Information-Theoretic Comparison of Channel Capacity for FDMA and DS-CDMA in a Rayleigh Fading Environment

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Abstract: - In this paper, a comparative estimate of the channel capacity assigned to each user for frequency-division multiple-access (FDMA) and direct-sequence code-division multiple-access (DS-CDMA) schemes operating in a Rayleigh fading channel is presented. Then, the achievable channel capacity per user (in the Shannon sense) is estimated in an average sense considering the inherent diversity potential that both schemes provide in a Rayleigh fading channel. The non-equal user rate for the DS-CDMA case is studied, considering the ratio of two neighboring user channel capacities. Hence, it is shown that, in contrast to the equal user rate, for non-equal user rate case in DS-CDMA and under normalized conditions, the average channel capacity per user, in FDMA, is higher than that of DS-CDMA and this result is quantitatively evaluated in terms of a defined average capacity gain factor.

Key-Words:- Channel capacity, code-division multiple-access, frequency-division multiple-access, Rayleigh fading.

1 Introduction

In information theory, ([1-3]), and assuming equal power users and operation in an ideal non-fading additive white Gaussian noise (AWGN) channel, the Shannon capacity region provided by a system, that utilizes FDMA or DS-CDMA schemes, is the same, since it does not matter whether the spectrum is divided into frequencies, or codes. However, in this case, DS-CDMA is in the information-theoretic sense, where distinct codes are used for different user and the receiver decodes them one by one.

For a time-variant channel, as it is the case in mobile radio, its capacity, i.e., the maximum rate, at which data can be transmitted with arbitrarily small bit error rate (BER), can be obtained by finding the best distribution of the transmitted signal power as a function of the instantaneous signal-to-noise power ratio (SNR) and then averaging over the SNR distribution, [4]. However, its capacity can also be estimated in an average sense indicating the average best rate for the least possible errors on the average. These average capacity formulas would indeed provide the true channel capacity, with channel state information (CSI) available to the receiver only, [5,6].

In a previous work, [7], we compared FDMA and DS-CDMA in terms of channel capacity assigned to each user, following the method described in [8]. However, we restricted our attention to the equal user rate case (i.e. for $C_1=C_2=C_3=\ldots$) for the DS-CDMA scheme. From a multi-user information theory, this is only a special case of the capacity region, which is achieved under the respective schemes, [9]. In contrast, here, we investigate the channel capacity assigned to each user, in a DS-CDMA scheme, for the non-equal user rate case. Then, assuming, in DS-CDMA case, a relationship between the neighboring user channel capacities, we derive a general expression useful to compare, under normalized conditions, FDMA and DS-CDMA schemes operating in a Rayleigh fading channel, in terms of channel capacity per user.

The paper is organized as follows. Section 2 and 3 describes the operation of FDMA and DS-CDMA schemes in an ideal non-fading AWGN and a Rayleigh fading channel, respectively. The numerical results and their comparison are given in Section 4. Final conclusion are outlined in the last section.

2 Operation in a non-fading awgn environment

It is well kwon, that channel capacity expressed by Shannon's formula establishes an upper limit for reliable information transmission over a bandlimited AWGN channel. Then, for the AWGN channel case, the capacity region achievable by FDMA is given by the Shannon-Hartley theorem, [10],

$$R_{i,F} \le C_{i,F} = B_i \log_2\left(1 + \frac{P_i}{N_0 B_i}\right) \quad (1)$$

where B_i is the signal bandwidth assigned to i-th user, i=[1,…,K], P_i being the received signal power, and N_0 is the power spectral density of the bandlimited AWGN channel.

