating a class of Special networks of given networks. Because the structure of this class of new networks which need to be enumerated was relativity small, the new algorithm's performance was better than Satyaranayanna's algorithm .Finally an example illustrates our conclusion.

*Key-Words:* - Reliability, Reliability expression, inclusion exclusion principle; wireless network

## 1 Introduction

Along with wireless networks apply from extensive to extensive, Wireless networks have become importance. it not only inexpensive, but also suited to some Special occasion ,for example: Mobile networks, Ad hoc networks and Data collection system etc. The people attach importance to the reliability of wireless networks, the reliability of networks have been lucubrated, but the research on the reliability of wireless networks is less. AboEIFotoh and Colbourn refer to a probability graph model, prove the problem computing the reliability of wireless networks is a NP hard problem, give an approximate algorithm. Some Special wireless networks have been studied[16,17], Fanjia-Kong[18,19]give a Factoring algorithm of K-terminal reliability of wireless networks .

In this paper, we derive a new topological formula for the ST reliability of wireless network G. The formula is a combination of IE and SDP, and the SP formula is its special case. For given mutually disjoint paths, the formula can be divided into two parts: the first part is in SDP form and the second part is in IE form. The terms of the second part, not counting a factor, correspond one-to-one with certain acyclic subgraphs of G which contain none of the given mutually disjoint paths (i.e. sp-acyclic subgraphs of G). In general, the number of sp-acyclic subgraphs of G is much less than that of p-acyclic subgraphs of G. An algorithm for computing the ST reliability of G is given in this paper. The reliability expressions obtained by this algorithm are shorter than those obtained by the SP algorithm. The algorithm follows the philosophy of the SP algorithm: a search tree controls the search for the sp-acyclic subgraphs. The formula can be applied to networks with directed as well as undirected links.

## 2 Probabistic Graph Models for Wireless Networks

### 2 .1 Assumptions

1. A wireless network model is a directed graph G = G( V E ) , where every site is represented by a node and an edge exists between two nodes if and only if the corresponding sites can directly communicate with each other. Nodes are either operating or failed. Every node has a stated probability of operating. Operation or failure of nodes are mutually statistically independent.

2. The surrounding medium is perfect for radio transmission within the range under consideration. Therefore, the existence of the edge between two nodes depends only on the following:

a. the distance between the corresponding sites,

b. the orientation of their antennas,

c. the power of their transmitter/receivers, and

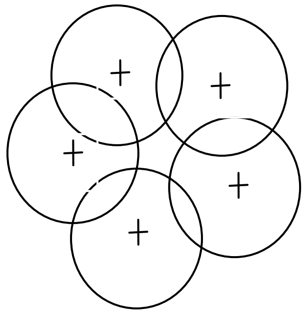
d. the absence of physical obstacles (eg, mountains, high buildings).

3. Sites are static during the communication period. Therefore, an edge between two nodes is either existent or nonexistent, and any existing edge is perfectly reliable.

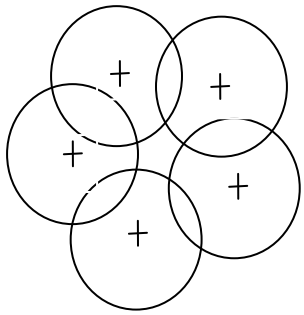
4. Any node that is failed (operable) remains failed(operable) throughout the entire communication period. Therefore, the model represents a relatively short time compared with the mean time between node failures.

A wireless network model under these assumptions is an arbitrary graph.

5. All sites are equipped with similar transmitters and receivers in terms of their antenna power and height, and all the antennas are directional. Hence each site can communicate to all the sites within a circle centered at that site having radius equal to the range of transmission. More formally, let the distance between two sites i and j , and r = the range of the transmitter/receiver. Site i can communicate directly with site j if and only if. Under assumptions 6, a wireless network model is an directed graph , where every site is represented by a node , and an edge if and only if . This graph is a unit-disk graph (UDG); UDGs are the intersection graphs of unit-radius circles in the plane (taking the unit-distance as half of r).An example of wireless network and the corresponding UDG are shown in figure 1. The circles in figure 1-a outline the range of the transmitter/receiver of each station. The circles in figure 1-b are the unit-radius circles of the intersection graph representation. Figure 1-c is the UDG model.



(1-a)



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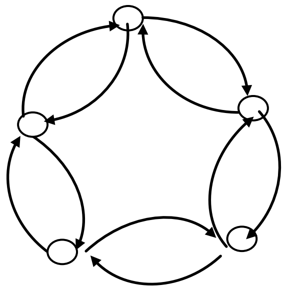

(1-c)

Fig. 1 a wireless network and its UDG model

There is 2 Special nodes s&t in the nodes set of G(V,E), the reliability of G is the probability source s and terminal t can communicate each other.2.1.1

## 3 Definition and Notation

### 3.1 Definition

$P_{ij}$ : A path from i to j.

$Sp_{ij}$ : A path from i to j, and the set of nodes on the path hasn't subset that can build up $P_{ij}$ , we name the simplest path $Sp_{ij}$ .

