

The Performances of the SSC Combiner Output Signal in the Presence of Log-Normal Fading

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Abstract: - Level crossing rate, outage probability and average time of fade duration of the SSC combiner output signal are determined in this paper. Log-normal fading at the input is present. The results are shown graphically for different variance values, decision threshold values and fading parameters values.

Key-Words: - Diversity reception, Fade Duration, Level Crossing Rate, Log-normal fading, Outage Probability, SSC Combining

1 Introduction

Many of the wireless communication systems use some form of diversity combining to reduce multipath fading appeared in the channel [1]. Among the simpler diversity combining schemes, the two most popular are selection combining (SC) and switch and stay combining (SSC). SSC is an attempt at simplifying the complexity of the system but with loss in performance. In this case the receiver selects a particular antenna until its quality drops below a predetermined threshold. When this happens, the receiver switches to another antenna and stays with it for the next time slot, regardless of whether or not the channel quality of that antenna is above or below the predetermined threshold.

In the paper [2] Alouini and Simon develop, analyze and optimize a simple form of dual-branch switch and stay combining (SSC). The consideration of SSC systems in the literature has been restricted to low-complexity mobile units where the number of diversity antennas is typically limited to two ([3], [4] and [5]). Furthermore, in all these publications, only predetection SSC has thus far been considered wherein the switching of the receiver between the two receiving antennas is based on a comparison of the instantaneous SNR of the connected antenna with a predetermined threshold. This results in a reduction in complexity relative to SC in that the simultaneous and continuous monitoring of both branches SNRs is no longer necessary.

In [6] the moment generating function (MGF) of the signal power at the output of dual-branch switch-and-stay selection diversity (SSC) combiners is derived.

The joint probability density function of the SSC combiner output signal at two time instants in the presence of Rayleigh fading is determined in [8]. The level crossing rate, outage probability and average time of fade duration of the SSC combiner output signal in the presence of Nakagami- m fading are calculated in [9]. In this paper level crossing rate, outage probability and average time of fade duration of the SSC combiner output signal in the presence of log-normal fading will be determined. The results will be shown graphically for different variance values, decision threshold values and fading parameters values.

2 System Model

The model of the SSC combiner with two inputs, considered in this paper, is shown in Fig. 1.

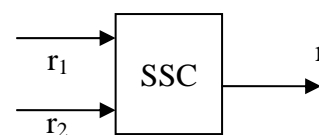


Fig. 1: Model of the SSC combiner with two inputs

The signals at the combiner input are r_1 and r_2 , and r is the combiner output signal. The predetection combining is observed.

The probability of the event that the combiner first examines the signal at the first input is P_1 , and for the second input is P_2 . If the combiner examines first the signal at the first input and if the value of the signal at the first input is above the threshold, r_T , SSC combiner forwards this signal to the circuit for the decision. If the value of the signal at the first input is below the threshold r_T , SSC combiner forwards the signal from the other input to the circuit for the decision, regardless it is above or below the predetermined threshold.

If the SSC combiner first examines the signal from the second combiner input it works in the similar way. The probability for the first input to be examined first is P_1 and for the second input to be examined first is P_2 .

The determination of the probability density of the combiner output signal is important for the receiver performances determination.

3 System Performances

Derivation of system performances is very important for telecommunication systems. Because of that we determine first the probability density functions (PDFs) of the combiner input signals, r_1 and r_2 , in the presence of log-normal fading. They are:

$$p_{r_1}(r_1) = \frac{1}{\sqrt{2\pi}\sigma_1 r_1} e^{-\frac{(\ln r_1 - \mu_1)^2}{2\sigma_1^2}} \quad r_1 \geq 0 \quad (1)$$

$$p_{r_2}(r_2) = \frac{1}{\sqrt{2\pi}\sigma_2 r_2} e^{-\frac{(\ln r_2 - \mu_2)^2}{2\sigma_2^2}} \quad r_2 \geq 0 \quad (2)$$

The cumulative probability densities (CDFs) are given by:

$$F_{r_1}(r_T) = \int_0^{r_T} p_{r_1}(x) dx \quad (3)$$

$$F_{r_2}(r_T) = \int_0^{r_T} p_{r_2}(x) dx \quad (4)$$

r_T is the threshold of the decision.

