# OFDM System under the Combined Effect Analysis of the Phase Noise, Carrier Frequency Offset, Doppler Spread, and Amplifier Nonlinearity of the SEL

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*Abstract:* - In this paper, we provide an exact closed-form expression of the effective signal-to-noise ratio (SNR) for orthogonal frequency division multiplexing (OFDM) performance under the combined effect of the phase noise, frequency offset, and amplifier nonlinearities of the soft envelope limiter (SEL). A simplified approximate expression is also obtained for the effective SNR under certain conditions. This formula describes explicitly the various parameters with the system performance. After deriving the effective SNR, we evaluate the bit-error-rate (BER) for slowly fading Rayleigh channel for QPSK and 16-QAM modulation formats. We also obtain the minimum total degradation (TD) versus phase noise rate for different values of frequency offset rate as a criterion of performance evaluation. We show that the degradation is increased as the phase noise levels and frequency offset levels are increased. For 16-QAM modulation format, we have obtained even worse results compared with QPSK modulation format.

*Key-Words:* - OFDM, Phase Noise, CFO, Doppler Spread, and Nonlinear Amplifier.

## 1 Introduction

Orthogonal frequency division multiplexing (OFDM) is considered a practical scheme to combat multipath channel fading (eg.[1],). OFDM is adopted OFDM has been adopted as the basis in wire and wireless communications, such as high-bit-rate digital subscriber lines (HDSL), digital audio broadcasting (DAB),terrestrial digital video broadcasting (DVB-T), a standard for wireless local networks (WLANs), such as IEEE 802.11, HIPERLAN II. It is also used for forth generation and IEEE 802.16 (WiMax) Broadband Fixed Wireless Access (BFWA) channel [2], [3].

However, OFDM has some disadvantages. One of the main disadvantages is the distortion generated by the high power amplifier (HPA) at the transmitter end. This imperfection leads to nonlinear distortion (NLD).Other source of impairments are the effect of the phase noise which is a random process caused by the fluctuation of the receiver and the transmitter oscillators. Other source of impairment is the frequency offset which is due to the deviation between the transmitter and the receiver, or by Doppler shifts which is known as Doppler spread. Phase noise, carrier frequency offset (CFO), and Doppler spread destroy the orthogonalities among the subcarriers. There are two effects of phase noise, CFO, and Doppler spread on the subcarriers: common error interference (CEI) and intercarrier interference (ICI). CEI does not change within an OFDM symbol period while ICI introduces interferences to any subcarriers of a certain symbol from all other subcarriers of that symbol.

Several studies have been proposed to analyze the effect of amplifier nonlinearities [4]-[11], the effect of phase noise [12]-[23], the effect of CFO [12],[24]-[26], and the effect of the Doppler spread [27]-[33] separately. However, few papers deal with the combined effect of these problems such as the studies of [34] and [35]. The joint effect of both amplifier nonlinearity and the phase noise in M-QAM OFDM system was analyzed in [34] while the combined effect of both phase noise and CFO was analyzed in [35]. However, the previous two studies [34], [35] do not provide, even for additive white Gaussian noise (AWGN) channel, a closed-form analytical expression that shows the exact quantitative relations between various parameters and system performance. In this paper, we derive a closed-form formula of the effective SNR of the combined effect analysis of phase noise, CFO, Doppler spread, and amplifier nonlinearity on the OFDM system performance. This expression is provided as a function of various system parameters that describe these parameters with the system performance.

The paper is organized as follows. In section 2, the system model is presented including nonlinear amplifier model, phase noise model, Doppler spectrum model, channel model, and OFDM system model. In section 3 , performance analysis is provided by evaluating the exact effective SNR, the approximations of the desired power, interference powers, and effective SNR , then, the BER and the TD of the system. The numerical results of the system performance are presented in section 4. Finally, the paper is discussed and concluded in section 5.

# 2 System Model

## 2.1 Nonlinear amplifier model

The nonlinear distortion at the transmitter causes some interference both inside and outside the signal bandwidth [6],[7], and [9]. In this paper, we concentrate our attention on the in-band interference of the SEL only. The AM/AM (amplitude modulation / amplitude modulation) function is given by

$$A(r) = \begin{cases} r, r \le A_s \\ A_s, r \ge A_s \end{cases} \quad (1)$$

where $A_s$ is the input amplitude of the maximum amplifier output power.