In the followed analysis, for both FDMA and DS-CDMA schemes, we consider no dynamic resource among the K users of the system but the equal-power and equal-bandwidth cases i.e., $B_i=B_s$ and $P_i=P_s$, i=[1,…,K], and that all channel inputs are subjected to equal average power constraints. Then, the capacity region for FDMA is defined by, [1],

$$R_{i,F} \le C_{i,F} = B_s \log_2\left(1 + \frac{P_s}{N_0 B_s}\right) = B_s \log_2\left(1 + \Gamma_i\right) \quad (2)$$

where $R_{i,F}$ is the user' s data rate, $\Gamma_i=\Gamma=(P_s/N_0B_s)$ is the received SNR over signal bandwidth B_s by each user i, i=[1,…,K]. The suffices F and C refer to FDMA and DS-CDMA schemes respectively.

Respectively, the capacity region achieved, in theory, by DS-CDMA is:

$$R_{i,C} \le C_{i,C} = B_C \log_2\left(1 + \frac{P_s}{N_0 B_C}\right) = B_C \log_2\left(1 + \Gamma_{ss}\right) \quad (3)$$

where $\Gamma_{ss}=(P_s/N_0B_C)=(P_s/N_0G_pB_s)=(\Gamma_i/G_p)=(\Gamma/G_p)$ is the spread received SNR reflecting the lowered signal power spectral density due to spreading the transmitted signal power P_s by some factor, known as processing gain G_p, defined as:

$$G_p = \frac{B_C}{B_s} \quad (4)$$

for a full coded DS-CDMA system with bandwidth B_C. Following eq.(3), the sum rate, (achievable rate region, [1]), $R_{t,C}$ in DS-CDMA operating in an ideal non-fading AWGN channel, will be equal to:

$$R_{t,C} = \sum_{i=1}^{K} R_{i,C} \le \sum_{i=1}^{K} C_{i,C} = C_C = B_C \sum_{i=1}^{K} \log_2(1+\Gamma_{ss}) \quad (5)$$

where $C_C=\sum_{i=1}^{K} C_{i,C}$, i=[1,..,K], is the total channel capacity available to all K users. Moreover, based on the fact that, in practice, Γ_{ss} is well below unity (in linear scale), eq.(5) can be approximated by:

$$C_C \cong B_C \cdot \log_2(1 + K \cdot \Gamma_{ss}) \quad (6)$$

In the case considered, it is assumed that both FDMA and DS-CDMA schemes accommodate the same and fixed number K of users that occupy the radio channel simultaneously. However, in common models for communication systems, a user access the channel randomly, as it gets a message to be transmitted, but the random access of users is a fundamental issue which is not yet satisfactorily treated in terms of information-theoretic aspects. In addition, in eq.(3), it is considered the cooperative model for DS-CDMA scheme, according to which joint demodulation and detection of all users' signals can be assumed. This implies knowledge of all spreading codes by the receiver, and, thus multiple access interference can be assumed negligible and ignored.

As already mentioned, in information theory and in an ideal non-fading AWGN channel, the required channel capacity per user of the utilized physical (in FDMA) or "logical" (in DS-CDMA) channel is eventually the same, i.e.:

$$C_{i,F}=C_{i,C} \quad (7)$$

It must be noticed that, in the DS-CDMA case, we use the term "logical channel capacity" to indicate the exact amount of information of the de-spread signal of a particular user, which, after spreading, is transmitted with no errors over the entire physical channel.

3 Operation in a rayleigh fading environment

In the following, the Rayleigh fading channel is modeled as a slowly fading, time-invariant and discrete multipath channel, [11]. The capacity of a single-user flat fading channel with perfect CSI known only to the receiver has been extensively studied, [12-15]. Since a pure FDMA scheme reduces multiple-access channel to K single-user Rayleigh fading channels, the average channel capacity has information-theoretic meaning for the FDMA case. In this case, the power of the transmitted signal is distributed along a channel bandwidth B_s that, in practice, is never greater than the coherence bandwidth B_{coh} of the Rayleigh fading channel, meaning that no physical diversity potential is offered, [7]. Hence, assuming that each user of the FDMA system experiences the same Rayleigh fading conditions, the average channel capacity $\langle C_i \rangle_{F,R}$, i=[1,..,K], available to each user, indicating the average best rate for error-free transmission, is given in [12] as:

