Design of Optimal Space-Time Codes in TDD/TDMA 4G systems

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Abstract: - Under the scenarios of fast mobile frequency selective channels, it is a very intractable issue to design the optimal space time codes (ST) that can achieve the optimal diversity and multiplexing gains tradeoff with full-diversity full-rate (FDFR) constrains. Therefore, an optimal space time design scheme is given in this paper for one novel 4G cellular communication systems with time division duplex (TDD) and time divisions multiple access (TDMA). The system is designed to overcome the great channel estimation overhead and the complexity of multiple user detectors in the cases of multiple antennas at transceivers. Here, one FDFR code given in recent literatures is adopted for the proposed TDD/TDMA 4G systems, which was proved in this paper to achieve the optimal diversity-multiplexing gain tradeoff. In typical cellular channel profiles, several space time schemes with optimal or sub-optimal tradeoff are tested with TDD/TDMA 4G systems with 8x2 antenna configurations at base and mobile stations respectively. Numerical results show that the FDFR design could achieve the maximal system throughout with better adaptive capacity to wireless link states than that of space time block codes (STBC) and Diagonal Bell Laboratories Layered Spare-Time Codes (D-BLAST). Furthermore, the performance of the proposed FDFR design for TDD/TDMA 4G systems could achieve better tradeoff for throughput and system symbol error rates than that of the cyclic-division-algebra (CDA) space time codes.

Key-words: - MIMO, diversity and multiplexing tradeoff, 4G, space time codes, full-diversity and full-rate

1 Introduction

Since the third-generation (3G) wireless mobile communication networks are deployed throughout the world, the designs of 3G beyond or the fourth-generation (4G) wireless communications systems under frequency selective fast fading channel scenarios have recently attracted a lot of attention. Many design schemes based MIMO and OFDM are given by recent literatures [1-3], such as VSF-OFCDM by DoCoMo Japan [1], TDD-MIMO-OFDM by FuTURE project of China [2], TDD-CDM-OFDM by Datang Mobile [3], and so on. However, code-division multiple access (CDMA) or orthogonal frequency-division multiple access (OFDMA) are generally adopted by these 4G schemes. When user terminals equipped with multiple transmit antennas in faster varying environments, two intractable issues [4] appear, i.e., the great channel estimation overhead [5] and the extreme complexity of multiple user detectors (MUD) [6]. So, we design one full time division 4G mobile communication system with time division duplex (TDD) and time divisions multiple access (TDMA), i.e., the TDD/TDMA 4G systems.

Configured with multiple antennas at transceivers, great performance improvements for wireless communication systems can be achieved by space-time code technologies. Generally, MIMO systems can provide two kinds of gain [7], i.e., diversity gain and multiplexing gain. Most existing MIMO space time code schemes, including space-time coding and layered space-time, aim at maximizing either of them. For example, the space time block codes (STBC), the space time trellis codes and the space time turbo trellis codes are aiming at maximizing diversity, while the layered space time codes at maximizing multiplexing gain, i.e., V-BLAST, D-BLAST and T-BLAST, and so on. Therefore, it is desirable to design one scheme to achieve the better tradeoff between the multiplexing and diversity gain. For given MIMO channels, both the two gains can be simultaneously obtained, as quantified by Zheng and Tse [8]. However, there exists one fundamental tradeoff between the two gains achieved by one given code scheme, i.e., the diversity-multiplexing tradeoff. So, many space time schemes in many literatures [9-13] are devoted to design a universal space-time code with the optimal tradeoff, such as algebraic space-time codes [9-11], cyclic division algebras (CDA) codes [12-13], and so on. Furthermore, one full-diversity with full-rate (FDFR) space-time code [14-16] was proposed for arbitrary antenna configurations to simultaneously achieve full rate and full diversity. However, another untraceable problem is to design an optimal tradeoff space time code with full-diversity full-rate, as the optimal diversity-multiplexing tradeoff is the essential property of wireless MIMO channels [8]. So, in the scenarios of the TDD/TDMA 4G systems with 8x2 antenna configuration at base stations and mobile stations, we detail the design of FDFR space-time codes and prove that the FDFR space-time codes can also achieve the optimal diversity-multiplexing tradeoff.

The rest of this paper is organized as following. The system model of TDD/TDMA 4G systems is outlined in Section 2, which is followed by the design of FDFR space-time codes in Section 3. Subsequently, performance analysis is discussed, and then the space time schemes are proved to be optimal for diversity-multiplexing tradeoff. Numerical compared results of the FDFR scheme and other schemes with sub-optimal diversity-multiplexing tradeoff are displayed in Section 5. Finally, Section 6 presents our conclusions.

2 TDD/TDMA 4G System Model

In order to be deployed in fast fading cellular mobile wireless channels, the proposed TDD/TDMA 4G system adopts MIMO and OFDM technology at its transceivers. For an universal space-time code, its architecture can be shown in Fig.1.

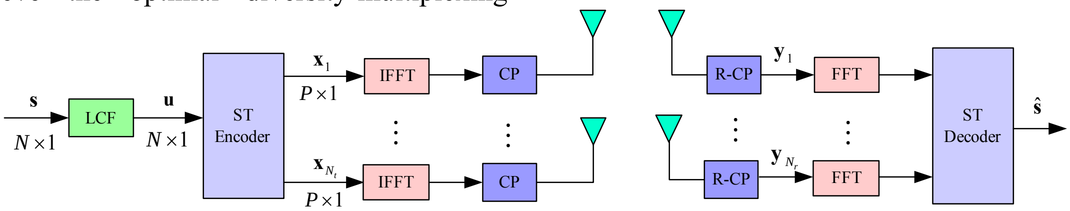

Fig.1 Functional Block Diagram of TDD/TDMA 4G system with universal space-time codes.

Suppose that N_t transmit antennas and N_r receive antennas are equipped at transmitters and receivers, respectively. The transmitted data symbols $\{s(i), i=1,\cdots,N,\cdots\}$ are firstly parsed into blocks as $\mathbf{s}=[s(1),\cdots,s(N)]^T$, and then each block $\boldsymbol{s}$ is input into linear complex field (LCF) encoders. Subsequently, one space-time word ($\mathbf{X}$) is obtained by space time (ST) encoder according to its input data vector ($\mathbf{u}$) from the LCF encoders. Finally, every row of the ST word is transmitted by its correspondent transmit antennas through one OFDM modulator. The inverse demodulators at receivers are used to restore the transmitted data symbols. It can be found that the space time encoder consists of two components, i.e., LCF encoders and ST mappers.

