Optimum Control Using Signal Processing in Integrated Aerospace navigation systems

AYMAN AL-LAWAMA[*] , ABDEL-RAHMAN AL-QAWASMI[**] and OMAR AL-AYASRAH[*]

[*]Electrical Engineering Department
Mutah University
Karak– 61710 P.O. Box (7)
JORDAN
ay_lawama@yahoo.com http://www.mutah.edu.jo
[**] Communications and Electronics Department
Philadelphia University
P.O. Box: 1 Amman 19392
JORDAN
qawasmi@philadelphia.edu.jo http://www.philadelphia.edu.jo

Abstract:-The global – air – navigation telecommunication network is an integrated system which joins telecommunication, navigation, and observation. This integrated system provides communication in real-time scale between any two inverse points .Direct and indirect measurement (DIM) of basic navigational parameters of aircraft movement (position , velocity , acceleration ,….. etc) should be done accurately and precision . This paper provides general theoretical fundamentals of Optimum Control (OC) of processed signal in integrated navigational systems to obtain relatively simple and practically convenient relations for calculating maximum accuracy.

The well known maximum probability method is used to obtain Optimum Complex Evaluation (OCE) for the processed results of Unequal Accuracy (UA) measurements .Two lemmas are considered for absolute concepts and ratio errors in DIM. Features of the proposed method are illustrated using the simplest case of two measurement (m=2) and then generalized using the method of induction for ($m>2$). The essential conditions of existence theorem of optimum integration (OI) are proved for these problems. Theoretical regulations of OI are illustrated using the results of experiment, numerical examples and graphics. Through the results of investigations, conclusions are formulated and practical recommendations are developed for the use of OI.

Key-Words: - aerospace navigation systems, Optimum control, DIM measurements, complex evaluation.

1 Introduction

The approach on air-lines of global aircraft liners as Boing 777-200 LR and the practical realization of the concept of global control of air traffic ("seamless sky conception") open principally new capabilities for enhancing safety flights, quality of passenger's service and commercial efficiency of aircraft companies. Global aircraft liners with 300 passengers on broadside support distance with nonstop flights equals to 17000 km. The use of global Air-navigation Telecommunication Network (ATN), which joins systems of telecommunications, navigation and observations (CNS/ATM) allows provide communication in real –time scale between any two inverse points. All of this creates important conditions for global control of air traffic.

One of main directions of CNS/ATM ATN application is to increase the measurement precision of basic navigational parameters of aircraft movement (Position, velocity and acceleration), airborne, ground-based and satellite navigational systems. Later, such systems shortly named PVA systems. The accuracy can be increased using methods of integrated systems (IS) [1-3]. Theoretically, this designates the application of optimum processing of UA results. When sensors (designed on different physical principles and have different errors of measurements) are used in measuring, the problem of experimental processed results, control, diagnosing and other many cases will be actual even in the theory of approximated calculations.

In all these cases, it is important to have a theoretical justification about how to get the best general evaluation of measurable parameter due to results of measurements of inaccurate sensors. Elements of this theory are developed by many specialists in the area of mathematical statistics and theory of statistical solutions, where the considered problem is known as a problem of processing of samples from nonhomogeneous -serial statistics [4]. However, obtained results, as a rule, are oriented on the prove of asymptotic characteristic of derivable evaluations- property to give more accurate evaluations by increasing the number of samples i.e. (sensors). Practically, we usually have to work with relatively small number of sensors –from 3 to 10, which give unpractical results.

The concept of integration of nonhomogeneous results of PVA measurements in aviation is not new. The simplest example of integration of two systems is the complex «GPS/INS integration» [1-3]. An example of more complicated situation is the measurement of flight altitude of aircraft depending on the results of measurement of its altitude using six to seven different PVA systems such as airborne sensor of barometrical altitude, airborne radio altimeter, earth radio navigation system "Omega", satellite Global Position System (GPS), Primary Surveillance Radar (PSR), Secondary surveillance radar (SSR) and Aeronautical Mobile Satellite Service (AMSS).

As shown by works [4-6], the use of redundant number of measurements (essentially redundant volume of signals) allows successfully to create high-precision structure - redundant info-measurement from relatively inaccurate systems.

All well-known methods of integration can be conditionally divided into three classes:

-Majority processing method of redundant inaccurate measurements;

-Methods using averaging value without considering the different precisions from sensors;

-Methods of substituting results of measurement using more-accurate sensors by the results of measurement using less-accurate sensors.

Each class has its advantages and disadvantages. Selection of suitable integration method in every specific case without theoretical justification presents laboriousness and insufficiently explored problem.

Brief examination prehistory of PVA integration shows actuality, theoretical value and practical concernment of solution of optimum-controlled problems of processed signals in integrated navigational systems.

2 Work objective and Problem definition

The work objective is to create general theoretical fundamentals of OC of processed signal in integrated navigational systems to get practically convenient and relatively simple relations for complex and Optimum Evaluation (OE) using criterion of maximum accuracy; and to show asymptotic convergence to true values of evaluations derivable as a result of OI.

The following well-known information is presumed:

1- The mathematical model of results taken from Indirect Measurement (IM) Y_i i-th of real- value system X_0 of navigational parameter at instant time t of measurements can be defined as

$$Y_i(t) = X_0(t) + \xi_i(t), \qquad (1)$$

where m ($i = 1,2,\ldots, m$) is the total number of systems forming the complex integrated system and ξ(t) is the random absolute error of Direct Measurements (DMs) (See below lemmas 1, 2).

