Extended Kalman Filtering and Phase Detector Modeling for a Digital Phase-Locked Loop

Tsai-Sheng Kao[1], Sheng-Chih Chen[2], Yuan-Chang Chang[1], Sheng-Yun Hou[1], and Chang-Jung Juan[1]
[1]Department of Electronic Engineering,
Hwa-Hsia Institute of Technology.
No. 111 Gong Jhuan Road, Chung Ho,
Taipei County 23568, Taiwan, R.O.C.
TsaiShengKao@gmail.com
[2]Graduate Program of Digital Contens and Technologies,
ChengChi University.
No. 64, Sec. 2, ZhiNan Rd., Wenshan District,
Taipei City 11605, Taiwan, R.O.C.

Abstract: The realization of a digital phase-locked loop (DPLL) requires to choose a suitable phase detector and to design an appropriate loop filter; these tasks are commonly nontrivial in most applications. In this paper, the phase detector is examined, and a simple model is given to describe the characteristics of the timing function. The DPLL system is then formulated as a state estimation problem; then an extended Kalman filter (EKF) is applied to realize this DPLL for estimating the sampling phase. Therefore, the phase detector and loop filter are simply realized by the EKF. The proposed DPLL has a simple structure and low realization complexity. Computer simulations for a conventional DPLL system are given to compare with those for the proposed timing recovery system. Simulation results indicate that the proposed realization can estimate the input phase rapidly without causing a large jittering.

Key–Words: Digital phase-locked loop (DPLL), phase detector, state estimation, extended Kalman filter (EKF)

1 Introduction

Phase-locked loop (PLL) which constitutes a basic building block for many synchronizers like carrier recovery or timing recovery is essential in most digital communication systems[1, 2, 3]. Owing to the continued advancement in VLSI, all digital phase-locked loop (DPLL) has been under extensive investigation for several years [4, 5]. To realize a DPLL system, however, the selection of a phase detector [6, 7, 8] is crucial and the design of a loop filter is nontrivial.

A DPLL is, in general, a nonlinear system due to the nonlinear behavior of the phase detector. Unfortunately, few studies have been published on modeling a phase detector. Hence, the loop filter design often ignores the dynamics of the phase detector, causing the performance of the DPLL less reliable. The conventional loop filter design involves in selecting the order of the loop filter and determining its loop gains such that the performance of a DPLL satisfies fast phase acquisition and small phase jitter. However, the two characteristics of conventional DPLL systems with fixed loop gains are contradictory since fast phase acquisition requires wide loop bandwidth and small phase jitter requires narrow loop bandwidth [9, 10]. Moreover, the determination of the loop gains is difficult using the transfer function approach, especially when the order of the loop filter is high. A Kalman filter (KF) was realized as a loop filter to fulfill the above characteristics together with time-variant loop gains [11, 12, 13, 14], and these Kalman gains were shown to be equivalent to the time-variant loop gains of a DPLL. The performance of this DPLL bit synchronizer is significantly improved by using these time-variant loop gains in place of the fixed gains of a conventional DPLL.

Although Driessen [11] used a KF to realize the loop filter of a DPLL, this realization did not take the phase detector into account and the timing information was assumed to be known in advance. In this paper, we use an extended Kalman filter (EKF) to realize the loop filter as well as the phase detector of a DPLL, and the loop gains are easily obtained via the extended Kalman filtering techniques. The proposed system has a simple structure and low realization complexity.

The rest of the paper is organized as follows. In section II, the channel model is described and the function of a DPLL is briefly reviewed. The phase detector is also examined and modeled in section III. In section IV, we formulate the DPLL system as a state estimation problem and apply an EKF to realize this

DPLL. In section V, phase domain models of both a conventional DPLL and the proposed EKF-based DPLL are described. In section VI, simulations are shown to verify the proposed DPLL. Finally, conclusions are given in section VII.

2 Channel Model and DPLL System Overview

The baseband model of a synchronous data transmission system is shown in Fig. 1. The information sequence $\{a_k\}$ is independently chosen from the set of $\{1, -1\}$ with equal probability, and the data bit a_k is transmitted through a transmission channel at time instant $t_k = (k - \epsilon_k)T$, where T is the bit interval and ϵ_k is the input phase, normalized with respect to T. Owing to the channel imperfections, an equalizer is commonly included to eliminate the intersymbol interference. Thus, the overall impulse response $h(t)$ encompasses the transmission channel and an equalizer; the output of the equalizer can then be described as:

$$r(t) = \sum_{i=-\infty}^{\infty} a_i h(t - (i - \epsilon_i)T) + n(t) \tag{1}$$

where $n(t)$ is the filtered noise.