$$\langle C_i \rangle_{F,R} = -B_s \log_2 e e^{\frac{1}{\Gamma}} \mathrm{Ei}\left[-\frac{1}{\Gamma}\right] \quad (8)$$

where ⟨.⟩ indicates average value, $\Gamma=\langle \gamma \rangle$ is the average received SNR over signal bandwidth B_s by each user i, i=[1,..,K], Ei[–x] is the exponential integral, [16], and the new suffice R used refers to the Rayleigh fading channel.

It is important to notice, that in the followed analysis, we assume that each user's data rate $R_{i,F,R}$, in a Rayleigh fading channel, is restricted by the average channel capacity $\langle C_i \rangle_{F,R}$ available from the physical channel of bandwidth B_s so that

$$R_{i,F,R} \le \langle C_i \rangle_{F,R} \quad (9)$$

At this point it must noted that since the communication channel is time-varying, $\langle C_i \rangle_{F,R}$ is the average channel capacity per user, achieved by a FDMA system, and not the maximum channel capacity achieved over such a channel, [6].

In [7], for the DS-CDMA scheme operating in a Rayleigh fading channel, we examined the case where all users' information rates are the same. In particular, we assumed that all users share equally the capacity $\langle C_C \rangle_{C,R}$ of the entire physical channel of bandwidth B_C. However, this is only a special case of the capacity region of all {R_i} K-tuples, which is achieved, [9,15]. Then, in this work, we consider the case where the user average channel capacities are as follows:

$$\langle C_1 \rangle_{C,R}, \langle C_2 \rangle_{C,R} = n\langle C_1 \rangle_{C,R}, \ldots, \langle C_K \rangle_{C,R} = n^{K-1}\langle C_1 \rangle_{C,R} \quad (10)$$

where n>1. Thus, $\langle C_1 \rangle_{C,R}$ is the minimum average user channel capacity and the ratio of two neighboring user channel capacities is n. Clearly, the sum of average user channel capacities equals to the average capacity $\langle C_C \rangle_{C,R}$ available from the entire physical bandwidth B_C in a Rayleigh fading channel. Thus, we can write that:

$$\langle C_1 \rangle_{C,R} + \langle C_2 \rangle_{C,R} + \ldots + \langle C_K \rangle_{C,R} = \langle C_C \rangle_{C,R} \quad (11)$$

Combining eqs (10) and (11) and using the sum of geometric progression to K terms, [16], we obtain:

$$\langle C_1 \rangle_{C,R} \left(\frac{1-n^K}{1-n} \right) = \langle C_C \rangle_{C,R} \quad (12)$$

or equivalently,

$$\langle C_i \rangle_{C,R} = n^{i-1}\langle C_1 \rangle_{C,R} = n^{i-1}\left(\frac{1-n}{1-n^K} \right)\langle C_C \rangle_{C,R} \quad (13)$$

for i=[1,..,K].

Considering a DS-CDMA system with bandwidth B_C greater than the coherence bandwidth B_{coh} of the Rayleigh fading channel, the wide physical channel will appear to be frequency-selective to the transmitted signals and the maximum number L of uncorrelated resolvable paths ("inherent diversity branches") will be given by, [11]:

$$L = [B_C T_m] + 1 = \left[\frac{B_C}{B_{coh}} \right] + 1 \quad (14)$$

where T_m is the maximum delay spread or total multipath spread of the fading channel (assumed known or measurable) and [.] returns the largest integer less than, or equal to, its argument. Although the number of resolvable paths L may be a random number, it is bounded by eq.(12). Thus, eq.(12) indicates the maximum number of branches of a physical frequency diversity scheme that consists of the transmitted signal frequency elements which exceed B_{coh} and fade independently to each other.

