Redundant Data Transmission and Nonlinear Codes

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Abstract: - In their paper the authors investigate redundant data transmission protected by arbitrary (not necessarily linear) codes. They generalize results known so far for linear codes, in particular for cyclic redundancy checks (CRCs). They use the concept of the Generalized Erasure Channel (GEC) to determine the probability of undetected error for some binary symmetric and non-symmetric channels protected by appropriate encoding procedures. In detail, they investigate communication via different Binary Symmetric Channels (BSCs), Generalized Erasure Channels and Symmetrized Binary Non Symmetric Channels (BNSCs). Simple upper bounds are given, relating the new formulas to that one of the BSC, and the channel capacities are calculated. Finally the results in connection with an inequality for proper codes are applied to multi channel coloured transmission via laser beams propagating through Polymer Optical Fiber (POF).

Key-Words: - nonlinear code, CRC, Binary Symmetric Channel, Erasure Channel, Non Symmetric Channel, Probability of Undetected Error, Weight Distribution, Channel Capacity, Proper Code, Wavelength Division Multiplexing, Semiconductor Lasers

1 Introduction

Safety related systems can get strongly affected due to electromagnetic disturbances. These disturbances can be caused by high frequency transmitters, mobile phones, or walky-talkies etc. Especially in high safety critical applications, like oil and gas extraction, or Ethylene and Phenol processes, disturbances can have fatal consequences.

To implement a safety related system of redundant processors, it is of paramount importance to have undisturbed communication channels with a transmission rate in the Gbps scope.

Basically, there are two attributes in order to avoid those difficulties:

- reduction of the bit error probability
- reduction of the probability of undetected error

An approved method to achieve the first aim is the usage of fiber optics. The bit error probability of channels using fiber optics has been investigated in [3], [4], [7], [8], [12], [13], [15], [16], [18], [24], and [25].

In case of a poor (large) bit error probability, a frequently used procedure to achieve the second aim is redundant data transmission, protected by appropriate checksum and monitoring procedures. In [23] Wacker&Boercsoek investigated the case of transmitting each message block via the same channel or at least via channels with the same bit error probability ε. They then found formulas and upper bounds for the probability of undetected error. Hillmer et al. in [9] and [10] suggested an approach by killing two birds with one stone: Fiber optics together with multi channel coloured transmission by Wavelength Division Multiplexing (WDM). With this approach it would be possible to implement a feedback-free, redundant and safe transmission with only one single fiber.

The aim of this paper is to determine equations for the probability of undetected error in the case redundant transmission via different channels, and

then to apply it to laser beams, thus bringing together both of the above features. Actually, there is a second way of generalizing the results of [23]: In [23] the authors restricted themselves to linear codes. In this paper we consider arbitrary (not necessarily) linear codes. The results turned out to remain valid in this more general case.

2 The Probability of Undetected Error for Non Linear Codes

2.1 The Distance Distribution of a Code

Let there be given a binary transmission channel and a transmission procedure protected by a (n, K) code. A (n, K) code is a subset C of the n- dimensional linear space $GF(2)^n$ of all n-tuples consisting of K elements. The space $GF(2)$ is the Galois field of order 2, consisting of the elements 0 and 1. The number n is called the block length. Given the n-tuples $\boldsymbol{x} = (x_1,\ldots,x_n)$ and $\boldsymbol{y} = (y_1,\ldots,y_n)$, $x_l, y_l = \{0, 1\}$, the Hamming distance of $\boldsymbol{x}$ and $\boldsymbol{y}$ is defined as the number of components in which $\boldsymbol{x}$ and $\boldsymbol{y}$ differ:

$$d(\boldsymbol{x},\boldsymbol{y}) =|\ \{l =1,\ldots,n: x_l \neq y_l\}\ |.$$

The Hamming distance induces a metric on the space $GF(2)^n$ of all n-tuples. The minimum distance of the code is defined as

$$d = \min\{d(\boldsymbol{x},\boldsymbol{y}): \boldsymbol{x}, \boldsymbol{y} \in C\}.$$

The distance distribution of an (n, K) code is the vector

$$D(C) = (D_0, D_1,\ldots,D_n),$$

where

$$D_l = \frac{1}{K}|\ \{(\boldsymbol{x},\boldsymbol{y}): \boldsymbol{x}, \boldsymbol{y} \in C \text{ and } d(\boldsymbol{x},\boldsymbol{y}) = l\}\ |$$

is the average number of code words being at distance l from a codeword of C (see [14]).
The Hamming weight $w(\boldsymbol{x})$ of a vector $\boldsymbol{x}$ is defined as the distance of $\boldsymbol{x}$ from $\boldsymbol{0}$:
$$w(\boldsymbol{x}) = d(\boldsymbol{x},\boldsymbol{0})$$

The weight distribution of an (n, K) code is the vector

$$A(C) = (A_0, A_1,\ldots,A_n),$$

where

$$\boldsymbol{A}_l =|\ \{\boldsymbol{x} \in C : w(\boldsymbol{x}) = l\}\ |$$

is the number of code words being at distance l from $\boldsymbol{0}$.
A linear $[n, k]$ code C is a k-dimensional linear subspace $GF(2)^n$ of all n-tuples. For linear codes the weight distribution and the distance distribution coincide.

2.2 The Binary Symmetric Channel (BSC)

Let there be given a binary symmetric channel without memory with a bit error probability ε and a transmission procedure protected by a (n, K) code C, for example a cyclic redundancy check (CRC). The probability of undetected error is given by (see [14] or [17]):

$$p_{ue}(\varepsilon, C) = \sum_{l=1}^{n} D_l \varepsilon^l (1 - \varepsilon)^{n-l},$$

where

ε = bit error probability (bit error ratio, BER) of the channel.