The analysis can be applied to other kinds of power amplifiers such as traveling wave tube amplifier (TWTA) and solid-state power amplifier (SSPA).

## 2.2 Phase noise model

Phase noise, $\phi(t)$ generated at both transmitter and receiver oscillators, can be described (eg. [36], [37], and [38]) as a continuous-path Brownian motion ( or Wiener process ) with zero mean and variance $\sigma_\phi^2$ which is given by

$$\sigma^2_{\phi_{Wiener}} = 2\pi\beta t \quad (2)$$

where the parameter $\beta$ represents the two-sided 3-dB linewidth of Lorentzian power spectral density of the local oscillator. This model is used when the local oscillator (LO) is frequency-locked (i.e. a free-running oscillator) [15], [20]-[23]. The Wiener process is nonstationary and infinite-power since $\phi(t)$ can grow without limit. This process has a Gaussian distribution. Although the Wiener model is nonstationary model, the autocorrelation function of the phase noise is stationary (see the Appendix).

In principle, the phase noise of both transmitter and receiver oscillators should be accounted for. However, in practice, the oscillator used in the transmitter is sufficiently stable to disregard its phase noise. So, in this paper, we analyze the effect of receiver phase noise only without loss of generality [35].

When the LO is phase-locked, phase-locked loop (PLL) model is used which is stationary model and has finite-power to analyze the effect of the phase noise [13],[19], and [22].

## 2.3 Doppler spectrum model

We have various spectrum models to describe the effect of the Doppler spread. These models are the uniform model, classical (Jakes) model, first-order Butterworth model, Gaussian model, and Rice model [29],[32], [39], and [40].

In this work, we use the worst extreme case of the Doppler spectrum. It is the two-path model [27], [29], and [32] where its autocorrelation function is given by

$$R_D(\tau) = \cos(2\pi f_d \tau) \quad (3)$$

where $f_d$ is the maximum Doppler frequency**.**

It should be noted that the two-path model corresponds to an OFDM with a fixed frequency offset of $f_d$. So, this model can be used to analyze the effect of both CFO and Doppler spread.

## 2.4 Channel model

We assume a multipath fading channel which is modeled in the time domain by M delayed impulses

$$h(t,\tau) = \sum_{r=0}^{N_p-1} \alpha_r(t)\delta(\tau - \frac{\tau_r(t)T}{N}) \quad (4)$$

Where $N_p$ is the number of paths and $\delta(\tau)$ is a Dirac delta function. $\alpha_r(t)$ and $\tau_r(t)$ are the attenuation coefficients and delay times of the rth path, respectively. $N$ is the number of the OFDM subcarriers and $T = T_s - T_g$ is the duration of the useful OFDM symbol where $T_s$ is the transmitter OFDM symbol and $T_g$ is the duration of the guard interval, respectively. $\{\alpha_r(t)\}_{r=0}^{N_p-1}$ are modeled as zero-mean complex Gaussian random variables. Also, $\{\tau_r(t)\}_{r=0}^{N_p-1}$ are modeled as independent uniform random variables in $(o, T_g)$ where the parameter t can be omitted in a slow fading channel. For arbitrary symbol of an N-subcarrier OFDM system, the corresponding channel gains in the frequency domain are expressed by $\{H_k\}_{k=0}^{N-1}$ with autocorrelation function $E\left(|H_k|^2\right) = 1$ , as given in [20].

### 2.5 OFDM system model

Due to the presence of the amplifier nonlinearities, phase noise, and the frequency offset the OFDM transmitted signal is given by ( eg. [20], [26]. [32], and [34] )

$$s(t) = \alpha e^{j2\pi\Delta ft} \sum_{k=0}^{N-1} a_k e^{j\phi(t)} e^{j2\pi k\frac{t}{T}} + \sum_{k=0}^{N-1} D_k e^{j2\pi\Delta ft} e^{j\phi(t)} e^{j2\pi k\frac{t}{T}} \tag{5}$$

where $\alpha$ is the complex gain factor due to the amplifier nonlinearity and $\Delta f$ is the frequency offset which is equal to $f_d + f_0$ where $f_d$ is the Doppler spread and $f_0$ is the CFO. $a_k$ is the data symbol which is assumed to be zero-mean random variable with variance $E_s = E\left(|a_k|^2\right)$ where $E_s$ is the transmitted energy per data symbol and $E(.)$ denotes the expectation of the argument. $D_k$ is the NLD due to HPA with variance $\sigma_D^2$.

