# Energy balancing by combinatorial optimization for wireless sensor networks

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*Abstract*: - In this paper novel protocols are developed for wireless sensor networks (WSNs) in order to ensure reliable packet transmission and maximize lifespan at the same time. The optimal transmission energies are derived which guarantee that the packets are received by the Base Station (BS) with a given reliability subject to achieving the longest possible lifespan. These protocols can be applied in biomedical applications where energy consumption and longevity is of crucial importance. The optimization has been carried out for the chain protocol (when nodes are forwarding the packets toward the BS via the neighbouring nodes) and for the shortcut type of protocols (when a packet may get to the BS by being first transferred in the chain up to a certain node which then sends it directly to the BS).

The new results have been tested by extensive simulations which also demonstrated that the lifespan of WSN can significantly be increased by the new protocols.

*Keywords:* Communication systems, Communication system routing, Network reliability, Protocols.

## 1. Introduction

Due to the recent advances in electronics and wireless communication, the development of low-cost, low-power, multifunctional sensors have received increasing attention [1]. These sensors are compact in size and besides sensing they also have some limited signal processing and communication capabilities. However, these limitations in size and in energy make WSNs different from other wireless and ad-hoc networks [2]. As a result, new protocols must be developed with special focus on energy balancing in order to increase the lifetime of the network which is crucial in case applications, where recharging of the nodes is out of reach (e.g. medical applications, military field observations, living habitat monitoring …etc., for more details see [3]).

The paper addresses reliable packet transmission in WSN when packets are to be received by the Base Station (BS) with a given reliability, in terms of keeping the error probability under a given threshold [4]. Since the success of every individual packet transmission depends on the distance and the transmission energy [5,6], the probability of correct reception will diminish exponentially with respect to the number hops, in the case of multihop packet transfers [7]. As a result to increase reliability two different protocols will be investigated:

- *Chain protocol* when packet transfer takes place over a 1D chain of nodes (created some routing algorithm running prior to the packet transfer), and the nodes send each packet to their neighbours on the nearside of the BS.
- *Shortcut protocol* when the packet travels in the chain up to a certain node *i* which then sends it directly the BS (shortcut).

Our concern is to derive the optimal transmission energies for each scenario needed to achieve a given reliability (i.e. a packet is received correctly by the BS with a given probability regardless of its source node) and yielding the longest possible lifespan of the network. As demonstrated in the paper this leads to a constrained optimization problem which is solved by combinatorial optimization tools. The solution yields the optimal energy matrix **G**, the $G_{ij}$ element of which indicates what is transmission energy node *j* should applied to transmit the received packet when it is originated from node *i* (the destination of the transmission depends on the specific protocol, e,g. in the case of chain protocol node *j* must retransmit the packet to its neighbour, while in the shortcut protocol node *j* may choose to send the packet directly to the BS).

## 2. The model

After the routing protocol (e.g. PEDAP [8]) has found the path to the base station the subsequent nodes participating in the packet transfer can be regarded as a one dimensional chain labeled by $i = 1,...,N$ and depicted by the Fig. 1.

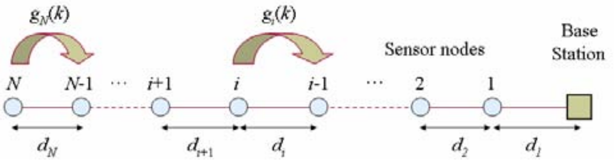

Fig. 1 One dimensional chain model for WSN

The system is characterized as follows:

- the topology is uniquely defined by a distance vector $\mathbf{d} = (d_1,...,d_N)$, where $d_i, i = 1,...,N$ denotes the distance between node $i$ and $i$-1, respectively;
- the energy needed to transmit packet over distance $d$ is given as $g = \frac{d^\alpha \Theta \sigma_Z^2}{-\ln p} + g_0$ dictated by the Rayleigh model, where $d$ is the distance, $\alpha$ depends on the propagation type, $p$ is the reliability of correct reception, $\Theta$ is the threshold, $\sigma_Z^2$ is the noise energy, while $g_0$ represents the consumption of the electronics during transmitting and receiving;
- the function connecting the reliability parameter $p$ of the transmission with the transmission energy $g$ is denoted by $p = \Psi(g)$ for short, furthermore if a packet is transmitted from node $i$ to the neighbouring node $i$-1 then the corresponding transmission power is denoted by $p_i$ and the reliability of this single transmission is $p_i = \Psi(g_i)$;
- we assume that only one node generates a packet at a time;
- the nodes operate in a time synchronous manner where the discrete time (clock signal) is denoted by $k = 0,1,2,...$;
- the initial battery power on each node is the same and denoted by $C$ and the energy state of the nodes at the $k^{\text{th}}$ time instant is expressed by vector $\mathbf{c}(k) = (c_1(k),....,c_N(k))$;

As a result, a WSN is fully characterized by vectors **g**, **p**, and **c** respectively.