Parallel path of edge (u,v): (u,v)is a edge of G, If there is a path of G from u to v, the path is a parallel path of edge (u,v).

p-graph: Every edge of the graph must be in a $P_{st}$ of G.

sp-graph : a subgraph is a P graph and every path from s to t in the subgraph is Sp path

p-acyclic graph: a subgraph of G is a p-graph and acyclic.

sp-acyclic graph: a subgraph of G is a sp-graph and acyclic.

d-set: a set of specified mutually link-disjoint (or vertex disjoint) minpaths without cycles of a network with reliable vertices (or unreliable vertices).

Sequence (in a graph): a non-empty finite sequence ($v_0$, $e_l$, $v_l$,. . . . $v_{n-1}$, $e_n$, $v_n$) of vertices ($v_0$, $v_l$ . . . . . $v_{n-1}$, $v_n$) and links ($e_l$,. . . . . $e_n$,) of a graph such that ei is incident out of $v_{i-1}$ into $v_i$ (i = 1 . . . . . n).

Formation of a p-graph: a subset of paths from s to t whose union yields the p-graph

Neutral sequence (in a p-acyclic graph): a sequence in which all internal vertices have in-degree and out-degree exactly equal to one, and whose deletion from the graph results in a subgraph which is also p-acyclic.

disjoined product (sum of disjoined product): a form of system reliability analysis ,where both the logical polynomial for success and corresponding probability polynomial which calculates the reliability are term wise 1:1 identical with one (+).

Assumption of s-independent (ASI): each node of a network is in either a good or failed state, and all states of vertices of the network are mutually statistically in dependent.

### 3.2 Notion

a, b, c….: a Boolean(0-1) variable.

A, B, C….: a term of a Boolean polynomial or a Boolean polynomial.

+ (addition): logical exclusive or arithmetic addition.

. (multiplication): logical intersection or arithmetic multiplication).

G (V, E): a wireless networks, V is the set of the nodes of G, E is the set of the edges of G.

s, t : The source node of G and the terminal node if G.

$v_i$ ($\overline{v_i}$) : The state of node is working(not working).

Kp: The set of the nodes in path p.

$p_i$ ($\overline{p_i}$) : The working (failure) state probability of node i.

R(G) : The ST reliability of wireless networks .

## 4 Background and Literature Review

Many methods have been developed for reliability analysis of networks [1]. These methods fall roughly into three broad classes: inclusion-exclusion (IE), sum of disjoint products (SDP) and pivotal decomposition(PD).

### 4.1 Inclusion-exclusion

If there are m minimal paths $A_1$, $A_2$ . . . , $A_m$ in a network G, using IE, the direct expression of reliability of G has $2^n-1$ terms. In most cases, some of these terms cancel each other. Satyanarayana and Prabhakar(SP)[2] proposed an algorithm for source-to-terminal(ST) reliability which generates only the non-cancelling terms. They demonstrated a fundamental fact for ST reliability: there is a one-to-one correspondence between the p-acyclic subgraphs of G and the non-cancelling terms of the expression of ST reliability of G. They introduced the concept of neutral sequences in p-acyclic graphs. Deleting a neutral sequence from a p-acyclic graph yields a subgraph which again is p-acyclic. A tree search identifies the subgraphs of G and finds all the p-acyclic subgraphs of G without duplications. Research on network reliability by the IE method since the publication of Ref. [2] has continued in the topological and graph theory direction [3-5].

### 4.2 Sum of disjoint products

The economies of SDP are due to a simple principle. If the terms of the logic function are disjoint, then the logic function and the probability formula are one-to-one identical with one another. The first algorithm of SDP was presented by Fratta and Montanari [6]. An algorithm of SDP by Aggarwal etal. [7] was published in 1975, followed by Abraham[8]. Locks [9] and Beichelt and Spross [10]improved Abraham's algorithm. Heidtmann [11], Veeraraghavan and Trivedi [12] and Wilson [13] found new algorithms which resulted in shorter computation time and fewer disjoint products.

#### 4.2.1 Classical Boolean Logic

Logical algebra emanate from Boole's 1854 classic, *The Law of Thought[15]*. Boole created the notation and the algebraic structure of set manipulations, and introduced concepts such as inversion, also called complementation, and minimization, which are essential for SDP and other types of system reliability analysis. De Morgan proved the two well known inversion theorems, Wittgenstein [20] prepared a philosophic work showing the relationship between Boolean logic and analysis of proposition.

#### 4.2.2 Rule of Boolean Algebra

Let the lower case a, b, c,… denote 0-1 variable, each variable representing a node of the wireless network, and let the capital letters A, B, c,…be either terms or polynomial, with all 0-1 variables, For each variables a, the corresponding node a either succeed(a=1),or else fail(a=0). Similarly for a polynomial: A or A=1 is the positive or primary value, and $\overline{A}$ OR A=0 is the negative or inverted value .In the discussion below ,all algebraic operation operations are logical rather than arithmetic; for convenience ,the usual addition and multiplication symbols are used for Boolean operations instead symbology for unions and intersections.