In the presence of log-normal fading CDFs are:

$$F_{r_1}(r_T) = \int_0^{r_T} \frac{1}{\sqrt{2\pi}\sigma_1 x} e^{-\frac{(\ln x - \mu_1)^2}{2\sigma_1^2}} dx = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_T - \mu_1}{\sigma_1 \sqrt{2}}\right) \quad (5)$$

$$F_{r_2}(r_T) = \int_0^{r_T} \frac{1}{\sqrt{2\pi}\sigma_2 x} e^{-\frac{(\ln x - \mu_2)^2}{2\sigma_2^2}} dx = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_T - \mu_2}{\sigma_2 \sqrt{2}}\right) \quad (6)$$

where $\operatorname{erfc}(x)$ is the error function and it is defined as [7]:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

The joint probability densities of the combiner input signals, r_1 and r_2 , and their derivatives \dot{r}_1 and \dot{r}_2 , in the presence of log-normal fading, are:

$$p_{r_1 \dot{r}_1}(r_1, \dot{r}_1) = \frac{1}{\sqrt{2\pi}\sigma_1 r_1} e^{-\frac{(\ln r_1 - \mu_1)^2}{2\sigma_1^2}} \cdot \frac{1}{\sqrt{2\pi}\beta_1 r_1} e^{-\frac{\dot{r}_1^2}{2\beta_1^2 r_1^2}} \quad r_1 \geq 0 \quad (7)$$

$$p_{r_2 \dot{r}_2}(r_2, \dot{r}_2) = \frac{1}{\sqrt{2\pi}\sigma_2 r_2} e^{-\frac{(\ln r_2 - \mu_2)^2}{2\sigma_2^2}} \cdot \frac{1}{\sqrt{2\pi}\beta_2 r_2} e^{-\frac{\dot{r}_2^2}{2\beta_2^2 r_2^2}} \quad r_2 \geq 0 \quad (8)$$

The probabilities P_1 and P_2 are:

$$P_1 = \frac{F_{r_2}(r_T)}{F_{r_1}(r_T) + F_{r_2}(r_T)} = \frac{1 + \operatorname{erf}\left(\frac{\ln r_T - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_T - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_T - \mu_2}{\sigma_2 \sqrt{2}}\right)} \quad (9)$$

$$P_2 = \frac{F_{r_1}(r_T)}{F_{r_1}(r_T) + F_{r_2}(r_T)} = \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \quad (10)$$

The expression for the joint probability density function of the SSC combiner output signal and its derivative will be determined first for the case: $r < r_T$:

$$p_{r\dot{r}}(r\dot{r}) = P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2\dot{r}_2}(r\dot{r}) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1\dot{r}_1}(r\dot{r}) \quad (11)$$

and then for $r \geq r_T$:

$$p_{r\dot{r}}(r\dot{r}) = P_1 \cdot p_{r_1\dot{r}_1}(r\dot{r}) + P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2\dot{r}_2}(r\dot{r}) + P_2 \cdot p_{r_2\dot{r}_2}(r\dot{r}) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1\dot{r}_1}(r\dot{r}) \quad (12)$$

We have now the case $r < r_T$:

$$p_{r\dot{r}}(r\dot{r}) = \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right)\right) \frac{1}{\sqrt{2\pi}\sigma_2 r} e^{-\frac{(\ln r - \mu_2)^2}{2\sigma_2^2}} \cdot \frac{1}{\sqrt{2\pi}\beta_2 r} e^{-\frac{i^2}{2\beta_2^2 r^2}} + \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)\right) \frac{1}{\sqrt{2\pi}\sigma_1 r} e^{-\frac{(\ln r - \mu_1)^2}{2\sigma_1^2}} \cdot \frac{1}{\sqrt{2\pi}\beta_1 r} e^{-\frac{i^2}{2\beta_1^2 r^2}} \quad (13)$$

and for $r \geq r_T$:

$$p_{r\dot{r}}(r\dot{r}) = \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \frac{1}{\sqrt{2\pi}\sigma_1 r} e^{-\frac{(\ln r - \mu_1)^2}{2\sigma_1^2}} \cdot \frac{1}{\sqrt{2\pi}\beta_1 r} e^{-\frac{i^2}{2\beta_1^2 r^2}} + \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right)\right) \frac{1}{\sqrt{2\pi}\sigma_2 r} e^{-\frac{(\ln r - \mu_2)^2}{2\sigma_2^2}} \cdot \frac{1}{\sqrt{2\pi}\beta_2 r} e^{-\frac{i^2}{2\beta_2^2 r^2}} + \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)\right) \frac{1}{\sqrt{2\pi}\sigma_2 r} e^{-\frac{(\ln r - \mu_2)^2}{2\sigma_2^2}} \cdot \frac{1}{\sqrt{2\pi}\beta_2 r} e^{-\frac{i^2}{2\beta_2^2 r^2}} + \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)\right) \frac{1}{\sqrt{2\pi}\sigma_1 r} e^{-\frac{(\ln r - \mu_1)^2}{2\sigma_1^2}} \cdot \frac{1}{\sqrt{2\pi}\beta_1 r} e^{-\frac{i^2}{2\beta_1^2 r^2}} \quad (14)$$

For the channels with identical parameters it is,
for $r < r_T$:

$$p_{r\dot{r}}(r\dot{r}) = \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\ln r_t - \mu}{\sigma\sqrt{2}} \right) \right) \cdot \frac{1}{\sqrt{2\pi}\sigma r} e^{-\frac{(\ln r - \mu)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\beta r} e^{-\frac{\dot{r}^2}{2\beta^2 r^2}} \quad (15)$$

and for $r \geq r_T$:

$$p_{r\dot{r}}(r\dot{r}) = \left(\frac{3}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\ln r_t - \mu}{\sigma\sqrt{2}} \right) \right) \cdot \frac{1}{\sqrt{2\pi}\sigma r} e^{-\frac{(\ln r - \mu)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\beta r} e^{-\frac{\dot{r}^2}{2\beta^2 r^2}} \quad (16)$$

The level crossing rate is:

$$N(r_{th}) = \int_0^\infty \dot{r} p_{r\dot{r}}(r_{th}, \dot{r}) d\dot{r} \quad (17)$$

For the channels with identical parameters it is valid:

for $r_{th} < r_T$:

$$N(r_{th}) = \frac{1 + \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}} \right)}{2 + \operatorname{erf} \left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}} \right) + \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}} \right)} \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}} \right) \right) \cdot \frac{1}{\sqrt{2\pi}\sigma_2 r_{th}} e^{-\frac{(\ln r_{th} - \mu_2)^2}{2\sigma_2^2}} \frac{\beta_2 r_{th}}{\sqrt{2\pi}} +$$

$$+ \frac{1 + \operatorname{erf} \left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}} \right)}{2 + \operatorname{erf} \left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}} \right) + \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}} \right)} \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}} \right) \right) \cdot \frac{1}{\sqrt{2\pi}\sigma_1 r_{th}} e^{-\frac{(\ln r_{th} - \mu_1)^2}{2\sigma_1^2}} \frac{\beta_1 r_{th}}{\sqrt{2\pi}} \quad (18)$$

and for $r_{th} \geq r_T$:

$$N(r_{th}) = \frac{1 + \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}} \right)}{2 + \operatorname{erf} \left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}} \right) + \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}} \right)} \cdot \frac{1}{\sqrt{2\pi}\sigma_1 r_{th}} e^{-\frac{(\ln r_{th} - \mu_1)^2}{2\sigma_1^2}} \cdot$$

$$\frac{\beta_1 r_{th}}{\sqrt{2\pi}} + \frac{1 + \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}} \right)}{2 + \operatorname{erf} \left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}} \right) + \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}} \right)}$$

$$\cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}} \right) \right) \cdot \frac{1}{\sqrt{2\pi}\sigma_2 r_{th}} e^{-\frac{(\ln r_{th} - \mu_2)^2}{2\sigma_2^2}} \frac{\beta_2 r_{th}}{\sqrt{2\pi}} + \frac{1 + \operatorname{erf} \left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}} \right)}{2 + \operatorname{erf} \left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}} \right) + \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}} \right)}$$

$$\begin{aligned} & \cdot \frac{1}{\sqrt{2\pi}\sigma_2 r_{th}} e^{-\frac{(\ln r_{th}-\mu_2)^2}{2\sigma_2^2}} \frac{\beta_2 r_{th}}{\sqrt{2\pi}} + \\ & + \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\ & \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right) \right) \cdot \\ & \cdot \frac{1}{\sqrt{2\pi}\sigma_1 r_{th}} e^{-\frac{(\ln r_{th}-\mu_1)^2}{2\sigma_1^2}} \frac{\beta_1 r_{th}}{\sqrt{2\pi}} \end{aligned} \quad (19)$$