In the case of CRCs, bounds on the distance distribution and the probability of undetected error can be found in [20], [21], and [22].

2.3 The Generalized Erasure Channel (GEC)

In the present paper we shall use the concept of the GEC to extend the results of Wacker&Boercsoek in [23] to the case described in the introduction: Redundancy with channels of different physical properties and non linear codes.
To get an approach to redundant data transmission, we shall take advantage of the concept of the Generalized Erasure Channel (GEC), introduced in [23] by Wacker & Boercsoek, and which we shall present now in short. Consider a channel with 2 input variables 0 and 1, but 3 output variables 0, 1, and e. The output e stands for the situation of receiving a symbol which cannot be decoded, i.e. can not be identified as a "0" or a "1". Hence, the symbol e may be considered as the error

symbol. We supposed the transition probabilities to be given by

$$p(e|0) = p(e|1) = \zeta,$$
$$p(1|0) = p(0|1) = \eta$$
$$p(0|0) = p(1|1) = \theta$$

($\zeta \geq 0$, $\eta \geq 0$, and $\theta \geq 0$, $\zeta + \theta + \eta = 1$, see Figure 1 below). Here $p(y|x)$ is the probability of receiving y given x is sent.

We called a channel defined by this properties "Generalized Erasure Channel" (GEC), because for $\eta = 0$ it is about the customary erasure channel. For $\zeta = 0$ it is the BSC. For linear codes the probability of undetected error of the GEC turned out to be given by

$$(1) \quad p_{ue}(\zeta,\eta,\theta,C) = \sum_{l=1}^{n} A_l \eta^l \theta_l^{-}$$

In this subsection we shall prove that (1) is true for arbitrary codes. In the course of the proof we shall make use of Bayes' theorem of the total probability

$$p(B) = \sum_{j=1}^{N} p(B \mid C_j) p(C_j),$$

Here $\{ C_j : j = 1, \ldots , N\}$ is a partition of the probability space Ω

$$\Omega = \bigcup_{j=1}^{N} C_j, \quad C_j \cap C_k = 0,$$

and $p(B| C_j)$ is the conditional probability of B given C_j.

Theorem 1: The probability of undetected error of the generalized erasure channel for an arbitrary (n, K) code is given by

$$p_{ue}(\zeta,\eta,\theta,C) = \sum_{l=1}^{n} D_l \eta^l \theta_l^{-}$$

Proof: We may assume that each code word is sent with equal probability. Therefore the probability $p(x)$ that a certain code word is sent is given by

$$p(x) = \frac{1}{K}.$$

Let further $p(y|x)$ denote the probability of receiving the message vector y, given the code word x is sent. Then, by Bayes' theorem of the total probability,

$$p_{ue}(\zeta,\eta,\theta,C) = \sum_{x \in C} (\sum_{\substack{y \in C \\ y \neq x}} p(y|x)) \cdot p(x)$$

$$= \sum_{x \in C} (\sum_{\substack{y \in C \\ y \neq x}} p(y|x)) \cdot \frac{1}{K}$$

$$= \sum_{x \in C} (\sum_{\substack{y \in C \\ y \neq x}} \eta^{d(y,x)} \theta^{n-d(y,x)}) \cdot \frac{1}{K}$$

$$= \sum_{x \in C} (\sum_{l=1}^{n} (\sum_{\substack{y \in C \\ d(y,x)=l}} 1) \eta^l \theta^{n-l}) \frac{1}{K}$$

$$= \sum_{l=1}^{n} \sum_{x \in C} (\sum_{\substack{y \in C \\ d(y,x)=l}} 1) \eta^l \theta^{n-l} \frac{1}{K}$$

$$= \sum_{l=1}^{n} \{\frac{1}{K} \sum_{x \in C} (\sum_{\substack{y \in C \\ d(y,x)=l}} 1)\} \eta^l \theta^{n-l}$$

$$= \sum_{l=1}^{n} D_l \eta^l \theta^{n-l}.$$

■

As for the channel capacity of the GEC we got in [23]

$$(2) \quad c(\zeta,\eta,\theta) = \eta\log\eta + \theta\log\theta - (\eta+\theta)\log(\eta+\theta) + \eta + \theta.$$

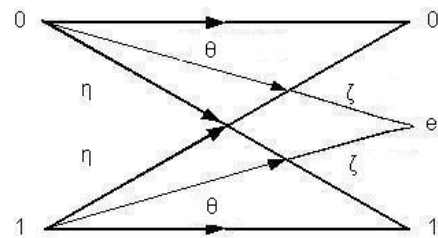Fig. 1 The GEC

3 Data Protection by Transmission via μ Different Channels$\varnothing$

3.1 μ BSCs Considered as one GEC
Suppose there is a transmission procedure transmitting each message block via μ different

binary symmetric channels without memory, each one with an individual bit error probability $\varepsilon_1,...,\varepsilon_\mu$. Assume further that the receiving device is performing a cross check between all μ message blocks having been received: A message block will be accepted by the receiver if and only if all of the μ blocks coincide bit by bit, otherwise it will be rejected as an error. The situation is cleared by table 1.

