A unified approach for heterogeneity and node fault robustness in dynamic sensor networks

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Abstract: The paper deals with the area coverage problem for a network of mobile sensors. Continuous mobility is used for getting measures over the area of interest with a reduced number of sensors units that do not cover all the area at any time but, moving, assure the measure at any point within a prefixed time interval. In addition, the case of heterogeneous network is addressed, in the sense that each node can be equipped with a different set of sensors. A centralized formulation of the control problem for the motion of the agents is given. It is also shown that the same problem formulation can be used to address the problem of nodes faults robust coverage for over dimensioned networks. Constraints introduced by the actuator limits as, communication connection and collision avoidance are clearly considered. A global solution is proposed and some simulations are reported to show its effectiveness.

Key–Words: Sensor network, dynamic configuration, optimal motion, network communications, heterogeneous sensors, robust coverage.

1 INTRODUCTION

Collecting data from large areas is basic requirement for many different monitoring or surveillance tasks; Some major examples are the measurement of chemical and physical parameters in relevant natural ecosystems (farms, lakes, woods, rivers, national parks, etc.) as well as in sensible and potentially hazardous structures (factories, chemical plants, refineries, etc.), or the detection of presence and distribution of people in particular areas or buildings for example during critical events, and so on.

Recent technological advances in wireless communication networking and in miniaturization of electronic devices have suggested the use of sets of smart sensors, able to perform simple elaborations an to exchange data over a communication network ([2, 17]). This kind of distributed sensors systems have been called, by the scientific and engineering community, *sensors networks* ([13, 20]).

In general, a sensor can detect an event or perform measurement within a given area that depends on sensor configuration. The problem of maximizing the number of detectable events or in general the field of measurement of a sensor network is known in literature as the *area coverage* problem.

Considering static sensors, the coverage problem has been addressed in terms of optimal usage of a given set of sensors, randomly deployed, in order to assure full coverage and minimizing energy consumption ([3, 24, 20]), or in terms of optimal sensors deployment on a given area, such as optimizing sensors locations ([18, 19, 6, 16, 25]).

In [21, 14, 8] the use of self-deploying mobile sensors has been proposed in order to make the sensors network able to configure according to the environment and to sensors faults. There, motion is used for the first sensors allocation and for occasional reconfiguration tasks.

Mobile sensors can also be used to dynamically cover the area of interest, that is moving continuously in order to cover all the field within a prefixed time interval Θ with a reduced number of sensor nodes that, at any given time t, can cover only a portion of the whole area. In this *dynamic sensor networks* the lost of pointwise continuous measurements can be a reasonable cost to pay for a strong reduction of the number of sensors.

On the other hand, it is quite usual that, in monitoring or measurements over large areas, almost all the physical quantities are required to be acquired with given space and time discretization. Then, it is not necessary that a sensor is fixed at a point with the measurement performed at prefixed times, letting it unused for all the remaining time.

It seems more fruitful that, after a measurement, the sensor moves to a different place to perform a new measurement. The only requirement is that it, or another equivalent sensor, can return to any point of interest within the prefixed sampling time.

The effect of the application of such an idea is to have continuously moving sensor units that, within a given time, collect measures over the whole area being ready to repeat the operation.

Then, under the assumption of dynamic network, the area coverage problem is posed in terms of looking for optimal trajectories for the N moving sensors, in presence of some constraints like communication connection preservation, motion limitations, energetic considerations and so on, that cover all the area.

In [22, 4] the dynamic coverage problem for multiple sensors is studied, with a variational approach, in the level set framework. Obstacles occlusions are considered and suboptimal solutions are proposed also in three dimensional environments ([5]).

A survey of coverage path planning algorithms for mobile robots moving on the plane is presented in [7].

In [1] the dynamic coverage problem for one mobile robot with finite range detectors is studied and an approach based on space decomposition and Voronoi graphs is proposed.

In [15], a distributed control law is developed in order to guarantee the coverage goal with multiple mobile sensors under the hypothesis of communication network connection. Collisions avoidance is also considered.

Various problems associated with optimal path planning for mobile observers such as mobile robots equipped with cameras to obtain maximum visual coverage in the three-dimensional Euclidean space are considered in [23]. Numerical algorithms for solving the corresponding approximated problems are there proposed.