#### 4.2.3 Boolean operations for variable, terms, or polynomial

Idempotency: $a+a=a, a=aa$

Negation: $a\overline{a}=\Phi$

Inversion (De.Morgans's law):

$$\overline{a+b}=\overline{a}\,\overline{b}$$

$$\overline{ab}=\overline{a}+\overline{b}$$

Disjoint sets: $a+b=a+\overline{a}b$

By induction, it can be shown that these rules extend to networks with indefinite number of variable; for example:

$$\overline{a+b+c}=\overline{a}\,\overline{b}\,\overline{c}$$

$$a+b+c=a+\overline{a}b+\overline{a}\,\overline{b}c$$

The rules also hold if the lower-case letters, representing variable, are replaced by terms or polynomial; for example :

$$\overline{A+B+C}=\overline{A}\,\overline{B}\,\overline{C}$$

$$=\overline{A}+A\overline{B}+AB\overline{C}$$

#### 4.2.3 Composite Boolean operations with variable and terms

Multiplication of a polynomial by a sum of variables:

$$(A+B+C)(a+b+c)=Aa+Ab+Ac$$
$$+Ba+Bb+Bc+Ca+Cb+Cc$$

Minimized multiplication of a term by a variable when the variable in the term:

$$(Aa)a=Aa$$

Absorption:

$$ab+a=a$$

$$aB+a=a$$

Minimized multiplication of a term by a sum of variables when one of the variables is in the term:

$$Aa(a+b+c)=Aa$$

Minimized recursive inversion (example)

Let $A=abc; B=acd$

$$\overline{A}B=\overline{abc}acd=(\overline{a}+\overline{b}+\overline{c})acd$$

$$=(\overline{b}+\overline{c})acd=\overline{bc}acd$$

$$=\overline{A-B}B$$

## 5 The New Topological Formula of Reliability of Wireless Networks

We consider a wireless networks G(V,E) , V =(1,2,3,...n) ,E=($e_1$,$e_2$…..$e_m$) , n =|V|, m=| E| , $p_i$ is the probability of the node i working,1- $p_i$ is the probability of the node i failure.

The definition of the reliability of wireless networks [1]:

R(G)=the probability that have a Pst in G that every nodes in the Pst is working

$$=\Pr(\bigcup_{allP_i inG} Pi)$$

According to the Boolean algebra axiom Pr(x+xy) =Pr(x) , if Pst isn't the simple path Spst , then $K_{pa}$ (the nodes set of Pst ) have a subset $K_{Sp}$, that can make up a $Sp_{st}$ ,

$K_{Sp}\subset K_P, \Pr(Sp+P)=\Pr(P)$, hence:

**Theorem1**. The reliability of wireless Networks

$$R(G)=\Pr(\sum_{i=1}^{m} Sp_i), 1\le i\le m \qquad (1)$$

m is the number of Spst of G

Example 1, Consider the directed graph $G_0$ of Fig. 1. For simplicity, we assume that all edges of $G_0$ are perfectly operational. The working probability of $v_i$ is $p_i$ (i= 1 , 2 , . . . , 5 ) and working events are s-independent. There are two $Sp_{st}$ path in the example , we have the ST reliability expression of G:

$$R(G)=\Pr(v_2+v_3)=p_2+q_2p_3$$

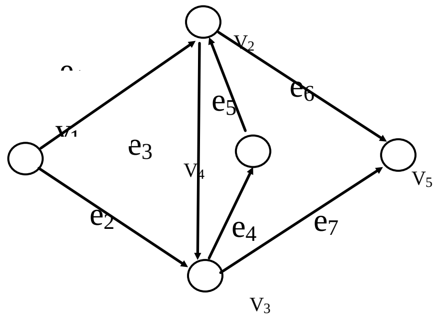


Fig.2 a graph of example 1($v_1$,$v_5$ is source and terminal node)

**Theorem 2.** If Sp is a st simple path, then it is one by only confirmed by its nodes set $K_{Sp}$

**Prove**: The length of Sp is | $K_{Sp}$ | -1=k, suppose Sp= s$w_1$ $w_2$ …$w_{k-1}$$w_k$ t, because there isn't parallel edge in wireless networks, if Sp isn't exclusive, suppose the other $Sp_2$= s$w_{i1}$ $w_{i2}$… $w_{ik}$ t, there is at least one the node that its locality in $Sp_2$ move along relative to in Sp , suppose it is $w_i$ ,the father of $w_i$ is $w_j$ , j <i-1, $w_i$ is neighbor to $w_j$ in graph G, then s$w_1$ $w_2$ …$w_j$ $w_i$ $w_{i+1}$ …$w_k$ t is a path Spst . Its length is less than k-1, it's conflicting to Sp is a simple path, so Sp is a st simple path, it is one by only confirmed by its nodes set $K_{Sp}$ .