## 3. Reliable packet transfer with a given probability

In this section we investigate reliable packet forwarding as a constrained optimization problem. This problem arises from the fact that in the case of multihop communication, the probability of correct packet reception at the BS is $P_C = \prod_{j=1}^{i} p_j$, and the associated energies are $g_j = \frac{d_j^\alpha \Theta \sigma_Z^2}{-\ln p_i} + g_0$, $j = 1,...,i$. As a result, a given $P_C$ can be achieved by several choices of $p_j$-s (i.e. several factorizations) yielding different energy consumptions. Our concern is to pick the optimal factorization which will yield minimal energy consumption in terms of maximizing the remaining energy on the bottleneck node (the node which has the smallest energy).

We will treat this constrained optimization problem in two different scenarios:

- *chain protocol* (packets are transferred downward in the chain towards the BS in a node-by-node fashion);
- *shortcut protocol* (first the packet is traveling in the chain from node to node, then from a certain node it is directly transmitted to the BS).

In the first case, we are concerned with identifying the optimal energy scenario to yield maximum longevity. In the second case, not only the optimal energy vector but also the optimal node index must be found where the shortcut to the BS will take place.

### 3.1. Reliable packet transfer by chain protocol

In this case, we assume that every node retransmits the received packet to its neighbour towards the BS. For a packet generated at node $i$ and traveling to the BS the reliability constraint is given as

$$P_C = \prod_{j=1}^{i} \Psi(g_j) = 1 - \varepsilon \quad (1)$$

In order to ensure maximum longevity, our objective is to enforce a packet transfer which maximizes the minimum remaining energy expressed as follows:

$$\mathbf{g}_{\text{opt}}:\max_{\mathbf{g}}\min_{j} c_j(k+1), \tag{2}$$

where $c_j(k+1)=c_j(k)-g_j$ and $\mathbf{g}=\left(g_1,...,g_i,0...0\right)$. Here $k$ denotes the number of packets sent to the BS. The optimization method to solve (2) is described in Section 5. Once the optimal solution $\mathbf{g}_{\text{opt}}^{(i)}=\left(g_{1\text{opt}},...,g_{i,\text{opt}},0...0\right)$, $i=1,...,N$ has been found the solution can be downloaded to each node (in our example to node $l$) in the Table 1.

Table 1. Routing table of chain protocol

| *Source node index* | *Transmission energy which maximizes the lifespan* |
|---|---|
| $N$ | $g_{l,\text{opt}}^{(N)}$ |
| $N$-$1$ | $g_{l,\text{opt}}^{(N-1)}$ |
| : | : |
| $l$+1 | $g_{l,\text{opt}}^{(l+1)}$ |
| $l$ | $g_{l,\text{opt}}^{(l)}$ |

When a packet arrives at node $l$ from source node $i$ then it can be retransmitted to from node $l$ to node $l$-1 with the corresponding energy $g_{l,\text{opt}}^{(i)}$ read out form the table. In this way, $G_{ij,\text{opt}}=g_{j,\text{opt}}^{(i)}$ and the tables downloaded to the node are obtained from the corresponding column of $\mathbf{G}_{\text{opt}}$.

### 3.2. Reliable packet transfer by shortcut protocol

In this case, the packet emitted by node $i$ traverses down in the chain to node $j$ in a node-by-node fashion and from node $j$ it gets directly transmitted to the BS. The energy needed to transmit the packet from node $l$ to node $l$-1 in the chain is denoted by $g_l, j+1\le l\le i$, whereas the energy needed to get the packet from node $j$ directly to the BS is denoted by $G_j$.