The detected values at the input of a decision element is given by

$$Z_k = \alpha I_0 a_k H_k + I_0 D_k H_k + \sum_{r=0, r\neq k}^{N-1} I_{k-r}(\alpha a_r + D_r) H_r + N_k \tag{6}$$

Where $I_k$ is given by

$$I_k = \frac{1}{T}\int_0^T e^{j2\pi\Delta ft} e^{j\phi(t)} e^{-j2\pi k\frac{t}{T}} dt \tag{7}$$

Equation (6) contains four components. The first component is the desired signal where $I_0$ is the common error (CE) due to the presence of phase noise and frequency offset. The second component is the nonlinear interference (NLI) signal. The third component is the (ICI) component and the last component is an AWGN component.

## 3 Performance analysis

### 3.1 Exact effective SNR derivation

In order to derive the effective SNR, it is convenient to separate the CE in its mean value and in its varying part [17], [26]. So, (6) will be

$$Z_k = \alpha E(|I_0|) a_k H_k + \alpha a_k (I_0 - E(|I_0|)) + I_0 D_k H_k + \sum_{r=0, r\neq k}^{N-1} I_{k-r}(\alpha a_r + D_r) H_r + N_k \tag{8}$$

where the second term is termed now, common error interference (CEI). Hence, the powers of the desired signal, CEI, NLI, ICI and AWGN is given by ( see the Appendix )

$$P_{des} = E_s|\alpha|^2 E^2(|I_0|)$$

$$= E_s|\alpha|^2 e^{-\frac{\sigma_\phi^2}{2}} \frac{\cos^2(\pi\Delta fT)\sinh^2(\sigma_\phi^2/2) + \sin^2(\pi\Delta fT)\cosh^2(\sigma_\phi^2/2)}{\frac{\sigma_\phi^4}{16} + \pi^2(\Delta f)^2 T^2} \tag{9}$$

where $\sin c(x) \equiv \sin(\pi x)/\pi x$.

$$P_{CEI} = E_s|\alpha|^2 Var(I_0) = E_s|\alpha|^2 [\frac{\sigma_\phi^2}{\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2 T^2} - \frac{2(\frac{\sigma_\phi^2}{4} - 4\pi^2(\Delta f)^2 T^2)}{\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2 T^2} + 2e^{-\frac{\sigma_\phi^2}{2}} \times \frac{(\frac{\sigma_\phi^4}{4} - 4\pi^2(\Delta f)^2 T^2)\cos(2\pi\Delta fT) - 2\pi\Delta fT\,\sigma_\phi^2 \sin(2\pi\Delta fT)}{(\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2 T^2)^2} - e^{-\frac{\sigma_\phi^2}{2}} \times \frac{\cos^2(\pi\Delta fT)\sinh^2(\sigma_\phi^2/2) + \sin^2(\pi\Delta fT)\cosh^2(\sigma_\phi^2/2)}{\frac{\sigma_\phi^4}{16} + \pi^2(\Delta f)^2 T^2} \tag{10}$$

$$P_{NLI} = E\left(|I_0 D_k|^2\right) = \sigma_D^2 [\frac{\sigma_\phi^2}{\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2 T^2} - \frac{2(\frac{\sigma_\phi^2}{4} - 4\pi^2(\Delta f)^2 T^2)}{\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2 T^2} + 2e^{-\frac{\sigma_\phi^2}{2}} \times \frac{(\frac{\sigma_\phi^4}{4} - 4\pi^2(\Delta f)^2 T^2)\cos(2\pi\Delta fT) - 2\pi\Delta fT_s\sigma_\phi^2 \sin(2\pi\Delta fT)}{(\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2 T^2)^2}] \tag{11}$$

$$P_{ICI} = E\left(\left|\alpha a_r + D_r\right|^2 \sum_{r=0,r\neq k}^{N-1}\left|I_{k-r}\right|^2\right) = (E_s|\alpha|^2 + \sigma_D^2)$$

$$\times$$

$$[1 - \frac{\sigma_\phi^2}{\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2T^2} + \frac{2(\frac{\sigma_\phi^2}{4} - 4\pi^2(\Delta f)^2T^2)}{\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2T^2} - 2e^{-\frac{\sigma_\phi^2}{2}} \times$$