In [11, 9, 10, 12] a general formulation of dynamic coverage is given by the authors, a sensor network model is proposed and an optimal control formulation is given. Suboptimal solution are computed by discretization. Sensors and actuators limits, geometric constraints, collisions avoidance and communication network connectivity maintenance are considered.

The approaches introduced up to now also by the authors ([11, 9, 10, 12]), are referred to homogeneous sensor networks, that is each node in the network is equivalent to any other one in terms of sensing capabilities (same sensor or same set of sensors over each node). A more real case is represented by several kind of sensors for different measurements, with different sensing range, collected in different sets and hosted on moving platforms. In this case, there are several sets of different sensors as nodes in the network. We will refer to as heterogeneous sensor networks.

In this paper the heterogeneous case is addressed. The mathematical model needed to introduce heterogeneity in the sensor networks is then formulated and it is shown that the same formulation can easily be extended to face one of the most important problem for sensors networks: the node fault robustness of the control strategy.

In details, some definitions useful when dealing with heterogeneous dynamic sensor networks are given in section 2. The mathematical model used is described in section 3. In section 4 the coverage area problem is introduced and described in terms of an optimal control problem, whose solution is computed by space and time discretization, so obtaining a nonlinear programming problem that gives a suboptimal solution. Simulations results are reported and discussed in section 6. Some concluding remarks in section 7 end the paper.

2 General Formulation

2.1 Dynamic Sensors Networks

Let W be the compact subset of the real Euclidean space $\mathbb{R}^n$ ($n = 2$ in the present paper) representing the *workspace*. A point $p \in W$ is denoted by $x_p = (x_{p_1}, \ldots, x_{p_n})$ with respect to a given orthonormal basis for $\mathbb{R}^n$.

Let $\Xi = \{\xi_1, \ldots, \xi_n\}$ be the set of quantities of interest defined on W.

A dynamic sensor network can be seen as a set $\Sigma = \{\sigma_1, \sigma_2, \ldots \sigma_m\}$ of *m mobile sensors*. Each mobile sensor can be represented by

$$\sigma_j =< \mathscr{C}^{(j)}, f^{(j)}, \Xi^{(j)}, \{M_\xi^{(j)} \,|\, \xi \in \Xi^{(j)}\}, \kappa^{(j)} >$$

where

- $\mathscr{C}^{(j)}$ is the sensor configuration space;
- $f^{(j)}$ is the sensor dynamic function that describes the evolution of the sensor configuration depending on a control input $u^{(j)}$ according to the dynamics
$$\dot{q}^{(j)} = f^{(j)}(q^{(j)}, u^{(j)})$$
- $\Xi^{(j)} \subseteq \Xi$ is the subset of magnitudes that sensor σ_j can measure.

- $M_\xi^{(j)} = M_\xi^{(j)}(q^{(j)}) \subseteq W$ is the subset of W in which the sensor σ_j, in configuration $q^{(j)}$, can measure the magnitude $\xi \in \Xi^{(j)}$. This fact will be addressed saying that sensor σ_j in configuration $q^{(j)}$ $\xi - covers$ the set $M_\xi^{(j)}(q^{(j)})$

- $\kappa^{(j)} = \kappa^{(j)}(\sigma_h)$ is the sensor communication function, that is σ_j can communicate with a sensor σ_h if and only if $\kappa^{(j)}(\sigma_h) > 0$

Once the whole network is considered, the m functions $\kappa^{(j)}$ can be collected into one function κ with two arguments, according to

$$\kappa\left(\sigma^{(j)}, \sigma^{(h)}\right) = \kappa^{(j)}(\sigma_h)$$

At the same time, it is possible to define generalized configuration $q(t)$ and generalized input $u(t)$. Generalized dynamic can then be written as

$$\dot{q}(t) = f(q(t), u(t))$$

2.2 Coverage

Let us indicate with $q^{(j)}(\Theta)$ the configuration evolution of sensor σ_j during a given a time interval $\Theta = [0, t_f]$. It is possible to define the subset of W $\xi - covered$ by σ_j during Θ as

$$M_\xi^{(j)}(q^{(j)}(\Theta)) = \bigcup_{t \in \Theta} M_\xi^{(j)}(q^{(j)}(t)) \qquad (1)$$