Table 1 Transmission via μ BSCs

Transmitter		Receiver		Probability
0	(0,...,0)	(0,...,0)	0	$(1-\varepsilon_1)\cdots(1-\varepsilon_\mu)$
1	(1,...,1)	(1,...,1)	1	
0	(0,...,0)	(1,...,1)	1	$\varepsilon_1\cdots\varepsilon_\mu$
1	(1,...,1)	(0,...,0)	0	
0	(0,...,0)	(...,1,...)	e	$1-(1-\varepsilon_1)\cdots(1-\varepsilon_\mu)$
1	(1,...,1)	(...,0,...)	e	$-\varepsilon_1\cdots\varepsilon_\mu$

The symbol e means that the received symbol cannot be identified. I.e. transmission via μ BSCs turns out to be transmission via a single GEC with

$$(3) \qquad \eta=\prod_{j=1}^{\mu}\varepsilon_j, \theta=\prod_{j=1}^{\mu}(1-\varepsilon_j), \zeta=1-\eta-\theta.$$

3.2 The Probability of Undetected Error for Transmission via μ Different BSCs

Suppose now the transmission procedure via each of the μ channels to be protected additionally by an arbitrary (n, K) code C (same code applied separately to each channel,). Let $p_{ue}(\varepsilon_1,...,\varepsilon_\mu,C)$ denote the probability of undetected error of C considered under this transmission procedure. What we expect is the inequality

$$(4) \qquad p_{ue}(\varepsilon_1,...,\varepsilon_\mu,C)\le\prod_{j=1}^{\mu}p_{ue}(\varepsilon_j,C).$$

By Theorem 1 and (3) we get

Theorem 2: In the case of μ BSCs with individual bit error probabilities $\varepsilon_1,...,\varepsilon_\mu$, the probability of undetected error of the (n, K) code C is given by

$$p_{ue}(\varepsilon_1,...,\varepsilon_\mu,C)=\sum_{l=1}^{n}D_l(\prod_{j=1}^{\mu}\varepsilon_j)^l(\prod_{j=1}^{\mu}(1-\varepsilon_j))^{n-l}.$$

Corollary 3 states that the performance of the code C considered on the GEC of table 1 at bit error probabilities of $\varepsilon_1,...,\varepsilon_\mu$ is at least as good as the performance of the same code C considered on the BSC at a bit error probability of $\varepsilon_1\cdot...\cdot\varepsilon_\mu$.

Corollary 3: For each (n, K) code C and $0\le\varepsilon_j\le1$

$$p_{ue}(\varepsilon_1,...,\varepsilon_\mu,C)\le p_{ue}(\prod_{j=1}^{\mu}\varepsilon_j,C).$$

Proof: Firstly, by induction, we prove the inequality

$$(5) \qquad (1-\varepsilon_1)\cdots(1-\varepsilon_\mu)\le1-\varepsilon_1\cdots\varepsilon_\mu$$

for $\mu=1, 2, 3, ...$
For $\mu=1$ inequality (5) reduces to the most trivial statement

$$1-\varepsilon_1\le1-\varepsilon_1$$

If (5) is true for some natural number μ, then, because of $0\le\varepsilon_j\le1$,

$$\varepsilon_1\cdots\varepsilon_\mu\ge\varepsilon_1\cdots\varepsilon_\mu\cdot\varepsilon_{\mu+1},$$

and

$$\varepsilon_{\mu+1}\ge\varepsilon_1\cdots\varepsilon_\mu\cdot\varepsilon_{\mu+1},$$

and therefore, by induction hypothesis,

$$(1-\varepsilon_1)\cdots(1-\varepsilon_{\mu+1})=(1-\varepsilon_1)\cdots(1-\varepsilon_\mu)\cdot(1-\varepsilon_{\mu+1})$$
$$\le(1-\varepsilon_1\cdots\varepsilon_\mu)\cdot(1-\varepsilon_{\mu+1})$$
$$=1-\varepsilon_1\cdots\varepsilon_\mu-\varepsilon_{\mu+1}+\varepsilon_1\cdots\varepsilon_\mu\cdot\varepsilon_{\mu+1}$$
$$\le1-\varepsilon_1\cdots\varepsilon_\mu\cdot\varepsilon_{\mu+1}-\varepsilon_1\cdots\varepsilon_\mu\cdot\varepsilon_{\mu+1}+\varepsilon_1\cdots\varepsilon_\mu\cdot\varepsilon_{\mu+1}$$
$$=1-\varepsilon_1\cdots\varepsilon_{\mu+1}.$$

Hence

$$p_{ue}(\varepsilon_1,...,\varepsilon_\mu,C)=\sum_{l=1}^{n}A_l(\prod_{j=1}^{\mu}\varepsilon_j)^l(\prod_{j=1}^{\mu}(1-\varepsilon_j))^{n-l}$$
$$\le\sum_{l=1}^{n}A_l(\prod_{j=1}^{\mu}\varepsilon_j)^l(1-\prod_{j=1}^{\mu}\varepsilon_j)^{n-l}$$
$$=p_{ue}(\prod_{j=1}^{\mu}\varepsilon_j,C).$$

■

Corollary 4 is the well known result mentioned at the beginning of this subsection (see (4)):

Corollary 4: For each (n, K) code C and $0 \leq \varepsilon_j \leq 1$

$$p_{\text{ue}}(\varepsilon_1, \ldots, \varepsilon_\mu, C) \leq \prod_{j=1}^{\mu} p_{\text{ue}}(\varepsilon_j, C).$$

