Functioning the Intelligent Programming to find Minimum Dispersion Wavelengths

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Abstract: A new method based on genetic concepts is suggested to analyze and determine the transmission (propagation and dispersion) characteristics for conventional fibers as well as photonic crystal fibers. The power and the advantage of this work, lie in the fact that it could be applied to any optical fibers type with any refractive index profile and the method computes its proper and accurate propagation constant which leads to investigate the desired dispersion and minimizes it to the least possible value.

Key-Words: optical fibers, PCF, dispersion and genetic algorithm.

1 Introduction

Development of fibers and devices for optical communications began in the early 1960s and continues strongly today. But the real change came in the 1980s. During this decade, an optical communication in public communication networks was developed from the status of a curiosity into being the dominant technology.

It is important to remember that optical communications are not like electronic communications. While it seems that light travels in a fiber much like electricity does in a wire, this is very misleading. Light is an electromagnetic wave and optical fiber is a waveguide. Anything has to do with transport of the signal even to simple things like coupling (joining) two fibers into one is very different from what happens in the electronic world. The two fields (electronics and optics) while closely related employ different principles in different ways [1].

Photonic Crystal Fibers (PCFs) offer increased flexibility in the dispersive and polarization properties of fiber. Index-guided PCFs, which have one hole missing in the center, guide light through total internal reflection. Air-core PCFs, or photonic bandgap fibers (PBGFs), guide light through the photonic band-gap effect, which is a completely different effect from the usual index guiding in optical fiber. Photonic crystals are periodic structures made of dielectric materials. Regions with different dielectric constants alternate periodically and the period is of the order of the wavelength of light [2].

2 Theory of Optical Fiber

There are three kinds of optical fiber cable: Multimode Step-Index, Multimode Graded-Index and Single-Mode Step-Index. The difference between them is in the way light travels along the fiber. A multimode stepindex fiber has a core of radius (a) and constant refractive index n_{co} . A cladding of slightly lower refractive index n_{cl} surrounds the core. The value of n_{cl} which is related with the value of wavelength (λ) in micrometer by Sellmeier equation, in both the step index and the graded index fibers of silica class materials is given by:

$$n_{cl}^{2} = 1 + \sum_{j=1}^{3} \frac{S_{j} \lambda^{2}}{\lambda^{2} - L_{j}^{2}}$$
(1)

where S_j are coefficients related to material oscillator strength while L_j are the corresponding oscillator wavelengths.

A multimode graded-index fiber has a core of radius (a). Unlike step-index fibers, the value of the refractive index of the core (n_{co}) varies according to the radial distance (r). Single mode step-index fiber has a core of radius (a) and a constant refractive index n_{co} . A cladding of slightly lower refractive index surrounds the core.

One often quoted and very useful measure of a fiber is usually called the "V". In some texts it is called the "normalized frequency" others and in just the "dimensionless fiber parameter". V summarizes all the of important characteristics of a fiber in a single number. It can be used directly to determine if the fiber will be single-mode or not at a particular wavelength and also to calculate the number of possible bound modes. In addition, it can be used to calculate the spot size, the cutoff wavelength and even chromatic dispersion. However, it is important to note that V incorporates the wavelength that we are using on the fiber and so to some extent it is a measure of a fiber within the context of a system rather than the fiber alone [1, 3].

$$V = \frac{2 \pi a}{\lambda} \sqrt{n_{co}^2 - n_{cl}^2}$$
(2)

where a is the core radius, λ wavelength, n_{co} and n_{cl} are the core and cladding refractive index respectively.

The normalized propagation constant (also called the normalized propagation parameter) is defined by $(k=2\pi/\lambda, \text{ it is known} as wave number)$:

$$B = \frac{(\beta / k)^{2} - n_{cl}^{2}}{n_{co}^{2} - n_{cl}^{2}}$$
(3)

This formula can also be used to define a normalized propagation constant for other optical waveguides. The propagation constant (β) is confined to range:

$$kn_{cl} \le \beta \le kn_{co} \tag{4}$$

The general Maxwell's equation which determines the light propagation within the optical fiber for both type of refractive index profile step index or graded index is given as follows [4]:

$$\frac{d^{2}E}{dR^{2}} + \frac{1}{R}\frac{dE}{dR} + \left[k^{2}n^{2}(r) - \beta^{2} - \frac{l^{2}}{r^{2}}\right]E = 0 \quad (5)$$

where: E: the electrical field, r: radial increment from the center of the fiber, n(r): refractive index with respect to r, *l*: mode number and R=r/a, normalized radial distance.