$$\frac{(\frac{\sigma_\phi^4}{4} - 4\pi^2(\Delta f)^2T^2)\cos(2\pi\Delta fT) - 2\pi\Delta fT\,\sigma_\phi^2\sin(2\pi\Delta fT)}{(\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2T^2)^2}] \tag{12}$$

where in all previous derivations, we use the fact that $E\left(\left|H_k\right|^2\right) = 1$ and $\sigma_\phi^2 = 2\pi\beta T$ where $\sigma_\phi^2$ is the Wiener phase noise variance, $\sigma_{\phi_{Wiener}}^2$ at the useful OFDM symbol, $T$. So, this analysis can be used for AWGN only and for slow fading channel corrupted by AWGN also. Finally, $P_{N_k}$ is given by

$$P_{N_k} = P_{AWGN} = \sigma^2 \tag{13}$$

Thus, the effective SNR is given by

$$SNR_{eff} = \frac{P_{des}}{P_{CEI} + P_{NLI} + P_{ICI} + P_{N_k}} \tag{14}$$

and after some algebra, we get

$$SNR_{eff} = \frac{|\alpha|^2 E^2(|I_0|).SNR}{1 + \sigma_{N_D}^2 + |\alpha|^2(1 - E^2(|I_0|).SNR}, \tag{15}$$

where SNR is the signal-to-noise ratio which is equal to $E_s/\sigma^2$ and $\sigma_{N_D}^2$ is the NLD variance $\sigma_D^2$ normalized to AWGN variance, $\sigma^2$.

Inspection of (9)-(12) and (15) yields exact calculations of the desired power, interference powers, and as a result the effective SNR. We get exact calculations at all levels of phase noise (without linearization of phase noise). Although small phase noise effect assumption may be true in practice, with *general* phase noise effect, it is possible to determine an acceptable level of phase noise for certain OFDM system and how it can be designed to avoid severe degradation in the presence of such noise [20].

In our former paper [41], (9) and (10) were in error due to the error in calculating [41, (22)] (see the Appendix ;(33)) ; now (9) and (10) are correctly presented here for evaluating the desired power and CEI power respectively.

In the absence of the phase noise and the frequency offset, we obtain the effective SNR *exactly* as was obtained in [5] and [7] for the effect of the amplifier nonlinearity only. Similarly, in the absence of amplifier nonlinearity and phase noise, the effective SNR is *exactly* as was given for CFO only [26]. In the presence of the phase noise only, we get the effective SNR *very closed* to that was obtained in [17]. The difference comes from that the analysis of [17] used discrete-time model while we use continuous-time model in our analysis.

The normalized NLD variance, $\sigma_{N_D}^2$ for the SEL is given by [7]

$$\sigma_{N_D}^2 = SNR(1 - e^{-(IBO)} - \alpha^2) \tag{16}$$

$$\alpha = 1 - e^{-(IBO)} + \frac{\sqrt{\pi(IBO)}}{2} erfc(\sqrt{IBO}) \tag{17}$$

which is a real value. IBO is the input backoff which represents the ratio between the input saturation power, $A_s^2$ and the input mean power and $erfc(x) \equiv \int_x^\infty e^{-y^2}dy.$

From (16) and (17), it is clear that as IBO decreases (i.e. the effect of amplifier nonlinearity increases), $\sigma_{N_D}^2$ will increase and $\alpha$ will decrease and vice versa.

In the previous analysis, we used Gaussiuan approximation method to get $SNR_{eff}$ . Although Gaussian approximation is not accurate, it is acceptable for small values of frequency offset which are the practical values in OFDM system [25]. Also, although the ICI due to phase noise is nongaussian [22], [23] and tends to Gaussian at high values of normalized linewidth $(\beta T \approx 1)$ [17], [22], and [23] Gaussian approximation can be used and gets acceptable results for moderate SNR even in small values of linewidth.

### 3.2 Approximations of calculations of powers and effective SNR

To get insight into the system performance, the calculations of the desired power and the interference powers are approximated under some certain conditions. Under small phase noise effect (i.e. $\sigma_\phi^2 << 1$) and small linewidth to frequency offset ratio (i.e. $\beta/\Delta f << 1$), the powers of the desired signal, CEI signal, NLI signal, and ICI signal will take the following simple forms:

$$P_{des} \cong E_s|\alpha|^2 e^{-\pi\beta T}\sin c^2(\Delta fT) \tag{18}$$

$$P_{CEI} \cong 0 \tag{19}$$

$$P_{NLI} \cong \sigma_D^2 e^{-\pi\beta T}\sin c^2(\Delta fT) \tag{20}$$

and

$$P_{ICI} \cong (E_s|\alpha|^2 + \sigma_D^2)[1 - e^{-\pi\beta T}\,\text{sinc}^2(\Delta fT)] \quad (21)$$