Proof: The prove will again be done by induction. For $\mu = 1$ the statement is evident.
Let now be

$$p_{\text{ue}}(\varepsilon_1, \ldots, \varepsilon_\mu, C) \leq \prod_{j=1}^{\mu} p_{\text{ue}}(\varepsilon_j, C)$$

for some natural number μ, then surely for each $l = 1, \ldots, n$

$$B_l = (\varepsilon_{\mu+1})^l (1 - \varepsilon_{\mu+1})^{n-l}$$
$$\leq \sum_{l=1}^{n} A_l (\varepsilon_{\mu+1})^l (1 - \varepsilon_{\mu+1})_l^{-}$$
$$= p_{\text{ue}}(\varepsilon_{\mu+1}, C).$$

Hence, by induction hypothesis,

$$p_{\text{ue}}(\varepsilon_1, \ldots, \varepsilon_{\mu+1}, C)$$
$$= \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu+1} \varepsilon_j)^l (\prod_{j=1}^{\mu+1}(1 - \varepsilon_j))^{n-l}$$
$$= \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu} \varepsilon_j)^l (\prod_{j=1}^{\mu}(1 - \varepsilon_j))^{n-l} B_l$$
$$\leq \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu} \varepsilon_j)^l (\prod_{j=1}^{\mu}(1 - \varepsilon_j))^{n-l} \cdot p_{\text{ue}}(\varepsilon_{\mu+1}, C)$$
$$\leq \prod_{j=1}^{\mu} p_{\text{ue}}(\varepsilon_j, C) \cdot p_{\text{ue}}(\varepsilon_{\mu+1}, C)$$
$$\leq \prod_{j=1}^{\mu+1} p_{\text{ue}}(\varepsilon_j, C)$$

■

The last three results generalize Theorem 1, Corollary 2 and Corollary 3 of Wacker&Boercsoek in [23].

3.3 The Probability of Undetected Error for Transmission via µ Different GECs

In this subsection we consider transmission via µ different GECs together with a cross check and protected by a (n, K) code C. To this end, let ζ_1, θ_1, η_1 ,..., ζ_μ, θ_μ, η_μ be the transition probabilities corresponding to each of the µ channels.

Table 2 Transmission via µ GECs

Transmitter		Receiver		Probability
0	(0,…,0)	(0,…,0)	0	$\theta_1 \cdots \theta_\mu$
1	(1,…,1)	(1,…,1)	1	
0	(0,…,0)	(1,…,1)	1	$\eta_1 \cdots \eta_\mu$
1	(1,…,1)	(0,…,0)	0	
0	(0,…,0)	(…,1,…)	e	$1 - \theta_1 \cdots \theta_\mu - \eta_1 \cdots \eta_\mu$
1	(1,…,1)	(…,0,…)	e	
0	(0,…,0)	(…,e,…)	e	
1	(1,…,1)	(…,e,…)	e	

As in subsection 3.1 the procedure is described by a table (table 2) and turns out to be equivalent to transmission via a single GEC with

$$\eta = \prod_{j=1}^{\mu} \eta_j, \theta = \prod_{j=1}^{\mu} \theta_j, \zeta = 1 - \eta - \theta.$$

Finally, let $p_{\text{ue}}(\zeta_1, \theta_1, \eta_1, \ldots, \zeta_\mu, \theta_\mu, \eta_\mu, C)$ denote the probability of undetected error of C considered on this channel. Then in a completely analogous manner as for the BSC in subsection 3.2 we get from Theorem 1:

Theorem 5: In the case of µ GECs with individual transition probabilities ζ_1, θ_1, η_1 ,..., ζ_μ, θ_μ, η_μ, the probability of undetected error of (n, K) code C is given by

$$p_{\text{ue}}(\zeta_1, \eta_1, \theta_1, \ldots, \zeta_\mu, \eta_\mu, \theta_\mu, C)$$
$$= \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu} \eta_j)^l (\prod_{j=1}^{\mu} \theta_j)^{n-l}.$$

The probability of undetected error via µ GECs is related to that one via µ BSCs by a simple inequality:

Corollary 6: For each (n, K) code C

$$p_{\text{ue}}(\zeta_1, \eta_1, \theta_1, \ldots, \zeta_\mu, \eta_\mu, \theta_\mu, C)$$
$$\leq \min\left(p_{\text{ue}}(\eta_1, \ldots, \eta_\mu, C), p_{\text{ue}}(1 - \theta_1, \ldots, 1 - \theta_\mu, C)\right)$$

Proof: By Theorem 5

$$p_{ue}(\zeta_1,\eta_1,\theta_1,\dots,\zeta_\mu,\eta_\mu,\theta_\mu,C)$$

$$= \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu}\eta_j)^l (\prod_{j=1}^{\mu}(1-\eta_j-\zeta_j))^{n-l}$$

$$\leq \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu}\eta_j)^l (\prod_{j=1}^{\mu}(1-\eta_j))^{n-l}$$

$$= p_{ue}(\eta_1,\dots,\eta_\mu,C)$$

and

$$p_{ue}(\zeta_1,\eta_1,\theta_1,\dots,\zeta_\mu,\eta_\mu,\theta_\mu,C)$$

$$= \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu}(1-\theta_j-\zeta_j))^l (\prod_{j=1}^{\mu}\theta_j)^{n-l}$$

$$\leq \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu}(1-\theta_j))^l (\prod_{j=1}^{\mu}\theta_j)^{n-l}$$

$$= p_{ue}(1-\theta_1,\dots,1-\theta_\mu,C). \qquad \blacksquare$$

This means that transmission via μ GECs is at least as good as transmission via μ BSCs at bit error probabilities of $\eta_1,\dots,\eta_\mu$ and $1-\theta_1,\dots,1-\theta_\mu$.