Assume ϵ_k is slowly time-variant, and write the sampling data of (1) at time instant $\hat{t}_k = (k - \hat{\epsilon}_{k|k-1})T$ as:

$$r_k = \sum_{i=-L}^{L} a_{k-i} h_i((\epsilon_k - \hat{\epsilon}_{k|k-1})T) + n_k \tag{2}$$

where $r_k = r((k - \hat{\epsilon}_{k|k-1})T)$, $n_k = n((k - \hat{\epsilon}_{k|k-1})T)$, and L is chosen such that the term, $h_i((\epsilon_k - \hat{\epsilon}_{k|k-1})T) = h(iT + (\epsilon_k - \hat{\epsilon}_{k|k-1})T)$, is negligible for $|i| > L$. In a compact form, (2) is rewritten as:

$$\begin{aligned} r_k &= \boldsymbol{a}_k^T \boldsymbol{h}((\epsilon_k - \hat{\epsilon}_{k|k-1})T) + n_k \\ &= y_k + n_k \end{aligned} \tag{3}$$

where the data vector $\boldsymbol{a}_k = [a_{k+L} \cdots a_k \cdots a_{k-L}]^T$, the channel parameter vector $\boldsymbol{h}((\epsilon_k - \hat{\epsilon}_{k|k-1})T) = [h_{-L}((\epsilon_k - \hat{\epsilon}_{k|k-1})T) \cdots h_0((\epsilon_k - \hat{\epsilon}_{k|k-1})T) \cdots h_L((\epsilon_k - \hat{\epsilon}_{k|k-1})T)]^T$, and the superscript T denotes the transpose. Thus, y_k denotes the noise-free data of r_k. The DPLL processes the measurement data r_k to adjust the sampling time $\hat{t}_k$ such that the timing error $e_k = t_k - \hat{t}_k = (\epsilon_k - \hat{\epsilon}_{k|k-1})T$ is approaching zero, where $\hat{\epsilon}_{k|k-1}$ is the predicted estimate of ϵ_k. Specifically, the information sequence $\{a_k\}$ is either known as a prior in the training mode or replaced by its estimate $\{\hat{a}_k\}$ in the tracking mode, and this class of DPLLs is classified as the data-aided (DA) structure.

3 Mueller and Müller's PD

A baseband model of a synchronous data transmission system is shown in Fig. 2 where the input data a_k may be 1 or -1 with equal probability, $h(t)$ denotes the impulse response of the cascade of the transmission channel and the equalizer, and $n(t)$ is the measurement noise. Denote the timing error as τ_k. Then the received sample r_k can be expressed by the following equation,

$$r_k = \boldsymbol{a}_k{}^T \boldsymbol{h}_k + n_k \tag{4}$$

where $\boldsymbol{a}_k = [a_{k+L}, \ldots, a_k, \ldots, a_{k-L}]^T$, $\boldsymbol{h}_k = [h(-LT_s - \tau_k), \ldots, h(0T_s - \tau_k), \ldots, h(LT_s - \tau_k)]^T$, $n_k = n(kT_s - \tau_k)$, the superscript T denotes the transpose, T_s is the period of input data, and L is normally chosen such that $h(iT_s)$ is negligible for $i > L$.

A data-aided baudrate PD uses the received sample r_k and the input data a_k for deriving the timing error. Mueller and Müller [7] presented a class of data-aided PDs by showing that a timing function of the timing error can be obtained as the linear combination of the channel impulse response, i.e.,

$$\rho(\tau_k) = \boldsymbol{u}^T \boldsymbol{h}_k \tag{5}$$

where the coefficient vector is $\boldsymbol{u} = [u_{-L}, \ldots, u_0, \ldots, u_L]^T$. Mueller and Müller also developed two particular classes of PD, type A and type B, with each class corresponding with a coefficient vector. These PDs have been useful in most DPLL applications.

3.1 LMS-realized Mueller and Müller's PD

This realization first uses the received sample r_k and the input data a_k to estimate the channel impulse response by the LMS algorithm; then the timing information is derived via (5). Therefore, the timing function is obtained as follows:

$$\hat{\boldsymbol{h}}_k = \hat{\boldsymbol{h}}_{k-1} + \mu(r_k - \boldsymbol{a}_k^T \hat{\boldsymbol{h}}_{k-1})\boldsymbol{a}_k \tag{6}$$

$$\hat{\rho}(\tau_k) = \boldsymbol{u}^t \hat{\boldsymbol{h}}_k \tag{7}$$

where μ is the step size.

3.2 PD Modeling

The timing function, $\rho(\tau_k)$, in general is a nonlinear function of the timing error τ_k. To design the loop filter and analyze the DPLL performance, $\rho(\tau_k)$