**Lemma 1.**[2] G is a digraph then the reliability of G , $R(G)=\sum d_i \Pr(G_i)$ (2)

,all the p-acyclic graph of G ,

$d_i=(-1)^{n_i+e_i+1}$ ,

$n_i$ is the number of the node of Gi, ei is the number of the edge of Gi .

**Theorem 3.** G is a digraph then the reliability of G

$$R(G)=\sum d_i \Pr(G_i) \tag{3}$$

$G_i$ is the sp-acyclic graph of G ,

$d_i=(-1)^{n_i+e_i+1}$ ,

$n_i$ is the number of the node of $G_i$, $e_i$ is the number of the edge of $G_i$ .

**Prove:** If Gi is a p-acyclic graph of G, let the set of all p path of G made by no Sp path of $G_i$ is{$p_1$,$p_2$,……$p_l$},the edge set A={$e_1$,$e_2$,……$e_m$} is the set of edges on the path $p_k$ $1\le k\le l$ , and $A\subseteq E(G), A\cap E(G_i)=\varnothing$ ,

$$\left\{e_{i1},e_{i_2},.....e_{i_k}\right\}\subseteq\left\{e_1,e_2,.....e_m\right\},1\le k\le m$$

Then

Gi + $\left\{e_{i1},e_{i_2},.....e_{i_k}\right\}$ is p-acyclic graph. Otherwise if Gi + $\left\{e_{i1},e_{i_2},.....e_{i_k}\right\}$ is p-cyclic graph. then there is a circle C in the graph Gi + $\left\{e_{i1},e_{i_2},.....e_{i_k}\right\}$ ,if $e_{i_j}$ =(u,v) is in the circle C, replaced by the parallel path Puv in the Gi , therefore there is a circle in the subgraph Gi, it's conflicting to Gi is a p-acyclic graph, so $G_i+\left\{e_{i1},e_{i_2},.....e_{i_k}\right\}$ is p-acyclic graph. According to Lemma 1 we know the domination of $G_i+\left\{e_{i1},e_{i_2},.....e_{i_k}\right\}$ d*=$(-1)^{n^*+e^*+1}$, n*=$n_i$,e*=$e_i$+k, then d*=$(-1)^{ni+ei+k+1}$=$d_i(-1)^k$, the number of p-acyclic graph like $G_i+\left\{e_{i1},e_{i_2},.....e_{i_k}\right\}$ is $C_n^k, 1\le k\le m$ ,as $\sum_{k=0}^{m}C_n^k(-1)^k=0$ ,so $R(G)=\sum d_i \Pr(G_i)$ , $G_i$ is the sp-acyclic graph.

We assume that the d-set D={$Sp_1$, $Sp_2$,..., $Sp_s$ }where $s\le m$ . Then, based on SDP [14], according to formula (1) we have

$$\begin{aligned}R(G)&=\Pr(Sp_1+Sp_2+...+Sp_m)\\&=\Pr(Sp_1)+\Pr(\overline{Sp_1}(Sp_1+Sp_2+...+Sp_m)\\&=\Pr(Sp_1)+\Pr(\overline{Sp_1}(Sp_2))+....\Pr(\overline{Sp_1}\,\overline{Sp_2}.....\overline{Sp_{s-1}}Sp_s)\\&\quad+\Pr(\overline{Sp_1}\,\overline{Sp_2}.....\overline{Sp_s}(Sp_1+Sp_2+...+Sp_m))\end{aligned} \tag{4}$$

Let

$$R_1(\mathrm{G})=\sum_{i=1}^{s}\Pr(\overline{Sp_1}\,\overline{Sp_2}...\overline{Sp_{i-1}}Sp_i) \tag{5a}$$

If the ASI hold , then

$$R_1(\mathrm{G})=\sum_{i=1}^{s}\Pr(\overline{Sp_1})\Pr(\overline{Sp_2})...\Pr(\overline{Sp_{i-1}})\Pr(Sp_i)) \tag{5b}$$

And

$$R_2(G)=\Pr(\overline{Sp_1}\,\overline{Sp_2}.....\overline{Sp_s}(Sp_1+Sp_2+...+Sp_m)) \tag{6}$$

Then

$$R(G)=R_1(G)+R_2(G) \tag{7}$$

Using equation (3) and (6), we have

$$R_2(G) = \Pr(\overline{Sp_1}\,\overline{Sp_2}.....\overline{Sp_s}(Sp_1 + Sp_2 + ... + Sp_m))$$
$$= \sum_i d_i \Pr(\overline{Sp_1}\,\overline{Sp_2}.....\overline{Sp_s}G_i)$$
$$= \sum_i d_i \Pr(G_i)\Pr(\overline{Sp_1}\,\overline{Sp_2}.....\overline{Sp_s} \mid G_i) \quad (8)$$

If $G_i$ contains $Sp_j, (j \le s)$, $G_i$ is sp-acyclic graph of G, then

$$\Pr(G_i)\Pr(\overline{Sp_1}\,\overline{Sp_2}.....\overline{Sp_s} \mid G_i) = 0$$