This is the scalar wave equation which is represented by a form of modified Bessel functions; and it has to be solved to determine the main propagation characteristics for the optical fiber.

Propagation characteristics are important since they lead to dispersion properties and they are specified for a fiber when refractive index measurements of both core and cladding are carried out correctly.

PCF consists of a thread of silica with a lattice of microscopic air capillaries running along the entire length of the fiber, see Fig.1. The optical properties of such fibers are quite unlike those of conventional fibers, for example, PCFs can exhibit unique dispersion characteristics, achieve high birefringence, provide single-mode operation for very short operating wavelengths, and remain singlemode for large scale fibers. In general, there are two different classes of PCFs, classified by the light guiding mechanism. There are several parameters characterizing the PCF: number of hole layers, N, lattice pitch (or hole-to-hole spacing) Λ , air hole diameter d, air hole shape, refractive index of the glass n_{co} , and type of lattice. The normalized air hole diameter, or air filling factor, d/Λ , describes the spatial period of the holes in the fiber section plane and characterizes the PCF cladding morphology.

The proposed definition of the V parameter for a PCF is given by [5]:

$$V_{PCF}(\lambda) = \frac{2\pi}{\lambda} \Lambda \sqrt{n_{co}^{2}(\lambda) - n_{eff}^{2}(\lambda)} \qquad (6)$$

where λ is the free-space wavelength and n_{co} is the refractive index associated with the fundamental mode while n_{eff} is the effective

refractive index corresponding to the correct solution.



Fig.1: Schematic Illustrations of the Fiber Cross Section.

It has been confirmed both numerically and experimentally that the single-mode regime is characterized by $V_{PCF} < \pi$. The V_{PCF} approaches a constant value, dependent on d/Λ , for increasing Λ/λ , and since the number of modes generally increases with the V parameter, this asymptotic behaviour of V_{PCF} is consistent with the endlessly single-mode property.

Dispersion is spreading out of light pulses as they travel along a fiber. It occurs because the speed of light through a fiber wavelength on its depends and the propagation mode. The differences in speed are slight, but like attenuation they are accumulated with distance. The four main types of dispersion arise from multimode transmission, the dependence of refractive index on wavelength, variation in waveguide properties with wavelength, and transmission of two different polarizations of light through single mode fiber [6].

Material (Chromatic) Dispersion: This is caused by the fact that the refractive index of the glass used varies (slightly) with the wavelength. Some wavelengths therefore have higher group velocities and so travel faster than others. Since every pulse consists of a range of wavelengths it will spread out to some degree during its travel.

All optical signals consist of a range of wavelengths. This range may be only a fraction of a nanometer wide but there is always a range involved. Typically optical pulses used in communications systems range from about 0.2 nm wide to 5 nm wide for systems using single-mode fiber (with lasers) [1]. The material dispersion can be expressed as in [5]:

$$D_{M} = -\frac{\lambda^{2}}{2\pi c} \frac{d^{2}\beta}{d\lambda^{2}}$$
(7)

Waveguide Dispersion: The shape (profile) of the fiber has a very significant effect on the group velocity. This is because the electric and magnetic fields that constitute the pulse of light extend outside of the core into the cladding. The amount of the fields overlapping between cores and cladding depends strongly on the wavelength. The longer wavelength extends the electromagnetic wave into the cladding.