For the channels with identical parameters it is valid for $r_{th} < r_T$:

$$\begin{aligned} N(r_{th}) &= \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu}{\sigma \sqrt{2}}\right) \right) \cdot \\ & \cdot \frac{1}{\sqrt{2\pi}\sigma r_{th}} e^{-\frac{(\ln r_{th}-\mu)^2}{2\sigma^2}} \frac{\beta r_{th}}{\sqrt{2\pi}} \end{aligned} \quad (20)$$

and for $r_{th} \geq r_T$:

$$\begin{aligned} N(r_{th}) &= \left(\frac{3}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu}{\sigma \sqrt{2}}\right) \right) \cdot \\ & \cdot \frac{1}{\sqrt{2\pi}\sigma r_{th}} e^{-\frac{(\ln r_{th}-\mu)^2}{2\sigma^2}} \frac{\beta r_{th}}{\sqrt{2\pi}} \end{aligned} \quad (21)$$

The outage probability $P_{out}(r_{th})$ is defined as:

$$P_{out}(r_{th}) = \int_0^{r_{th}} p_r(r) dr \quad (22)$$

For $r < r_T$ probability density function is:

$$p_r(r) = P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2}(r) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1}(r) \quad (23)$$

and for $r \geq r_T$

$$\begin{aligned} p_r(r) &= P_1 \cdot p_{r_1}(r) + P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2}(r) + \\ & + P_2 \cdot p_{r_2}(r) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1}(r) \end{aligned} \quad (24)$$

In the presence of log-normal fading and for $r < r_T$ the probability density function is:

$$\begin{aligned} p_r(r) &= \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\ & \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) \right) \frac{1}{\sqrt{2\pi}\sigma_2 r} e^{-\frac{(\ln r - \mu_2)^2}{2\sigma_2^2}} + \\ & + \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\ & \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right) \right) \frac{1}{\sqrt{2\pi}\sigma_1 r} e^{-\frac{(\ln r - \mu_1)^2}{2\sigma_1^2}} \end{aligned} \quad (25)$$

For $r \geq r_T$:

$$\begin{aligned} p_r(r) &= \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\ & \cdot \frac{1}{\sqrt{2\pi}\sigma_1 r} e^{-\frac{(\ln r - \mu_1)^2}{2\sigma_1^2}} + \\ & + \frac{1 + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\ & \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_t - \mu_1}{\sigma_1 \sqrt{2}}\right) \right) \frac{1}{\sqrt{2\pi}\sigma_2 r} e^{-\frac{(\ln r - \mu_2)^2}{2\sigma_2^2}} + \end{aligned}$$

$$\begin{aligned}
 & + \frac{1 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\
 & \quad \cdot \frac{1}{\sqrt{2\pi\sigma_2 r}} e^{-\frac{(\ln r - \mu_2)^2}{2\sigma_2^2}} + \\
 & + \frac{1 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\
 & \quad \cdot \left(\frac{1}{2} + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)\right) \frac{1}{\sqrt{2\pi\sigma_1 r}} e^{-\frac{(\ln r - \mu_1)^2}{2\sigma_1^2}}
 \end{aligned} \tag{26}$$

The outage probabilities $P_{out}(r_{th})$ are defined as:
for $r_{th} < r_T$:

$$\begin{aligned}
 P_{out}(r_{th}) = & \frac{1 + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\
 & \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right)\right) \left(\frac{1}{2} + \operatorname{erf}\left(\frac{\ln r_{th} - \mu_2}{\sigma_2 \sqrt{2}}\right)\right) + \\
 & + \frac{1 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\
 & \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)\right) \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_{th} - \mu_1}{\sigma_1 \sqrt{2}}\right)\right)
 \end{aligned} \tag{27}$$

and for $r_{th} \geq r_T$:

$$\begin{aligned}
 P_{out}(r_{th}) = & \frac{1 + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\
 & \cdot \left(\frac{1}{2} \operatorname{erf}\left(\frac{\ln r_{th} - \mu_1}{\sigma_1 \sqrt{2}}\right) - \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right)\right) + \\
 & + \frac{1 + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\
 & \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right)\right) \left(\frac{1}{2} + \operatorname{erf}\left(\frac{\ln r_{th} - \mu_2}{\sigma_2 \sqrt{2}}\right)\right) + \\
 & + \frac{1 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\
 & \cdot \left(\frac{1}{2} \operatorname{erf}\left(\frac{\ln r_{th} - \mu_2}{\sigma_2 \sqrt{2}}\right) - \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)\right) + \\
 & + \frac{1 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right)}{2 + \operatorname{erf}\left(\frac{\ln r_i - \mu_1}{\sigma_1 \sqrt{2}}\right) + \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)} \cdot \\
 & \cdot \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_i - \mu_2}{\sigma_2 \sqrt{2}}\right)\right) \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_{th} - \mu_1}{\sigma_1 \sqrt{2}}\right)\right)
 \end{aligned} \tag{28}$$