In general, the multipath-intensity profile (MIP) in a Rayleigh fading channel is exponential, but, here, MIP is assumed finite, discrete and constant, so that the "resolvable" path model can be considered to have equal path strengths on the average. In a conventional maximal-ratio combining (MRC) RAKE receiver, the output's decision variable is identical to the decision variable which corresponds to the output of a L-branch space diversity maximal-ratio combining technique, with L branches [17]. It is well-known that MRC shows the best performance in a fading environment, [17]. Consequently, the maximal-ratio coherently combining reception of DS-CDMA spread signals, achieved by the considered RAKE receiver, is equivalent to a L-branch space diversity maximal-ratio combining technique. Therefore, the probability density function (p.d.f.) of the combined instantaneous SNR $\gamma_{m,t}$ of the spread signal over the bandwidth B_C, with no correlation among the L branches, will follow the Erlang distribution, [17], i.e.,

$$p^L(\gamma_{m,t}) = \frac{1}{(L-1)!} \frac{(\gamma_{m,t})^{L-1}}{(\Gamma_{m,t})^L} \exp\left(-\frac{\gamma_{m,t}}{\Gamma_{m,t}} \right) \quad (15)$$

where $\Gamma_{m,t} = \langle\gamma_{m,t}\rangle$ is the totally received average spread SNR value in the m-th, m=[1,…,L], diversity branch from all K users, L is obtained from eq.(14) and the subscripts 't' and 'm' refer to the totally received average spread SNR from all users and to the m-th diversity branch, respectively. Thus, an expression for the average capacity $\langle C_C\rangle_{C,R}$ of the channel B_C over all spread SNR levels, can be written as:

$$\langle C_C\rangle_{C,R} = B_C\int_0^\infty \log_2(1+\gamma_{m,t})\frac{(\gamma_{m,t})^{L-1}}{(L-1)!(\Gamma_{m,t})^L}\exp\left(-\frac{\gamma_{m,t}}{\Gamma_{m,t}}\right)d(\gamma_{m,t}) \quad (16)$$

Clearly, this capacity estimation, is based on the equivalence described above and indicates the average channel capacity that appears at the receiver output in the form of the average best recovered data rate from all users. However, the performance of the coherent maximal-ratio RAKE receiver depends on the number of the employed taps and the fading channel estimation. If the number of taps is less than the resolvable paths' number, the receiver performance will substantially be degraded because the power of the remaining "branches" will appear at the receiver output as self-noise power. In this work, we consider the optimum operation of the coherent maximal-ratio RAKE receiver where the number of taps employed is equal to the number L of resolvable paths as given by eq.(14).

If we consider that each user's signal appears at the receiver input with the same average spread signal-to-noise power ratio, the received average spread SNR from all users will be equal to $K\cdot\Gamma_{ss}$, and $\Gamma_{m,t}$ can be written as:

$$\Gamma_{m,t} = K\Gamma_{ss} \quad (17)$$

Then, combining eqs (16) and (12), the *i*-th user's average channel capacity of a DS-CDMA system operating in a Rayleigh fading channel can be expressed as following:

$$\langle C_i\rangle_{C,R} = n^{i-1}\left(\frac{1-n}{1-n^K}\right)\frac{G_p\cdot B_s}{(L-1)!\cdot(K\Gamma_{ss})^L}\cdot \int_0^\infty \log_2(1+\gamma)\gamma^{L-1}\exp\left(-\frac{\gamma}{K\Gamma_{ss}}\right)d\gamma \quad (18)$$

where the notation of the combined instantaneous SNR $\gamma_{m,t}$, used in eq.(16), has been changed to γ.

4 Comparison bewteen fdma and ds-cdma in a rayleigh fading environment

In this section, we proceed to the comparison of the average capacity per user offered by the considered DS-CDMA and FDMA schemes, under normalized conditions. i.e., for $B_C=G_pB_s=B_F=KB_s$ where B_F is the totally allocated bandwidth in the FDMA system. Also it is assumed that the delay spread T_m of the Rayleigh fading channel equals to 3 μsec.