The RI experienced by the wave is an average of the RI of core and cladding depending on the relative proportion of the wave that travels there. Thus since a greater proportion of the wave at shorter wavelengths is confined within the core, the shorter wavelengths, a higher RI than do longer wavelengths (Because the RI of the core is higher than that of the cladding). Therefore, shorter wavelengths tend to travel more slowly than longer ones to. Thus, signals are dispersed (because every signal consists of a range of wavelengths) [1]. The waveguide dispersion can be fitted by [5]:

$$D_{W} = -\frac{\lambda}{\pi c} \frac{d\beta}{d\lambda}$$
(8)

$$D_T = D_M + D_W \tag{9}$$

where: D_M : Material dispersion D_W : Waveguide dispersion D_T : Total dispersion

The advantageous effect here is that these two forms of dispersion have opposite signs, so they tend to counteract one another. Fig.2 shows the wavelength dependent dispersion characteristics of "standard" single-mode fiber. Notice that the two forms of dispersion cancel one another at a wavelength of 1310 nm. Thus if the signal is sent at 1310 nm, dispersion will be minimized.



Fig.2: Total Dispersion vs. the Waveguide Dispersion and Material Dispersion.

The general equation which describes the dispersion in optical fibers is as follows [7]:

$$D_{T} = \frac{-\lambda^{2}}{2\pi c} \left[\frac{d^{2}\beta(\lambda)}{d\lambda^{2}} \right]$$
(10)

where the $\left(\frac{d^2\beta(\lambda)}{d\lambda^2}\right)$ represent the second derivative of propagation constant equation (β) with respect to wavelength (λ).

Dispersion (from all causes) is often grouped under the name "Group Velocity Dispersion" (GVD). On standard single-mode fiber, usually there are two GVD regimes to be considered: the "normal dispersion regime" and the "anomalous dispersion regime" [1 and 5].

In multimode fibers. waveguide dispersion and material dispersion are basically separate properties. Multimode waveguide dispersion is generally small compared to material dispersion. Waveguide dispersion is usually neglected. However, in single mode fibers, material and waveguide dispersion are interrelated. The total dispersion present in single mode fibers may be minimized by trading material and waveguide properties depending on the wavelength of operation.

3 The Proposed Method

The proposed method is extremely based on genetic concepts, i.e. both genetic algorithms and genetic programming. The GAs will be applied to solve numerically the transmission characteristics firstly for the conventional fibers and then to the most recently manufactured PCFs.

The GAs are applied in this research in three steps; each one is confined to produce an optimum result to prepare it to the next step and they are as follows:

First step: Select the best form of the electrical field from a suggested set of equations which represents the most appropriate transformed power through the fiber cable. The original set number of these equations is saved and an appropriate equation would be elected arbitrarily. As it is known, the normalized field distribution of the step index fiber for the fundamental mode is almost similar to Gaussian distribution [1]. To determine the field distribution by the adopted GAs, a number of equations as mentioned earlier were derived by analytical methods and other equations were proposed directly. Hence, by applying the GAs it is possible to produce better equations that represent the electrical field distribution of the fiber whose parameters were already known. The fitness function is a reference function suggested by the programmer, i.e. for example, Gaussian in single mode fiber. From the set of equations a comparison is made by applying them to find out which one of them gives acceptable approximate values that suite the corresponding fiber under investigation. The GAs cannot deal with forms and equations directly; so, the first step is applying the process of encoding of these equations in general in a certain way such that it converts the above equations into the binary system to easily be delt with and to coincide programming with language. Every mathematical operation and mathematical function or expressions in the set of equations were given a certain code number as shown in Table 1. To adopt Elitism selection technique, two equations were elected and a crossover operation on them will produce two new equations. The process of generation of new equation may lead to the correct solution and it's considered as one among other advantages of the GAs.

Table	1:	Binary	Representation	of	the
Mather	natio	cal Opera	tion or Function.		

Hexadecimal Code		
F0F0		
F0F1		
F0F2		
F0F3		
F8F0		
F8F1		
F8F2		
F8F3		
FCF0		
FCF1		
FCF2		
FCF3		
FEF0		
FEF1		
FEF2		
FEF3		
FFF0		
FFF1		
FFF2		
FFF3		
FFF8		
FFF9		
FFFA		
FFFB		
FFFF		

Fig.3 and fig.4 shows the equations in binary form and how to get new equations.



Fig.3: Equations in Binary Form.



