Analytic Solution of Nadir Attitude Pointing for LEO Microsatellite

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Abstract: An Analytic solution of Nadir attitude pointing equation of gravity gradient satellite stabilised is presented. The attitude equation is Euler linearised equation for near Nadir pointing axially symmetric satellite including only gravity gradient torque and assuming other torques such as magnetic torque, aerodynamic torque, solar radiation pressure torque and controller are constants. The obtained analytical solution was compared to numerical solution of satellite attitude equation.

Keywords:- LEO, Microsatellite, Alsat-1, Analytic, Nadir, Attitude, Simulation.

1 Introduction

A spacecraft in orbit always needs to stabilise the attitude against the external disturbance torques acting on it. Attitude control usually needs to be autonomous or semi-autonomous. On Alsat-1 [1], [6] the available actuators are reaction/momentum wheels and magnetic torquing. A mixture of attitude estimation and control algorithms is needed: these take the sensor measurements as inputs, compute the attitude and rates of the satellite, and then send commands to the actuators to maintain or stabilise that attitude, or direct the satellite to a new attitude.

Alsat-1 exploited the passive gravity gradient torque [4]. A substantial amount of literature has studied the technical problems of ADCS in many different areas.

The motion of a spacecraft presents two dynamic aspects of interest. Classical dynamics allows, under certain general conditions, for the motion of a body to be treated as the combination of two motions: a translational motion of the centre of mass and a rotation of the body about the centre of mass. The theory of attitude control generally considers only the second effect and ignores the first. The application of any force can only be interpreted as the resultant torque that would exist around the centre of mass and ignores any change to the translational velocity [8].

The equations of motion of a spacecraft can be divided into two parts: The dynamic equations of motion and kinematic equations of motion. The dynamic equations of motion express the relationship between the spacecraft body angular rate and the applied torque. These are necessary for dynamic simulations and for attitude prediction, whenever gyroscopic measurements of the angular rate is unavailable. The kinematic equations of motion are a set of first-order differential equations expressing the relationship between the attitude parameters and the rate [9].

2 Analytical Solution:

From [2], [3] and [5] the linearised Euler equation for near Nadir pointing axially symmetric satellite is given as follows

$$\ddot{\phi} + 4(1-k)\omega_0^2\phi - k\omega_0\dot{\psi} = n_x \tag{1.a}$$

$$\ddot{\theta} + 3(1-k)\omega_0^2\theta = n_y \tag{1.b}$$

$$\ddot{\psi} + \omega_0\dot{\phi} = n_z \tag{1.c}$$

Initial conditions is given as follows

$$\begin{aligned} &\phi(t_0) = \phi_0, \theta(t_0) = \theta_0, \psi(t_0) = \psi_0 \\ &\dot{\phi}(t_0) = \dot{\phi}_0, \dot{\theta}(t_0) = \dot{\theta}_0, \dot{\psi}(t_0) = \dot{\psi}_0 \end{aligned} \tag{2}$$

Where

ω_o : orbital rate;

θ : roll angle in rad ;

φ : pitch angle in rad ;

ψ : yaw angle in rad;

I = diag [I_T I_T I_z] moment of inertia tensor of the Spacecraft, $k = \frac{I_z}{I_T}$;

$\mathbf{N} = [n_x \ n_y \ n_z]^T$ normalized torque induced by controller or unmodelled disturbances torque.

We want to find tout he analytic solution of Equation (1) by assuming n_x, n_y and n_z constants. In order to make the solution simpler k<<1 is assumed which is true for Alsat-1 microsatellite [3].

2.1 Pitch Equation:

The solution of the pitch equation is given as follows

$$\theta(t) = \frac{n_y}{3\omega_0^2}(1+k) + A_1 \cos\theta\omega_\theta t + A_2 \sin\theta\omega_\theta t \tag{3}$$

Where

$\omega_\theta^2 = 3(1-k)\omega_0^2$

A_1, A_2 integral constant.