Thus, the effective SNR will be

$$SNR_{eff} = \frac{SNR|\alpha|^2 e^{-\pi\beta T}\sin c^2(\Delta fT)}{1 + \sigma_{N_D}^2 + SNR|\alpha|^2(1 - e^{-\pi\beta T}\sin c^2(\Delta fT))} \quad (22)$$

We see that (22) comes from substituting (18) - (21) in addition to (13) into (14), or it can be obtained from approximating (15) *directly* by approximating (9). These equations can be further approximated using

$$e^{-\pi\beta T} \approx 1 - \pi\beta T, \quad (23)$$

$$\sin c^2(\Delta fT) \approx 1 - \frac{\pi^2}{3}(\Delta fT)^2, \quad (24)$$

In (23), we take the linear effect of phase noise into consideration only. This approximation can be applied when $\beta T \le 0.0064$ [22]. Equation (24) is much closed to the exact value for $\Delta f \le 0.2$ [32].

Using (23) and (24) into (18) and (19)-(22), and making further approximation, we get

$$P_{des} \cong E_s|\alpha|^2(1 - \pi\beta T - \frac{\pi^2}{3}(\Delta fT)^2 \quad (25)$$

$$P_{NLI} \cong \sigma_D^2(1 - \pi\beta T - \frac{\pi^2}{3}(\Delta fT)^2 \quad (26)$$

$$P_{ICI} \cong (E_s|\alpha|^2 + \sigma_D^2)[\pi\beta T + \frac{\pi^2}{3}(\Delta fT)^2] \quad (27)$$

and

$$SNR_{eff} = \frac{SNR|\alpha|^2(1 - \pi\beta T - \frac{\pi^2}{3}(\Delta fT)^2)}{1 + \sigma_{N_D}^2 + SNR|\alpha|^2(\pi\beta T + \frac{\pi^2}{3}(\Delta fT)^2)} \quad (28)$$

provided that $\Delta f \le 0.2$.

It is clear to see that (28) comes *directly* from using (23) and (24) into (22) or substituting (25)-(27) in addition to (13) into (14).

It is interesting to see that in all the previous expressions, (18)-(28), the effects of the various parameters (i.e. $\alpha$, $\sigma_D^2$ for amplifier nonlinearity effect, $\beta T$ for phase noise effect and $\Delta fT = f_0T + f_DT$ for CFO and Doppler spread effects respectively) on the system performance are described explicitly.

These approximations are useful in some practical applications. As an example, we take the IEEE 802.11a, which specifies carrier frequency $f_C = 5.2GHz$ and useful symbol duration $T = 3.2\mu\sec$ [1], [2]. So, the Doppler frequency and normalized Doppler frequency can take the values $f_d = 52Hz$ and $f_dT = 1.664 \times 10^{-4}$ respectively [2]. The typical values of normalized linewidth in this standard are $\beta T = 2 \times 10^{-4} - 13 \times 10^{-4}$ [21]. For oscillator accuracy with values ranging between 0.6-6 ppm**,** the CFO and the normalized CFO will take the values $f_0 = 3.12 - 31.2kHz$ and $f_0T = 9.984 \times 10^{-3} - 9.984 \times 10^{-2}$ respectively. Thus, all the previous approximations can be applied to this standard which gives insight into the performance of the system.

### 3.3 BER calculations

Under slowly Rayleigh fading channel, the BER for QPSK and 16QAM modulation formats with coherent detection technique and Gray encoding are given by [42]

$$BER_{QPSK,Ray.} = \frac{1}{2}\left[1 - \frac{1}{\sqrt{1 + 1/SNR_{eff}}}\right] \quad (29)$$

$$BER_{16QAM,Ray.} = \frac{3}{8}\left[1 - \frac{1}{\sqrt{1 + 5/(2SNR_{eff})}}\right] \quad (30)$$

### 3.4 Total degradation measure

IBO is one of the two parameters used to identify the operating point of the amplifier. Another parameter is the output backoff (OBO) which is the ratio between the saturation power to the mean output power of the amplifier. The effect of nonlinear power amplifier can be reduced by working with high backoff which means the operating point of the amplifier is moved to the linear region. On the other hand, this leads to a loss in the power efficiency of the HPA.