2- The real value of measurable parameter in (1) is a deterministic and constant value, where the error is presented as a Gaussian stationary signal with known numerical characteristics.

3- The mathematical expectation of error

$$M[\xi_i(t)] = 0, \qquad (2)$$

and dispersion

$$D[\xi_i(t)] = \sigma_i^2, i = \overline{1, m}. \qquad (3)$$

In this paper we need to:

1- create an OC method to process *m* signals (1) (optimum integration method of UA systems) using criterion of maximum probability,

2- find function of optimum integration using criterion of maximum probability.

$$Z_m = f(Y_1, Y_m), \qquad (4)$$

with arguments presented by results of IMs (1).

3- determinate the necessary conditions of non-bias, opulence and asymptotic effectiveness of evaluation (4),

4- evaluate the comparative effectiveness of OE application, advantages and disadvantages of proposed optimum integration method and to

5- bring out convenient practical applications relative to simple calculated recurrent relations for OE (4) and its numerical characteristic.

3 Program and procedure of analysis

First, two lemmas about properties of absolute error under DIMs are proved. Second, the well-known method of maximum probability (4), in convenient way, is used to get OEs. The proposed method can be illustrated in simple case when *m=2*. Later on, this method is used for main set of convenient recurrent relations. Results of this method, by induction, will be generalized into a complex problematical situation for *m>2*. For these cases, theories about essential conditions of OI are proved. Calculated theoretical relations are introduced for various indices of effectiveness of OI. Theoretical situation of OI is illustrated using numerical and graphical examples, depending on the results of analysis, conclusions and practical recommendations are formulated to use OI.

In order to theoretically substantiate the fairness of used mathematical model of IMs result in the form (1), two lemmas are considered to prove the properties of random absolute and relative errors of results of DMs and the necessary conditions for using the mathematical model (1).

We remind that in theory of approximate calculations traditionally the term "error" is used, whereas in measurement theory more often the term "mistake" is used. In future these terms are used as convertible terms, preference return to standard terms of mathematical statistics.

The first Lemma is about random properties of absolute and relative errors of DMs results.
If the true value of parameter X_0 is measured with random absolute error of DMs

$$\Delta = X - X_0 \tag{5}$$

and the random result of DM X has Gaussian distribution with numerical characteristics

$$M[X] = X_0, D[X] = \sigma_x^2, \tag{6}$$

then the absolute and relative errors of DMs results also have Gaussian distribution with numerical characteristics

$$M[\Delta] = \Delta_0 = 0, D[\Delta] = \sigma_x^2, \tag{7}$$

$$M[\Delta / X_0] = M[\delta] = 0,$$

$$D[\Delta / X_0] = D[\delta] = V_x^2 = \frac{\sigma_x^2}{X_0^2} \tag{8}$$

Relations (7) denote that the absence of so-called "Systematic error of DMs" is accomplished due to the absence of bias of evaluation of DMs (6).

Relations (7) show that the absolute error (5) of DM (while condition (6) is implemented) has the same dispersion as the result of DMs.

Relations (8) show that while condition (6) is achieved, the mathematical expectation of relative error δ equals to zero, but its dispersion equals to the square of variation coefficient *X*.

$$V_X = \frac{\sigma_x}{x_0} \tag{9}$$

The second Lemma is about necessary conditions of using mathematical model of the IMs result in the form (1).
If the variation coefficient (9) is

$$V_x << 1 \tag{10}$$

and at the same time, the following necessary conditions are satisfied

1. $M[X] = X_0$ (11)

2. $M[\Delta] = \Delta_0 = 0$ (12)

3. $D[\Delta] = D[X] = \sigma_x^2$ (13)

4. $\left[\left[\frac{dy(x_0,\Delta_0)}{dx}\right]^2 + \left[\frac{dy(x_0,\Delta_0)}{d\Delta}\right]^2\right] = 1$ (14)

where *M[.]* and *D[.]* are the mathematical expectation and dispersion respectively. The result of IMs *Y(X, Δ)*, as function of two variables using (6) approximately described by three terms of Taylor's series at point(x_0, Δ_0):

$$Y(X,\Delta) = Y(x_0,\Delta_0) + \frac{dx(x_0,\Delta_0)}{dx}(X - x_0)$$

$$+\frac{dx(x_0,\Delta_0)}{d\Delta}(\Delta - \Delta_0) \tag{15}$$

then and only then IMs result can be approximately presented in the form of sum of the DMs result and the absolute error of measurements

$$Y(X,\Delta) = X + \Delta \tag{16}$$

This can be proved using the mathematical expectation and dispersion of IMs results that can be determined using standard ensemble averaging method of multiple realizations. Thus

$$M[Y(X,\Delta)] = M[Y(x_0,\Delta_0)] +$$

$$M\left[\frac{dx(x_0,\Delta_0)}{dx}(X - x_0) + \frac{dx(x_0,\Delta_0)}{d\Delta}(\Delta - \Delta_0)\right] =$$

$$x_0 + M[\Delta]\left\{\frac{dx(x_0,\Delta_0)}{dx} + \frac{dx(x_0,\Delta_0)}{d\Delta}\right\} = x_0 \tag{17}$$

and

$$D[Y(X,\Delta)] = D[x_0] + \left[\frac{dx(x_0,\Delta_0)}{dx}\right]^2 D[X]$$

$$+\left[\frac{dx(x_0,\Delta_0)}{d\Delta}\right]^2 D[\Delta] =$$

$$D[X]\left\{\left[\frac{dx(x_0,\Delta_0)}{dx}\right]^2 + \left[\frac{dx(x_0,\Delta_0)}{d\Delta}\right]^2\right\} = \sigma_x^2 \tag{18}$$