From the initial condition, the solution of the pitch equation will be

$$\begin{aligned} \theta(t) = &\frac{n_y}{3\omega_0^2}(1+k) + (\theta_0 - (1+k)\frac{n_y}{3\omega_0^2})\cos\theta\omega_\theta t \\ &+ (1+\frac{k}{2})\frac{\dot{\theta}_0}{\sqrt{3}\omega_0}\sin\theta\omega_\theta t \end{aligned} \tag{4}$$

2.2 Roll Equation:

Equation (1.c) can be integrated as

$$\dot{\psi} + \omega_0\phi = n_z t + A_3 \tag{5}$$

From the initial condition, the above equation will take the following form

$$\dot{\psi} + \omega_0\phi = n_z t + \dot{\psi}_0 + \omega_0\phi_0 \tag{6}$$

Substituting equation (6) in roll equation (1.b) yields

$$\ddot{\phi} + (4-3k)\omega_0^2\phi = n_x + k\omega_0^2(\frac{\dot{\psi}_0}{\omega_0} + \phi_0) + k\omega_0 n_z t \tag{7}$$

Regarding to the initial condition, the solution is as follows

$$\begin{aligned} \phi = &\frac{1}{4}(1+\frac{3}{4}k)\frac{n_x}{\omega_0^2} + \frac{k}{4}(\frac{\dot{\psi}_0}{\omega_0} + \phi_0) + \frac{k}{4}\frac{n_z}{\omega_0}t + \\ &\left[\phi_0 - \frac{1}{4}(1+\frac{3}{4}k)\frac{n_x}{\omega_0^2} - \frac{k}{4}(\frac{\dot{\psi}_0}{\omega_0} + \phi_0)\right]\cos\omega_\phi t + \\ &\left[\frac{1}{2}(1+\frac{3}{8}k)\frac{\dot{\phi}_0}{\omega_0} - \frac{k}{8}\frac{n_z}{\omega_0^2}\right]\sin\omega_\phi t \end{aligned} \tag{8}$$

2.3 Yaw Equation:

Substituting roll equation (8) into equation (5), yields

$$\begin{aligned} \psi = &\psi_0 - \frac{1}{4}((1+\frac{3}{4}k)\frac{\dot{\phi}_0}{\omega_0} - \frac{k}{4}\frac{n_z}{\omega_0^2}) \\ &-\left[\frac{1}{4}(1+\frac{3}{4}k)\frac{n_x}{\omega_0^2} - (1-\frac{k}{4})(\frac{\dot{\psi}_0}{\omega_0} + \phi_0)\right]\omega_0 t \\ &-\frac{1}{2}\left[(1+\frac{3}{8}k)\phi_0 - \frac{1}{4}(1+\frac{9}{8}k)\frac{n_x}{\omega_0^2} - \frac{k}{4}(\frac{\dot{\psi}_0}{\omega_0} + \phi_0)\right]\sin\omega_\phi t \\ &+\frac{1}{4}\left[(1+\frac{3}{4}k)\frac{\dot{\phi}_0}{\omega_0} - \frac{k}{4}\frac{n_z}{\omega_0^2}\right]\cos\omega_\phi t + \frac{1}{2}(1-\frac{k}{4})n_z t^2 \end{aligned} \tag{9}$$

3 Numerical Solution:

From [5], [7] and [10] The dynamic of the spacecraft in the inertial space is governed by Euler's equations of motion. With the added influence of the gravity gradient boom and reaction wheel angular momentum, the equation in vector form can be expressed as

$$\mathbf{I}\dot{\boldsymbol{\omega}}_B^I = \mathbf{N}_{GG} + \mathbf{N}_D + \mathbf{N}_M - \boldsymbol{\omega}_B^I \times (\mathbf{I}\boldsymbol{\omega}_B^I + \mathbf{h}) - \dot{\mathbf{h}} \quad (10)$$

where,

$\boldsymbol{\omega}_B^I = [\omega_x \quad \omega_y \quad \omega_z]^T$: inertially referenced body angular rate vector;

$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$: moment of inertia tensor of spacecraft (MOI);

$\mathbf{h} = [h_x \quad h_y \quad h_z]^T$: reaction wheel angular momentum vector;