Considering generalized configuration $q(\Theta)$ it is possible to define the network field of measure during Θ, with respect to magnitude ξ, as

$$M_\xi(q(\Theta)) = \bigcup_{\sigma_j \,|\, \xi \in \Xi^{(j)}} M_\xi^{(j)}(q^{(j)}(\Theta)) \qquad (2)$$

Looking at the whole set of magnitudes, the subset of W $\Xi - covered$ by the network can be defined as

$$M_\Xi(q(\Theta)) = \bigcap_{\xi \in \Xi} M_\xi(q(\Theta)) \qquad (3)$$

The area $\Xi - covered$ by the sensor network during Θ is then the measure of $M(q(\Theta))$

$$A_\Xi(\Theta) = \mu(M_\Xi(q(\Theta))) \qquad (4)$$

2.3 Communication

According with their communication capabilities, sensors can be seen as nodes of a dynamic communication network. This network can be represented by a dynamic graph

$$\mathscr{G}(t) = < N_\mathscr{G}, E_\mathscr{G}(t) >$$

where

$$N_\mathscr{G} = \{\sigma_j\}$$

indicates the nodes set and

$$E_\mathscr{G}(t) = \{(\sigma_j, \sigma_h) \in N_\mathscr{G} \times N_\mathscr{G} \,|\, \kappa(\sigma_j, \sigma_h) > 0\}$$

indicates the edges set. It is important to stress that the edge set is time varying because it depends on the network generalized configuration $q(t)$, since $\kappa(\sigma_j, \sigma_h)$ can vary according to $q(t)$.

3 The dynamic sensor network model

In this paper the mathematical model for dynamic sensor networks is introduced in terms of **L**inear dynamic-**P**roximity measure-**P**roximity communication (**LPP**) Model. A short description is here given.

3.1 Sensors Dynamics

Each sensor σ_j is modeled, from the dynamic point of view, as a material point of mass m_j moving on $\mathbb{R}^2$. For sake of simplicity, the motion of each mobile unit is assumed to satisfy the classical simple equations

$$\ddot{x}^{(j)}(t) = \frac{u^{(j)}(t)}{m_j} \qquad (5)$$

where $x^{(j)}(t)$ is the sensor position on $\mathbb{R}^2$ at time t.

Clearly, more realistic dynamics can be used without any change in the formulation introduced in the sequel. The only difference is in the numerical implementation of the problem solution.

Sensor configuration is then represented by

$$q^{(j)}(t) = \begin{pmatrix} \dot{x}_1^{(j)}(t) & x_1^{(j)}(t) & \dot{x}_2^{(j)}(t) & x_2^{(j)}(t) \end{pmatrix}^T$$

and for the configuration space one has

$$\mathscr{C}^{(j)} \subseteq \mathbb{R}^4 \quad \forall j$$

From now on, *sensor trajectory* will mean *sensor position evolution*.

3.2 Proximity Measure Model

It is assumed that at any time t sensor σ_j can take measures on magnitude $\xi \in \Xi^{(j)}$ in a circular area of radius ρ_ξ around its current position $x^{(j)}(t)$. The sensor field of measure with respect to ξ is then a disk of center $x^{(j)}(t)$ and radius ρ_ξ

$$M_\xi^{(j)}(q^{(j)}) = \{p \in W : \|x^{(j)} - x_p\| \leq \rho_\xi \quad \xi \in \Xi^{(j)}\} \tag{6}$$

As seen in subsection 2.2, starting from $M_\xi^{(j)}(q^{(j)})$ it is possible to define the area $\xi - covered$ and the area $\Xi - covered$ by the sensor network during a given time interval Θ.

3.3 Proximity Communication Model

The communication network is modeled as an Euclidean graph. Two mobile sensors are assumed to communicate each other at time t if the distance between them is smaller than a given communication radius ρ_C.

The communication function is then given by

$$\kappa(\sigma_j, \sigma_h) = \kappa\left(x^{(j)}, x^{(h)}\right) = \rho_C - \|x^{(j)} - x^{(h)}\| \tag{7}$$

It is easy to see that this communication function makes the network graph $\mathscr{G}$ undirected. In fact

$$\kappa(\sigma_j, \sigma_h) = \kappa(\sigma_h, \sigma_j) \qquad \forall j, h \in [1, \ldots, m]$$

4 Coverage Problem Formulation

According to the mathematical model introduced above, it is possible to formulate the coverage problem as an optimal control problem. The idea is to maximize the area covered by sensors in a fixed time interval according to the constrains defined in the sequel.