Therefore, on accomplishment of necessary conditions (6)-(10) the IMs result represents non-bias evaluation, which has Gaussian distribution with parameters

$$M[Y(X,\Delta)] = x_0 \quad (19)$$

$$D[Y(X,\Delta)] = \sigma_x^2 \quad (20)$$

and can be represented in the form (16), which required to be proved.

If the DMs are unbiased (7) and the systematical error entire them is absented (12), then the IMs using formula (12) give unbiased evaluation.

Derivatives in (14) characterize transmission coefficients of converted DMs results into IMs taking into account their absolute errors.

Condition (14) essentially represents the condition of such normalization of transmission coefficients of converters, which maintain adequacy of DIMs

Since

$$\frac{dx(x_0,\Delta_0)}{dx} = \frac{dx(x_0,\Delta_0)}{d\Delta}, \quad (21)$$

from condition (14) followed that

$$2\left[\frac{dx(x_0,\Delta_0)}{dx}\right]^2 = 1, \ \frac{dx(x_0,\Delta_0)}{d\Delta} = \sqrt{1/2} \quad (22)$$

If in DMs the absolute precision is achieved and represented by references, then the DMs result of corresponds to the real value

$$X = X_0, \sigma_x^2 = 0 \quad (23)$$

and the transmission coefficient of transducers in IMs is

$$\frac{dx(x_0,\Delta_0)}{dx} = 1. \quad (24)$$

4 Analysis of optimum complex evaluation of indirect measurements

Considering the case $m = 2$, the founded results will be generalized into the case $m > 2$. This case can be analyzed using theorem 1, which contains necessary conditions of unbiasedness and effectiveness of OCE in UA of IMs.

Theorem 1 states that if in UA of IMs the following necessary conditions are accomplished:

A. Measuring transducers of all channels that have linear Gaussian characteristics transformation of DMs results into IMs results.

B. Numerical characteristics of initial real value of parameter X_{00} is defined by the relation

$$M[X_{00}] = 0, D[X_{00}] = 0 \quad (25)$$

C. Numerical characteristics of current real value X_0 are defined by the relations

$$M[X_0] = X_0, D[X_0] = 0 \quad (26)$$

D. Result of *i-th* IM in compliance with statement Lemma 1 can be represented in the form

$$Y_i(t) = X_0(t) + \xi_i(t), \ i = 1, 2, \quad (27)$$

where ξ_i is the Absolute error of *i-th* DMs result.

E. Numerical characteristics of *i-th* error are defined by

$$M[\xi_i] = 0, D[\xi_i] = D[Y_i], \ i=1, 2 \quad (28)$$

F. Numerical characteristics of *i-th* IMs result in (27) of initial point Y_{io} are determined by

$$M[Y_{io}] = 0, D[Y_{io}] = 0, i = 1,2 \quad (29)$$

G. Numerical characteristics of complex evaluation $Z(Y_1,Y_2)$ of IMs at initial point Y_{1o}, Y_{2o} are determined by

$$M[Z[Y_{1o},Y_{2o}]] = 0, D[Z[Y_{1o},Y_{2o}]] = 0. \quad (30)$$

H. The Condition of normalization of transmission coefficients of channel transducers is:

$$\sum_{i=1}^{2}\frac{dz}{dy_{i0}} = \sum_{i=1}^{2} g_i = 1, \quad (31)$$

where $\frac{dz}{dy_{i0}}$ is the mathematical expectation of *i-th* derived at initial point Y_{io}.

I. The condition of OCE $Z(Y_1,Y_2)$

$$\frac{dD[Z]}{dg_i} = 0, i = 1,2 \quad (32)$$

where D[z] is the dispersion of complex evaluation

J. The condition of minimum achievement D[z] at point Z_{opt}

$$\frac{d^2D[Z]}{dg_i^2} > 0, i = 1,2. \quad (33)$$

,then and only then the complex evaluation

$$Z(Y_1,Y_2) = \sum_{i=1}^{2} g_{iopt} Y_i \quad (34)$$

will be optimum using criterion of maximum accuracy, unbiasedness and effectiveness. Optimum coefficients can be determined by relations

$$g_{1opt} = \frac{D(Y_2)}{D(Y_1) + D(Y_2)} = \frac{1}{1 + D(Y_1)/D(Y_2)}, \quad (35)$$

$$g_{2opt} = \frac{D(Y_1)}{D(Y_1) + D(Y_2)} = \frac{1}{1 + D(Y_2)/D(Y_1)}, \quad (36)$$

and the minimum value of dispersion of OCE (34)

$$D_{\min}[Z_{opt}] = g_{iopt} D[Y_i] = 1/\sum_{i=1}^{2} 1/D[Y_i] \quad (37)$$

To prove that we use condition A and represent $Z(Y_1,Y_2)$ in the form of expansion in Maclaurain series, in which we keep only the linear terms as:

$$Z(Y_1,Y_2) = f(Y_1,Y_2) + \frac{df}{dy_1}(Y_1 - Y_{10})$$
$$+\frac{df}{dy_1}(Y_2 - Y_{20}). \qquad (38)$$