As already stated, in information theory, when these schemes operate in an ideal non-fading AWGN channel, under the assumptions set, they exhibit exactly the same channel capacity per user. However, in a Rayleigh fading channel, the situation is different. In order to account for the effect of a different user's rates in DS-CDMA scheme, we introduce the term "average capacity gain" per user, $\langle C_G\rangle_i$, *i*=[1,..,K], defined as the ratio of $\langle C_i\rangle_{F,R}$ to $\langle C_i\rangle_{C,R}$, as given by eqs (8) and (16), respectively. Hence, $\langle C_G\rangle_i$ can be expressed as:

$$\langle C_G\rangle_i = \frac{\langle C_i\rangle_{F,R}}{\langle C_i\rangle_{C,R}} = \quad (19)$$

$$= \frac{-\log_2 e\, e^{\frac{1}{\Gamma}}\mathrm{Ei}\left[-\frac{1}{\Gamma}\right]}{n^{i-1}\left(\frac{1-n}{1-n^K}\right)\frac{G_pB_s}{(L-1)!(K\Gamma_{ss})^L}\int_0^\infty \log_2(1+\gamma)\gamma^{L-1}\exp\left(-\frac{\gamma}{K\Gamma_{ss}}\right)d\gamma}$$

Eq.(19) serves as a general expression for estimating the average capacity gain per user between the FDMA and DS-CDMA schemes accommodating the same number K of simultaneous transmitting users when Rayleigh fading is present. The integral in the numerator of eq.(19) is calculated numerically as it can not be expressed in closed form. Then, average capacity gain $\langle C_G\rangle_i$ is plotted as a function of the average received SNR Γ (in dB) for a Rayleigh fading channel in Fig.1 for n=2, *i*=2, $K=G_p$=41, B_C=1.25MHz, and B_s=30KHz. It is easy to see that, for the non-equal user rate case, in contrast to the equal user rate case, [7], FDMA scheme provides greater average channel capacity per user than DS-CDMA under normalized conditions, when operating in a Rayleigh fading channel.

the i-th user, according to eq.(19), for n=2, B_C=1.25MHz, B_s=30KHz, K=G_p=41 and Γ=20dB.

Finally, in Fig. 3, average capacity gain $\langle C_G \rangle_i$ is plotted against the number L of inherent diversity branches, given by eq.(14), for n=2, i=3, B_s=30KHz and Γ=20dB.

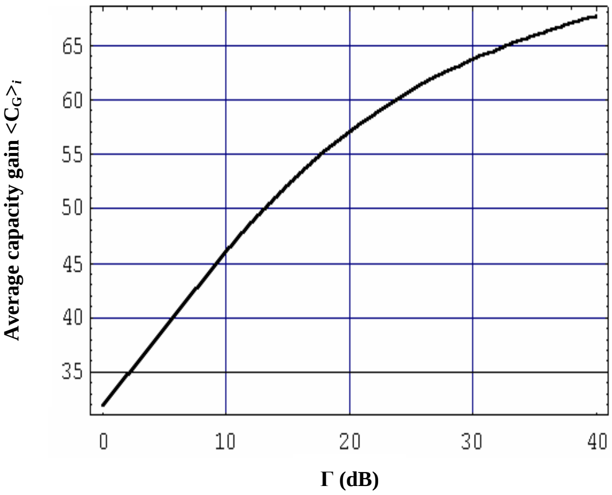

Fig.1. Average capacity gain $\langle C_G \rangle_i$ between FDMA and DS-CDMA in a Rayleigh fading channel against Γ(dB), according to eq.(19), for n=2, i=2, K=G_p=41, B_C=1.25MHz and B_s=30KHz.