Let one OFDM word is carried by K consecutive sub-carriers. The data symbol block ($\mathbf{u}$) is converted into one space time word in T sample times, which consists of the data symbols and their conjugates. At receivers, one least squared space-time decoder is used to restore the transmitted data symbols by decoding the received space-time signals. In order to delineate the system model compactly, we omit the time indicator of MIMO OFDM words and neglect symbol timing errors and frequency offsets. Assume one MIMO OFDM word can only carry one space-time codeword, so the receive signals in one MIMO OFDM word period can be given by

$$y(n,k)=\sum_{m=1}^{N_t} H[n,m](k)\,x(m,k)+w(n,k)$$
$$n=1,\cdots,N_r\ ,\ k=1,\cdots,T \qquad (1)$$

where $y(n,k)$ is the received data at the k-th carrier of the n-th receive antenna, $H[n,m](k)$ represents the fading coefficient at the k-th carrier of the spatial channel between the n-th receive antenna and the m-th transmit antenna, $x(m,k)$ denotes the element at m-th row and k-th column of one space-time codeword matrix $\boldsymbol{X}$, and $w(n,k)$ is the channel noise at the k-th carrier for the n-th receive antenna.

Substituting the sum term in (1) by its matrix form, we rewrite (1) as

$$y(n,k)=\mathbf{H}[:,n](k)\,\mathbf{x}(:,k)+w(n,k) \qquad (2)$$

where $\mathbf{x}(:,k)$ is the k-th column of space-time codeword $\boldsymbol{X}$, $\mathbf{H}[:,n](k)$ is the n-th column of the

MIMO channel fading coefficient matrix $\mathbf{H}(k)$ at *k*-th carriers, which can be delineated by (3).

Let $\mathbf{y}(:,n)$ denote the received signal vector at the *n*-th receive antenna in one MIMO OFDM word period, we can rewrite (2) into matrix form as (4), where $\mathbf{w}(:,n)$ is the channel white noise corresponding to $\mathbf{y}(:,n)$, and $\tilde{\mathbf{x}} = vec(\mathbf{X})$, where $vec(\cdot)$ denotes the column vector by stacking the columns of one matrix into one column vector.

Thus, in one MIMO OFDM word period, assembling the received signals from all the receive antennas into one matrix form, we can get

$$vec\left(\mathbf{y}^T\right) = \tilde{\mathbf{H}} vec\left(\mathbf{X}\right) + vec\left(\mathbf{w}^T\right) \quad (5)$$

where $\mathbf{y}=[\mathbf{y}(:,1)^T,\mathbf{y}(:,2)^T,\ldots,\mathbf{y}(:,N_r)^T]^T$, $\mathbf{w}=[\mathbf{w}(:,1)^T,\mathbf{w}(:,2)^T,\ldots,\mathbf{w}(:,N_r)^T]^T$, $\tilde{\mathbf{H}} = \left[\tilde{\mathbf{H}}(:,1)^T, \tilde{\mathbf{H}}(:,2)^T, \cdots \tilde{\mathbf{H}}(:,N_r)^T\right]^T$.

$$\mathbf{H}(k) = \begin{pmatrix} H[1,1](k) & H[1,2](k) & \cdots & H[1,N_t](k) \\ H[2,1](k) & H[2,2](k) & \cdots & H[2,N_t](k) \\ \vdots & \vdots & & \vdots \\ H[N_r,1](k) & H[N_r,2](k) & \cdots & H[N_r,N_t](k) \end{pmatrix} \quad (3)$$

$$\left[\mathbf{y}(:,n)\right]^T = \underbrace{\begin{pmatrix} \mathbf{H}[:,n](1) & & & \\ & \mathbf{H}[:,n](2) & & \\ & & \ddots & \\ & & & \mathbf{H}[:,n](T) \end{pmatrix}}_{\tilde{\mathbf{H}}(:,n)} \underbrace{\begin{pmatrix} \mathbf{x}(:,1) \\ \mathbf{x}(:,2) \\ \vdots \\ \mathbf{x}(:,T) \end{pmatrix}}_{\tilde{\mathbf{x}}} + \left[\mathbf{w}(:,n)\right]^T \quad (4)$$

3 FDFR ST Codes

Following the design of universal space time codes [14] and FDFR codes [15-16], the FDFR space-time codes in TDD/TDMA 4G systems can be detailed as followings.

3.1 Design of FDFR ST Codes

Assume the number of receive antennas to be larger than that of transmit antennas, and the length of data symbol block ($\mathbf{s}$) to be N_t^2. The block ($\mathbf{s}$) is further truncated into N_t sub-blocks ($\mathbf{s}_i$). Thus, the i-th sub-block ($\mathbf{u}_i$) of the output data vector ($\mathbf{u}$) for the LCF encoder, is achieved by the i-th sub-block ($\mathbf{s}_i$) of one data symbol block ($\mathbf{s}$) via one transformation matrix ($\mathbf{\Theta}_i$), i.e.,

$$\mathbf{u}_i = \mathbf{\Theta}_i \mathbf{s}_i, \quad i = 1, \cdots, N_t \quad (6)$$

where $\mathbf{\Theta}_i$ is one T dimensional complex square matrix in the form of

$$\mathbf{\Theta}_i = \beta^{i-1}\mathbf{\Theta} \quad (7)$$

Here, β is one scale factor and $\mathbf{\Theta}$ is one unitary Vander monde matrix.

Thus, full diversity can be achieved by the above space-time code with elaborately designed parameters, i.e., β and $\mathbf{\Theta}$. Furthermore, the unitary Vander monde matrix can be given as

$$\mathbf{\Theta} = \frac{1}{\sqrt{N_t}} \mathbf{F}_{N_t}^H diag\left[1, \alpha, \cdots, \alpha^{N_t - 1}\right] \quad (8)$$

where $\mathbf{F}_{N_t}$ is N_t points fast Fourier transform (FFT) matrix. Notice that in (8) the unitary Vander monde matrix is parameterized by a single parameter α.