3.4 The Probability of Undetected Error for Transmission via μ Different BNSCs

A Binary Non Symmetric Channel (BNSC) is characterized by the fact that the transition probability from "0" to "1" is different from the transition probability from "1" to "0". Thus the situation is described by the equations

$p(1|0) = \delta,\qquad p(0|1) = \varepsilon,$
$p(0|0) = 1 - \delta,\qquad p(1|1) = 1 - \varepsilon.$
$(\varepsilon \geq 0, \delta \geq 0, \varepsilon \neq \delta).$

In [23] Wacker&Boercsoek symmetrized the BNSC by transmitting each block twice: the original block and the second one with an inverted bit pattern. If a cross check is performed in the receiving device, the resulting channel is a GEC with transition probabilities given by:

$p(0|0) = (1 - \delta)\cdot(1 - \varepsilon),\qquad p(0|1) = \delta\cdot\varepsilon,$
$p(1|0) = \delta\cdot\varepsilon,\qquad p(1|1) = (1 - \delta)\cdot(1 - \varepsilon),$
$p(e|0) = \delta + \varepsilon - 2\cdot\delta\cdot\varepsilon,\qquad p(e|1) = \delta + \varepsilon - 2\cdot\delta\cdot\varepsilon.$

They then calculated the probability of undetected error of this transmission procedure, when it is protected by a linear code.

In this subsection we shall consider transmission via μ BNSCs with different transition probabilities δ_1, ε_1 ,..., δ_μ, ε_μ protected by a (n, K) code. When each of the μ channels is symmetrized in the way described above, we get transmission via μ GECs. Therefore Theorem 5 yields

Theorem 7: In the case of μ symmetrized BNSCs with individual transition probabilities δ_1, ε_1,..., δ_μ, ε_μ, the probability of undetected error of the (n, K) code C is given by

$$p_{ue}(\delta_1,\varepsilon_1,\dots,\delta_\mu,\varepsilon_\mu,C)$$

$$= \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu}\delta_j\varepsilon_j)^l (\prod_{j=1}^{\mu}(1-\delta_j)(1-\varepsilon_j))^{n-l}.$$

Similar to subsection 3.3, the undetected error probability of μ BNSCs is upper bounded by that one of transmission via μ BSCs:

Corollary 8: For each (n, K) code C and $0\leq \varepsilon_j,\delta_j \leq 1$.

$$p_{ue}(\delta_1,\varepsilon_1,\dots,\delta_\mu,\varepsilon_\mu,C) \leq p_{ue}(\delta_1\varepsilon_1,\dots,\delta_\mu\varepsilon_\mu,C)$$

Proof: Firstly,

$$(1-\delta_j)(1-\varepsilon_j) = 1-\delta_j-\varepsilon_j+\delta_j\cdot\varepsilon_j$$

$$\leq 1-2\sqrt{\delta_j\cdot\varepsilon_j}+\delta_j\cdot\varepsilon_j$$

$$\leq 1-\delta_j\cdot\varepsilon_j,$$

and then by Theorem 7 and Theorem 2

$$p_{ue}(\delta_1,\varepsilon_1,\dots,\delta_\mu,\varepsilon_\mu,C)$$

$$= \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu}\delta_j\varepsilon_j)^l (\prod_{j=1}^{\mu}(1-\delta_j)(1-\varepsilon_j))^{n-l}.$$

$$\leq \sum_{l=1}^{n} A_l (\prod_{j=1}^{\mu}\delta_j\varepsilon_j)^l (\prod_{j=1}^{\mu}(1-\delta_j\cdot\varepsilon_j))^{n-l}$$

$$= p_{ue}(\delta_1\varepsilon_1,\dots,\delta_\mu\varepsilon_\mu,C). \qquad \blacksquare$$

3.5 The Channel Capacity of μ Different Channels

The capacity of a channel with the input alphabet X and the output alphabet Y is defined by

Cap:= $\sup\{I_p(X,Y): p$ probability measure on $X\}$,

where

$$I_p(X,Y) = \sum_{y \in Y}\sum_{x \in X} p(y|x)p(x)\log\frac{p(y|x)}{p(y)}$$

is the mutual information between X and Y. All logarithms are related to the base 2. In [23] Wacker&Boercsoek used the concept of the GEC and the formula for its channel capacity (cf. (2)) to calculate the channel capacity of the symmetrized BNSC

$$\begin{aligned}c(\delta,\varepsilon) = {}& \delta\varepsilon\log(\delta\varepsilon)+(1-\delta)(1-\varepsilon)\log(1-\delta)(1-\varepsilon)\\ &-(1-\varepsilon-\delta+2\varepsilon\delta)\log(1-\varepsilon-\delta+2\varepsilon\delta)\\ &+1-\varepsilon-\delta+2\varepsilon\delta.\end{aligned}$$

And so, by (2) and by the results of the preceding section we get:

Theorem 9: For a channel consisting of μ different channels, the channel capacity turns out to be equal to:
a) In case of μ BSCs with bit error probabilities $\varepsilon_1,...,\varepsilon_\mu$

$$\begin{aligned}c(\mu*\mathrm{BSC}) = {}& \prod_{j=1}^{\mu}\varepsilon_j\log\prod_{j=1}^{\mu}\varepsilon_j + \prod_{j=1}^{\mu}(1-\varepsilon_j)\log\prod_{j=1}^{\mu}(1-\varepsilon_j)\\ &-(\prod_{j=1}^{\mu}\varepsilon_j + \prod_{j=1}^{\mu}(1-\varepsilon_j))\log(\prod_{j=1}^{\mu}\varepsilon_j + \prod_{j=1}^{\mu}(1-\varepsilon_j))\\ &+\prod_{j=1}^{\mu}\varepsilon_j + \prod_{j=1}^{\mu}(1-\varepsilon_j).\end{aligned}$$

b) In case of μ GECs with transition probabilities ζ_1, $\theta_1, \eta_1,..., \zeta_\mu, \theta_\mu, \eta_\mu$

$$\begin{aligned}c(\mu*\mathrm{GEC}) = {}& \prod_{j=1}^{\mu}\eta_j\log\prod_{j=1}^{\mu}\eta_j + \prod_{j=1}^{\mu}\theta_j\log\prod_{j=1}^{\mu}\theta_j\\ &-(\prod_{j=1}^{\mu}\eta_j + \prod_{j=1}^{\mu}\theta_j)\log(\prod_{j=1}^{\mu}\eta_j + \prod_{j=1}^{\mu}\theta_j)\\ &+\prod_{j=1}^{\mu}\eta_j + \prod_{j=1}^{\mu}\theta_j.\end{aligned}$$

c) In case of μ symmetrized BNSCs with transition probabilities $\delta_1, \varepsilon_1,..., \delta_\mu, \varepsilon_\mu$

$$\begin{aligned}c(\mu*\mathrm{BNSC}) = {}& \prod_{j=1}^{\mu}(\delta_j\varepsilon_j)\log\prod_{j=1}^{\mu}(\delta_j\varepsilon_j)\\ &+\prod_{j=1}^{\mu}(1-\delta_j)(1-\varepsilon_j)\log\prod_{j=1}^{\mu}(1-\delta_j)(1-\varepsilon_j)\\ &-\prod_{j=1}^{\mu}(\delta_j\varepsilon_j)\log(\prod_{j=1}^{\mu}(\delta_j\varepsilon_j) + \prod_{j=1}^{\mu}(1-\delta_j)(1-\varepsilon_j))\\ &-\prod_{j=1}^{\mu}(1-\delta_j)(1-\varepsilon_j)\log(\prod_{j=1}^{\mu}(\delta_j\varepsilon_j) + \prod_{j=1}^{\mu}(1-\delta_j)(1-\varepsilon_j))\\ &+\prod_{j=1}^{\mu}(\delta_j\varepsilon_j) + \prod_{j=1}^{\mu}(1-\delta_j)(1-\varepsilon_j)\end{aligned}$$

3.6 Redundancy with Proper Linear Codes

The aim of this subsection is an upper bound on the probability of undetected error for redundant transmission protected by proper linear codes. A linear code C is said to be proper if and only if the probability of undetected error $p_{ue}(\varepsilon,C)$ is an increasing function of ε in the interval $[0, 1/2]$. In [20] and [21] Wacker&Boercsoek proved that for proper linear codes and for all $0 \le \varepsilon \le 1/2$ the inequality

$$(6)\quad p_{ue}(\varepsilon,C) \le \frac{72}{121}\sqrt{2\pi}\frac{\sqrt{n}}{2^r}\frac{1}{d!}n^d\varepsilon^d + 2(\sqrt{2\varepsilon})^{n-1}$$

holds, where d is the minimum distance of C.

Therefore by the Corollaries 3, 6, and 8 for proper linear codes and redundant transmission via μ channels the probability of undetected satisfies the upper bounds of

Theorem 10: Let C be a proper linear code, then
a) In the case of μ BSCs

$$p_{ue}(\varepsilon_1,...,\varepsilon_\mu,C)$$
$$\le \frac{72}{121}\frac{\sqrt{2\pi n}}{2^r}\frac{1}{d!}n^d\varepsilon_{max}^{\mu\cdot d} + 2(\sqrt{2\varepsilon_{max}^{\mu}})^{n-1}$$

where $0 \le \varepsilon_1, ... , \varepsilon_\mu \le 1/2$ and $\varepsilon_{max} = \max(\varepsilon_1, ... , \varepsilon_\mu)$.
b) In the case of μ GECs

$$p_{ue}(\zeta_1,\eta_1,\theta_1,...,\zeta_\mu,\eta_\mu,\theta_\mu,C)$$
$$\le \min(\frac{72}{121}\frac{\sqrt{2\pi n}}{2^r}\frac{1}{d!}n^d\eta_{max}^{\mu\cdot d} + 2(\sqrt{2\eta_{max}^{\mu}})^{n-1},$$
$$\frac{72}{121}\frac{\sqrt{2\pi n}}{2^r}\frac{1}{d!}n^d(1-\theta_{min})^{\mu\cdot d} + 2(\sqrt{2(1-\theta_{min})^{\mu}})^{n-1}$$

where $0 \le \eta_1, \dots, \eta_\mu, \theta_1, \dots, \theta_\mu \le 1/2$, $\eta_{max} = \max(\eta_1, \dots, \eta_\mu)$ and $\theta_{min} = \min(\theta_1, \dots, \theta_\mu)$.

c) In the case of μ symmetrized BNSCs

$$p_{ue}(\delta_1, \varepsilon_1, \dots, \delta_\mu, \varepsilon_\mu, C)$$
$$\le \frac{72}{121} \frac{\sqrt{2\pi n}}{2^r} \frac{1}{d!} n^d (\varepsilon_{max} \delta_{max})^{\mu \cdot d}$$
$$+ 2(\sqrt{2(\varepsilon_{max} \delta_{max})^\mu})^{n-1}$$

where $0 \le \varepsilon_1, \dots, \varepsilon_\mu, \delta_1, \dots, \delta_\mu \le 1/2$, $\varepsilon_{max} = \max(\varepsilon_1, \dots, \varepsilon_\mu)$ and $\delta_{max} = \max(\delta_1, \dots, \delta_\mu)$.