From equation (8),we deduce that

$$R_2(G) = \sum_i d_i \Pr(G_i)\Pr(\overline{Sp_1}\,\overline{Sp_2}.....\overline{Sp_s} \mid G_i)$$
$$= \sum_i d_i \Pr(G_i)\Pr(\overline{Sp_1 - G_i}\,\overline{Sp_2 - G_i}.....\overline{Sp_s - G_i})$$
$$= \sum_i d_i \Pr(G_i)\Pr(\overline{B_{i1}}\,\overline{B_{i2}}.....\overline{B_{is}}) \quad (9a)$$

And if ASI holds then

$$R_2(G) = \sum_i d_i \Pr(G_i)\Pr(\overline{B_{i1}})\Pr(\overline{B_{i2}}).....\Pr(\overline{B_{is}}))$$

Where $G_i$ is the ith sp-acyclic graph of G, $d_i$ is the domination of $G_i$, and the sum is over all the sp-acyclic graph of G.

Combining the equation (5),(7) and (9),we have the following theorem.

**Theorem 4.** Let G be a probabilistic directed graph and d-set of G

D = {$Sp_1$,$Sp_2$ … $Sp_s$}

where $s \le m$, then the ST reliability of G

$$R(G) = R_1(G) + R_2(G)$$
$$= \sum_{i=1}^{s} \Pr(\overline{Sp_1}\,\overline{Sp_2}...\overline{Sp_{i-1}}Sp_i) +$$
$$\sum_i d_i \Pr(G_i)\Pr(\overline{Sp_1 - G_i}\,\overline{Sp_2 - G_i}.....\overline{Sp_s - G_i})$$
$$= \sum_{i=1}^{s} \Pr(\overline{Sp_1}\,\overline{Sp_2}...\overline{Sp_{i-1}}Sp_i)$$
$$+ \sum_i d_i \Pr(G_i)\Pr(\overline{B_{i1}}\,\overline{B_{i2}}.....\overline{B_{is}}) \quad (10a)$$

And if ASI holds, then

$$R(G) = R_1(G) + R_2(G)$$
$$= \sum_{i=1}^{s} \Pr(\overline{Sp_1}\,\overline{Sp_2}...\overline{Sp_{i-1}}Sp_i) +$$
$$\sum_i d_i \Pr(G_i)\Pr(\overline{Sp_1 - G_i})\Pr(\overline{Sp_2 - G_i}).....\Pr(\overline{Sp_s - G_i}))$$
$$= \sum_{i=1}^{s} \Pr(\overline{Sp_1})\Pr(\overline{Sp_2})...\Pr(\overline{Sp_{i-1}})\Pr(Sp_i))$$
$$+ \sum_i d_i \Pr(G_i)\Pr(\overline{B_{i1}})\Pr(\overline{B_{i2}}).....\Pr(\overline{B_{is}})) \quad (10b)$$

The second sum is taken over the sp-acyclic graph of G for a given d-set D. The first part of formula is in SDP form ,and the second part is in IE form. If the d-set D of G is empty, then the equation (10) become the equation (3), Obviously, it is very simple to compute $R_1(G)$. For computing $R_2(G)$, we shall develop an algorithm to generate all sp-acyclic subgraphs of G. Now we illustrate formula (10) by using Example 1of Ref. [2].

Example 1 [2]. There exist mutually disjoint paths

$Sp_1 = v_1v_2v_5$ and $Sp_1 = v_1v_3v_5$. Let D = { $Sp_1$, $Sp_2$ }. Figure 2 gives the rooted directed tree generated by the SP algorithm for G. The subgraphs encircled with dashed lines do not appear in the rooted directed tree generated by the algorithm of this paper. From eqn (10b) we have the ST reliability expression of G:

$$R(G) = p_2 + \overline{p_2}p_3$$

This result is same as that by formula (2)

## 6 A Rapid algorithm of Reliability of Wireless Networks

Now we present an algorithm for generating all sp-acyclic subgraphs of $G_0$. The algorithm generates a rooted directed tree T which identifies the subgraphs of $G_0$ and determines how the subgraphs contribute to the reliability formula. Nodes of T represent non-empty subgraphs of $G_0$, the root node being $G_0$. A weight is associated with each edge from node i to node j of T. The weight is the set of links deleted from Gi to obtain $G_j$. In T, node i is the father of j and j is the child of i if there exists an edge directed from i to j. Node i is an ancestor to j if there exists a directed path from i to j in T. Two or more nodes with the same father are brothers. A node i is the younger (elder) brother of node j if the algorithm generates the children of i later (earlier) than that of

j. We shall use the above definitions in the same sense.

The algorithm of this paper consists of the following five stages:

1. Successively decycle (in all possible ways) Go to obtain its acyclic subgraphs of G (by the following Rule 1).

2. Obtain maximal p-acyclic subgraphs of these acyclic subgraphs (by the following Rule 2).

4. Obtain maximal ap-acyclic subgraphs of these p-acyclic subgraphs by deleting neutral sequences(by the following Rule 3).

5. Obtain all sp-acyclic subgraphs of these maximal sp-acyclic subgraphs by deleting neutral sequences(by the following Rules 4 or 5).