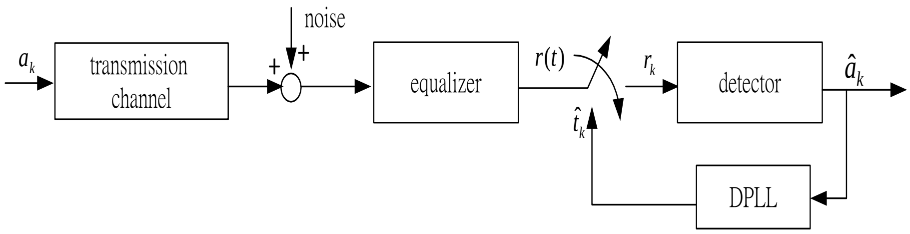

Figure 1: Model for the synchronous data transmission system

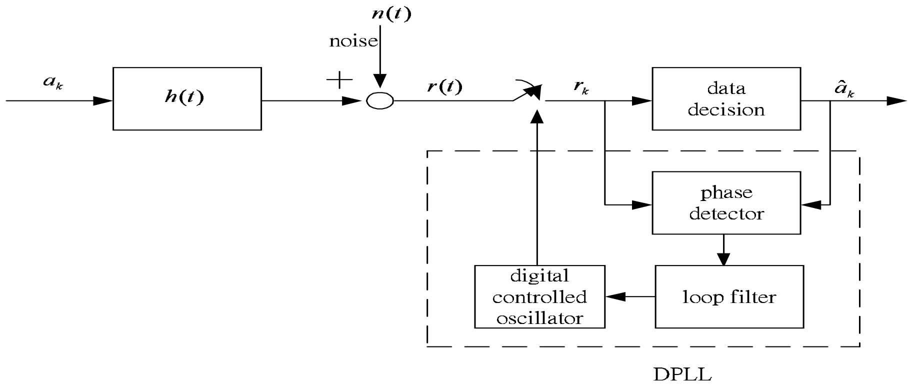

Figure 2: The baseband model of a synchronous data transmission system

is conventionally approximated by its linearization at $\tau_k = 0$, yielding

$$\rho(\tau_k) \simeq G_{pd}\tau_k \tag{8}$$

where $G_{pd} = \frac{\partial\rho(\tau_k)}{\partial\tau_k} \mid_{\tau_k=0}$ is called the PD gain. This PD model considers only the steady-state behavior but neglects the inherent dynamic property. It is known that the LMS adaptive algorithm is a dynamic system and can be analyzed by its averaging behavior. Hence, we apply this technique to develop a simple one-pole model for expressing the PD dynamics. The one-pole PD model, $P(z)$, is given below

$$P(z) = \frac{2\mu G_{pd}}{1-(1-2\mu)z^{-1}} \tag{9}$$

Note that the model pole depends only on the LMS step size and the steady-state response is identical to that of the conventional model.

The derivation of this model is explained as follows. For an FIR filter of $2L+1$ taps, as shown in [**?**], the LMS has $2L+1$ modes, with the time constant of each mode equal to $1-2\mu\lambda_i, i = 0, \cdots, 2L$, where λ_i is the $i-$th eigenvalue of the correlation matrix of input data. This correlation matrix, $E[\boldsymbol{a}_k\boldsymbol{a}_k^T]$, is exactly an identity matrix because the input a_k is random, uncorrelated, and is 1 or -1 with equal probability. Thus, all eigenvalues are equal to 1; all modes have the same time constants of $1-2\mu$. Consequently, the PD dynamics can be characterized by a single-pole model with its pole at $1-2\mu$. Since its steady-state response is G_{pd}, the one-pole model (13) is therefore derived.

3.3 Computer PD Simulation

The impulse response of the raised cosine channel is of the form, $h(t) = \frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}}\frac{\cos\frac{\beta\pi t}{T}}{1-(\frac{2\beta t}{T})^2}$, where $\beta \in [0,1]$ is the roll-off factor. The coefficient vector, as shown in [7], can be $u_1 = 1, u_{-1} = -1$, $u_i = 0$ for $\mid i \mid \neq 1$. Let the timing error τ_k be 0 for the first 500 samples, 0.1 for $500 < k \leq 2000$, and 0.05 for $2000 < k \leq 3500$. Choose $L = 4$ such that 9 channel parameters are estimated by the LMS algorithm. The initial parameter values are zeros and the first 500 iterations are used to eliminate the effect of initial settings. Fig. 3 depicts the measured timing function, $\hat{\rho}(\tau_k)$, the predicted output of the model, and the settings for $\beta = 0.5, \mu = 0.005$, and SNR $= 30$dB. Note that the settings are derived from the timing error τ_k and the channel response. The one-pole model is then $G_{pd}\frac{0.01}{1-0.99z^{-1}}$ where $G_{pd} = -1.5708$. This simulation demonstrates that the predicted outputs follow the measured data closely. Hence the one-pole model characterizes the LMS-realized PD dynamics accurately. The small discrepancy between the model output and the measured timing error arises because of the measurement noise and the unmodeled PD nonlinearity. Thus, as the SNR decreases, the ripple increases. Note that although it is not illustrated here, the roll-off parameter β also influences the ripple. The ripple increases as β increases because the PD gain, G_{pd}, will decrease.

4 Extended Kalman Filter for a DPLL System

In this section, we formulate the DPLL system as a state estimation problem, and an EKF is applied to realize this DPLL for estimating the sampling phase due to the nonlinear relationship between the input phase and the noise-free data y_k. To establish the model [11], define the state vector $\boldsymbol{x}_k = [\epsilon_k \ \dot{\epsilon}_k]^T$, where $\dot{\epsilon}_k$ is the timing offset; write the process equation as:

$$\boldsymbol{x}_{k+1} = \boldsymbol{\Phi}\boldsymbol{x}_k + \boldsymbol{v}_k \tag{10}$$

where the state transition matrix $\boldsymbol{\Phi} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\boldsymbol{v}_k = \begin{bmatrix} u_k \\ w_k \end{bmatrix}$ representing a zero mean phase jitter u_k and zero mean timing offset jitter w_k. Since the noise-free data y_k of the measurement equation (3) is a nonlinear function of the state vector $\boldsymbol{x}_k = [\epsilon_k \ \dot{\epsilon}_k]^T$, it is linearized about the predicted estimate $\hat{\boldsymbol{x}}_{k|k-1}$ of $\boldsymbol{x}_k$ as:

$$y_k = \boldsymbol{a}_k^T \boldsymbol{h}(0) + \boldsymbol{H}_k(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1}) \tag{11}$$