To prove the unbiasedness of evaluation (34) we define the mathematical expectation (38) and considering the conditions B-F for corresponding mathematical expectations, which yields:

$$M[Z(Y_1,Y_2)] = M\left[\frac{df}{dy_1}(Y_1 - Y_{10}) + \frac{df}{dy_1}(Y_2 - Y_{20})\right]$$
$$= M\left[\frac{df}{dy_1} + \frac{df}{dy_1}\right]X_0, \qquad (39)$$

If the condition of normalization G (31) is accomplished

$$M[Z(Y_1,Y_2)] = X_0, \qquad (40)$$

, then the evaluation (38) represents an unbiased evaluation.

To prove the effectiveness of evaluation (38) we will define its dispersion and considering at the same time conditions B-F for corresponding dispersions, we will get

$$D[Z(Y_1,Y_2)] - D[f(Y_1,Y_2)] + D\left[\frac{df}{dy_1}(Y_1 - Y_{10})\right] +$$
$$D\left[\frac{df}{dy_1}(Y_2 - Y_{20})\right] = \sum_{i=1}^{2}\left(\frac{df}{dy_{i0}}\right)^2 D(Y_i) =$$
$$\sum_{i=1}^{2} g_i^2 D(Y_i). \qquad (41)$$

Considering (41) in condition (31) and choosing $g_1 = g$, we get

$$D_z(g) = g^2 D_1 + (1-g)^2 D_2 \qquad (42)$$

The optimum value of g at which condition H (32) is satisfied when the minimum value of dispersion is reached:

$$g_{opt} = \arg\min_{g\in[0,1]} D_z(g) \qquad (43)$$

A standard method can be used to find an extremum of one-variable function. First, we find the first derivative of function (42) on g and second, we equate the result of derivation to zero. The optimized value will be

$$2g_{opt}D_1 - 2(1-g_{opt})D_2 = 0. \qquad (44)$$

Solving equation (44), we get

$$g_{opt} = D_2/(D_1 + D_2) = \frac{1}{1 + D_1/D_2} \qquad (45)$$

As the second derivative of function (42) is

$$\frac{\delta^2}{\delta g^2} D_z(g) = 2(D_1 + D_2) > 0, \qquad (46)$$

The condition of minimum achievement (I) is carried out and the founded value g_{opt} gives the minimum of (42) that equals to

$$D_{2\min}(g_{opt}) = \frac{D_1D_2}{D_1 + D_2} = g_{opt}D_1 = (1-g_{opt})D_2, \quad (47)$$

hence, the evaluation is effective

The received result allows entering three obvious parameters of efficiency of OI of PVA systems:

An index parameter of maximum value of efficiency of OI

$$W_1 = \frac{\max(D_1,D_2)}{D_{2\min}} = \frac{1}{g_{opt}} = 1 + \frac{D_1}{D_2}, \qquad (48)$$

where *D1>D2*

An Index parameter of average value of efficiency of OI when instead of an OE

$$Z_{2opt} = g_{opt}Y_1 + (1-g_{opt})Y_2 \qquad (49)$$

an arithmetic-mean value of results of two measurements is used

$$X_{20} = (Y_1 + Y_2)/2 \qquad (50)$$

$$W_2 = \frac{D_{20}}{D_{2\min}} = \frac{(D_1 + D_2)^2}{4D_1D_2} = \frac{(1 + D_1/D_2)}{4D_1/D_2} \qquad (51)$$

An Index parameter of the minimal value of efficiency (Minorant) of OI when instead of evaluation (49) the measured value on an output of that sensor with a smaller error is used

$$W_3 = \frac{\min(D_1,D_2)}{D_{2\min}} = 1 + \frac{D_1}{D_2}, \text{ D1<D2} \qquad (52)$$

Let's designate dimensionless relation *D1/D2* through *U*, *U*. $\in[1,\infty)$. We shall express parameters of efficiency of OI as function of *U*:

$$W_1 = 1 + U, W_2 = \frac{(1+U)^2}{4U}, W_3 = 1 + \frac{1}{U}. \qquad (53)$$

Let's establish interrelations between these parameters. As $U=W_1-1$:

$$W_1 = W_2/(W_2 - 1), W_2 = W_1^2/4(W_1 - 1),$$
$$W_3 = W_1/(W_1 - 1). \qquad (54)$$

On figure 1 graphics of functions (53) depending on *U* are shown. By increasing *U* from 1 (a case equally accurate measurements) and up to ∞ (the case of integration with reference system at which $D_2 = 0$), a proportional increasing in W_1 is observed. It designates that the use of a complex evaluation (49) gives the maximal gain on the accuracy, proportional to the relation of dispersions.