Fig.4: Two New Equations as a Result of Crossover Operation

Second step: Apply the boundary condition between any adjacent materials (core and cladding), which states that the field and its first derivative at the core-cladding boundary should be continuous. Although the fitness function, which will be used in this stage of work, depends on some conditions which were applicable at the boundary (i.e. in optical fiber: core-clad boundary), it also depends on the first derivative of the resultant field equation. Optical fiber boundary condition may be given by the following relation:

$$\frac{1}{E(1)} \left[\frac{dE}{dR} \right]_{R=1} = \frac{1}{K_{l}(w)} \left[\frac{d}{dR} K_{l}(wR) \right]_{R=1} (11)$$

where the K_l is the *l*th order of the second kind Bessel function (Hankel function) and *w* is the usual fiber parameter.

Third step: The final step of the work that related with the GAs determines the propagation characteristics that contain the normalized propagation constant, or in the other words the propagation constant. Propagation characteristics will be taken into account to determine the dispersion factor which is the main goal in every research concerning any kinds of fiber.

As a mater of fact the relation between the normalized propagation constant (B) and normalized frequency (V) should be computed; then the propagation constant (β) versus the wavelength (λ) will be determined for given fiber parameters. Other important

factor of these relations is to show the cut-off frequency and the mode of optical fibers operations (either it is single-mode or multimode). Determine the propagation characteristics by applying the GAs as a search technique. The normalized propagation constant (B) versus the normalized frequency (V) is the most important relation that must be determined carefully.

The processes which are performed by GAs to find the propagation characteristics are as follows:

a. The general search area in which the solution (propagation constant) is included depends on the following equation:

$$kn_{cl} \le \beta \le kn_{co} \tag{12}$$

The most important point in GAs is to determine the searching area accurately to avoid wasting time and getting rid of the solutions that are out scope of study.

- b.Dividing the above general range given in Equation (12) into n sections for simplicity and then giving each section area certain fitness value; see Fig.5a. The given fitness value for the available sections will be arbitrary chosen and then the fitness percentage value of each section area will computed by dividing the fitness numbers of each section area on the total sum of fitnesses of all section areas. Fig.5b is similar to Fig.5a but with normalized propagation constant, B.
- c. Among the set of equations saved in the matrix as the solution group, the first field equation is adopted primarily. The selected equation may be replaced by another next equation from the same group if the current process search not recognizes the desired propagation constant, and so on.





Fig.5: General rang of β which contains the True Solution.

d Select the desired section which has the biggest fitness value, refer to Fig.5a. Within the selected section assign arbitrary two numbers to be the main items (parents), to use them in crossover operation, see Fig.6. Notice the change in the number's values before and after crossover. During the crossover operation, there are new two numbers produced and added to the main range of " β " (if they are not included formerly in the range of " β ") and there is a chance for these numbers to be the correct solution. The four numbers, the parents and produced numbers (offspring); are tested to observe if they are included in the true solution or not, this test is achieved by substituting each one of them in the boundary limits given by Equation (12). In case no one of the available parents and offspring satisfy the mentioned criteria, the next two arbitrary numbers from the same section will be selected. Then the same crossover is applied and two new generations will appear and this will repeatedly carried out for a fixed number of trials.



Fig.6: Production of New Numbers by the Crossover Operation.

- e. The error value, which is primarily fixed, or even less leads to the correct solution, β , and the whole process should be stopped.
- f. In case of not finding the true solution, whole process (from 2 to 5) is repeated to

find another biggest fitness section to test whether the correct solution is included or not.

Note that what is meant by satisfying the general equation of the proposed fiber is to find the equation root which represents the equation solution. In this part the root of the equation is the propagation constant (β) which is regarded as the solution. It is worth noting here that GAs technique are capable to consider fibers with any kind of profiles, such as graded index as well as multi step index profile.

In the whole process of finding the solution using GAs, mutation operation has not been used since it affects the value of selected intermediate elements which may move them from inside the correct range to the outside; see Fig.7. On the contrary the crossover has a very significant effect to find out the solution and it can be easily controlled by determining the exchanging section areas inside the same range of values.



Fig.7: Mutation Effects on the Number.

4 Dispersion Computation

Minimum dispersion is the most important characteristics which must be controlled perfectly through fiber's geometry and parameters as well as operating wavelengths to reach the optimum fiber design.