The vector $\boldsymbol{h}(0)$ contains the samples of the overall channel impulse response without a phase error, i.e., $\boldsymbol{h}(0) = [h_{-L}(0) \cdots h_0(0) \cdots h_L(0)]^T$. Notably, the transpose of the Jacobian matrix plays the role of the measurement matrix in the regular Kalman filtering and is given by

$$\begin{aligned} \boldsymbol{H}_k &= \begin{bmatrix} \frac{\partial y_k}{\partial \epsilon_k} & \frac{\partial y_k}{\partial \dot{\epsilon}_k} \end{bmatrix}_{\boldsymbol{x}_k = \hat{\boldsymbol{x}}_{k|k-1}} \\ &= \begin{bmatrix} \boldsymbol{a}_k^T \boldsymbol{h}'(0) & 0 \end{bmatrix}_{\epsilon_k = \hat{\epsilon}_{k|k-1}} \end{aligned} \tag{12}$$

where $\boldsymbol{h}'(0) = [h'_{-L}((\epsilon_k - \hat{\epsilon}_{k|k-1})T) \cdots h'_0((\epsilon_k - \hat{\epsilon}_{k|k-1})T) \cdots h'_L((\epsilon_k - \hat{\epsilon}_{k|k-1})T)]^T_{\epsilon_k = \hat{\epsilon}_{k|k-1}}$ and $h'_i((\epsilon_k - \hat{\epsilon}_{k|k-1})T) = \frac{\partial h_i((\epsilon_k - \hat{\epsilon}_{k|k-1})T)}{\partial \epsilon_k}$ for $i = -L, \ldots, 0, \ldots, L$. Thus, the EKF for a DPLL system can be described as:

$$\begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{\Phi}\boldsymbol{x}_k + \boldsymbol{v}_k \\ z_k = \boldsymbol{H}_k(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k-1}) + n_k \end{cases} \tag{13}$$

where the data $z_k = r_k - \boldsymbol{a}_k^T \boldsymbol{h}(0)$.

Furthermore, assume $\boldsymbol{v}_k$ and n_k are white Gaussian noise with zero mean and their corvariance matrices are given by

$$E[\boldsymbol{v}_k \boldsymbol{v}_i^T] = \begin{cases} \boldsymbol{Q}_k, & i = k \\ 0, & i \neq k \end{cases} \tag{14}$$

and

$$E[n_k n_i] = \begin{cases} R_k, & i = k \\ 0, & i \neq k \end{cases} \tag{15}$$

where $E[.]$ denotes the expectation operation. Denote $\boldsymbol{P}_k = E[(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k)(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k)^T]$ and $\boldsymbol{P}_{k+1|k} = E[(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k})(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1|k})^T]$. The extended Kalman filter algorithm for estimating the phase and timing offset of a DPLL for $k \geq 0$ is described in the following with the initial state vector $\hat{\boldsymbol{x}}_{0|-1}$ and the corvarinace matrix $\boldsymbol{P}_{0|-1}$ [15, 16]. The extended Kalman gain equation is

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T(\boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T + R_k)^{-1} \tag{16}$$

The estimation equation is

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k(r_k - \boldsymbol{a}_k^T \boldsymbol{h}(0)) \tag{17}$$

The error covariance equation is

$$\boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{k|k-1} \tag{18}$$

The prediction equations are

$$\hat{\boldsymbol{x}}_{k+1|k} = \boldsymbol{\Phi}\hat{\boldsymbol{x}}_k \tag{19}$$

and

$$\boldsymbol{P}_{k+1|k} = \boldsymbol{\Phi}\boldsymbol{P}_k\boldsymbol{\Phi}^T + \boldsymbol{Q}_k \tag{20}$$

The computational steps using equations (12) and (16-20) are depicted in Fig. 4.

5 Phase Domain Analyses of a DPLL

In this section, the proposed EKF approach is further shown to resemble a 2nd-order DPLL with time-variant loop gains. Before doing this, we briefly describe a conventional 2nd-order DPLL and then compare it with the proposed EKF-based DPLL.

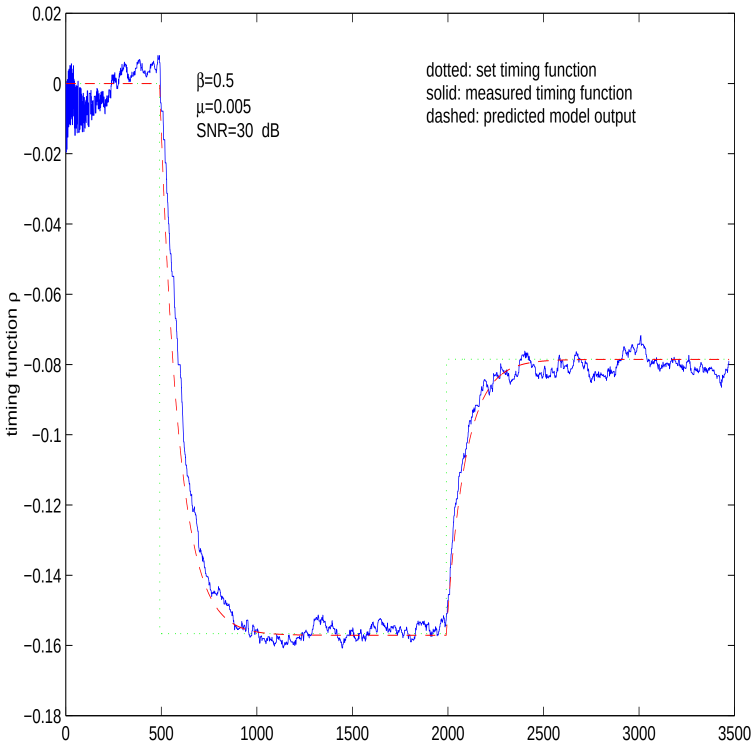

Figure 3: The measured timing function, the model output, and the settings for $\beta = 0.5$, $\mu = 0.005$, and SNR = 30dB