Finally, the ST encoders assemble $\mathbf{u}_i$ ($i = 1, \cdots, N_t$) into space-time word ($\mathbf{c}$), i.e.,

$$\mathbf{c} = \begin{bmatrix} u_1(1) & u_{N_t}(2) & \cdots & u_2(N_t) \\ u_2(1) & u_1(2) & \cdots & u_3(N_t) \\ \vdots & \vdots & & \vdots \\ u_{N_t}(1) & u_{N_t-1}(2) & \cdots & u_1(N_t) \end{bmatrix} \quad (9)$$

Here, $u_i(n)$ delineate the n-th element of $\mathbf{u}_i$.

For the case that the number of transmit antennas is larger than that of receive antennas, N_r blocks ($\mathbf{s}_i$) are obtained via truncating $\mathbf{s}$ with N_r*N_t symbols, and the similar space-time encoding process is performed as shown above. According to the results [14], the FDFR space time codes can achieve the data rate min (N_t,N_r), consistent with system capacity under the same antennas configuration [8].

Clearly, the designed FDFR space time code is actually determined by the parameter pairs (α, β), and three different design methods are given as following.

Design A: Select α such that the minimum polynomial of over the field $\mathbb{Q}(j)$ has degree greater than or equal to N_t. Given α, choose β such that the minimum polynomial β^{N_t} in the field $\mathbb{Q}(j)\left(e^{j2\pi/N_t}\right)(\alpha)$ has degree greater than or equal to N_t. For example, when $N_t = 2^k$ ($k \in \mathbb{N}$), we select $\alpha = e^{j2\pi/(2N_t)}$, $\beta^{N_t} = e^{j2\pi/(4N_t^2)}$; When $N_t = 3$, we select $\alpha = e^{j\pi/9}$ and $\beta^{N_t} = e^{j\pi/54}$; when $N_t = 5$, we select $\alpha = e^{j\pi/25}$ and $\beta^{N_t} = e^{j\pi/250}$.

Design B: Fixing $\beta^{N_t} = \alpha$, select α such that the minimum polynomial α in the field $\mathbb{Q}(j)\left(e^{j2\pi/N_t}\right)$ has degree greater than or equal to N_t^2. When $N_t = 2^k$, $k \in \mathbb{N}$, we select $\mathbb{Q}(j)\left(e^{j2\pi/N_t}\right)$; when $N_t = 3$, we select $\alpha = e^{j\pi/54}$; when $N_t = 5$, we select $\alpha = e^{j\pi/250}$.

Design C: First, select α such that the minimum polynomial α in the field $\mathbb{Q}(j)$ has degree greater than or equal to N_t. Based on α, we can find one transcendental number in the field $\mathbb{Q}(j)\left(e^{j2\pi/N_t}\right)(\alpha)$. Alternatively, we can find a transcendental number α directly for the field $\mathbb{Q}(j)\left(e^{j2\pi/N_t}\right)$. When N_t is given, we select $\mathbf{\Theta}$ according to design A, then let $\beta^{N_t} = e^{j/2}$; for given N_t , we select $\beta^{N_t} = \alpha$ and $\alpha = e^{j/2}$.

Aiming at the antennas configuration of 8x2 at BS and MS respectively, the proposed FDFR ST coder for TDD/TDMA 4G systems, can be constructed according to design A, i.e., we select $\alpha = e^{j\pi/16}$, $\beta^{N_t} = e^{j\pi/256}$ for down links, and $\alpha = e^{j\pi/4}$, $\beta^{N_t} = e^{j\pi/16}$ for up links.

3.2 Decoding of FDFR ST Codes

Stacking the receive vectors into one vector, we can rewrite input–output relationship (5) into

$$\mathbf{y} = \left(\mathbf{I}_{N_t} \otimes \mathbf{H}\right)\begin{bmatrix} \mathbf{c}(:,1) \\ \vdots \\ \mathbf{c}(:,N_t) \end{bmatrix} + \mathbf{w} \tag{10}$$

Where $\otimes$ denotes the kronecker product and $\mathbf{c}(:,n)$ is the n-th column of space time code word **c**. Meanwhile, $\mathbf{c}(:,n)$ can be further rewrite into

$$\mathbf{c}(:,n) = \left[\left(\mathbf{P}_n\mathbf{D}_\beta\right)\otimes\boldsymbol{\theta}_n^T\right]\mathbf{s} \tag{11}$$

Here, the permutation matrix $\mathbf{P}_n$ and the diagonal matrix $\mathbf{D}_\beta$, are defined respectively as

$$\mathbf{P}_n = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{n-1} \\ \mathbf{I}_{N_t-n+1} & \mathbf{0} \end{bmatrix}, \mathbf{D}_\beta = diag\left[1, \beta, \cdots, \beta^{N_t-1}\right] \tag{12}$$

Where $\boldsymbol{\theta}_n^T$ denotes the n-th row of $\mathbf{\Theta}$.

Then, by defining $\boldsymbol{\mathcal{H}} = \mathbf{I}_{N_t} \otimes \mathbf{H}$ and the unitary matrix $\mathbf{\Phi}$, i.e.,

$$\mathbf{\Phi} = \begin{bmatrix} \left(\mathbf{P}_1\mathbf{D}_\beta\right)\otimes\boldsymbol{\theta}_1^T \\ \vdots \\ \left(\mathbf{P}_{N_t}\mathbf{D}_\beta\right)\otimes\boldsymbol{\theta}_{N_t}^T \end{bmatrix} \tag{13}$$

Thus, we can rewrite (10) into as

$$\mathbf{y} = \boldsymbol{\mathcal{H}}\mathbf{\Phi}\mathbf{s} + \mathbf{w} \tag{14}$$

According to (14), the maximum likelihood (ML) decoding can be employed to optimally detect the transmitted data symbols from the received signals at receivers but with high complexity, regardless of receive antenna numbers. The sphere decoding (SD) [17-19] or the semi-definite programming [20] algorithms can also be used to achieve near-optimal performance. The decoding complexity depends on the length of $\mathbf{s}$, i.e., $N = N_t^2$.

4 Performances of FDFR ST Coders

Designed to achieve full diversity and full rate, the proposed FDFR ST coders can gain the optimal diversity-multiplexing (DM) tradeoff, which will be verified by the following performance analysis. Furthermore, the proposed FDFR ST coder shows its consistency to the optimal DM tradeoff space time codes.