4.1 Objective Functional

In subsection 2.2 the area $\xi - covered$ by a set of m moving sensors was defined as the union of the measure sets of the sensors, with respect to the magnitude ξ, at any time t. From a computational point of view, this quantity is not easy to be computed, even for the simple measure set model introduced in 3.2. The interest of an online implementation, with the well known limits on computation time, especially if low power devices are used, presses for the definition of an equivalent alternative performance measure.

Defining the distance between a point p of the workspace and a generalized trajectory $x(\Theta)$ within a time interval $\Theta = [0, t_f]$ as

$$d_\xi(x(\Theta), p) = \min_{t \in \Theta, j \in \{1,2,\ldots m\}} \left\|x_p - x^{(j)}(\Theta)\right\|, \quad \xi \in \Xi^{(j)} \tag{8}$$

and making use of the function

$$\text{pos}(\chi) = \begin{cases} \chi & \text{if} \quad \chi > 0 \\ 0 & \text{if} \quad \chi \leq 0 \end{cases} \tag{9}$$

that fixes to zero any non positive value, the function

$$\hat{d}_\xi(x(\Theta), p, \rho_\xi) = \text{pos}\left(d_\xi(x(\Theta), p) - \rho_\xi\right) \geq 0$$

can be defined. Then, a measure of how the generalized trajectory $q(\Theta)$ produces a good $\xi - coverage$ of the workspace can be given by

$$J_\xi(x(\Theta)) = \int_{p \in W} \hat{d}_\xi(x(\Theta), p, \rho_\xi) \tag{10}$$

The simplest way to evaluate how a given generalized trajectory $x(\Theta)$ $\Xi - covers$ the set of interest W w.r.t. the whole set of magnitudes Ξ is

$$J_\Xi(x(\Theta)) = \sum_{\xi \in \Xi} \alpha_\xi J_\xi(x(\Theta)) \tag{11}$$

With this choice, coverage level of different magnitudes could be sensibly different, being dependent on the arbitrary choice of the coefficients $\alpha^{(j)}$. On the other hand, it is possible to choose such weights $\alpha^{(j)}$ so that it can be possible to define a hierarchy of importance between magnitudes, setting $\alpha^{(i)} > \alpha^{(j)}$ when $\sigma^{(i)}$ is more important than $\sigma^{(j)}$, or $\alpha^{(i)} = \alpha^{(j)}$ when they are equally important ($i, j \in [1, m]$).

An alternative way to evaluate *global* coverage could be represented by

$$J_\Xi(x(\Theta)) = \max_{\xi \in \Xi} J_\xi(x(\Theta)) \tag{12}$$

Clearly, minimizing $J_\Xi(x(\Theta)$ means maximizing $\Xi - coverage$ of W. If $J_\Xi(x(\Theta)) = 0$ then $x(\Theta)$ $\Xi - covers$ completely the workspace.

4.2 Geometric Constraints

It is possible to constrain sensors to move inside a subset of $\mathbb{R}^2$. For a box-shaped subset, one can write for the constraints

$$x_{min} \leq x^{(j)}(t) \leq x_{max}$$

If required, it is possible to set the starting and/or the final state (positions and/or speeds)

$$q(0) = q_{start}$$
$$q(t_f) = q_{end}$$

This can be useful for the continuous measurement of the area according what discussed in the introduction. Once that a maximum time delay t_{MAX} between measurements of the same physical quantity at the same point of the space W is given, a solution that assures the respect of such a constraint is represented by periodic solution to the motion problem. In this case, periodic trajectories must be imposed, so bringing to the introduction of the additional constraints

$$q^{(j)}(0) = q^{(j)}(t_f)$$

with $t_f \leq t_{MAX}$, $j = 1, \ldots, m$.

4.3 Dynamic Constraints

Physical limits for the motion on the actuators, represented by maximum power and devices saturations, and/or on the sensors, mainly in terms of velocity of the measurement acquisition and, if present, of pre-elaboration of the data, suggest the introduction of the following additional constraints

$$|\dot{x}(t)| \leq v_{max}$$

for the velocities and

$$|u(t)| \leq u_{max}$$

for the actuators power.