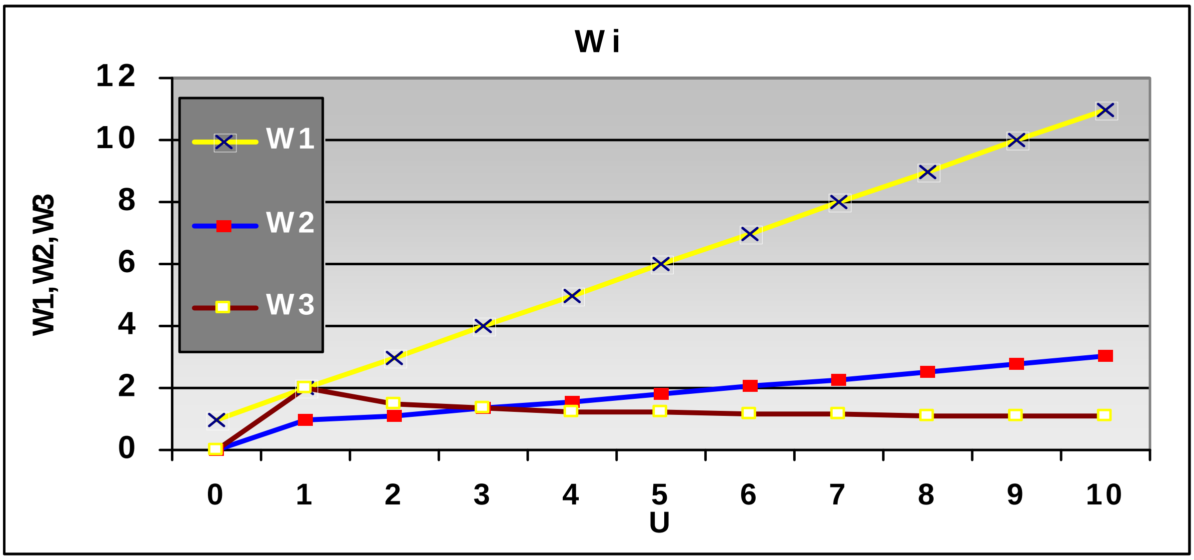

Figure1: Graphics of function 49

The use of OE (49) comparing to the mean-arithmetic value (50) gives an increasing in effectiveness parameter in number of times (curve 2), in how many times the square of arithmetic-mean value of dispersions of measurements exceeds the square of compound value of these dispersions. The application of substitution method (Curve 3) is more effective because there is more difference between dispersion measurements. When $U=1$ ($D_1=D_2$) measurements are to be equally accurate and formulas (53) result in known results $W_1(1)=W_3(1)=2$, $W_2(1)=1$. Thus, OI (Coarse) and (Exact) measurement systems with the application of unbiased evaluation (49) and coefficients (45) provide achievement of minimum value of general dispersion of measurements (47). The integration is more effective when the errors of complex system have more differences between each other.

We will observe two characteristic examples of OI of aerospace navigational systems.

In example1 we will observe what the OI of primary and secondary radiolocators gives, when considering the measurement of flight altitude of airplanes as control objects.

We will use the data of work [7]. Arithmetic mean error of measurement of distance R using radiolocators range at flight altitudes $H=(10\text{-}20)$ *км* equals to $\sigma_R = (0{,}2\text{-}0{,}25)$ *км*. Arithmetic mean error $\sigma_{н2}$ altitude transmission H by transponders of secondary radiolocators through readings of flight instruments equals to $\sigma_{н2} \approx 37{,}5$ *м*. We will perform through this data the OI and we will evaluate its effectiveness using formulas (53).

For definition of an arithmetic-mean error of measurement of airplanes altitude by radio locators range we shall consider, that between the range of operation R and height H of airplanes flight is a relation

$$H = R\sin\left|\arctan(H/\sqrt{R^2-H^2})\right| = R\sin\beta \quad (55)$$

At $R = 400$ *км*, $H = 10$ *км*, $tg\ (10/400) \approx 0.25$, $\beta \approx 14^o$, $\sin 14^o \approx 0.2419$. Applying linear approximation (55) for evaluation root mean square errors of measurement of altitude of radiolocators range we shall get

$$\sigma_{H2} = \sqrt{0.2419^2 \times 250^2} \approx 60.475\, M.$$

Thus, $D_{H1} \approx 3657$ $м^2$, $D_{H2} \approx 1406.25$ $м^2$, $U \approx 2.6$, $g_{opt} \approx 0.72226$, $1\text{-}g_{opt} \approx 0.27774$, $D_{2min} \approx 0.72226 \times 1406.25 \approx 0.27774 \times 3657 \approx 1015.82$ m^2, $\sigma_{2min} \approx 31.87$ *м*, $W_1(2{,}6) \approx 3.6$, $W_2(2.6) \approx 1.24615$, $W_3(2{,}6) \approx 1.38461$. From here follows, that in comparison with an arithmetic-mean evaluation OI allows to reduce approximately by 25 % a dispersion of radiolocators measurements of flight altitude ,that is especially actual at reduction of ranges separation and use the concept « free flight ».

In Example2, we shall examine features of OI systems GPS and INS [1]. For GPS root-mean-square error of Doppler, a measuring instrument of velocity $\sigma v2 \approx 0.01$ *m/s*. For INS root-mean-square error of an independent measuring instrument of velocity $\sigma v1 \approx 0.5$ *m/s* for one hour of flight. We shall determine the efficiency of OI-GPS and INS on measurement of velocity.