4.1 Asymptotic Information Lossless Design

Let $\mathbf{X}$ be one rate k/T, $N_t \times T$ design, and the system capacity of the equivalent systems in (14) can be given as

$$C_{\mathrm{X}}\left(N_t, N_r, \gamma, \mathbf{H}\right) = \frac{1}{T}\log\det\left(\mathbf{I}_N + \gamma\mathbf{\Phi}^H \mathrm{H}^H \mathrm{H}\mathbf{\Phi}\right) \tag{15}$$

Where γ is the average signal noise ratio (SNR), N delineates the length of input vector ($\mathbf{s}$) of ST coders, and $\mathbf{\Phi}$ is called the generation matrix for space time scheme (**X**) so that $vec(\mathbf{X}) = \mathbf{\Phi}\mathbf{s}$.

However, the ergodic capacity [21] of the MIMO systems in flat fading channels, is computed by

$$C\left(N_t, N_r, \gamma, \mathbf{H}\right) = \log\det\left(\mathbf{I}_{N_r} + \gamma\mathbf{H}\mathbf{H}^H\right) \quad (16)$$

One given space time design $\mathbf{X}$ is called as an asymptotic-information-lossless design, if it satisfies

$$\lim_{\gamma\to\infty}\frac{C\left(N_t, N_r, \gamma, \mathbf{H}\right)}{C_{\mathbf{X}}\left(N_t, N_r, \gamma, \mathbf{H}\right)} = 1 \quad (17)$$

That is to say, the design $\mathbf{X}$ is an information-lossless (ILL) design for N_r receive antennas if $C_{\mathbf{X}}\left(N_t, N_r, \gamma, \mathbf{H}\right) = C\left(N_t, N_r, \gamma, \mathbf{H}\right)$ for $\forall$ SNR.

Meanwhile, as pointed out in reference [19], AIIL design is the necessary conditions for one space time design to be an optimal diversity-multiplexing tradeoff design. So, the sufficient and necessary conditions can be given as

$$rank\left(\mathcal{H}\mathbf{\Phi}\right) \geq \min\left(N_t, N_r\right) \times T \quad (18)$$

Where $rank(\bullet)$ denotes the rank of one matrix. In the classical orthogonal designs for N_t transmit antennas, where one space time code word has to be transmitted in *T* symbol periods, the number of variables is not more than *T*. Thus, the orthogonal STBC codes can not achieve the optimal diversity-multiplexing tradeoff except for the Alamouti space time codes [13].

When one data symbol vector $\mathbf{s} \sim \mathrm{CN}\left(\mathbf{0}, \varepsilon_s / N_t \mathbf{I}_N\right)$ and $\varepsilon_{\mathbf{s}}$ is the given total transmit power, the system capacity achieved by the FDFR ST codes in (14), is given as

$$C_{FDFR}\left(N_t, N_r, \gamma, \mathbf{H}\right) \approx \log\det\left(\mathbf{I}_{N_r} + \gamma\mathbf{H}\mathbf{H}^H\right) \quad (19)$$

Here, $\gamma = \varepsilon_{\mathbf{s}} / \left(\varepsilon_w N_t\right)$ and ε_w denotes the power of the random noise in wireless channels. So, according to the above analysis, the FDFR ST codes can obtain the same system capacity as that in (16), and it's an AILL space time design.

4.2 Optimal Diversity-Multiplexing Tradeoff

As the results shown in literature [8, 22], the diversity-multiplexing curve is the essential property of MIMO wireless channels, and the limit achieved by one space time design. Thus, it can be used to measure the performance of one space time design by analyzing its difference to that achieved by the optimal space time codes.

4.2.1 Computing Diversity-Multiplexing Tradeoff

Firstly, according to (14), the capacity of equivalent system models is given for one specific space-time design. Subsequently, symbol error ratio (SER) under the rate $R = r\log_2\gamma$, is shown as

$$P_e\left(\gamma\right) \doteq P_{out,\mathbf{X}}\left(r\right) + P\left(error \mid no\ outage\right) \geq P_{out}\left(r\right) \quad (20)$$

Where $P_{out,\mathbf{X}}\left(r\right) = P\left(C_{\mathbf{X}}\left(N_t, N_r, \gamma\right) \leq r\log\gamma\right)$, $P_{out}\left(r\right) = P\left(C\left(N_t, N_r, \gamma\right) \leq R\right)$, and the second term in (20) denotes the SER caused completely by channel white noise. However, the operator $\doteq$ is called exponential equality, which can be defined as followings.

For two given functions, i.e., $f\left(\gamma, r\right)$ and $g\left(\gamma, r\right)$, if they satisfy

$$\lim_{\gamma\to\infty}\frac{\log f\left(\gamma, r\right)}{\log\left(\gamma\right)} = \lim_{\gamma\to\infty}\frac{\log g\left(\gamma, r\right)}{\log\left(\gamma\right)} \quad (21)$$

The relation can be denoted as $f\left(r\right) \doteq g\left(r\right)$. Similarly, we use $\dot{\leq}$ and $\dot{\geq}$ for exponential inequalities.

Clearly, when the rate is $R = r\log_2\gamma$, the channel capacity outage probability ($P_{out,\mathbf{X}}\left(r\right)$) for the design ($\mathbf{X}$) is greater than that of system capacity ($P_{out}\left(r\right)$), so the following relationships can be obtained according to (20).

$$d_{\mathbf{X}}\left(r\right) \leq d_{out,\mathbf{X}}\left(r\right) \leq d_{out}\left(r\right) \quad (22)$$

Where $P_{out,\mathbf{X}}\left(r\right) \doteq \gamma^{-d_{out,\mathbf{X}}(r)}$, $P_{out}\left(r\right) \doteq \gamma^{-d_{out}(r)}$ and the tradeoff of design ($\mathbf{X}$) is bounded by the optimal tradeoff curve.

Finally, the limitation of the logarithmic ratio between system SER and SNR is regarded as the tradeoff curve achieved by the space time design ($\mathbf{X}$).At high SNR, the SER caused by channel white noise can be ignored, and the tradeoff curve can be simply inferred.

4.2.2 Relative Lemma

As the base of tradeoff analysis, three lemmas are detailed as followings, and their proof can be found in literatures [8, 14].

Lemma 1: Assume γ denotes the average SNR at every receive antenna and $r \leq \min\left(N_t, N_r\right)$. When one data rate is given as $R = r\log_2\gamma$, the outage probability of system capacity satisfies

$$P_{out}\left(r\right) \doteq \gamma^{-d_{out}(r)} \quad (23)$$

Where $d_{out}\left(r\right) = d^*\left(r\right)$ is the optimal tradeoff of MIMO wireless systems. That is to say, when one space time design ($\mathbf{X}$) can achieve the same system capacity as shown in (16) under the data rate of $R = r\log_2 \gamma$, it will achieve the optimal tradeoff curves for MIMO systems.