4.4 Communication Constraints

Communication between mobile sensors is very important, since the mobile units constitute the communication network used for data exchange and transmission, but also for sensor localization, coordination and commands communication. Several works in literature deal with communication problems in multi agent systems, mainly form the protocols and energetic point of view ([26, 27] for example). In this case, due to the introduction of the mobility, in order to assure communication between sensors a full connection of the sensor network at any time is required. This can be imposed introducing some motion constraints.

As said before, the communication model introduced in subsection 3.3 makes the communication graph $\mathscr{G}(t)$ undirected. It is well known tha an undirected graph is connected if it contains a spanning tree.

Assuming that, at time $t = 0$, $\mathscr{G}$ is connected, it is possible to maintain network connection just maintaining links that belong to a spanning tree.

Assigning a weight at each edge of $E_{\mathscr{G}}$ it is possible to define the *Minimum Spanning Tree* of $\mathscr{G}$ as the spanning tree with minimum weight. In particular, being $\mathscr{G}$ an Euclidean graph, it comes natural to define the edges weights as

$$w(x^{(j)}, x^{(h)}) = \|x^{(j)} - x^{(h)}\|$$

In this case the minimum spanning tree is said *Euclidean* (EMST).

The EMST can be easily and efficiently computed by standard well known algorithms (for example Kruskal's algorithm, Prim's algorithm, etc.).

Indicating the EMST with $\mathscr{T}(t) = < V_{\mathscr{G}}, E_{\mathscr{T}}(t) >$, where $E_{\mathscr{T}}(t) \subseteq E_{\mathscr{G}}(t)$, to maintain the communication network connection it is necessary to satisfy the following constraints $\forall t \in \Theta$

$$\|x^{(j)}(t) - x^{(h)}(t)\| \leq \rho_C \quad \forall (\sigma_j, \sigma_h) \in E_{\mathscr{T}}(t) \quad (13)$$

4.5 Optimal Control Problem

The coverage problem can now be formulated as an optimal control problem

$$\min_{q(0), u(\Theta)} J_{\Xi}(x(\Theta))$$

$$x_{min} \leq x(t))) \leq x_{max} \quad \forall t \in \Theta$$
$$(q(0) = q_{start})$$
$$(q(t_f) = q_{end})$$

$$v_{min} \leq \dot{x}(t) \leq v_{max} \quad \forall t \in \Theta$$
$$u_{min} \leq |u(t)| \leq u_{max} \quad \forall t \in \Theta$$

$$\|x^{(j)}(t) - x^{(h)}(t)\| \leq \rho_C \quad \forall (\sigma_j, \sigma_h) \in E_{\mathscr{T}}(t)$$

This problem is, for general cases, very hard to be solved analytically. In order to get a solution, a discretization is performed, with respect to both space and time in all the time dependent expressions ([11, 9, 10, 12]). The workspace W is then divided into square cells, with resolution (size) l_{res}, so obtaining a grid in witch each point c_{rs} is the center of a cell, and the trajectories are discretized with sample time T_s.

Discretization allows to represent the coverage problem as a solvable *Nonlinear Programming Problem*

$$\min_{x(0),u_N} J_\Xi(x_N)$$

$$x_{min} \le x_N \le x_{max}\ , \forall n = 0,1,\cdots,N$$
$$(q^{(j)}(0) = q^{(j)}_{start}\ \forall j)$$
$$(q^{(j)}(NT_s) = q^{(j)}_{end}\ \forall j)$$

$$v_{min} \le \dot{x}_N \le v_{max}$$
$$u_{min} \le u_N \le u_{max}$$

$$\|x^{(j)}(nTs) - x^{(h)}(nTs)\| \le \rho_C \quad \forall(\sigma_j,\sigma_h) \in E_{\mathscr{T}}(nT_s)$$

Then, suboptimal solutions can be easily and fastly computed using numerical methods. In the simulations performed, the SQP (Sequential Quadratic Programming) method has been applied.

5 Robust Coverage

Robustness w.r.t node faults is, obviously, a very desirable characteristic for a sensor network. For dynamic sensor networks robustness can be achieved online, dynamically changing sensors trajectories when a node fault happens, or offline, over-dimensioning the sensor network and planning sensors trajectories in order to guarantee coverage performances in case of faults. The second approach is the one considered in this section, since it is possible to see how robust trajectories planning can be considered as a particular heterogeneous sensor network trajectory planning.