Then the reliability constraint can be expressed as

$$\Psi\left(G_j\right)\prod_{l=j+1}^{i}\Psi\left(g_l\right)=1-\varepsilon, \tag{3}$$

In order to ensure maximum longevity, our objective again is to enforce a packet transfer which maximizes the minimum remaining energy of participating nodes, given as:

$$\mathbf{g}_{\text{opt}}:\max_{\mathbf{g}}\min_{l} c_l(k+1), \tag{4}$$

where $c_l\left(k+1\right)=c_l\left(k\right)-g_l$ and $\mathbf{g}=\left(0,..,0,G_j,g_{j+1}...,g_i,0...0\right)$, where $k$ denotes the number of packets sent to the BS.

Once the optimal solution $\mathbf{g}_{\text{opt}}^{(i)}=\left(0,..,0,G_{j_{\text{opt}}},g_{j_{\text{opt}}+1,\text{opt}}...,g_{i,\text{opt}},0...0\right)$ has been found the solution can be downloaded to each node (in our example to node $l$).

When a packet arrives at node $l$ from a given source node then it can be retransmitted from node $l$ to node $l$-1 or it can be shortcut to the BS with the corresponding energies read out form the table.

## 4. Protocol optimization

In this section the optimization process is described to obtain the energy vectors which maximize the lifespan in the case of the different protocols. As was seen in the previous chapter, protocol optimization amounts to solving the optimization problems in (2) and (4).

The required transmission probability can be calculated by the following:

$$\prod_i p_i=P_{\text{C}}, \tag{5}$$

therefore the following initialized $p_{ri}$ can be calculated:

$$\forall i:\quad p_i=P_{\text{C}}^{\frac{1}{M}}. \tag{6}$$

Since (2) is defined over a discrete set the optimum is sought by stochastic search akin to the Mathias algorithm. The transmission probability optimization (TPO) algorithm is described in Fig. 2.

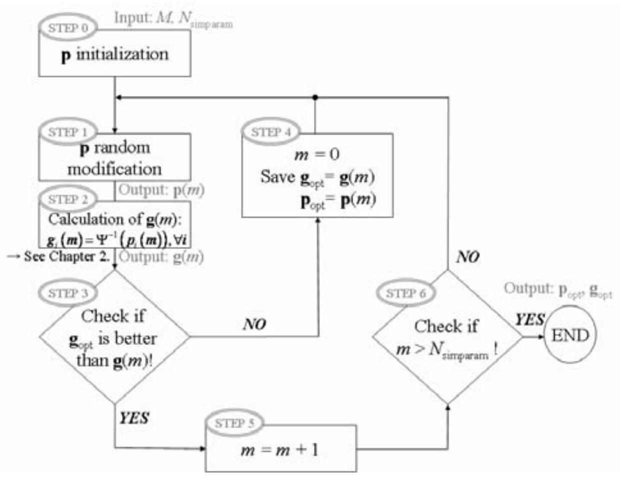

Fig. 2 The flow diagram of the TPO algorithm

The steps of the TPO algorithm are given as follows:

STEP 0. we set the initial **p** vector in uniform manner;

STEP 1. by matrix $\mathbf{P}_{\text{mod}}$, we modify vector **p** subject to the condition that the product of the components remains the same:

$$P_j = P_j \cdot P_{\text{mod}\,j,k} \text{ and } P_k = P_k / P_{\text{mod}\,j,k} \text{ , } \vee j,k \text{ ;}$$

STEP 2. we calculate the transmission energy vector $\mathbf{g}(m)$;

STEP 3. we evaluate the objective function (2) and check whether the newly obtained solution $\mathbf{g}(m)$ is better the previously stored $\mathbf{g}_{\text{opt}}$;

STEP 4. if yes then $\mathbf{g}(m)$ gets stored and we set $m = 0$;

STEP 5. if not then $\mathbf{g}(m)$ is discarded and $\mathbf{g}_{\text{opt}}$ remains unchanged while $m = m + 1$;

STEP 6. if $m$ equals the simulation parameter $N_{\text{simparam}}$ then the algorithm ends.

One must note that instead of the algorithm described above, any other methods of combinatorial optimization can be used, such as Genetic Algorithms or Simulated Annealing. However, based on the empirical studies and simulations we ran, we found that in the present case our algorithm converged faster to the optimal solution than other methods.

# 5. Numerical results

In this section the performance of the protocols described above are investigated by extensive simulations.