A useful system performance measure is the total degradation (TD) as a function of the HPA IBO. It is given by

$$TD_{dB} = SNR_{dB} - SNR'_{dB} + IBO_{dB} \quad (31)$$

where $SNR_{dB}$ is the required SNR in dB at the input of the threshold detector to obtain a fixed BER for a given value of the IBO, phase noise, and frequency offset. $SNR'_{dB}$ is the required SNR to obtain the same BER in the presence of AWGN only (i.e. in the absence of all interferences) and $IBO_{dB}$ is the IBO in dB.

In the other studies [4], [34], $OBO_{dB}$ was used in (31) instead of $IBO_{dB}$ where $OBO_{dB}$ is the OBO in dB.

The optimum operating point of the amplifier is obtained by minimizing (31). This results in minimum TD.

## 4 Numerical results

Due to the error in our former paper [41], (see section 3.1), all the figures in [41] can be used again, but with halving the values of normalized linewidth, $\beta T$.

Fig. 1 and Fig. 2 show the BER against SNR for different values of normalized linewidth, $\beta T$, normalized frequency offset, $\Delta fT$, and IBO under slowly Rayleigh fading channel for the two modulation formats QPSK and 16-QAM respectively. It is clear that the loss will be increased as one , some or all the previous parameters are increased.

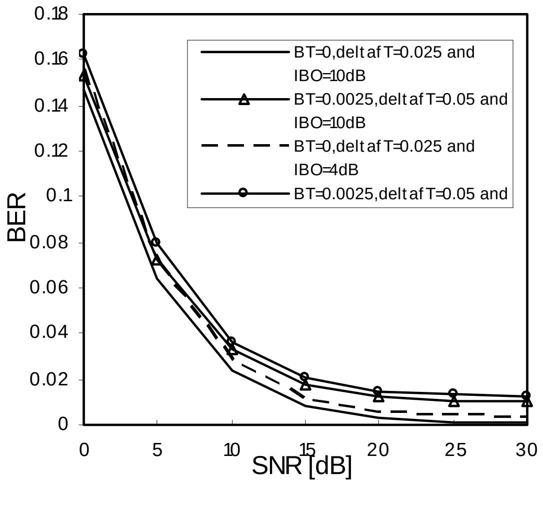


Fig. 1. BER versus SNR for QPSK under slowly Rayleigh fading channel for different values of normalized linewidth, frequency offset, and IBO.

Total degradation curves are obtained in Fig. 3 versus $IBO_{dB}$ for 16-QAM modulation format under slowly Rayleigh fading channel when $BER = 0.02$ for different values of $\beta T$ and $\Delta fT$. I t shows that $TD_{dB}$ will be 10.2 dB for $\beta T = 0$, $\Delta fT = 0.025$ at $IBO = 10dB$, and will take the value of 5.68dB for the same values of normalized linewidth and normalized frequency offset at $IBO = 4dB$. Also, $TD_{dB}$ will be 15.9 dB for $\beta T = 0.0025$, $\Delta fT = 0.025$ at $IBO = 10dB$, and will take the value of 17.5 dB for the same values of normalized linewidth and normalized frequency offset at $IBO = 4dB$.

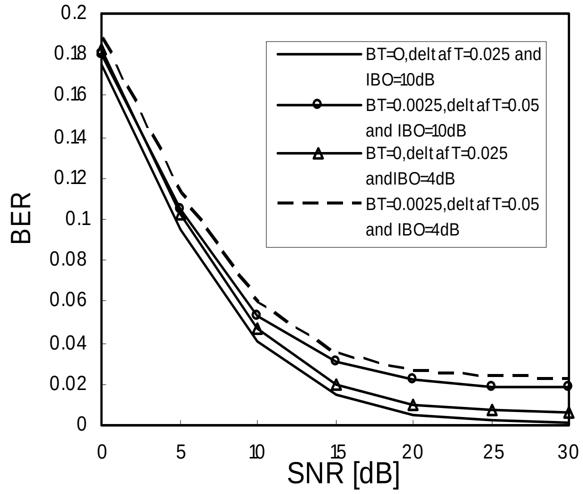


Fig. 2. BER versus SNR for QAM under slowly Rayleigh fading channel for different values of normalized linewidth, frequency offset, and IBO.