Consider a magnitude ξ defined on the sensor network workspace W, that can be measured within a radius ρ_ξ. Assume, now, that m sensors allow to reach the desired coverage performances w.r.t. the magnitude ξ. As said, it is possible to reach robustness over-sizing the sensor network, that is increasing the number of sensors. This means that if it is required to assure robustness w.r.t. the fault of h sensors, then $m+h$ sensors must be used.

To plan sensors trajectories it is possible to use the same formulation as in subsection 4.5, adding $\binom{m+k}{m} - 1$ auxiliary magnitudes, measurable within the same radius of ξ. Denote by $\hat{\xi}_i$, $i = 1,2,...,\binom{m+k}{m}$, the new set of magnitudes of interest. Consider all the combination of m sensors, calling them $\{\sigma\}_i$, $i = 1,2,...,\binom{m+k}{m}$. Finally, associate magnitudes to sensors according to the following law

$$\hat{\xi}_i \in \Xi^{(j)} \iff \sigma_j \in \{\sigma\}_i$$

The multiple magnitudes coverage problem formulated in 4.5, so modified, is equivalent to the robust coverage problem w.r.t the magnitude ξ.

The idea at the basis of this formulation is that if any subset of m sensors in a set of $m+k$ must be able to cover all the field, so being robust with respect of k faults, one can imagine to deal with $\binom{m+k}{m}$ fictitious heterogeneous sets of sensors. Then, the solution is represented by the area coverage of each of such set independently.

So, assuming the fault of $h \le k$ sensors, each of the remaining $\binom{m+k-h}{m}$ subsets still cover the field.

Clearly, in case of real multiple quantities of interest, the same operation must be performed with respect to each quantity.

Robustness in communications can be attained choosing an appropriate topology for the network and constraining sensors to maintain it ([10]).

6 Simulations

In this section simulations results are presented to show the effectiveness of the proposed methodology.

In the simulations, sensors are assumed to have unitary masses ($m_j = 1,\ j = 1,\dots,m$). Speeds and controls are constrained as follow

$$|\dot{x}^{(j)}(t)| \le 1$$

$$|u^{(j)}(t)| \le 1.5$$

$\forall j \in [1,m]$. All the sensors communication is assumed to be reliable within the communication radius $\rho_c = 5.5$.

The considered time interval is $\Theta = 15\ sec$.

In figure 1 it is shown a solution for the problem of measuring three quantities ($\Xi = (\xi_1,\xi_2,\xi_3)$) using four sensor units ($\Sigma = (\sigma_1,\sigma_2,\sigma_3,\sigma_4)$ equipped according the following subsets of Ξ

$$\Xi_1 = \{\xi_1,\xi_2\} \quad \Xi_2 = \{\xi_2,\xi_3\}$$

$$\Xi_3 = \{\xi_1,\xi_2\} \quad \Xi_4 = \{\xi_2,\xi_3\}$$

The workspace is assumed box shaped.

Magnitudes can be measured within the following radii:

$$\rho_{\xi_1} = 2 \quad \rho_{\xi_2} = 1 \quad \rho_{\xi_3} = 3$$

Suboptimal trajectories and the coverage status of the workspace are displayed (w.r.t each magnitude (a,b,c), and to the whole magnitudes set (d)). The figures show that the velocity constraints, together with the dimensions of the area to be measured and the time of 15 sec., do not allow a full area coverage. However, it is evident that the uncovered parts represent a small fraction of the whole area. Increasing the number of sensors m or the maximum time Θ the fraction can be reduced or even cancelled.

In figure 2 it is reported a solution for the problem of measuring a single magnitude with a dynamic sensor network robust with respect to the fault of one sensing node. It is assumed that the set is composed by 3 sensors. Then, according to the notations introduced in section 5, $m = 2$ and $k = 1$.

Also in this case the workspace is assumed to be a box subset of the Euclidean space $\mathbb{R}^2$.

Suboptimal trajectories and the coverage status of the workspace are displayed in the case of fault of one of the nodes (a,b,c), and without faults (d). In this case, the choices of Θ and of the number of sensors allow a full coverage of the given area.

In figure 3 the same task of the previous simulation is performed on a generic shaped convex workspace in order to show that the shape of the workspace does not affects the complexity of the computation but the solutions obtained are clearly different since different is the motion required to cover the different area. At the same time, it is evident from the figures that also in this case full area coverage is attained.