## 5.1. Network description and the propagation model

The simulations were carried out in three different scenarios all of them including 10 sensors and placing them:

- equidistantly;
- random I (their distances were selected subject to truncated Gaussian distribution);
- random II (their distances were selected subject to Poisson distribution);

We set the following parameters:

- The propagation model is determined by the Rayleigh fading, yielding $g = \dfrac{d^{\alpha}\Theta\sigma_Z^2}{-\ln p_r} + g_0$;
- Conditioning energy needed by the electronics $g_0 = 50\mu\text{W}$;
- Threshold: $\Theta = 10^{-6}$;
- Average noise energy: $\sigma^2 = 0.1$;
- Propagation parameter: $\alpha = 2$;
- Initial energy: $g_j(0) = 100\text{mW}$; $j = 1, \dots, N$.

The lifespan was defined as the number of steps until which each node has the energy to transmit packets complying with the given reliability parameter. As soon as, a node (the bottleneck node) goes flat (being not able to participate in the reliable packet transfer, because of falling short of the required energy), then network is considered dead.

## 5.2. Performance analysis

Based on the discussion above, one must note that the optimization should be carried out after each packet transmission as the remaining energy $\mathbf{c}(k)$ changes with respect to $k$.

Since the underlying optimization is time consuming, we let the system run $L$ steps after each optimization cycles without changes with respect to the transmission energies. As a result, energy optimization takes only every $L$ steps, where we investigated the case of $L$=1,3,5,7,15,20,50,100. In this way, the computational overhead is reduced on the expense of achieving suboptimal energy consumption (the lifespan of the system is slightly reduced).

Figure 4. shows the effect of increased optimization cycle on the lifespan (optimization takes place only after each $L$ steps). One can infer that the larger $L$ becomes the smaller gain can be obtained, which is explained by the fact that between two consecutive optimizations, the network must operates in non-optimized time windows.

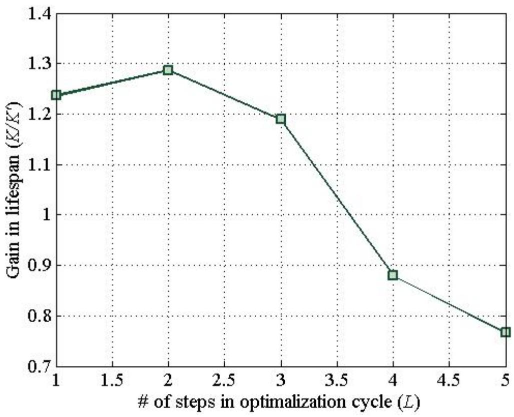

Fig. 4 The effect of increased optimization cycle on the lifespan

In the Fig. 5 the ratio of optimized lifespan versus the non-optimized lifespan is depicted, as a function of the reliability parameter and the number of nodes.

We run the simulations for different reliability parameters: $\varepsilon = 0.1; 0.09; 0.08; 0.05; 0.01$. One can see that the biggest gain in lifespan is obtained when $\varepsilon$=0.1 ($P_{\mathrm{req}}$ = 0.9). On the other hand, the smaller the reliability parameter, the more complex the optimization becomes. Furthermore, the gain is further increased, when the number of nodes grows.

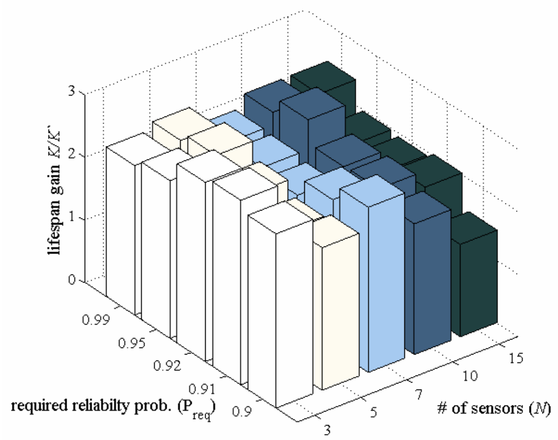

Fig. 5 Lifespan on the bottleneck node ($P_{\mathrm{req}} = P_C$)

The next figure depicts the lifespan achieved by the shortcut protocol compared with the optimized chain and non-optimized short-cut. The investigated WSN contained 10 nodes the locations of which were subject to Gaussian and the required reliability was set 0.91 ($\varepsilon = 0.09$). The figure exhibits the change of energy on the bottleneck node with respect to the number of sent packets.