Lemma 2: For the system MIMO system models as (5), when data rate is $R = r\log_2 \gamma$, the symbol error probability for arbitrary space time design is bounded by

$$P_e(\gamma) \dot{\geq} \gamma^{-d_{out}(r)} \quad (24)$$

Lemma 3: Let $\mathbf{R}$ is one $m \times n$ dimensional matrix, whose entries are independent with the same distributions $\mathrm{CN}(0,1)$ and $m > n$. If $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_n$ are the nonzero ordered singular values of $\mathbf{R}^H\mathbf{R}$, their joint distribution probability density functions is given as

$$p(\mu_1, \cdots, \mu_n) = K_{m,n}^{-1} \prod_{i=1}^{n} \mu_i^{m-n} \prod_{i<j} (\mu_i - \mu_j)^2 e^{-\sum_i \mu_i} \quad (25)$$

Where $K_{m,n}$ is one normalized constant. Meanwhile, define

$$\alpha_i = -\log \mu_i / \log SNR \quad \forall i \in [1, \cdots, n] \quad (26)$$

The joint probability density functions of random vector $\boldsymbol{\alpha} = [\alpha_1, \cdots, \alpha_n]$ are given by

$$p(\boldsymbol{\alpha}) = K_{m,n}^{-1} (\log SNR)^n \prod_{i=1}^{n} SNR^{-(m-n+1)\alpha_i}$$
$$\prod_{i<j} \left(SNR^{-\alpha_i} - SNR^{-\alpha_j} \right)^2 \exp\left[-\sum_{i=1}^{n} SNR^{-\alpha_i} \right] \quad (27)$$

4.2.3 Tradeoff of FDFR ST Codes

Assume the inputted data symbols to one space time encoder to be independent with the same distributions. For the data rate $R = r\log_2 \gamma$ and one space time design lossless scheme, the outage probability is equal to the outage probability of systems without space time codes.

Thus, the symbol error probability of the FDFR ST codes can be given by

$$P_e(\gamma) = P_{out}(r) P(error \mid outage) + P(error, no\ outage)$$
$$\leq P_{out}(r) + P(error, no\ outage) \quad (28)$$

Where $P_{out}(r)$ denotes the channel outage probability of MIMO systems at $R = r\log_2 \gamma$ without space time codes. Clearly, the upper bound of the second term can be obtained by its consistent bound. If $\mathbf{X}(0)$ and $\mathbf{X}(1)$ are the two transmitted space time code words, $\Delta\mathbf{X} = \mathbf{X}(1) - \mathbf{X}(0)$ denotes their difference matrix. When $\mathbf{X}(0)$ is the transmitted word, the error probability of received $\mathbf{X}(1)$, i.e., the pairwise error probability (PER) of the FDFR ST codes is described as

$$P(\mathbf{X}(0) \to \mathbf{X}(1) \mid \mathbf{H}) = P\left(\gamma \left\| \frac{1}{2} \mathbf{H}(\Delta\mathbf{X}) \right\|_F^2 \leq \|\mathbf{w}\|^2 \right) \quad (29)$$

Where $\mathbf{w}$ is the white Gaussian noise at direction $\mathbf{H}\Delta\mathbf{X}$ with variance 1/2.

In order to simplify the right side of (29), let λ_i , $i = 1, \cdots, \min(N_t, N_r)$ denote the nonzero singular values of $\mathbf{H}\mathbf{H}^H$, $\Delta\mathbf{x}_i \in C^T$ be the *i*-th row of $\Delta\mathbf{X}$, C^T be the *T*-dimension complex space. In respect of the equal orientation $\Delta\mathbf{X}$, i.e., after a unitary transformation, its distribution is kept unchanged. Thus, the following expression can be achieved,

$$\left\| H(\Delta X) \right\|_F^2 \xrightarrow{d} \sum_{i=1}^{\min\{N_t, N_r\}} \lambda_i \|\Delta\mathbf{X}\|^2 \quad (30)$$

Here $\xrightarrow{d}$ denotes its two sides have the same distributions. Now, consider

$$P\left(\gamma \left\| \frac{1}{2} \mathbf{H}(\Delta\mathbf{X}) \right\|_F^2 \leq 1 \right) = P\left(\sum_{i=1}^{\min\{N_t, N_r\}} \lambda_i \|\Delta\mathbf{X}\|^2 \leq 4\gamma^{-1} \right) \quad (31)$$

Its upper and low bounds can be described as

$$P\left(\lambda_i \|\Delta x_i\|^2 \leq \frac{4\gamma^{-1}}{\min(N_t, N_r)}, i = 1, \cdots, \min(N_t, N_r) \right)$$
$$\leq P\left(\sum_{i=1}^{\min(N_t, N_r)} \lambda_i \|\Delta\mathbf{x}_i\|^2 \leq 4\gamma^{-1} \right)$$
$$\leq P\left(\lambda_i \|\Delta\mathbf{x}_i\|^2 \leq 4\gamma^{-1}, i = 1, \cdots, \min(N_t, N_r) \right) \quad (32)$$

Clearly, its upper and low bounds have the same SNR exponents. So

$$P\left(\gamma \left\| \frac{1}{2} \mathbf{H}(\Delta\mathbf{X}) \right\|_F^2 \leq 1 \right)$$
$$\doteq P\left(\lambda_i \|\Delta\mathbf{x}_i\|^2 \leq 4\gamma^{-1}, i = 1, \cdots, \min(N_t, N_r) \right)$$
$$= P\left(\|\Delta\mathbf{x}_i\|^2 \leq 4(\gamma\lambda_1)^{-1}, i = 1, \cdots, \min(N_t, N_r) \right) \quad (33)$$

When $\lambda_i \geq \gamma^{-1}$, one could get

$$P\left(\|\Delta\mathbf{x}_i\|^2 \leq (\gamma\lambda_i)^{-1}, \forall i \right) \doteq \prod_{i=1}^{\min(N_t, N_r)} (\gamma\lambda_i)^{-T} \quad (34)$$

When $\lambda_i < \gamma^{-1}$, one obtains

$$P\left(\left\|\Delta \mathbf{x}_i\right\|^2 \leq \left(\gamma\lambda_i\right)^{-1}\right) \doteq \gamma^0 \quad (35)$$