$\mathbf{N}_{GG} = [N_{ggx} \quad N_{ggy} \quad N_{ggz}]^T$: gravity-gradient torque vector;

$$\mathbf{N}_{GG} = 3\frac{GM}{R_s^3}(I_{zz} - \frac{I_{xx} + I_{yy}}{2})(\mathbf{Z}_0.\mathbf{Z})(\mathbf{Z}_0 \times \mathbf{Z}) \quad (11)$$

R_s : geocentric position vector length;

$\mathbf{Z}$: principal body Z-axis unit vector;

$\mathbf{Z}_0 = [A_{13}\ A_{23}\ A_{33}]^T$: Nadir unit vector in body coordinates;

$\mathbf{N}_D = [N_{dx} \quad N_{dy} \quad N_{dz}]^T$: external disturbance torque vector such as aerodynamic torque and solar radiation pressuretorque ;

$\mathbf{N}_M = [N_{mx} \quad N_{my} \quad N_{mz}]^T$: applied torque vector by 3-axis magnetorquers.

For an axially symmetric satellite with Y/Z wheels and the principal moment of inertia axes along the body axes, the off-diagonal products of inertia elements in the MOI tensor will be zero. The deployed boom along the Z-axis also increases the MOI elements I_{xx} and I_{yy} to a much larger and equal value. This value is called the transverse MOI, I_T.

The complete set of dynamic equations of motion can then be written as follows

$$I_T\dot{\omega}_x = N_{mx} + N_{dx} - \frac{3GM}{R_s^3}(I_T - I_{zz})A_{23}A_{33} - \omega_y(I_{zz}\omega_z + h_z) + \omega_z(I_T\omega_y + h_y) \quad (12.a)$$

$$I_T\dot{\omega}_y = N_{my} + N_{dy} + \frac{3GM}{R_s^3}(I_T - I_{zz})A_{13}A_{33} + \omega_x(I_{zz}\omega_z + h_z) - \omega_z(I_T\omega_x) - \dot{h}_y \quad (12.b)$$

$$I_z\dot{\omega}_z = N_{mz} + N_{dz} - \omega_x h_y - \dot{h}_z \quad (12.c)$$

Since Euler angles are defined with regard to the local orbit coordinate, Euler angle equations are as follows:

$$\dot{\phi} = \omega_{ox}\cos\psi - \omega_{oy}\sin\psi \quad (13.a)$$

$$\dot{\theta} = (\omega_{ox}\sin\psi + \omega_{oy}\cos\psi)\sec\phi \quad (13.b)$$

$$\dot{\psi} = \omega_{oz} + (\omega_{ox}\sin\psi + \omega_{oy}\cos\psi)\tan\phi \quad (13.c)$$

where ϕ, θ and ψ are roll, pitch and yaw respectively and $\boldsymbol{\omega}_{LO} = [\omega_{ox}\ \omega_{oy}\ \omega_{oz}]^T$ is an orbit reference body angular velocity vector. This vector can be derived by

$$\boldsymbol{\omega}_{LO} = \boldsymbol{\omega}_{BY} - \mathbf{A}\boldsymbol{\omega}_0 \quad (14)$$

where $\boldsymbol{\omega}_0 = [0 - \omega_0\ 0]^T$ is an orbital rate vector and A as the Euler 213 direction cosine matrix (DCM). Using attitude matrix, equation (14) becomes

$$\omega_{ox} = \omega_x + \omega_0\cos\phi\sin\psi \quad (15.a)$$

$$\omega_{oy} = \omega_y + \omega_0\cos\phi\cos\psi \quad (15.b)$$

$$\omega_{oz} = \omega_z - \omega_0\sin\phi \quad (15.c)$$

Substituting equation (15) into equation (13)

$$\dot{\phi} = \omega_x\cos\psi - \omega_y\sin\psi \quad (16.a)$$

$$\dot{\theta} = (\omega_x\sin\psi + \omega_y\cos\psi)\sec\phi + \omega_0 \quad (16.b)$$

$$\dot{\psi} = \omega_z + (\omega_x\sin\psi + \omega_y\cos\psi)\tan\phi \quad (16.c)$$

Note that Euler 2-1-3 equation has a singularity when the roll angle ϕ equals 90 degrees.