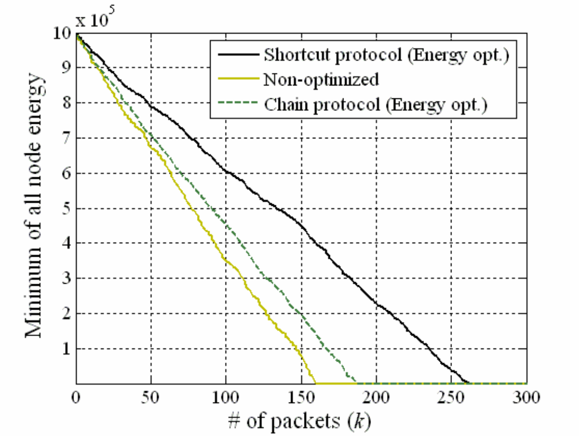

Fig. 6 Minimum node energy as function of packets ($P_{\mathrm{req}}$=0.91, $N$=10)

From the figure, one can see that the shortcut protocol is able to carry much more packet in its lifetime than the chain protocol.

In the following bar chart the energy dynamics of the bottleneck node of the chain and shortcut protocols are compared when the nodes are distributed subject to equidistant, Gaussian and Poissonian manner.

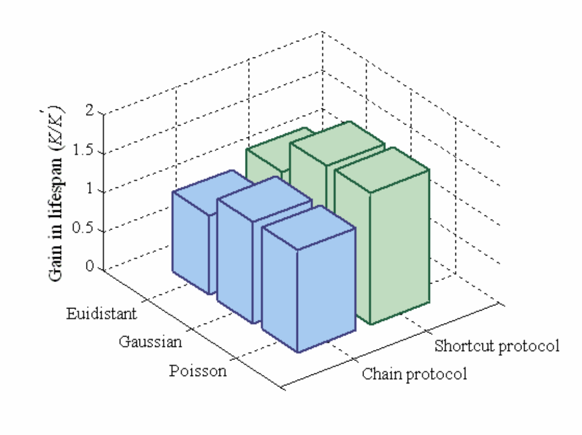

Fig. 7 (Gain in span in the case of development protocols in the equidistant, Gaussian and Poissonian located nodes ($P_{\mathrm{req}}$=0.91, $N$=10)

It can be seen that the shortcut protocol tends to be the most superior, especially in the case of nodes distributed in a Poissonian fashion.

# 6. Conclusions

In this paper, novel energy balancing packet forwarding methods have been developed to maximize the lifespan of WSNs and to ensure reliable packet transfer at the same time. We have optimized the transmission energies of the nodes in order to minimize the energy consumption of the bottleneck node (the node with the lowest available energy) subject to satisfying a given the reliability constraint. Two scenario has been studied extensively: (i) the traditional chain protocol (nodes are passing the incoming packet to their neighbours closer to the BS); (ii) the random shortcut protocol (nodes make random choices subject to an optimized probability mass function) whether to forward the packet to the neighbouring node or sending it directly to the BS).

The underlying protocol optimization was reduced to a constrained optimization problem which has been solved by a stochastic search algorithm.

The performance of the protocols have been analyzed and compared to each other in the case of a 10-node WSN where the nodes were distributed in equidistant or random fashion subject to Gaussian and Poisson distribution. The reliability constraint was set 0.91. Form the performance analysis one can infer, that the shortcut protocol provides longer lifespan (the obtained gain is approximately 1.2).

The results can directly be used in any application where energy consumption and lifespan are of concern. One primary target field is related to biomedical applications, where the energy consumption of sensorial implants must be minimized (in order to avoid the hazard of galvanic recharging via the human body). In this case, reliable but low-energy packet transfers of measurements from the body to the monitoring BS is indeed of crucial [9].

Our future plan is to extend the optimization to protocols where transmission from a node is allowed to any other nodes (not only to the neighbouring node or to the BS). Since this type of protocols has a huge combinatorial state space further work on the optimization algorithm is needed as well to make the algorithm faster.

*Acknowledgment*

The research reported here was supported by the National Office for Research and Technology in Hungary.

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