7 CONCLUSIONS

In this paper the case of heterogeneous mobile sensor networks has been considered. The mobility of the sensors is introduced in order to allow a reduced number of sensors to measure the same field, under the assumption that discontinuous measurements are acceptable. In addition, each mobile platform representing the nodes of the net has been considered equipped with different sets of sensors, so introducing a non homogeneity in the sensor network. A general formulation of the field coverage problem has been introduced in terms of optimal control techniques. A discretized formulation of the problem has been introduced in order to admit an online implementation, thanks to a reduced order of computation complexity, giving suboptimal solutions that, however, does work in a satisfactory way. With the same formulation, robust coverage with respect to the fault of up to a prefixed number k of the initial $m+k$ nodes has been addressed. Constraints introduced by kinematic and dynamic limits on mobility of the moving elements as well as by communications limits (network connectivity) have been also considered. Some simulation results showing the behavior and the effectiveness of the proposed solution have been reported. A global approach has been followed in the present paper. However, the possibility of using local approaches, that reduce the data transfer between network nodes and make possible fast local computations at each node, is under investigation.

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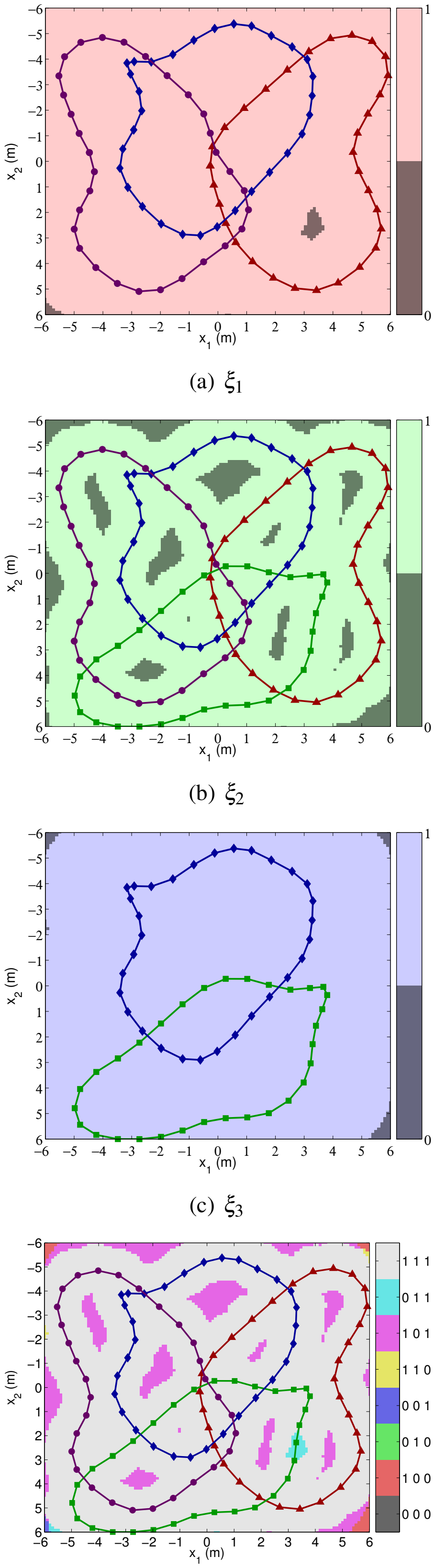