For the channels with identical parameters it is,
for $r_{th} < r_T$:

$$\begin{aligned}
 P_{out}(r_{th}) = & \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{\ln r_i - \mu}{\sigma \sqrt{2}}\right)\right) \cdot \\
 & \cdot \left(\frac{1}{2} + \operatorname{erf}\left(\frac{\ln r_{th} - \mu}{\sigma \sqrt{2}}\right)\right)
 \end{aligned} \tag{29}$$

and for $r_{th} \geq r_T$:

$$P_{out}(r_{th}) = \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\ln r_t - \mu}{\sigma \sqrt{2}} \right) \right) \cdot \left(\frac{1}{2} + \operatorname{erf} \left(\frac{\ln r_{th} - \mu}{\sigma \sqrt{2}} \right) \right) + \left(\frac{1}{2} \operatorname{erf} \left(\frac{\ln r_{th} - \mu_2}{\sigma_2 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erf} \left(\frac{\ln r_t - \mu_2}{\sigma_2 \sqrt{2}} \right) \right) \quad (30)$$

Finally, fade duration is obtain from the expression:

$$T(r_{th}) = \frac{P_{out}(r_{th})}{N(r_{th})} \quad (31)$$

4 Numerical Results

The joint probability density functions (PDF) of the SSC combiner output signal is shown in Figs. 2-3 for some values of r_T , σ , μ and β .

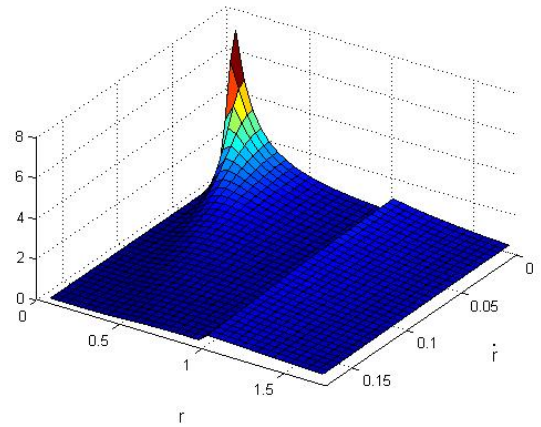


Fig. 3. The joint PDF of the SSC combiner output signal and its derivative $p_{rr}(r, \dot{r})$ for $r_T=1$, $\sigma=1.5$, $\mu=0.5$ and $\beta=0.15$

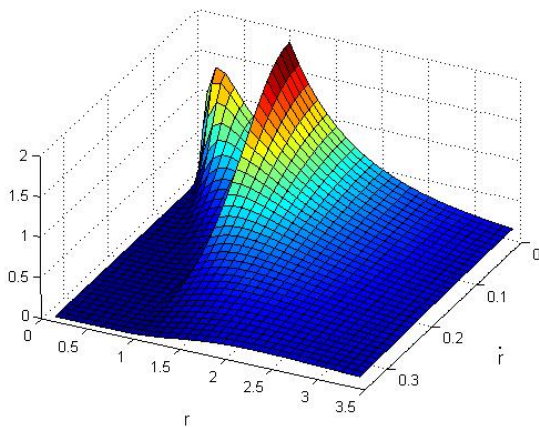


Fig. 2. The joint PDF of the SSC combiner output signal and its derivative $p_{rr}(r, \dot{r})$ for $r_T=1$, $\sigma=1$, $\mu=0.5$ and $\beta=0.1$