Proof: a) By Corollary 3 and (6)

$$p_{ue}(\varepsilon_1, \dots, \varepsilon_\mu, C) \le p_{ue}(\prod_{j=1}^{\mu} \varepsilon_j, C)$$
$$\le \frac{72}{121} \sqrt{2\pi} \frac{\sqrt{n}}{2^r} \frac{1}{d!} n^d \left(\prod_{j=1}^{\mu} \varepsilon_j\right)^d + 2(\sqrt{2\prod_{j=1}^{\mu} \varepsilon_j})^{n-1}$$
$$\le \frac{72}{121} \frac{\sqrt{2\pi n}}{2^r} \frac{1}{d!} n^d \varepsilon_{max}^{\mu \cdot d} + 2(\sqrt{2\varepsilon_{max}^\mu})^{n-1}.$$

b) By Corollary 3 and (6)

$$p_{ue}(\eta_1, \dots, \eta_\mu, C) \le p_{ue}(\prod_{j=1}^{\mu} \eta_j, C)$$
$$\le \frac{72}{121} \sqrt{2\pi} \frac{\sqrt{n}}{2^r} \frac{1}{d!} n^d \left(\prod_{j=1}^{\mu} \eta_j\right)^d + 2(\sqrt{2\prod_{j=1}^{\mu} \eta_j})^{n-1}$$
$$\le \frac{72}{121} \frac{\sqrt{2\pi n}}{2^r} \frac{1}{d!} n^d \eta_{max}^{\mu \cdot d} + 2(\sqrt{2\eta_{max}^\mu})^{n-1},$$

and

$$p_{ue}(1-\theta_1, \dots, 1-\theta_\mu, C)$$
$$\le \frac{72}{121} \sqrt{2\pi} \frac{\sqrt{n}}{2^r} \frac{1}{d!} n^d \left(\prod_{j=1}^{\mu} (1-\theta_j)\right)^d$$
$$+ 2(\sqrt{2\prod_{j=1}^{\mu} (1-\theta_j)})^{n-1}$$
$$\le \frac{72}{121} \frac{\sqrt{2\pi n}}{2^r} \frac{1}{d!} n^d (1-\theta_{min})^{\mu \cdot d}$$
$$+ 2(\sqrt{2(1-\theta_{min})^{\mu \cdot d}})^{n-1},$$

then, by Corollary 6

$$p_{ue}(\zeta_1, \eta_1, \theta_1, \dots, \zeta_\mu, \eta_\mu, \theta_\mu, C)$$
$$\le \min\left(p_{ue}(\eta_1, \dots, \eta_\mu, C), p_{ue}(1-\theta_1, \dots, 1-\theta_\mu, C)\right)$$
$$\le \min(\frac{72}{121} \frac{\sqrt{2\pi n}}{2^r} \frac{1}{d!} n^d \eta_{max}^{\mu \cdot d} + 2(\sqrt{2\eta_{max}^\mu})^{n-1},$$
$$\frac{72}{121} \frac{\sqrt{2\pi n}}{2^r} \frac{1}{d!} n^d (1-\theta_{min})^{\mu \cdot d} + 2(\sqrt{2(1-\theta_{min})^\mu})^{n-1}$$

b) By Corollary 8 and 3 and by (6)

$$p_{ue}(\delta_1, \varepsilon_1, \dots, \delta_\mu, \varepsilon_\mu, C) \le p_{ue}(\delta_1 \varepsilon_1, \dots, \delta_\mu \varepsilon_\mu, C)$$
$$\le p_{ue}(\prod_{j=1}^{\mu} \delta_j \varepsilon_j, C)$$
$$\le \frac{72}{121} \sqrt{2\pi} \frac{\sqrt{n}}{2^r} \frac{1}{d!} n^d \left(\prod_{j=1}^{\mu} \delta_j \varepsilon_j\right)^d$$
$$+ 2(\sqrt{2\prod_{j=1}^{\mu} \delta_j \varepsilon_j})^{n-1}$$
$$\le \frac{72}{121} \frac{\sqrt{2\pi n}}{2^r} \frac{1}{d!} n^d (\delta_{max} \varepsilon_{max})^{\mu \cdot d}$$
$$+ 2(\sqrt{2(\delta_{max} \varepsilon_{max})^\mu})^{n-1}.$$

∎

Even for relatively poor ε the remainder term

$$2(\sqrt{2\varepsilon})^{n-1}$$

is small compared with the other term on the right hand sight of (6). So Theorem 10 is very useful to determine maximum values of ε and n for meeting specific upper bound B on $p_{ue}(\varepsilon, C)$: Firstly choose ε_0 and n_0 such that

$$\frac{72}{121} \sqrt{2\pi} \frac{\sqrt{n_0}}{2^r} \frac{1}{d!} n_0^d \varepsilon_0^d < B$$

(or the respective terms of Theorem 10). then verify that

$$2(\sqrt{2\varepsilon_0})^{n_0-1} < B - \frac{72}{121} \sqrt{2\pi} \frac{\sqrt{n_0}}{2^r} \frac{1}{d!} n_0^d \varepsilon_0^d$$

Let us state, at this place, without proof, that the results of Theorem 10 remain valid for nonlinear

proper codes. But for our needs in section 4, this generalization is not essential.