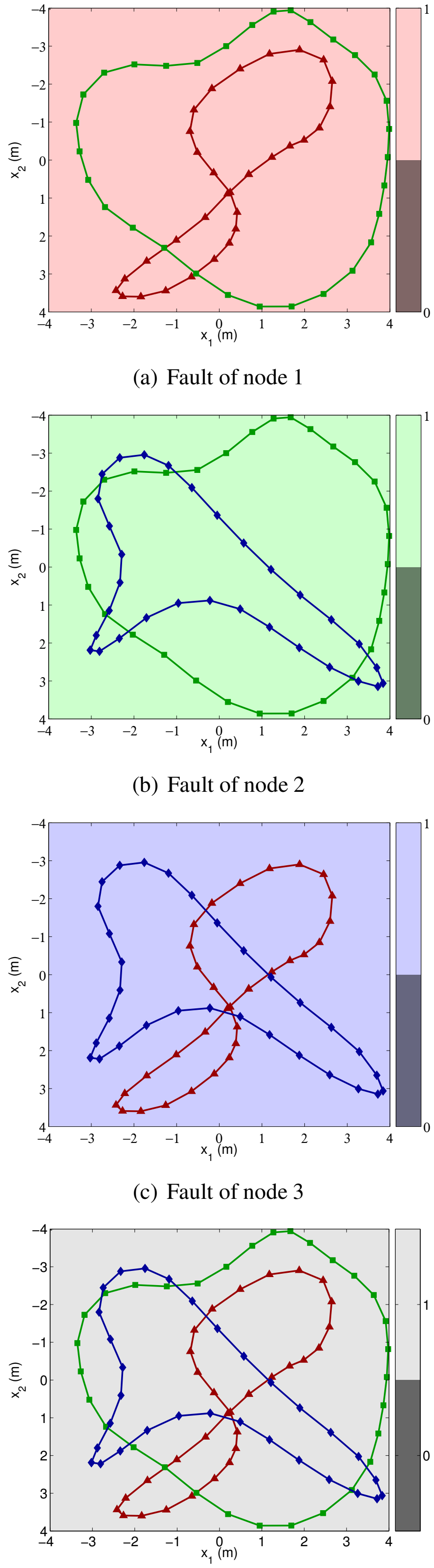
(a) ξ_1

(b) ξ_2

(c) ξ_3

(d) ξ_1 ξ_2 ξ_3

Figure 1: Multiple measures with an heterogeneous dynamic sensor network



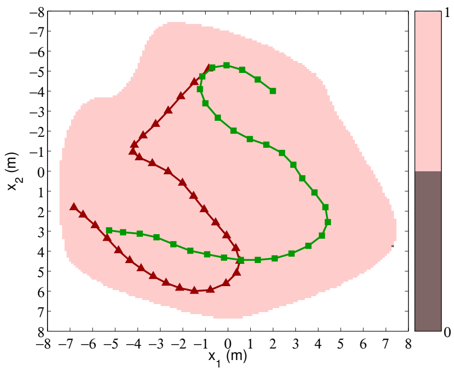
(a) Fault of node 1

(b) Fault of node 2

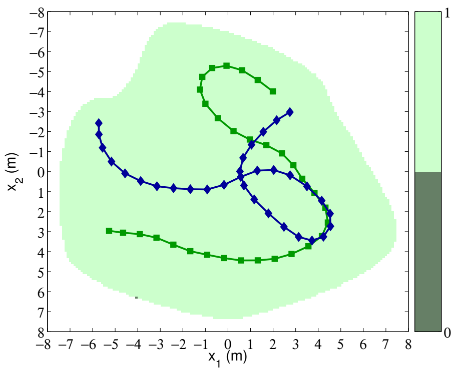
(c) Fault of node 3

(d) No faults

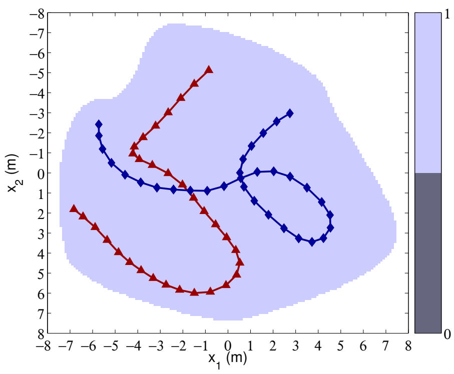
Figure 2: One node fault robust dynamic sensor network



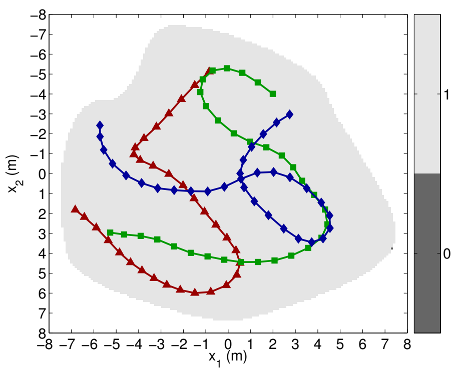
(a) Fault of node 1



(b) Fault of node 2



(c) Fault of node 3



(d) No faults

Figure 3: One node fault robust dynamic sensor network on generic shaped workspace