Starting from the root node $G_0$, the algorithm grows a rooted directed tree by progressively generating children on all possible nodes of the tree. The following five processing rules are applied for generating the children of node k of the tree, depending on the nature of $G_k$. Each rule must ncorporate the weight restriction.Let X be the weight of the edge directed into any elder brother of k or elder brother of an ancester of k.

*Rule 1.* $G_k$ is cyclic. Consider a cycle C in $G_k$. The cycle C contains links $e_1, e_2 \ldots\ldots e_a$, then $G_{kj} = G_k - e_j$ $(j = 1, 2 \ldots\ldots a)$ is a child of $G_k$, provided $e_j \cap X = \varnothing$.

*Rule 2.* $G_k$ is acyclic, but not p-acyclic. Suppose Y is the minimal set of links in $G_k$ whose deletion generates a non-empty p-acyclic subgraph and $Y \cap X = \varnothing$, then $Gj = G_k - Y$ is the only child of $G_k$.

*Rule 3.* $G_k$ is p-acylic, but not *sp-acylic*. Obtain all the neutral sequences $x_j$ (j = 1, 2 . . . . , b) of $G_k$ contained in the path. If $x_j \cap X = \varnothing$, then $G_{kj} = G_k - x_j$ is a child *o f Gk*.

*Rule 4.* $G_k$ is ap-acyclic, but its father is not. Obtain all neutral sequences $x_j$ (j = 1, 2 . . . . , c) of $G_k$, then $G_{kj} = G_k - x_j$ is a child of $G_k$ provided $x_j \cap X = \varnothing$.

*Rule 5. G k* is sp-acyclic and its father is sp-acyclic. Consider the weight Y of the link incident into ayounger brother of k. Suppose that a neutral sequence $x_j$ (j = 1, 2 . . . . . d) of $G_k$ contains Y. If $x_j \cap X = \varnothing$,then $G_{kj} = G_k - x_j$ is a child of $G_k$. Rules 1, 2, 4 and 5 of this paper are the same as Rules 1, 2, 3 and 4 of Ref. [2], respectively. A corollary to Rule of of this paper is that if $G_{kj}$ is p-acyclic but not sp-acyclic, continue applying Rule 3.

Rules 1, 2, 4 and 5 of this paper are the same as Rules 1, 2, 3 and 4 of Ref. [2], respectively. A corollary to Rule of of this paper is that if $G_{kj}$ is p-acyclic but not sp-acyclic, consider another path of d-set D and continue applying Rule 3.

***Algorithm***

Search out all the Sp path and a d-set D of $G_0$ where D = { $Sp_1, Sp_2, \ldots .Sp_s$ }.

1. Initialise $f_1 \leftarrow 0, f_2 \leftarrow 0$. Set $k \leftarrow 0$
(i.e. consider$G_k = G_0$).
2. If $G_k$ is not cyclic go to step 4.
3. Generate children of k using Rule 1.
(a) k has children: increment $f_1 \leftarrow f_1 + 1$. Set $k \leftarrow$ (first child of k). Go to step 2.
(b) k has no children: go to step 5.
4. (a) $G_k$ is p-acylic: go to step 6.
(b) $G_k$ is not p-acylic: generate the only child of k using Rule 2.
(i) $G_k$ has child: increment f $f_1 \leftarrow f_1 + 1$. Set $k \leftarrow$ (child of k). Go to step 6.
(ii) Gk has no child: continue.
5. Delete node k from the tree.
(a) k has younger brother;, set $k \leftarrow$ (younger brother of k). Go to step 2.
(b) k has no younger brother but has elder brother: set $k \leftarrow$ (father of k). Decrement $f_1 \leftarrow f_1 - 1$. Goto step 13.
(c) k has no younger brother and no elder brother:set $k \leftarrow$ (father of k). Decrement $f_1 \leftarrow f_1 - 1$. Go to step 5.
6. $G_k$ does not contain any path of D: go to step 9.
7. $G_k$ contains a path $Sp_r$ of D.
(a) $b_k = n_k - 1$: go to step 8.
(b) $b_k > n_k - 1$: generate children of k using Rule 3.
(i) k has children: $k \leftarrow$ (first child of k). Increment $f_2 = f_2 + 1$. Go to step 6.
(ii) k has no children: continue.
8. Delete node k from the tree.
(a) k has younger brother: set $k \leftarrow$ (younger brother of k).
(i) $f_2 \geq 1$: go to step 6.
(ii) $f_2 = 0$ : go to step 2.
(b) k has no younger brother but has elder brother. Set $k \leftarrow$ (father of k).
(i) $f_2 \geq 1$: $f_2 = f_2 - 1$ Go to step 12.

(ii) $f_2 = 0$: $f_1 \leftarrow f_1 - 1$. Go to step 13.

(c) k has no younger brother and elder brother. Set $k \leftarrow$ (father of k).

(i) $f_2 \geq 1$: $f_2 = f_2 - 1$. Go to step 8.