Let's calculate the dimensionless parameter U and weight factors (45) and evaluations (51):

$U = \sigma_{v1}^2/\sigma_{v2}^2 = 0.5^2/0{,}01^2 \approx 2500$, $g_{opt} = \sigma_{v2}^2/(\sigma_{v1}^2+\sigma_{v2}^2) = 0.01^2/(0.5^2+0.01^2) = 0.0339984$, $1-g_{opt} = 0.9360016$.

Let's determine values of D_{2min}, σ_{2min} and parameters of efficiency W_1 - W_3:

$D_{2min} \approx g_{opt}\ x\ \sigma_{v1} \approx 0.0339984\ x\ 0.25 \approx 0.0339984\ x\ 0.25 \approx 0.04936001$.

σ_{2min} $=0.009998$ m/s. $W_1 = 1+U = 1+2500=2501$. $W_2 = (1+U)^2/4U = (1+2500)^2/4\ x\ x2500 = 625.5$. $W_3 = 1+1/U = 1+1/2500 = 1.034$.

Let's calculate the contribution σ_{v12} in general dispersion D_{20}:

$\Delta D_1 = g_{opt}^2\ D_1 \approx (0.0339984)^2\ x\ 0.25 \approx 0.073996$.

Let's determine a relative error of a substitution method:

$\Delta = \Delta D_1/\ D_{20} \approx 0.073996/0.04936001 \approx (0.0339975986) \approx 0.04\%$.

Results of calculations allow doing an unequivocal conclusion that in this case the most effective is use of a method of substitution. It leads relative root-mean-square to an error, smaller than 0,2 %. In other words, results of measurements INS are expedient for adjusting approximately in each hour of flight by results of satellite measurements GPS which can be considered as reference.

For complexes with UA systems it is useful to enter characteristic number m_0 as follows. We shall designate dispersions of measurements so that the condition was satisfied

$$D[Y_1] > D[Y_2]. \tag{56}$$

Let's consider these dispersions as first two members of a decreasing arithmetic progression with a difference

$$d = D[Y_1] - D[Y_2], \tag{57}$$

Then from the condition

$$D[Y_1] - d(m_0 - 1) = 0, \tag{58}$$

It is possible to find such number m_0 « the virtual channel » which possesses a zero dispersion of measurement, that has a zero error:

$$m_0 = 1 + \frac{D[Y_1]}{d} = \frac{2u-1}{u-1}. \tag{59}$$

We investigate limiting properties of function $m_0(u)$

$$\lim_{u\to 1} m_0(u) \to \infty, \tag{60}$$

$$\lim_{u\to \infty} m_0(u) \to 2 \tag{61}$$

The singular case (60) corresponds to asymptotic approach of d to zero that is approach UA measurements. The singular case (61) corresponds to asymptotic approach d to D (Y1). It shows approach of a complex from large number of UA measurements to a complex of only two systems. Thus one system asymptotically comes nearer to reference so that its error approaches zero. It is a limiting case of s rough-exact measurements when one system allows defining the approached value of parameter, and the second system allows estimating value with high accuracy. $m_0 \in [2,\infty]$, is a number that can conveniently be used as integrated characteristic of various UA complexes which allows comparing them among themselves.

In turn, the size u can be considered as inverse function for (59), which is as a function of characteristic number of a complex.

$$u = \frac{m_0 - 1}{m_0 - 2}, \tag{62}$$

Relations (59) and (62) form pair transformations which completely describe complexes UA measurements in which attitudes of dispersions of the next measurements are identical.

We shall establish a relation and attitudes of the entered integrated characteristics of complexes UA measurements with parameters of accuracy [8]. A parameter of accuracy *hi i-th* measurements *Yi* and its dispersion *D [Yi]* are connected by relations:

$$h_i = 1/\sqrt{2D[Y_i]},\ D[Y_i] = 1/2h_i^2. \tag{63}$$

In the classical theory of UA measurements enter « a normal measure of accuracy »

$$h = \sqrt{\frac{2\sum_{i=1}^{m} g_i (X_0 - Y_i)^2}{m-1}} = \sqrt{2D[Z_m]}, \tag{64}$$

a weight coefficients g_i is to be found from the condition

$$g_i = \frac{h_i^2}{h^2},\ i = 1, m, \tag{65}$$

for evaluation of parameter that can be found in the form

$Z(m) = \sum_{i=1}^{m} g_i Y_i / \sum_{i=1}^{m} g_i$, but thus leaves open a question of a choosing h for a case when gi is unknown.