The phase domain model of a conventional 2nd-order DPLL is depicted in Fig. 5, which consisting of a phase detector, a loop filter and a numerical-controlled oscillator (NCO). Although the phase detector is, in general, a nonlinear device, it is often mathematically linearized as a constant gain. That is,

$$\tau_k = K_{pd}(\epsilon_k - \hat{\epsilon}_{k|k-1}) \tag{21}$$

where τ_k denotes the phase detector output and K_{pd} is called the phase detector gain. The conventional design approach has to choose a suitable phase detector and determine the fixed constants K_p and K_i of the loop filter [17] such that the estimated phase follows the input phase in some performance criteria. However, these tasks are usually difficult and time-consuming because the nonlinear dynamics of the phase detector has been ignored. In this study, an EKF is used to completely describe the DPLL with time-variant loop gains and these design parameters can be easily obtained by the Kalman filtering techniques.

The phase domain model of the EKF-based DPLL is derived as follows. By substituting (17) into (19), the state estimation equation is given by

$$\hat{\boldsymbol{x}}_{k+1|k} = \boldsymbol{\Phi}\hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{\Phi}\boldsymbol{K}_k(r_k - \boldsymbol{a}_k^T\boldsymbol{h}(0)) \tag{22}$$

We further define $\boldsymbol{K}_k = [\alpha_k \ \beta_k]^T$, and then rewrite (22) in the following. For $k = 0$,

$$\hat{\epsilon}_{1|0} = \hat{\epsilon}_{0|-1} + \hat{\dot{\epsilon}}_{0|-1} + \alpha_0 z_0 + \beta_0 z_0 \tag{23}$$

and

$$\hat{\dot{\epsilon}}_{1|0} = \hat{\dot{\epsilon}}_{0|-1} + \beta_0 z_0 \tag{24}$$

For $k = 1$ and using (24), write

$$\begin{aligned}\hat{\epsilon}_{2|1} &= \hat{\epsilon}_{1|0} + \hat{\dot{\epsilon}}_{1|0} + \alpha_1 z_1 + \beta_1 z_1 \\ &= \hat{\epsilon}_{1|0} + \hat{\dot{\epsilon}}_{0|-1} + \beta_0 z_0 + \alpha_1 z_1 + \beta_1 z_1 \\ &= \hat{\epsilon}_{1|0} + \alpha_1 z_1 + \Sigma_{i=0}^{1}\beta_i z_i \end{aligned} \tag{25}$$

Finally, the estimate of $\hat{\epsilon}_{k|k-1}$ is obtained recursively by

$$\hat{\epsilon}_{k|k-1} = \hat{\epsilon}_{k-1|k-2} + \alpha_{k-1} z_{k-1} + \Sigma_{i=0}^{k-1}\beta_i z_i \tag{26}$$

The phase estimate equation (26) is updated by inputting the phase information z_k through a filter with time-variant loop gains, α_k and β_k. An EKF that completely models the DPLL and governs the phase update equation (26) is depicted in Fig. 6.