4 Simulations Results:

The following initialization parameters were utilized

Normalized Torque

$n_x = n_y = n_z = 0$

Inertial Tensor (Satellite configuration I)

I [kgm^2] : diag [185 158 5]^t

Miscellaneous

Simulation time [orbit] : 2
Integration step [sec] : 1
Sampling time [sec] : 5

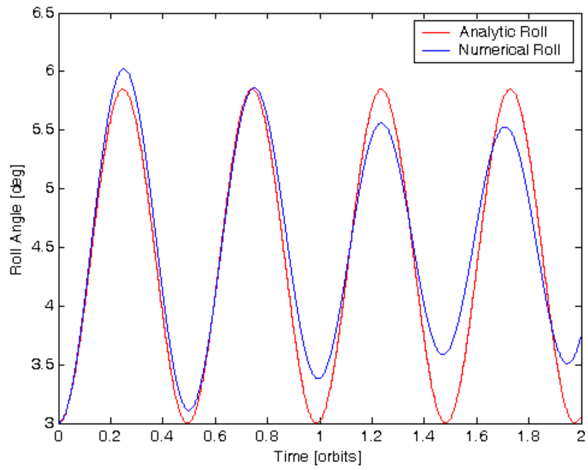

Fig. 1 Roll Angle

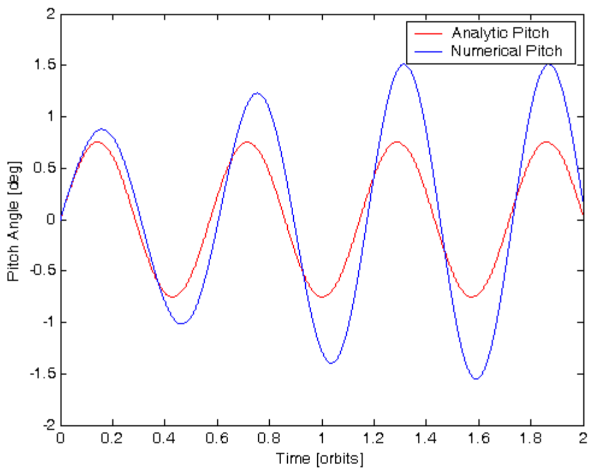

Fig. 2 Pitch Angle

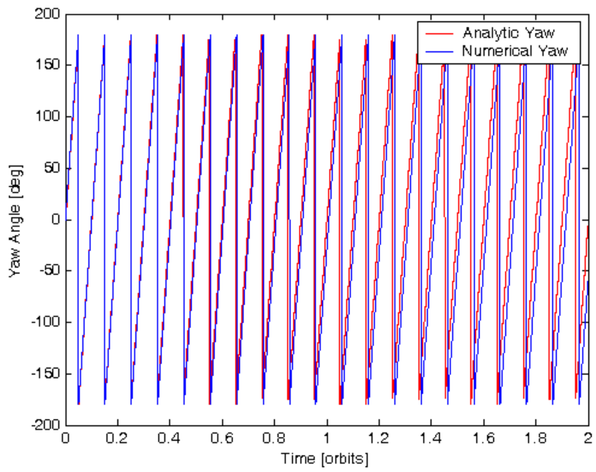

Fig. 3 Yaw Angle

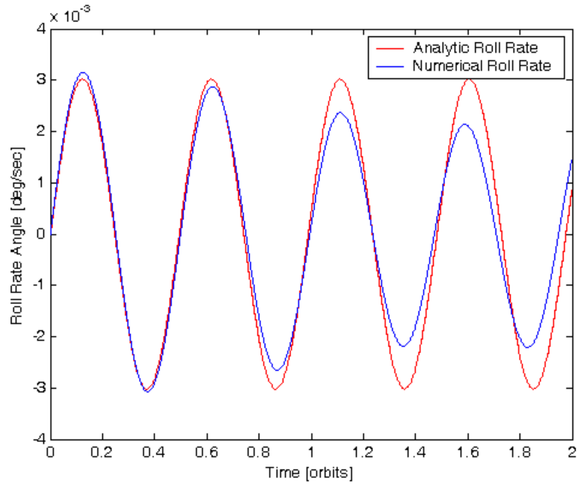

Fig. 4 Roll Rate Angle

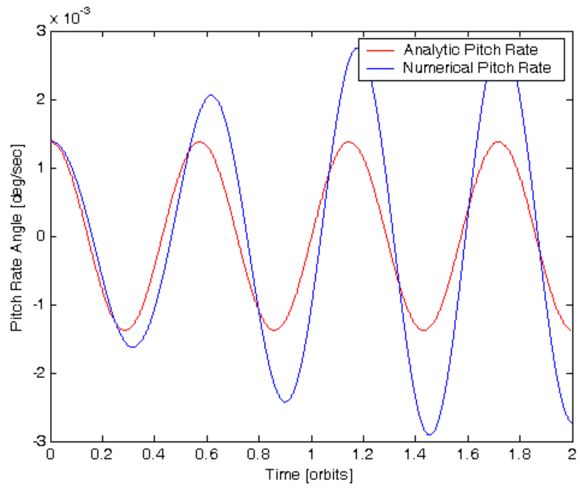

Fig. 5 Pitch Rate Angle

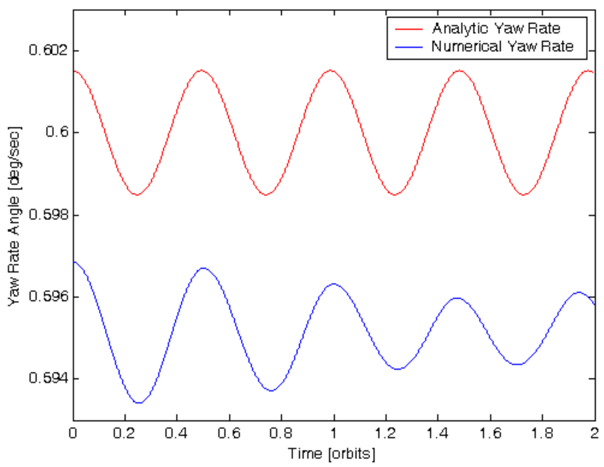

Fig. 6 Yaw Rate Angle

Table 1: Lists the angular error

	Roll [degree]	**Pitch [degree]**	**Yaw [degree]**
Average	-0.108	-0.039	0.353
STD [1-σ]	0.142	-0.395	-0.362
RMS	0.178	0.397	0.506

Table 2: Lists the rate error

	Roll Rate [deg/sec]	**Pitch Rate [deg/sec]**	**Yaw Rate [deg/sec]**
Average	$2.9*10^{-4}$	$-9.84*10^{-6}$	$48*10^{-4}$
STD [1-σ]	$-5.68*10^{-5}$	$-7.36*10^{-4}$	$1.68*10^{-4}$
RMS	$2.96*10^{-4}$	$7.36*10^{-4}$	$49*10^{-4}$

Table 3: Lists the error magnitude angles and rates

	Mag Error Average	**Mag Error STD [1-σ]**	**Mag Error RMS**
Angles [deg]	0.371	0.554	0.667
Rate [deg/sec]	$48*10^{-4}$	$7.5*10^{-4}$	$49.6*10^{-4}$

For the graphs presented above, notice that the magnitude of the RMS error results indicates that the angular error is approximately 0.66 degree and the rate error is about 0.005 degree/second.

5 Conclusion

This paper detailed the analytic solution of Nadir attitude pointing equation of gravity gradient LEO satellite.

The magnitude of the RMS error results indicates that the angular error is approximately 0.66 degree and the rate error is about 0.005 degree/second, both in degrees.

A low cost method of full satellite attitude propagator was proposed to be used for LEO microsatellite gravity gradient stabilised (small libration).

The version presented is only valid for LEO microsatellite gravity gradient stabilised including gravity gradient disturbance. The extension to aerodynamic disturbances is in progress.

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