Combining (34) and (35), one obtains the following result

$$P\left(\gamma\left\|\frac{1}{2}\mathbf{H}(\Delta\mathbf{X})\right\|_F^2 \leq 1\right) \doteq \prod_{i=1}^{\min(N_t,N_r)}\left(\min\left(1,\gamma\lambda_i\right)\right)^{-T} \quad (36)$$

Furthermore, when $\gamma\left\|\frac{1}{2}\mathbf{H}\left(\Delta\mathbf{X}\right)\right\|_F^2 \leq 1$, there always exits $P\left(\left\|w\right\|^2 > 1\right) > 0$, which leads to symbol detection errors. So, the error probability can be written in the form

$$P\left(\mathbf{X}(0)\to\mathbf{X}(1)\,|\,\mathbf{H}\right) \doteq P\left(\gamma\left\|\frac{1}{2}\mathbf{H}(\Delta\mathbf{X})\right\|_F^2 \leq 1\right) \quad (37)$$

Following the tail form of the normal distribution [17], i.e.,

$$Q(t) \leq \frac{1}{2}e^{-t^2/2}$$

Then, (37) is simplified by

$$P\left(\mathbf{X}(0)\to\mathbf{X}(1)\,|\,\mathbf{H}\right) \leq \exp\left(-\frac{\gamma}{4}\left\|\mathbf{H}\Delta\mathbf{X}\right\|^2\right) \quad (38)$$

Now, according to the space time codeword (**X**), we average (38) to achieve the PER for the TDD/TDMA 4G systems with one space time design (**X**).

$$P\left(\mathbf{X}(0)\to\mathbf{X}(1)\,|\,\mathbf{H}\right) \leq \det\left(I+\frac{\gamma}{2}\mathbf{H}\mathbf{H}^H\right)^{-T} \quad (39)$$

When data rate is $R = r\log_2\gamma$ (b/symbol), there are γ^{rT} space time code words for the FDFR ST codes, and the lower bound of $P(error, no\,outage)$ can be deduced by the following consistent bounds of PER in (39), i.e.,

$$P\left(error\,|\,\mathbf{H}\right) \leq \gamma^{rT}\det\left(I+\frac{\gamma}{2}\mathbf{H}\mathbf{H}^H\right)^{-T}$$
$$= \gamma^{rT}\prod_{i=1}^{\min(N_t,N_r)}\left(1+\frac{\gamma}{2}\lambda_i\right)^{-T} \quad (40)$$

Assume that $\lambda_i = \gamma^{-\alpha_i}$, $i = 1,\cdots,\min\left(N_t,N_r\right)$ are the nonzero singular values of $\mathbf{H}$, (40) is rewritten as

$$P\left(error\,|\,\boldsymbol{\alpha}\right) \dot{\leq} \gamma^{-T\left[\sum(1-\alpha_i)^+ - r\right]} \quad (41)$$

Here, the operator $(\cdot)^+$ denotes $\max\left(0,\cdot\right)$. Following lemma 3, we have

$$P\left(error, no\,outage\right) = \int_{(A')^c} p\left(\boldsymbol{\alpha}\right)P\left(error\,|\,\boldsymbol{\alpha}\right)d\boldsymbol{\alpha}$$

$$\dot{\leq}\int_{(A')^c} p\left(\boldsymbol{\alpha}\right)\gamma^{-T\left[\sum(1-\alpha_i)^+ - r\right]}d\boldsymbol{\alpha} \quad (42)$$

Here, $\left(\mathcal{A}'\right)^c$ is the complement of events when the FDFR ST code is outage, and it is defined as

$$A' = \left\{\boldsymbol{\alpha}\in R^{\min(N_t,N_r)+}\,|\,\alpha_1 \geq\cdots\geq\alpha_{\min(N_t,N_r)} \geq 0 \;\&\; \sum_i\left(1-\alpha_i\right)^+ < r\right\} \quad (43)$$

Thus, we further have

$$P\left(error, no\,outage\right)$$
$$\dot{\leq}\int_{(A')^c}\gamma^{-\sum_u(|N_t-N_r|+2i-1)\alpha_i}\gamma^{-T\left[\sum(1-\alpha_i)^+ - r\right]}d\boldsymbol{\alpha}$$
$$\doteq\int_{(A')^c}\gamma^{-d_G(r,\boldsymbol{\alpha})}d\boldsymbol{\alpha} \quad (44)$$

Where

$$d_G\left(r,\boldsymbol{\alpha}\right) = \sum_{i=1}^{\min\{N_t,N_r\}}\left(2i-1+\left|N_t-N_r\right|\right)\alpha_i$$
$$+T\left(\sum_{i=1}^{\min\{N_t,N_r\}}\left(1-\alpha_i\right)^+ - r\right)$$

As the error probability mainly depends on $\boldsymbol{\alpha}_{opt}$, which can maximize $d_G\left(r,\boldsymbol{\alpha}\right)$, so we have

$$P\left(error, no\,outage\right) \dot{\leq} \gamma^{-d_G(r)} \quad (45)$$

Here $d_G\left(r\right) = \min_{\boldsymbol{\alpha}\in\mathcal{A}'} d_G\left(r,\boldsymbol{\alpha}\right)$.

Finally, submitting (45) into (28), we get the system error probability, i.e.,

$$P_e\left(\gamma\right) \doteq \gamma^{-d_{out}(r)} + \gamma^{-d_G(r)} \dot{\leq} \gamma^{-d_{out}(r)} \quad (46)$$

This is the upper bound of the above error probability, and we obtain $P_e\left(\gamma\right) = \gamma^{-d_{out}(r)}$ by combining (46) and (24). Consequently, the tradeoff of the FDFR ST codes can be obtained

$$d_{FDFR}\left(r\right) = d_{out}\left(r\right) = d^*\left(r\right) \quad (47)$$

From the results in (47), the proposed FDFR ST codes can achieve the optimal tradeoff under the case of TDD/TDMA 4G systems. Furthermore, the flexibility of FDFR designs can also achieve the performance-rate-complexity tradeoffs.