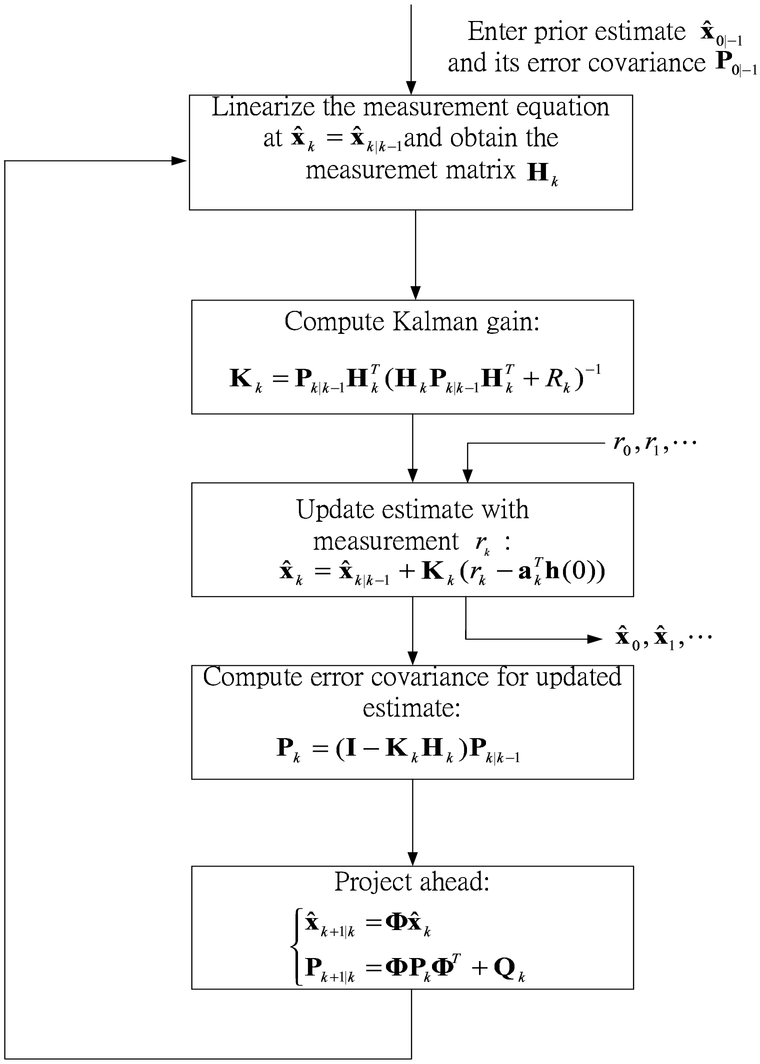

Figure 4: Extended Kalman filter algorithm for the DPLL

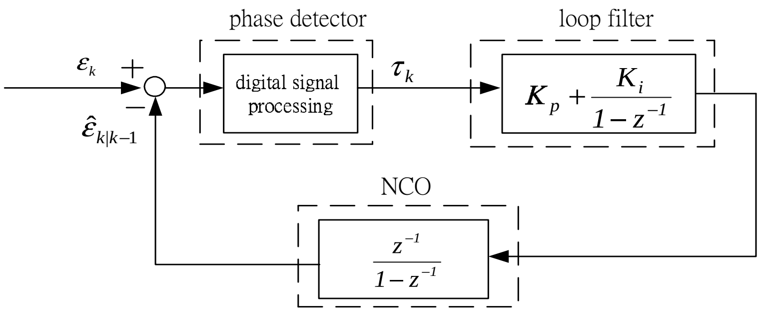

Figure 5: Phase domain model for a conventional 2nd-order DPLL

6 Computer Simulations

In this section, computer simulations for the conventional DPLL system are given to compare with those for the proposed timing recovery system. The overall channel impulse response is assumed to be a raise-cosine function with the roll-off factor $\alpha = 0$ and $L = 1$. The signal-to-noise ratio (SNR) is defined as $10 \log E[\frac{(r_k - n_k)^2}{n_k^2}]$; in this example, SNR is set

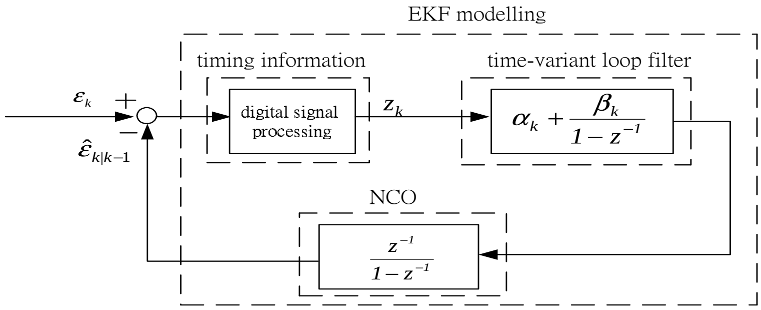

Figure 6: Phase domain model for an EKF-based DPLL

to be 20 dB. First, assume a constant phase delay between the transmission time instant and the sampling time instant, i.e., $\epsilon_k = 0.2$, and $\dot{\epsilon}_k = 0$ for $k > 0$. For the conventional DPLL design, Mueller and Müller's phase detector [7] is adopted, that is, $\tau_k = r_k a_{k-1} - r_{k-1} a_k$ and $K_{pd} = -2$. After several simulation trails, set $K_p = -2.75 * 10^{-2}$ and $K_i = -3.88 * 10^{-5}$ for the loop filter. The result is depicted in Fig. 7 (a), and the estimated phase converges for $k > 200$ with a slightly large jittering. To have a smooth response, set $K_p = -9.3 * 10^{-3}$ and $K_i = -4.93 * 10^{-5}$ and Fig. 7 (b) shows the simulation result. The estimated phase converges slowly for $k > 400$. Notably, both fast phase acquisition and small jitter cannot be simultaneously met for a conventional DPLL with fixed loop gains.

For the EKF-based DPLL, set the covariance matrices $\boldsymbol{Q}_k = 10^{-10}\boldsymbol{I}$ and $R_k = 0.01$, where $\boldsymbol{I}$ is the identity matrix. The initial settings $\hat{\boldsymbol{x}}_{0|-1}$ and $\boldsymbol{P}_{0|-1}$ are zeros and $0.1\boldsymbol{I}$, respectively. The estimated phase $\hat{\epsilon}_k$ and timing offset $\hat{\dot{\epsilon}}_k$ are respectively plotted in Fig. 8(a) and (b). Note that the proposed DPLL rapidly estimates the input phase and timing offset for $k \geq 60$ with a small jittering.

In the second simulation, the proposed DPLL is further demonstrated to track the input phase with a nonzero timing offset; in this case, $\epsilon_k = \epsilon_{k-1} + \dot{\epsilon}_{k-1}$, and $\dot{\epsilon}_k = 0.002$ for $k > 0$. Figure 9(a) and (b) plot the estimated phase $\hat{\epsilon}_k$ and timing offset $\hat{\dot{\epsilon}}_k$, and show that the estimated ones closely follow the true ones for $k \geq 70$.