(ii) $f_2 = 0$: $f_1 \leftarrow f_1 - 1$. Go to step 13.

9. Initialize $f_3 \leftarrow 0$. Set SIGN $\leftarrow \sigma_k$. Generate $B_{kr} = Sp_r - G_k (1 \leq r \leq s)$ and $g_k$. The sign of $g_k$ is SIGN.

(a) $b_k = n_k - 1$: go to step 11.

(b) $b_k > n_k - 1$: generate children of k using Rule 4.

(i) k has children: set $SIGN \leftarrow -SIGN$. (Suppose $k_1$, $k_2$ . . . . . $k_q$ are children of k. Generate $B_{k_j r} = Sp_r - G_{k_j} (1 \leq j \leq q, 1 \leq r \leq s)$, then $g_{k1}, g_{k2}, \ldots, g_{kq}$ constitute terms of $R_2(G)$, whose sign is given by SIGN.) Set $f_3 \leftarrow f_3 - 1$, $k \leftarrow$ (first child of k), $b_m \leftarrow b_k$ and $n_m \leftarrow n_k$ Continue.

(ii) k has no children: go to step 11.

10. (a) $f_3 = b_m - n_m + 2$. Decrement $f_3 \leftarrow f_3 - 1$. $k \leftarrow$ (father of k) and SIGN $\leftarrow$ -SIGN. Go to step 11.

(b) $f_3 < b_m - n_m + 2$. Generate children of k using Rule 5.

(i) k has children: set SIGN $\leftarrow$-SIGN.

(Suppose $k_1$, $k_2$ . . . . . $k_q$ are children of k. Generate $B_{k_j r} = Sp_r - G_{k_j} (1 \leq j \leq q, 1 \leq r \leq s)$, then $g_{k1}, g_{k2}, \ldots, g_{kq}$ constitute terms of $R_2(G)$ with sign given by SIGN.) Increment $f_3 \leftarrow f_3 - 1$. Set $k \leftarrow$ (first child of k). Go to step 10.

(ii) k has no children: go to step 11.

11.(a) $f_3 \neq 0$

(i) k has younger brother. Set $k \leftarrow$ (youngerbrother of k). Go to step 10

(ii) k has no younger brother. Set $k \leftarrow$ (fatherof k), SIGN $\leftarrow$ -SIGN. Decrement $f_3 \leftarrow f_3 - 1$. Go to step 11.

12. (a) $f_2 \neq 0$

(i) k has younger brother. Set $k \leftarrow$ (younger brother of k). Go to step 6.

(ii) k has no younger brother. Set $k \leftarrow$ (father of k). Decrement $f_2 = f_2 - 1$. Go to step12.(b) $f_2 = 0$. Continue.

13. (a) $f_1 \neq 0$

(i) k has younger brother. Set $k \leftarrow$ (younger brother of k). Go to step 2.

(ii) k has no younger brother. Set $k \leftarrow$ (father of k) Decrement $f_1 \leftarrow f_1 - 1$. Go to step13.(b) $f_1 = 0$. STOP.

**Theorem 5.** The algorithm generates all sp-acyclic subgraphs of $G_0$ without duplications.

**Prove:** Satyanarayana and Praghakar [2]proved that the SP algorithm generates all p-acyclicsubgraphs of $G_0$ without duplications. Now we prove that the algorithm of this paper generates all sp-acyclicsubgraphs of $G_0$ without duplications. The basic differencebetween these two algorithms is the use of Rule 3 of this paper.When the algorithm of this paper enters step 7 if p-acyclic subgraph $G_k$ of $G_0$ is a trivial sp-acyclic subgraph (i.e. path of D), then $G_k$ is deleted. Otherwise, using Rule 3, maximal sp-acyclic subgraphs of $G_k$ are obtained by deletingd-neutral sequences in the paths $Sp_1$, $Sp_2$ . . . . , $Sp_s$ of d-set D progressively. step 7 of the algorithm of this paper generates all maximal sp-acyclic subgraphs without duplications; and using Rules 4 and 5, steps 9 and 10 generated all sp-acyclic subgraphs of $G_0$. QED.

## 7 Example

We also use Example 1 to illustrate the algorithm. Figure 2 also shows the rooted directed tree generated by our algorithm. Consider the graph $G_0$. D={$v_2$},corresponding node 0 of the rooted directed tree T, contains a cycle C = {$e_3$, $e_4$, $e_5$}. By applying Rule 1, the children $G_1$, $G_2$ and $G_3$ (corresponding to nodes 1, 2 and 3, respectively) of Go are obtained by deleting $e_3$, $e_4$ and $e_5$ of cycle C. $G_1$ is a p-acyclic. Using Rule 3, v,$G_4$ (i.e. node 4) is obtained by deleting neutral $e_4e_5$, $G_4$ contains two Sp path $Sp_1 = e_1e_6$ and $Sp_2 = e_2e_7$. Using Rule 4, $G_5$ and $G_6$ ( is obtained by deleting neutral sequence $e_1e_6$ and $e_2e_7$ . is trivial sp-acyclic subgraph, go back to node 2 and then node 1 that it isn't a p-graph use Rule 2 deleting $e_5$ , $G_7$ being obtained is a p-acyclic, but isn't sp- acyclic, have parall path $e_1$and$e_1e_3$,by applying Rule 3, $e_3 \cap X \neq \varnothing$,then $G_7$ haven't child .