5 Numerical Results

This section presents some numerical results to illustrate the SER performance of the proposed FDFR space time designs, when the TDD/TDMA 4G systems in wireless cellular environments were considered. Equipped with 8x2 antennas at BS and MS respectively, the TDD/TDMA 4G systems has 20MHz bandwidth at carrier frequency of 4.5 GHz and 2048 carriers, where 1/4 OFDM word is used as guard interval. Furthermore, several space time codes are also evaluated to verify the performance of the FDFR ST designs, such as the CDA ST codes [12-13], the Tilted-QAM scheme [24-25] for two transmit antennas, the D-BLAST with optimal tradeoff [26], the Alamouti ST codes [20] for up links and the STBC8 scheme[27] for down links. As the Titled-QAM and the D-BLAST can only achieve full rate in up links and down links of TDD/TDMA 4G systems respectively, they are implemented only in the corresponding up and down links. Under the spatially uncorrelated ITU channels [28], we evaluate the system SER at different data rate and SNR.

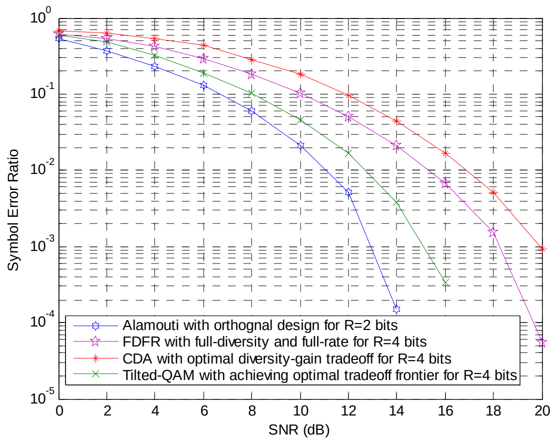

Fig.2 Symbol error probability of FDFR ST codes, CDA ST codes, Tilted-QAM and Alamouti codes for up links in TDD/TDMA 4G systems, where the date rate is 2bits/pcu for Alamouti codes and 4bits/pcu for other ST codes.

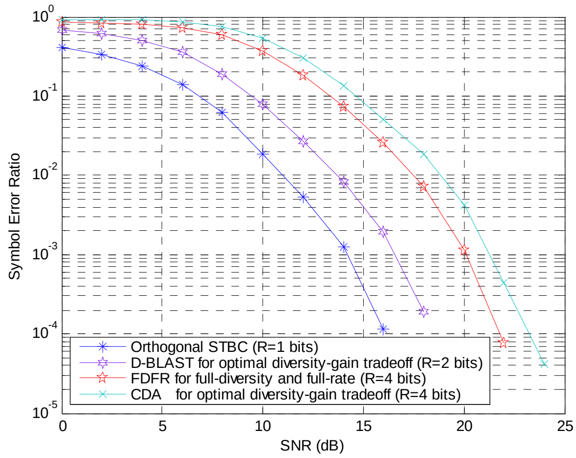

Fig.3 Symbol error probability of FDFR ST codes, CDA ST codes, D-BLAST and STBC8 codes for down links in TDD/TDMA 4G systems, where the date rate is 1bits/pcu for STBC8 codes, 2bits/pcu for D-BLAST and 4bits/pcu for other ST codes.

Firstly, when different space time schemes are used, the TDD/TDMA 4G systems are tested with QPSK modulation under ITU pedestrian A channel profiles [28]. The SER results for up and down links are shown in Fig.2 and Fig.3 respectively, where the data rate is 4bits per channel use (bits/pcu) for three full rate space codes, 2bits/pcu and 1bits/pcu for the STBC codes in the corresponding up and down links, respectively. At high SNR, the approximate slopes of SER curves in Fig.2 and Fig.3 show that these space time codes can achieve approximate diversity gains, i.e., the maximal diversity 16. The coherence among different transmit antennas would deteriorate the performance of these full rate ST codes, as the full rate ST codes are not orthogonal codes. Thus, without orthogonality, the full rate ST codes decrease the minimal distance of data symbols, and result in the reduction of anti-noise performance and the increase of self-interference. Therefore, this reflects that the Tilted-QAM has better performance, the FDFR ST design has inferior performance, and CDA ST codes has bad results, as disclosed in Fig.2 and Fig.3.

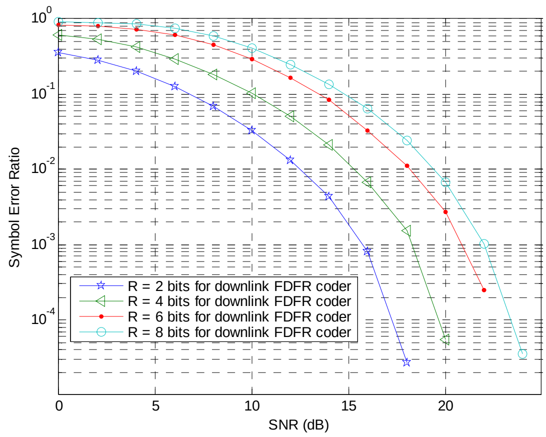

Fig.4 Symbol error probability of FDFR space time design at data rates of 2, 4, 6, 8bits/pcu for up links in TDD/TDMA 4G cellular scenarios, where the typical pedestrian A channel profiles are used.

Subsequently, the SER performance of the FDFR ST code is evaluated at data rate of 2, 4, 6, and 8bits/pcu respectively. Their SER curves are compared in Fig.4 and Fig.5 for up and down links respectively. As displayed in these figures, the performance of up links is better than that of down links, for the reason that the inference of different antennas for down links in TDD/TDMA 4G systems is greater than that for up links. This also indicates the inference of transmit antennas should be deal with properly in MIMO systems.

Then, in order to validate the FDFR ST code and the CDA ST code in cellular environments, their SER results with TDD/TDMA 4G systems at up and down links are revealed at date rate 4bits/pcu in Fig.6 and Fig.7 respectively, where the Doppler frequency for ITU indoor channel A, pedestrian channel A and vehicle channels are given as 0, 100 and 500Hz. Clearly, their performance is hardly effected on by channel profiles. Furthermore, the FDFR ST codes have better performance than the CDA ST codes in up links, while they have approximate performance in down links.

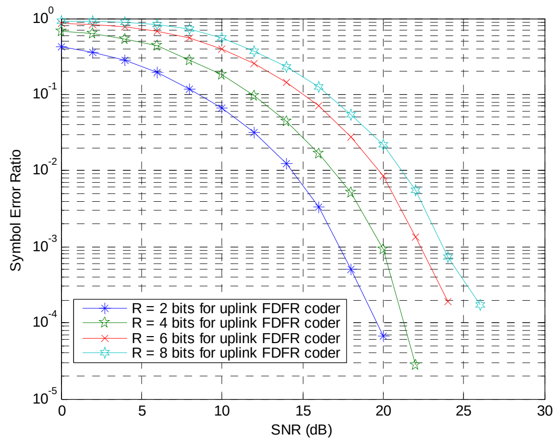

Fig.5 Symbol error probability of FDFR space time design at data rates of 2, 4, 6, 8bits/pcu for down links in TDD/TDMA 4G cellular scenarios, where the typical pedestrian A channel profiles are used.