Fig. 4 illustrates minimum TD in dB versus normalized linewidth for different values of normalized frequency offset $\Delta fT = 0.025$, $\Delta fT = 0.05$ under slowly Rayleigh fading channel at $BER = 0.02$ for QPSK and 16QAM modulation formats. For QPSK, TD are about 6.19 dB and 6.7 dB at $\beta T = 0.00125$, and are increased to about 7.7 dB and 8.4 dB at $\beta T = 0.0025$. For 16QAM modulation format, the degradations are increased which are about 7.65 dB and 9.3 dB at $\beta T = 0.00125$ and will be about 12.69 dB and 15.6 dB at $\beta T = 0.0025$.

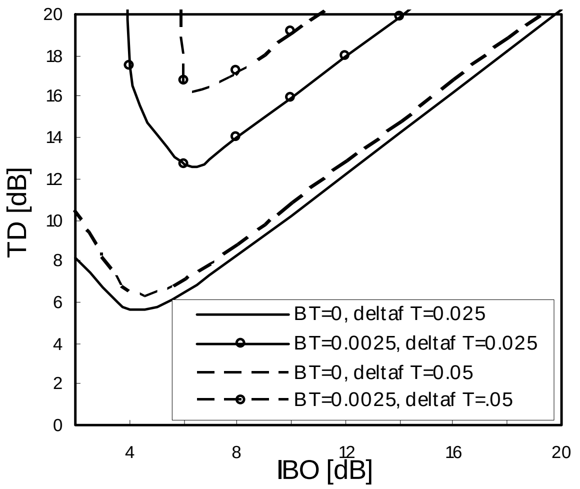


Fig. 3 .TD versus IBO for different values of normalized linewidth and normalized frequency offset for16QAM at BER=0.02.

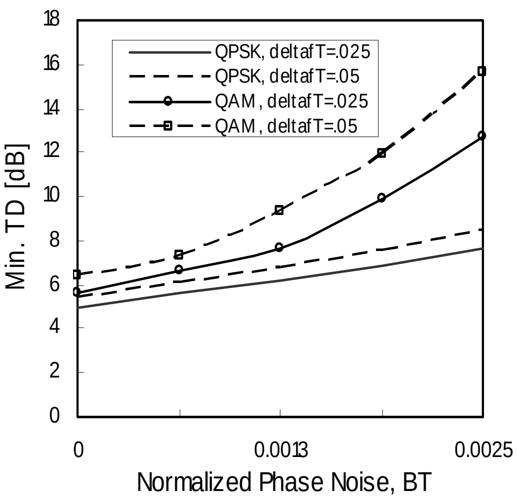


Fig. 4. Min. TD versus normalized linewidth for different values of normalized frequency offset for QPSK and 16QAM at BER=0.02.

The previous figures can be used to specify the imperfections parameters for designing the system by:

1) Obtain the requirements of fixed parameters such as the level of the SNR, BER, modulation format, normalized linwidths, normalized frequency offsets, and so on.

2) Specify the remaining parameters such as the minimum TD by optimizing (31).

## 5 Discussion and conclusion

We have obtained exact closed-form expressions for the desired power and the interference powers, CEI, NLI, and ICI, and as a result the effective SNR of the OFDM system. The effective SNR has been obtained under the combined effect of the Wiener phase noise, CFO, Doppler spread and nonlinear power amplifier effect of the SEL. These formulas have been obtained without linearization of phase noise levels. Although the assumption of small phase noise may be true in practice, with general phase noise effect, it enables one to determine under which condition phase noise can be treated as small, leading to adequate solutions, and what are proper parameter settings for a specific OFDM system to help mitigate the phase noise effect

We have also obtained simple closed-form expressions for the desired power, the interference powers, and the effective SNR of the OFDM under some certain conditions. These expressions are provided as a function of various system parameters, and give a quantitive understanding of how system behavior changes with a certain parameter (i.e. describe explicitly the various parameters with the system performance). These approximations can be applied to various OFDM systems such as IEEE 802.11 and HIPERLAN II to get insight into the system performance.

All the previous formula can be applied to both AWGN and slowly Rayleigh fading channels.

BER curves have been investigated versus SNR for different values of phase noise rate (i.e. normalized linewidth), normalized frequency offset, and IBO for QPSK and 16QAM modulation formats under slowly Rayleigh fading channel. The loss in the system performance will be increased as one , some or all the previous parameters are increased.