go back to node 3, $e_5 \cap X \neq \varnothing$ then Node 3 hasn't child and stop. Thus we have $R(G) = p_2 + \overline{p_2} p_3$

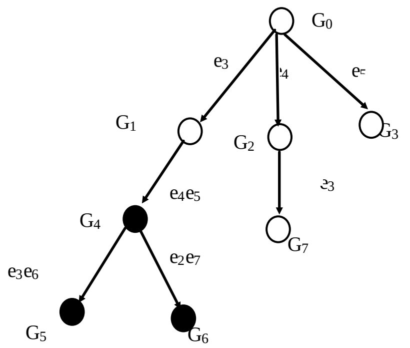


Fig. 3. Rooted directed tree for Go of Fig. 2.

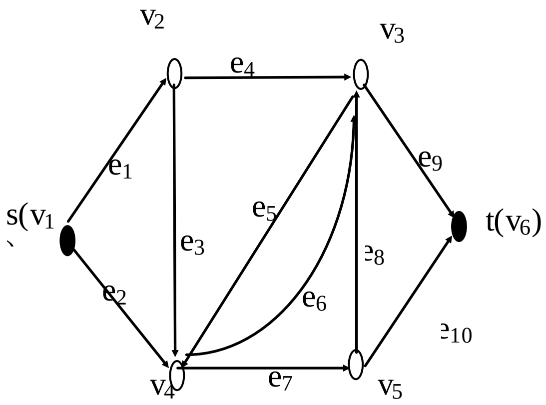


Fig 4 a graph of example 2

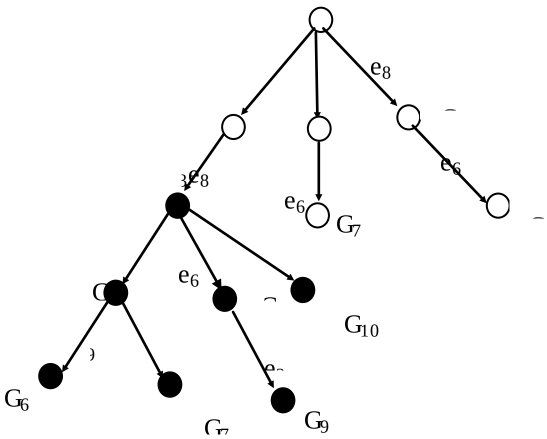


Fig. 5. Rooted directed tree for Go of Fig. 4.

Consider the example in Fig.3 of the paper[6] which contains a cycle C = {$e_4$, $e_5$, }. Figure 4 shows the rooted directed tree generated by the algorithm for the graph Go of Fig. 3 of this paper. The graph $G_o$ has 27 p-acyclic subgraphs and $R(G_0)$ has 27 terms by the algorithm. Table 1 shows sp-acyclic subgraphs in the rooted directed tree generated by the algorithm of this paper and the terms of $R(G_0)$, respectively. The graph $G_o$ has only 7 sp-acyclic subgraphs. The reliability $R(G_o)$ has only 7 terms. then we have

$$R(G) = R_1(G) + R_2(G) = (p_2 p_3 + \overline{p_2 p_3} p_4 p_5) + (\overline{p_2 p_5} p_3 p_4)$$

Table 1. Subgraphs of Go of Fig.4 and their weights, sign

| Sp-acycle subgrph | Subgraphs | weight | nodes | sign | $B_{i1}$ | $B_{i2}$ |
|---|---|---|---|---|---|---|
| 1 | $G_4$ | $e_1e_2e_4e_6e_7e_9e_{10}$ | $v_2v_3v_4v_5$ | + | | |
| 2 | $G_8$ | $e_1e_2e_4e_7e_9e_{10}$ | $v_2v_3v_4v_5$ | - | | |
| 3 | $G_5$ | $e_2e_6e_7e_9e_{10}$ | $v_2v_3v_4v_5$ | - | | |
| 4 | $G_{10}$ | $e_1e_2e_4e_6e_9$ | $v_2v_3v_4v_5$ | - | | |
| 5 | $G_9$ | $e_1e_4e_9$ | $v_2v_3$ | + | | |
| 6 | $G_6$ | $e_2e_7e_{10}$ | $v_4v_5$ | + | | |
| 7 | $G_7$ | $e_2e_6e_9$ | $v_3v_4$ | + | $v_2$ | $v_5$ |

## 8 Conclusion

The number of the ap-acyclic subgraphs is less than the number of the p-acyclic subgraphs ,R (G) can easy computed. The workload apply Satyaranayanna 's algorithm is very big, so the expression of R(G) is very long. Compare our algorithm the workload is less, and the expression is shorter. This shows the advantage of our algorithm.

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