A similar result can be obtained from Fig. 2 where average capacity gain $\langle C_G \rangle_i$ is plotted for the i-th user for n=2, K=G_p=41, B_C=1.25MHz, B_s=30KHz, and Γ=20dB.

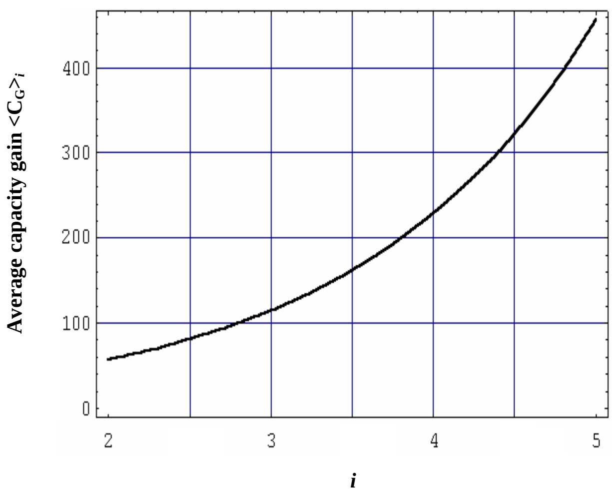

Fig.2. Average capacity gain $\langle C_G \rangle_i$ between FDMA and DS-CDMA in a Rayleigh fading channel for the i-th user, according to eq.(19), for n=2, B_C=1.25MHz, B_s=30KHz, K=G_p=41 and Γ=20dB.

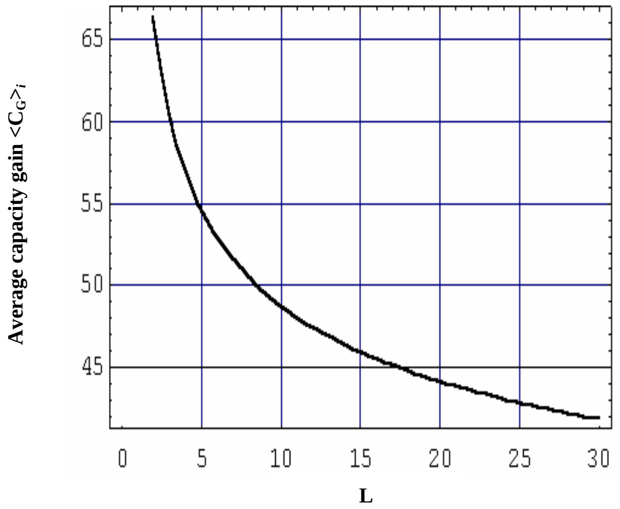

Fig.3. Average capacity gain $\langle C_G \rangle_i$ between FDMA and DS-CDMA in a Rayleigh fading channel against the number of inherent branches L of DS-CDMA, according to eq.(19), for n=2, i=3, B_s=30KHz and Γ=20dB.

As it can be seen as the number L of diversity branches increases, in a DS-CDMA scheme, average capacity gain is decreased following the respective decrease of channel capacity achieved by the corresponded DS-CDMA scheme.

5 Conclusion

In this paper, we considered FDMA and DS-CDMA schemes from a multi-user information theoretic point of view. In particular, we derived a general expression for the user channel capacity comparison between the previous multiple-access schemes when channel capacity (in the Shannon sense) is estimated in an average sense. It was shown that for different user channel capacities, in DS-CDMA case, and when operating in a Rayleigh fading channel, FDMA provides significantly greater channel capacity per user that DS-CDMA, under normalized conditions. However, for the equal users' information rate case, the situation is altered. It has been shown, in a previous work, [6], that channel capacity per user, in DS-CDMA scheme operating in a Rayleigh fading channel, exceeds that provided by FDMA. Thus, the equal and non-equal user-rate cases, in a DS-CDMA scheme, appear to behave very different. Our results are useful for the prediction of the information-theoretic comparison of previous schemes and then used as a figure of merit.

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