The dispersion in the conventional fiber is determined depending on the set of values of the propagation constants versus wavelengths. After obtaining a relation between β values and λ values by using one of the interpolation techniques such as curve fitting tool (CFTOOL) in the MATLAB program, and hence the exact relation of the propagation constants with respect to wavelengths can be drawn. A special GUI will be shown when CFTOOL instruction is called. The inputs for the GUI are two

matrices one of them contains the β values, which represents the y-axis, and other one is the λ values that represents the x-axis. The inputs values were considered when "Data" choice is selected. The related curve will be drawn inside the GUI; see Fig.8. Note that fiber parameters are: step-index, a=4µm, Δ =0.9%, λ =0.55-2.2µm and l=1.

To obtain the exact equation that corresponds to the curve which is shown in the previous figure, click the "Fitting" button (on the main CFTOOL window), new sub window appears as shown in Fig.9. In this sub window the fitting type and its polynomials must be determined, and then clicking the "Apply" button the equation will be got with all its coefficients and will be in the "Results" field.



Fig.8: The Main Window of CFTOOL GUI.

Fit Editor			
New fit	Copy fit		
Fit Name:	fit 1		
Data set:	pcon vs. wl	Exclusion ru	ile: (none) 💌
Type of fit:	Polynomial	Center :	and scale X data
Polynomial	1		
Sth degree 6th degree 7th degree	polynomial polynomial nolynomial		
Fit option	s 🛛 🗖 Imm	ediate apply Canc	el Apply
Results			
Press "Aj	oply" to save th	e changes to the	fit.

Fig.9: Sub Window for the CFTOOL GUI.

If the interpolation degree fixed to (n), the form of the equation will be as follow:

$$\beta(\lambda) = \sum_{i=0}^{n} a_i \lambda^i \tag{12}$$

$$\beta(\lambda) = a_0 + a_1\lambda^1 + a_2\lambda^2 + a_3\lambda^3 + \dots + a_n\lambda^n$$
(13)

$$\dot{\beta}(\lambda) = a_1 + 2a_2\lambda + 3a_3\lambda^2 + \dots + na_n\lambda^{n-1}$$
(14)

$$\beta''(\lambda) = 2a_2 + 6a_3\lambda + \dots + n(n-1)a_n\lambda^{n-2}$$
(15)

The coefficients a_0 , a_1 , a_2 ,, an are instantaneous constants which differ when the main parameters of the optical fiber are altered. The Equation (13) is ready to be applied now in dispersion computation after taking its first and second derivatives, Equations (14, 15), and applying the following equations:

Material Dispersion:
$$D_M = -\frac{\lambda^2}{2\pi c} \frac{d^2 \beta}{d\lambda^2}$$
 (16)

Waveguide Dispersion:
$$D_W = -\frac{\lambda}{\pi c} \frac{d\beta}{d\lambda} (17)$$

Total Dispersion $D_T = D_M + D_W$ (18)

5 Results

To compute the dispersion for the step index fibers, it should at first compute the relation between the $\beta \& \lambda$. Fig.10 show the shape of this relation, for an optical fiber with the following parameters: core radius, a= $4\mu m$; wavelength range $0.75 \le \lambda \le 3\mu m$; and the relative difference $\Delta = 0.9\%$.



Fig.10: The Relation Between (β) and (λ) for Step Index Fiber.

The dispersion curve; as shown in Fig.11, of step index fiber crosses the wavelengths axis at single point, means that there is only one value to satisfy the zero dispersion wavelengths.



Fig.11: Dispersion versus Wavelength for Step Index Fiber.

The dispersion of the previous types of optical fiber can be easily found in the same way of the step index fiber. The curves of the dispersion for the triangular and parabolic fiber are almost similar in shape to the step index fiber except that there is a valuable shift as shown in Fig.12 and Fig.13, for the same parameter.



Fig.12: Dispersion vs. Wavelength for Triangular Index Fiber.

W-type profile fiber represents an optical fiber with more than one clad region to optimize the signal delivery, and each one has a different refractive index. The profile of the W-fiber in general is shown in Fig.14. The core has higher refractive index, the first clad layer has the lowest refractive index, and the second clad layer has a refractive index in between them.