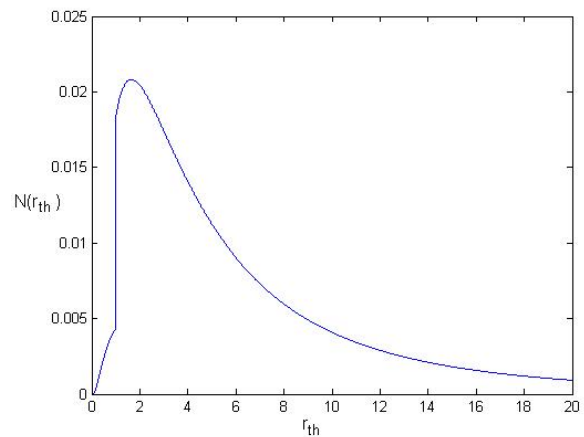


Fig. 4. Level crossing rate $N(r_{th})$ for $r_T=1$, $\sigma=1$, $\mu=0.5$ and $\beta=0.1$

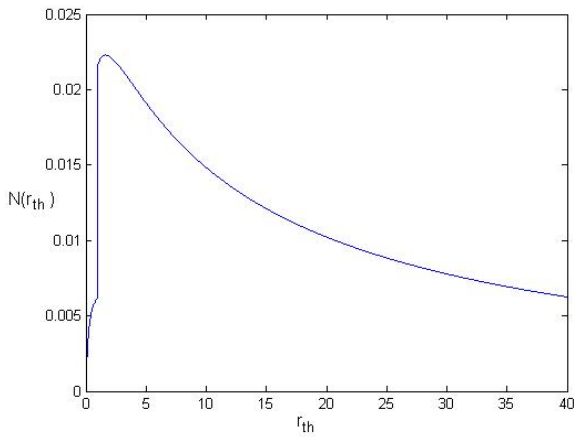


Fig. 5. Level crossing rate $N(r_{th})$ for $r_T = 1, \sigma = 2, \mu = 0.5$ and $\beta = 0.2$

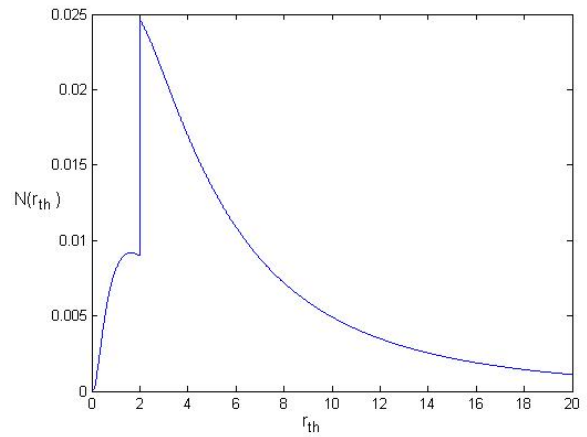


Fig. 7. Level crossing rate $N(r_{th})$ for $r_T = 2, \sigma = 1, \mu = 0.5$ and $\beta = 0.1$

The level crossing rate curves $N(r_{th})$ are given in Figs. 4-7. for different threshold values, for some parameters. We can note that in all cases the curves have the same shape, but numerical values of threshold determine the discontinuity moment appearance. These discontinuities can be observed very clear.

Fade duration curves $T(r_{th})$ are shown in Figs. 8. to 11. for different parameter values. If we compare these figures we can observe that all curves, $T(r_{th})$ dependant from threshold, have similar shape, but threshold numerical value influence to the discontinuity moment appearance. Larger rise of fade duration corresponds to smaller threshold value.

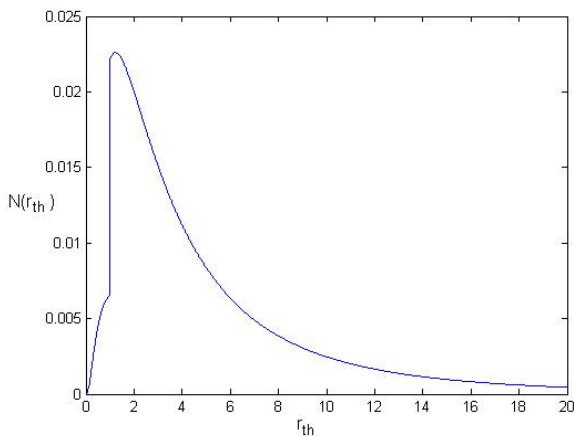


Fig. 6. Level crossing rate $N(r_{th})$ for $r_T = 1, \sigma = 1, \mu = 0.2$ and $\beta = 0.1$