To increase the accuracy of measurement, it is convenient to use the index

$$W_4 = \frac{\max h_i}{\min h_i} = \frac{\sigma_{\min}\lfloor Z_{opt}(m) \rfloor}{\sigma(Y_1)}. \tag{66}$$

The value (66) shows, what percentage makes minimal root mean square value of an OCE rather root mean square values of the worst measurement. It shows relative reduction of a field of the tolerance of an OE in comparison with a field of the tolerance for the first measurement.

$$W_4 = \sqrt{\frac{1}{W_1}} = \sqrt{1-\frac{1}{W_2}} = \sqrt{\frac{1}{1+u}} = \sqrt{\frac{m_0-2}{2m_0-3}} \quad (67)$$

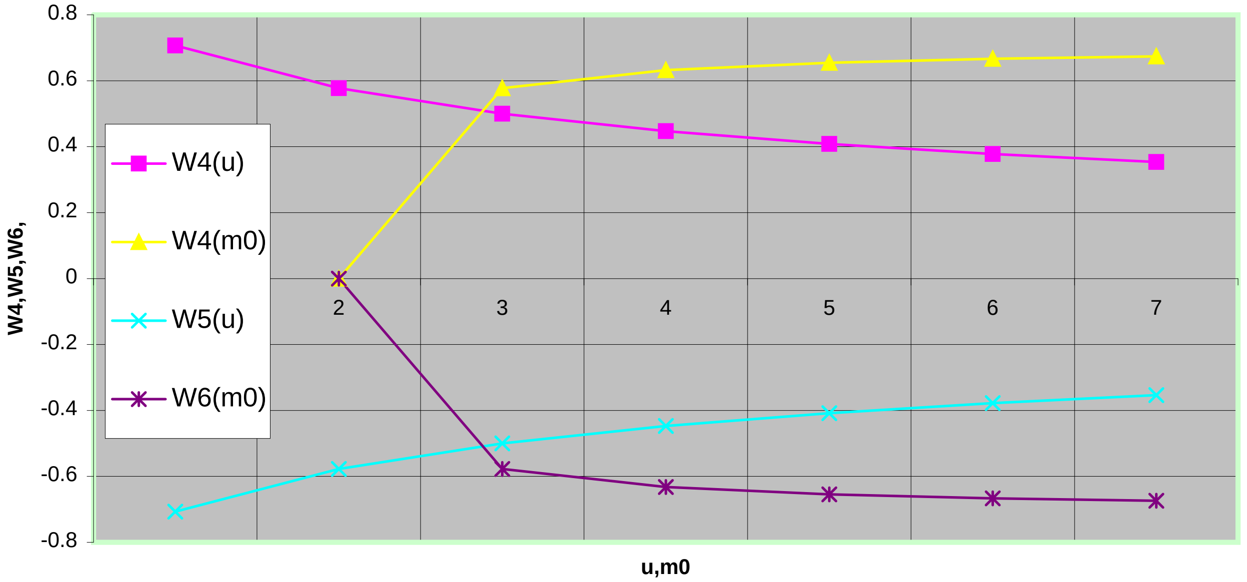

Fig.2. Graphics of functions $W_4(u)$, $W_4(m_0)$, $W_5(u)$, $W_6(m_0)$

It is convenient to consider both of root signs in (67) evidently to illustrate narrowing a field of the tolerance. On fig.2 graphics W_4 *(u)* W_4 *(*m_0*)* are shown, negative branch W_4 *(u)* is designated as W_5 *(u)*, negative branch W_4 *(*m_0*)* is designated as W_5 *(*m_0*)*.

It is possible to notice, that as function of arguments *u* and m_0 value W_4 has the following limiting properties:

$$\lim_{u\to 1} W_4(u) = 1/\sqrt{2}, \quad \lim_{u\to\infty} W_4(u) = 0, \qquad (68)$$

$$\lim_{m_0\to 2} W_4(m_0) = 0, \quad \lim_{m_0\to\infty} W_4(m_0) = 1/\sqrt{2}. \qquad (69)$$

Let's pass to consideration of a case *m> 2*. We shall number dispersions by a way of their decrease. We shall receive range sequence $D[Y_1]$, $D[Y_m]$, for which the condition is valid

$$D[Y_i] > D[Y_{i+1}], i = 1, m. \qquad (70)$$

We approximate decreasing sequence of dispersions as an arithmetic progression on a method of the least squares:

$$D_0(i) = D_0(1) + d_0(i-1), i = 1, m, \qquad (71)$$

where $D_0(i)$ - is an OE of *i-th* member of the progression, received by optimization $D_0(1)$and d_0 from a condition of minimization of the sum

$$S[D_0(1), d_0] = \sum_{i=1}^{m} [D_0(1) + d_0(i-1) - D(Y_i)]^2 \quad (72)$$

We Use (71) for the proof of conditions of existence and uniqueness of the optimum decision at of UA measurements.

Theorem 2 states that if:

A. Conditions A-J of the theorem 1 are carried out for all of UA measurements.

B. Range sequence of the known dispersions, satisfying to a condition (70), it is possible to approximate an arithmetic progression (71).

C. The Characteristic number m_0 of UA measurements satisfies samples of results to an inequality

$$m_0 = 1 + \frac{D_0(Y_1)}{d_0} \geq 3, \qquad (73)$$

That for all *m* which satisfy to an inequality

$$3 \leq m \leq m_0 \qquad (74)$$

There are optimum values of evaluation of a true value of parameter

$$Z_{opt}(Y_1, Y_m) = Z_{opt}(Y_1, Y_{m-1}) + \frac{D_{\min}[Z_{opt}(Y_1, Y_{m-1})]}{D_{\min}[Z_{opt}(Y_1, Y_{m-1})] + D(Y_m)} \times [Z_{opt}(Y_1, Y_{m-1}) - Y_m], \qquad (75)$$

that have the minimal values of dispersions

$$D_{\min} = [Z_{opt}(Y_1, Y_m)] = \frac{D(Y_m)}{D_{\min}[Z_{opt}(Y_1, Y_{m-1})] + D(Y_m)} \times$$