Fig.13: Dispersion vs. Wavelength for Parabolic Index Fiber.

The relation between the B and V for the W-fiber is shown in Fig.15 whose fiber parameters are: $a=3\mu m$, $b=4\mu m$, $\Delta 1=0.5\%$, $\Delta 2=-1\%$. This relation is nearly similar in shape to the step index fiber, nevertheless the W-fiber has different dispersion characteristics.

The dispersion characteristics are determined in the same manner as discussed with previous types of fibers, but the results shown different shape of those characteristics as shown in Fig.16. Here, two crossing points $(0.9\mu m-1.2\mu m)$ and a range of many wavelengths $(0.85\mu m-1.25\mu m)$ that have near zero dispersion values are obtained.



Fig.14: W-Fiber Refractive Index Profile.

The n_{eff} is approximated as a function of the wavelength depending on the PCF cross-section; see Fig.17, as follows:



Fig.15: The Relation Between (B) and (V) for W-Fiber.



Fig.16: Dispersion vs. Wavelength for W-Fiber.

The n_{eff} computation is carried out by using Equation (19). The fiber parameters which are taken into account during the computation fixed as follows: Λ =1.5µm, d/Λ =0.75 and N=5.

The effective refractive index n_{eff} of the clad region was determined in the PCF to be used in dispersion equation which related to the PCF.

$$D = -\frac{\lambda}{C} \frac{d^2 n_{eff}}{d\lambda^2}$$
(10)



Fig.17: Simple Section of PCF Cross-section.

Fig.18, shows the relation between the silica refractive index values and the cladding region, in the PCF, effective refractive index (neff), for a certain range of wavelengths.



Fig.18: The Relation between Refractive Index of Silica and (neff).

For a five layers PCF with Λ =1.5 and d/Λ =0.75 the dispersion is shown in Fig.19. More than one intersection point with the wavelength axis can be observed. This kind of fiber has ultra flattened zero-dispersion wavelengths.

There are many parameters which affects the total dispersion of PCF. The most important that directly affect the total dispersion are the number of air-hole layers N; see Fig.20, and the Λ value that is combined with d in d/Λ which is a relative function including the hole diameter d and hole-to-hole distance Λ , Fig.21 shows these effects.



Fig.19: Dispersion for the PCF.



Fig.20: Effect of No. of Air-hole Layers on the Total Dispersion (N=1, 3, 5, 7 and 9) from Top to Bottom.



Fig.21: Effect of (Λ) on the Total Dispersion, Optimum Case at Smallest Value.

5 Conclusion

In this paper, the GAs has been used, for the first time, as active method to solve the scalar wave equation of the optical fiber to find out the propagation characteristics and the dispersion parameters for both conventional fiber and photonic crystal fiber.

Based on the proposed method, optimal electrical field equation has been obtained from the set of equations which are prepared primarily to represent the field correctly. The propagation characteristics for the step index single mode circular cross-section fiber have been determined for the conventional fiber. Accordingly the zero dispersion wavelengths is determined accurately. The proposed method has a very small error compared with the analytical solution of the step index single mode fiber as well as with the traditional methods (for example finite difference method and power series method). The GAs can be used to find out the propagation characteristics for any fiber profile, for example the graded index and multi step fibers. In addition to the conventional fiber, the PCF the modern manufactured fiber has been studied using the same technique (GAs) find out the main transmission to characteristics (propagation and dispersion). The dispersion computation for the PCF is carried out depending on the n_{eff} equation which is, in this work, approximated based on the cable cross-section and contains the physical parameters (Λ , d and N) which are affect the dispersion characteristics in this type of fibers.

The proposed method has the following features:

- 1. Avoiding problems of falling into local minimum that is usually occurred in the case of traditional methods.
- 2. Reducing the complexity due to other methods that may be occurred in the final boundary condition.
- 3. The GAs can be simply modified and can easily consider any refractive index profile or any electrical field representation.
- 4. The GAs are applicable to any type of the optical fiber, that constructed from any material and any profile or different technology such as PCF and PBGF.

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