4 Laser Channels

4.1 Safety Integrity levels (SIL)

In this section we investigate the problem of achieving a certain Safety Integrity Level (SIL) by optical data transmission via fibers using multiple wavelength semiconductor laser sources (short: laser channels, in the following) protected by a CRC C.

Safety Integrity Levels are defined by bounds on the number Λ of undetected errors per hour, given by

(7) $\Lambda = 3600 \cdot p_{ue}(C) \cdot v \cdot (m-1) \cdot 100$

where

  v = number of safety related messages per second
  m = number of communicating devices
100 = 1% factor

For details see IEC 61508 2000, [11]. In [11] it is highly recommended that "the safety communication channel does not consume more than 1 % of the maximum PFH (Probability of Failure per Hour) of the target SIL for which the functional safety communication profile is designed". The 1%-factor is introduced to avoid the proof of this assumption. The respective bounds on Λ are shown in table 3.

Table 3 Safety Integrity Levels

SIL	4	3	2	1
Λ high demand	10^{-8}	10^{-7}	10^{-6}	10^{-5}

A more detailed analysis of safety networks and the used items can be found in [1] and [2]. In [19] a safety analysis of fieldbus systems is performed by means of a Markov model.

4.2 The Bit Error Probability of Laser Channels

A lot of publications have been dealing with laser channels and their respective bit error probabilities. Clearly the bit error probability depends on the material the waveguides are consisting of. Because of long-term low cost arguments we mainly studied waveguides based on polymers or fibers and multi

channel transmission techniques by Wavelength Division Multiplexing (WDM). Table 4 contains a list of POF waveguides together with the respective bit error probabilities reported in the literature. The example of Boom et al. [4] shows that WDM does not affect the bit error probability.

Table 4 Bit Error Probabilities

ref.	material	wave-length [nm]	transfer rate [Gbps]	BER
[3]	GI-POF	840	1.25	10^{-9}
[4]	GI-POF WDM	840/1310	2.5	10^{-9}
[7]	MMF	850	3.125	$<10^{-11}$
[8]	GI-POF	1300	11	10^{-10}
[12]	GI-POF	647	2.5	10^{-9}
[13]	PMMA GI-POF	645	2.5	10^{-9}
[15]	GI-POF MMF	935	3	10^{-11}
[15]	GI-POF	935	7	10^{-11}
[16]	GI-POF	850	2.5	10^{-11}
[18]	MMP	850	1.0625	10^{-11}
[24]	GI-POF	850	2.5	10^{-12}
[25]	GPF POF	1300	1	10^{-9}

4.3 Redundancy by 4 Laser Channels

For our application we consider 2 communicating devices, a redundant 2-processor system for example in a back to back arrangement (Fig.2), communicating at a maximum transfer rate of 10 Gigabit per second via 4 binary symmetric channels without memory: 2 waveguides, each one multiplexed by 2 laser beams. The
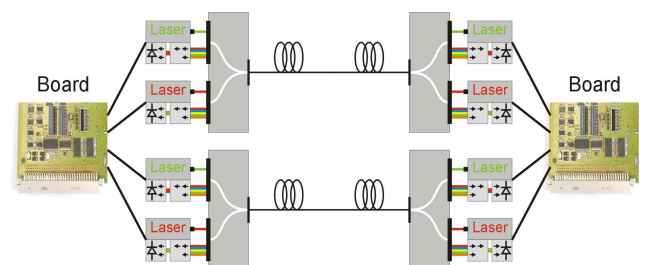

Fig. 2 Laser Channels

message length is supposed to be fixed at 1024 bytes, and data transmission should be protected by a CRC32. Take for example the CRC32/6 of Castagnoli et al. from [5] or the standard ethernet

polynomial IEEE 802.3 investigated by Fujiwara et al. in [6]. In any case we have a block length $n = 8224$. Then, by (7) and table 3, SIL 4 is achieved, if

$$(8) \qquad p_{ue}(C) < 10^{-20}$$

Now, by [5], at a block length of $n = 8224$, the CRC32/6 has a minimum distance of $d = 6$. And, by [6], at the same block length, the IEEE802.3 polynomial has $d = 4$. By [5] both CRCs are proper for this block length. Therefore by (8) and Theorem 10 a) SIL 4 is surely achieved, if the maximum bit error probability ε_{max} occurring in our example satisfies the bounds presented in table 5. Taking account of the possible failure of one waveguide, table 5 also includes the maximum values in the case of 2 channels.

Table 5 Maximum Bit Error Probability

CRC	ε_{max} for 4 channels	ε_{max} for 2 channels
CRC32/6	0.04	0.0017
IEEE 802.3	0.02	0.0004

A comparison of table 5 with the results mentioned in table 4 shows that in any case the bit error probability of our example is small enough to meet SIL4. Even in the case of a complete failure of one waveguide there is a safety margin of at least 10^{-5} between the bounds of table 5 and the measured values.

4 Conclusion

Formulas for the probability of undetected error of redundant data transmission via different channel types protected by an arbitrary (not necessarily linear) (n, K) code have been derived. The result is applied to ensure Safety Integrity Level 4 for multi channel coloured laser transmission protected by CRC32/6 of Castagnoli et al. or the standard Ethernet polynomial IEEE 802.3. As was to be expected because of the larger minimum distance d CRC32/6 performs slightly better than IEEE 802.3. In any case the performance of both polynomials is sufficient to guarantee SIL4 in the gigabit scope..

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