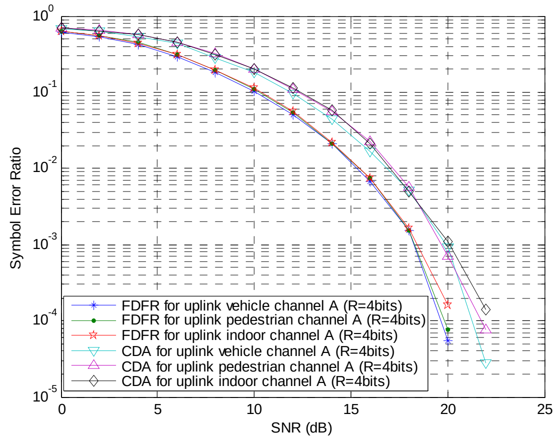

Fig.6 Symbol error probability of FDFT and CDA ST codes with data rate 4bits/pcu in up links of TDD/TDMA 4G cellular channel scenarios, where the typical indoor, pedestrian and vehicle channel profiles are used respectively.

Finally, we test the system SERs at data rate R=rlog$_2$(SNR) for the FDFR and CDA ST codes, where their multiplexing gains are given as 0.5, 1, 1.5 and 2 respectively. As described in Fig.8 and Fig.9 for up links and down links respectively, the slopes of SER curves at different multiplexing gains of FDFR and CDA ST codes can approximate the corresponding given multiplexing gains. This also verifies that the FDFR and CDA ST codes can achieve the optimal tradeoff in TDD/TDMA 4G systems. Meanwhile, the difference inference among transmit antennas in up and down links, is also reflected in Fig.8 and Fig.9, i.e., the performance of up links is better than down links.

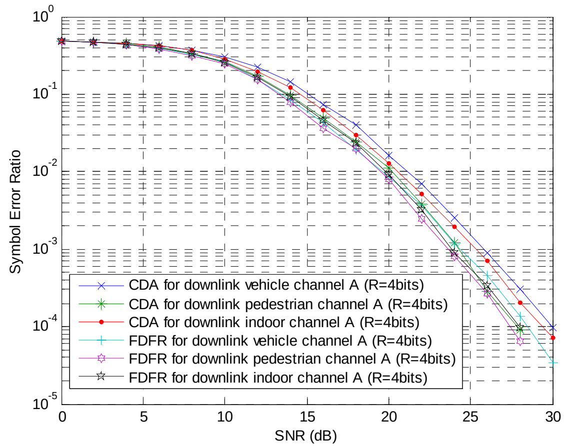

Fig.7 Symbol error probability of FDFT and CDA ST codes with data rate 4bits/pcu in down links of TDD/TDMA 4G cellular channel scenarios, where the typical indoor, pedestrian and vehicle channel profiles are used respectively.

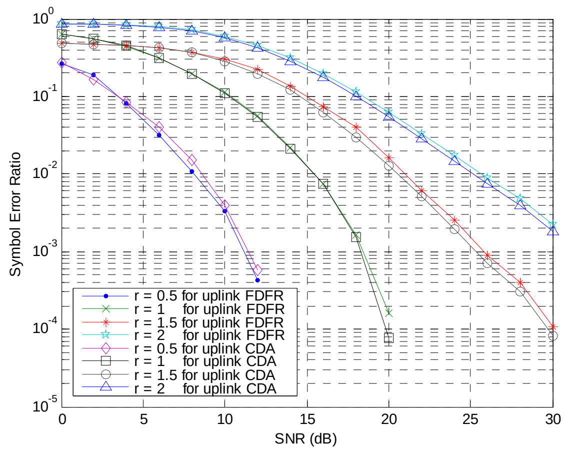

Fig.8 Symbol error probability of FDFT and CDA ST codes at different data rate R=rlog2(SNR) in up links of TDD/TDMA 4G systems, where the value r is given for 0.5, 1, 1.5 and 2 respectively.

In TDD/TDMA 4G systems, these numerical observations clearly validate that the FDFR ST codes can achieve the optimal tradeoff and have better performance than the CDA ST codes and other sub-optimal space time schemes. These sub-optimal space time codes, such as Tilted-QAM and D-BLAST, only fit for special number transmit antennas or receive antennas. Although the SER performance of the FDFR space time codes is inferior to that of STBC space time schemes, its throughput can compensate its performance loss. This fit for the cases of great throughput but low performance requirements.

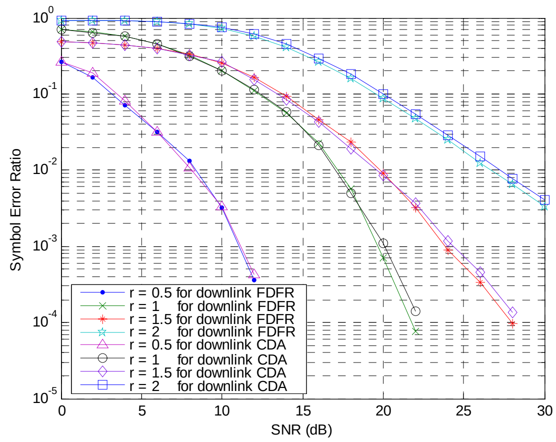

Fig.9 Symbol error probability of FDFT and CDA ST codes at different data rate R=rlog2(SNR) in down links of TDD/TDMA 4G systems, where the value r is given for 0.5, 1, 1.5 and 2 respectively.

6 Conclusions

For the wireless communication systems with multiple antennas, how to achieve the optimal diversity multiplexing tradeoff with full rate full diversity is one great challenge to be met in space time designs. Following the FDFR ST designs given recent literatures, we consider its implementation in TDD/TDMA 4G cellular communication system. The FDFR design is proved in this paper to be an asymptotic information lossless design, and can achieve the optimal tradeoff. In typical cellular wireless channel scenarios, several different space time schemes are numerically simulated to test the performance of the proposed FDFR space time design. Numerical results show the performance of the FDFR ST scheme is consistent with corresponding theoretic analysis.

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