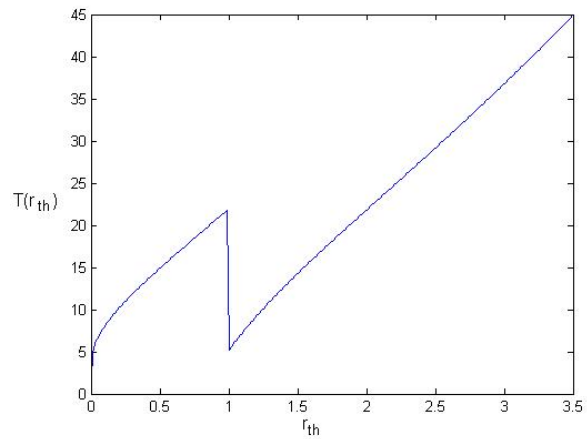


Fig. 8. Fade duration $T(r_{th})$ for $r_T = 1, \sigma = 1, \mu = 0.5, \beta = 0.1$

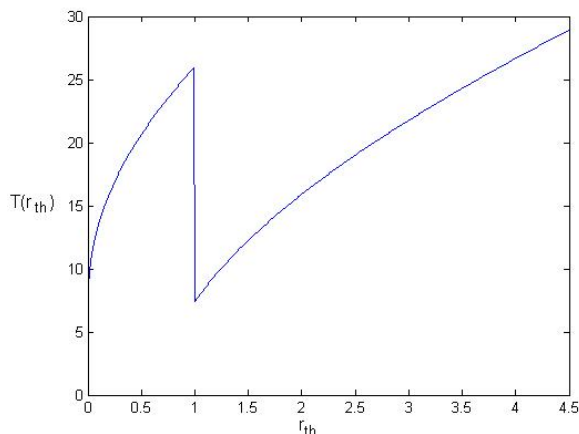


Fig. 9. Fade duration $T(r_{th})$ for
 $r_t = 1, \sigma = 2, \mu = 0.5, \beta = 0.2$

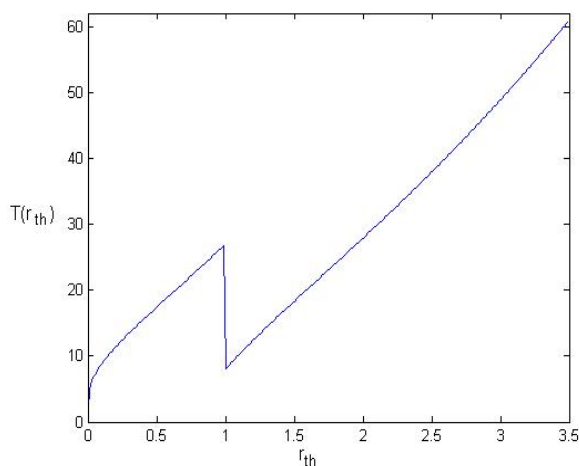


Fig. 10. Fade duration $T(r_{th})$ for
 $r_t = 1, \sigma = 1, \mu = 0.2, \beta = 0.1$

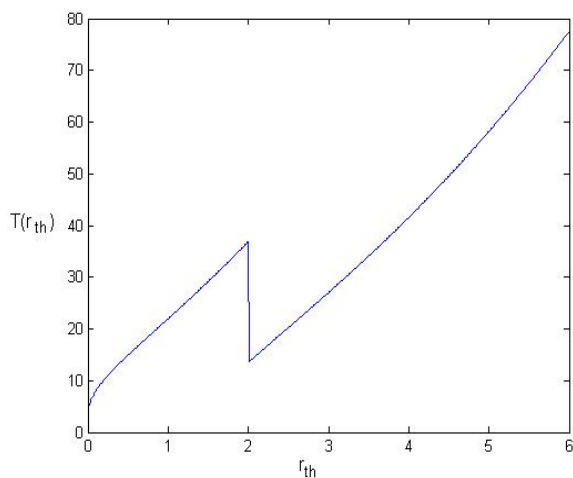


Fig. 11. Fade duration $T(r_{th})$ for
 $r_t = 2, \sigma = 1, \mu = 0.5, \beta = 0.1$

4 Conclusion

It is notable that level crossing rate, outage probability and average time of fade duration of the combiner output signal are very important system performances. In this paper the level crossing rate, outage probability and fade duration of the SSC combiner output signal are determined in the presence of log-normal fading. The results are shown graphically for different variance values, decision threshold values and parameters values.

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