TD has also evaluated against IBO for different values of normalized linewidth and normalized frequency offset. The minimum TD (i.e. the optimum operating point of the HPA) is obtained at the point which provides a good tradeoff between the output power and the degradation due to the NLD. Minimum TD has also plotted versus normalized linewidths for different values of normalized frequency offset for the two modulation formats QPSK and 16-QAM. It is taken as a measure of the system performance.

It is clear that the BER and TD curves will be further degraded as the modulation formats increase (i.e. 64-QAM and 256-QAM).

Although this paper focuses on the Wiener phase noise model, the analysis can be extended to describe the effect of the PLL LO model combined with the nonlinearity effect of the HPA and the frequency offset effect resulting from CFO and Doppler shifts. Also the analysis can be applied to other kinds of power amplifiers such as SSPA and TWTA.

Future work will includes the solutions of the previous combined imperfections.

# APPENDIX

## 1 Computation of the desired power, $P_{des}$:

From (7), $E(|I_0|)$ is given by

$$E(|I_0|) = \left| \frac{1}{T} \int_0^T E\left(e^{j\phi(t)}\right) e^{-j2\pi\Delta f t} dt \right| \quad (32)$$

where $E\left(e^{j\phi(t)}\right)$ is given by [36] and [38] as follows

$$E\left(e^{j\phi(t)}\right) = e^{-\pi\beta t} \quad (33)$$

which is corrected here, (compare it with [41, (22)])
From (33) into (32), we get

$$E^2(|I_0|) = e^{-\frac{\sigma_\phi^2}{2}} \frac{\cos^2(\pi\Delta fT)\sinh^2(\sigma_\phi^2/2) + \sin^2(\pi\Delta fT)\cosh^2(\sigma_\phi^2/2)}{\frac{\sigma_\phi^4}{16} + \pi^2(\Delta f)^2T^2} \quad (34)$$

Thus, from (34), we get (9).

It is interesting to see that at the absence of frequency offset (i.e. $\Delta fT = 0$), (34) reduces to [38, (3)].

## 2 Computation of the interference powers; $P_{CEI}$, $P_{NLI}$ and $P_{ICI}$

$$E\left(\sum_{r=0, r\neq k}^{N-1} |I_{k-r}|^2\right) = E\left(\sum_{r\neq 0} |I_r|^2\right) = \sum_{r\neq 0} E\left(|I_r|^2\right) \quad (35)$$

where $E\left(|I_r|^2\right)$ is given by [43, (10.27) ]

$$E\left(|I_r|^2\right) = \int_{-1}^{1} (1-|x|) e^{-j2\pi rx} R_\phi(T x) R_{\Delta f}(Tx) dx \quad (36)$$

where $R_\phi(\tau)$ is the autocorrelation function of the phase noise, which is given by [36]

$$R_\phi(T_s x) = e^{-\frac{\sigma_\phi^2 |x|}{2}} \quad (37)$$

and $R_{\Delta f}(\tau)$ is the autocorrelation function of the frequency offset which was given by (3).
Using the formula [44, (4-4-16), (4-4-17) ]

$$\sum_{r=-\infty}^{\infty} e^{-j2\pi rx} = \sum_{n=-\infty}^{\infty} \delta(x-n), \quad (38)$$

where $\delta(x)$ is the Dirac delta function.Thus, from (3), (36), (37), and (38) into (35), and after some algebra, we obtain

$$E\left(\sum_{r=0, r\neq k}^{N-1} |I_{k-r}|^2\right) = 1 - \frac{\sigma_\phi^2}{\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2T^2} - \frac{2\left(\frac{\sigma_\phi^2}{4} - 4\pi^2(\Delta f)^2T^2\right)}{\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2T^2} + 2e^{-\frac{\sigma_\phi^2}{2}} \times \frac{\left(\frac{\sigma_\phi^4}{4} - 4\pi^2(\Delta f)^2T^2\right)\cos(2\pi\Delta fT) - 2\pi\Delta fT_s\sigma_\phi^2\sin(2\pi\Delta fT)}{\left(\frac{\sigma_\phi^4}{4} + 4\pi^2(\Delta f)^2T^2\right)^2} \quad (39)$$

From (36) and (38), we get

$$E\left(|I_0|^2\right) = 1 - \sum_{r\neq 0} E\left(|I_r|^2\right) \quad (40)$$

Thus, from (34), (35), (39), and (40), we get the interference powers (10), (11), and (12).

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