7 Conclusions

In this article, the phase detector is examined and a simple model is given to describe the characteristics of the timing function. The EKF has been successfully applied for realizing a DPLL, which completely describes the phase detector and loop filter. Thus, the time-variant loop gains can be obtained by the extended Kalman filtering techniques. In addition, the proposed timing recovery system has been compared with a conventional DPLL system. Simulation results indicate that the proposed realization can estimate the input phase rapidly without causing a large jittering.

Acknowledgements: This work was supported in part by National Science Council, Taiwan, under Grant No. NSC 98-2221-E-146-004-.

References:

[1] D. R. Sulaiman, Design and analysis of a second order phase locked loops (PLLs), *Proceedings of the 5th WSEAS International Conference on Telecommunications and Informatics,* pp. 377-382, Istanbul, Turkey, May 27-29, 2006.

[2] M. A. Ibrahim and J. A. Hamadamin, Noise Analysis of Phase Locked Loops, *Proceedings of the 5th WSEAS International Conference on Telecommunications and Informatics,* pp. 479-484, Istanbul, Turkey, May 27-29, 2006.

[3] M. Stork, Fully digital fractional frequency synthesizer, *Proceedings of 12th WSEAS International Conference on SYSTEMS,* pp. 98-103, Heraklion, Greece, July 22-24, 2008

[4] W. C. Lindsey and C. M. Chie, A survey of digital phase-Locked loops, *Proceedings of the IEEE,* vol. 69, no. 4, Apr. 1981.

[5] J. W.M. Bergmans and H. Wong-Lam, A class of data-aided timing-recovery schemes," *IEEE Trans. Commun.,* vol. 43, no. 4, pp. 1819-1827, Mar. 1995.

[6] F. M. Gardner, A BPSK/QPSK timing error detector for sampled receiver," *IEEE Trans. Commun.,* vol. COM-34, no. 5, pp. 423-429, May 1986.

[7] K. H. Mueller and M. Müller, Timing recovery in digital synchronous data receivers, *IEEE Trans. Commun.,* vol. COM-24, no. 7, pp. 516-531, May 1976.

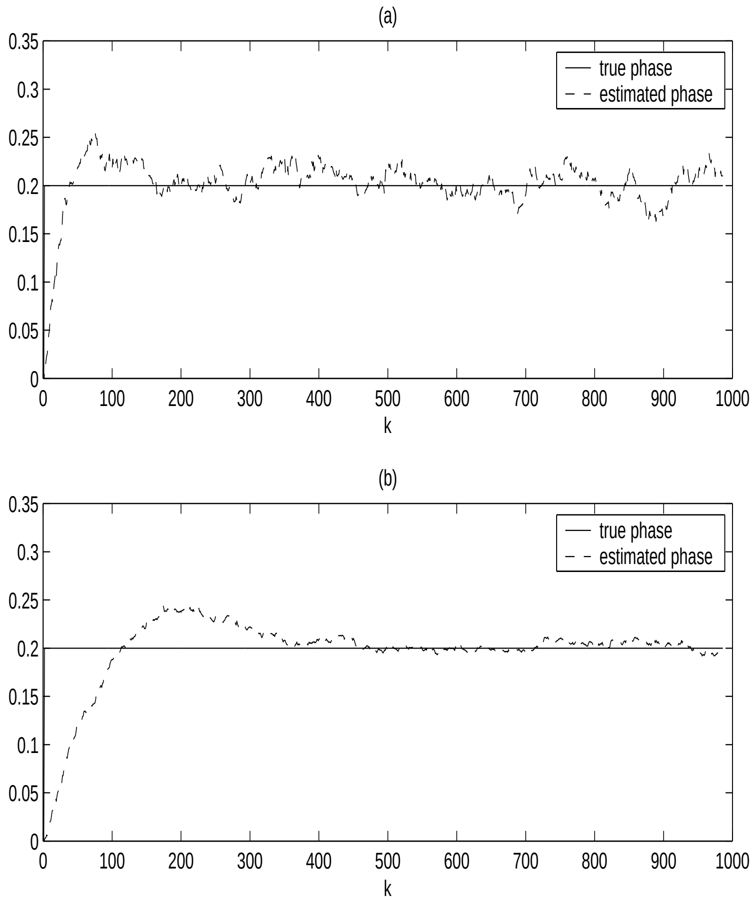

Figure 7: The responses of the conventional DPLL system: (a) fast acquisition, and (b) small jitter

[8] A. M. Gottlieb, P. M. Crespo, J. L. Dixon, and T. R. Hsing, The DSP implementation of a new timing recovery technique for high-speed digital data transmission," in *Proc. ICASSP*, New Mexico, USA, Apr. 1990, pp. 1679-1682.

[9] F. M. Gardner, *Phaselock Techniques,* Wiley-Interscience Publication, 1979.

[10] S. C. Gupta, Phase-locked loops," *Proceedings of the IEEE,* vol. 63, pp. 291-306, Feb. 1975.

[11] P. F. Driessen, DPLL bit synchronizer with rapid acquisition using adaptive Kalman filtering techniques, *IEEE Trans. Commun.*, vol. 42, no. 9, pp. 2673-2675, Sept. 1994.

[12] G. S. Christiansen, Modeling of a PRML timing loop as A Kalman filter, in *GlobeCom '94*, 1994, pp. 1157-1161.

[13] B. Chun, B. Kim, and Y. H. Lee, Adaptive carrier recovery using multi-order DPLL for mobile communication applications," *IEEE International Symposium on Circuits and Systems,* vol.2, pp. 1082-1085, 1995.