$$D_{\min}\left[Z_{opt}(Y_1, Y_{m-1})\right] \tag{76}$$

To prove this theorem its clear that as the arithmetic progression (71) is decreasing, for the proof of validity of recurrent relations (75), (76) it is enough to prove, that the minimal value of a dispersion of the OE, received in the previous step,

m	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Y_m	0,98	1,02	0,95	1,04	1,01	1,005	0,996	0,999	1,000
D(m)	0,04	0,035	0,03	0,025	0,02	0,015	0,01	0,005	0,000
V(m),%	20	18,7	17,32	15,81	14,1	12,25	10,0	7,07	0,000
Z(m)	0,98	0,998666	0,980000	0,998912	1,002046	1,002854	1,000889	1,000194	1,000
D_{min}(m)	0,04	0,01867	0,01151	0.00788	0,005653	0,004105	0.002911	0,00184	0,000
V_0(m),%	20,4	13,68	10,9	8,88	7,5	6,389	5,39	4,288	0,000
V_0(m)/V(m)	1,02	0,7355	0,62933	0,56167	0,53191	0,52155	0,539	0,6065	0,000
W_4,%	100	68,32	53,6	44,3	37,59	32,03	26,9	21,44	0,000
E_m,%	-2	2	-5	4	1	0,5	-0,4	-0,1	0,00
E_0,%	-2	-0,133	-1,999	-0,1088	0,204	0,285	0,0889	0,0194	0,000

Table 1

always there is less than dispersion of UA measurement on a following step.

We shall show validity of this statement for first three measurements. For this purpose it is necessary to prove validity of an inequality

$$D_{\min}\left[Z_{opt}(Y_1, Y_2)\right] = \frac{D_0(1)}{D_0(1) + D_0(2)} D_0(2) < D_0(3). \tag{77}$$

Let's consider that the approximated values of dispersions of UA measurements are connected by a condition (71) then from (77) we shall receive

$$D_0(1)\left[D_0(1) - d_0\right] < \left[D_0(1) - 2d_0\right] \times \left[2D_0(1) - d_0\right]. \tag{78}$$

Removing the brackets and carrying out the elementary transformations of this inequality, we shall receive finally

$$\left[D_0(1) - 2d_0\right]^2 > 0, D_0^2(3) > 0. \tag{79}$$

From validity of an inequality (79) validity of recurrent relations (75) and (76) follows at $m=3$,to use (45) in (47) and (49) at definition Z_{opt} (Y_1, Y_3). Applying a method of an induction for $m> 3$ for all m, which satisfy to an inequality (74), we shall get analogues of an inequality (79) for dispersions with these numbers as was shown.

We shall consider results of experimental digital imitating modeling of UA measurements. Optimum integration results are executed at the following modeling initial data:

$X_0=1.0$, $D_0(1)= 0.04$, $d_0= - 0.0025$; $m_0 = [D_0(1)/d_0] + 1 = 9$.

In table 1 which contains eleven lines and ten columns, results of realization of UA measurements and their OI are shown. The first line contains number of measurement m. Realizations of UA the measurements, the smoothed values of their dispersion and factors of a variation are displayed, accordingly, in the second - the fourth lines. In the fifth - the seventh lines results OI by means of recurrent parities (75), (76) are reflected, accordingly: Z_{opt} (m), D_{min} (m), V_0 (m). In the eighth line the attitude of factors of a variation of OEs and measurements is given. In the ninth line the change of a parameter of efficiency OI is shown. The tenth and eleventh lines show, how the values of relative errors in this realization of experiment is changed.

The analysis of experimental results shows, in the first, validity of the offered theoretical positions, and in the second, high efficiency OI even at rather low accuracy of primary measuring systems. The third and fifth lines evidently show speed asymptotic to convergence of the minimal dispersion to a zero with growth m.

5 Conclusions

1. Optimum integration aerospace of navigation systems is an important and actual problem. Solving it will promote an increase of accuracy of the approached calculations, approximations of the functions, scientifically use of redundant volumes of the measuring information, optimum construction of processing algorithms of UA measurements and the control and diagnosing of systems to construct exact complexes from inexact systems.

2. The Examined lemmas and the proved theorems about necessary conditions of OI results in UA measurements are a theoretical basis of construction of exact complexes from inexact systems through UA measurements.

3. In-depth, the study of the simplest case of two UA measurements allows entering a number of constructive offers to assess the capability of

efficiency of optimum indirect of UA measurements. The Use of concept of characteristic number m_0 (59) complexes of UA measurements deserves special attention. This number is unequivocally connected with all parameters of efficiency of a complex and allows not only to compare successfully such complexes among themselves, but also to determine a degree of their proximity to complexes of equal accuracy measurements.

4. Conditions ABC, (73), (74) theorems 2, existence of optimum decisions for the general case $2<m<m_0$ allow to construct recurrent relations (75), (76). They are useful not only for OC of processing of the results of UA measurements, but also can find application in the decision of problems of the measurement, the approached calculations, the control, diagnosing, evaluate of reliability and others. Obvious advantages of application (75), (76) consist in reduction of memory sizes of computers and in an opportunity of a choice of demanded number of systems from a condition of maintenance of the set accuracy of measurements.

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