[14] A. Patapoutuan, On phase-locked loops and Kalman filters, *IEEE Trans. Commun.*, vol. 47, no. 5, pp. 670-672, May 1999.

[15] J. M. Mendel, *Lesson in Estimation Theory for Signal Processing, Communication, and Control,* Prentice-Hall, 1995.

[16] Y. Bar-Shalom and X. R. Li, *Estimation and Tracking: Principle, Techniques and Software*, Artech House, Inc., 1993

[17] W. Li and J. Meiners, Introduction to phase-locked loop system modeling, *Analog Applications Journal*, pp. 5-10, May 2000.

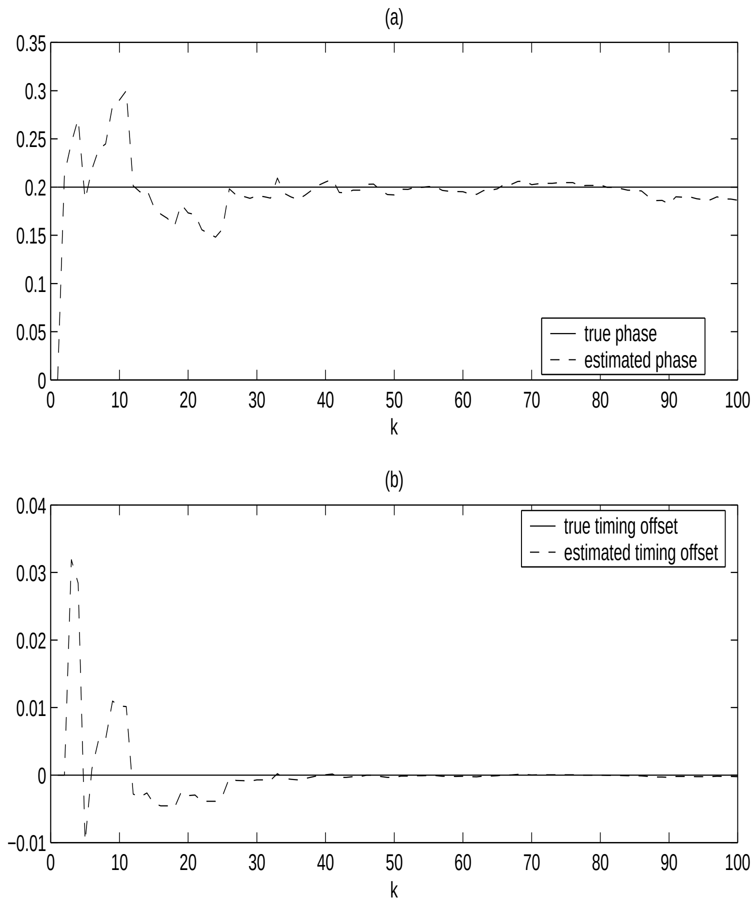

Figure 8: (a) The estimated phase $\hat{\epsilon}_k$, and (b) the estimated timing offset $\hat{\dot{\epsilon}}_k$

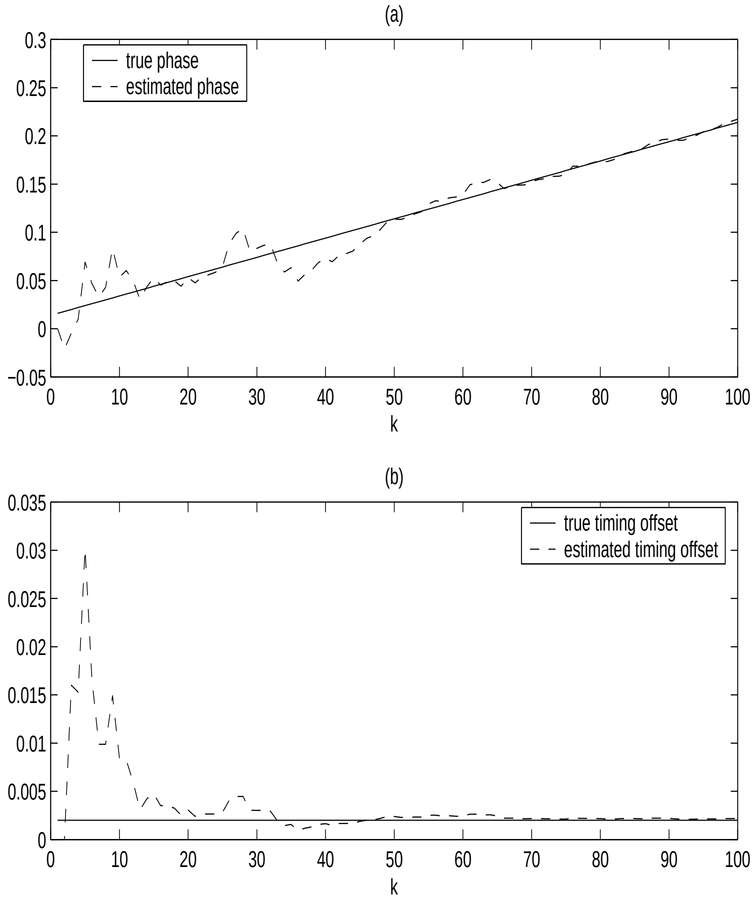

Figure 9: (a) The estimated phase $\hat{\epsilon}_k$, and (b) the estimated timing offset $\hat{\dot{\epsilon}}_k$