

Probability Density Function of M-ary FSK Signal in the Presence of Noise, Interference and Fading

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Abstract: - In this paper the receiver for the demodulation of M-FSK signals in the presence of Gaussian noise, intersymbol interference, impulse interference and fading is considered. The probability density function of M-ary FSK signal in the presence of noise, interference and fading is derived. These interferences can seriously degrade the communication system performances.

Key-Words: - M-ary Frequency Shift Keying, Gaussian Noise, Intersymbol Interference, Impulse Interference, Rayleigh Fading, Nakagami Fading, Probability Density Function, Joint Probability Density Function

1 Introduction

In this paper we consider a system for coherent demodulation of M -ary FSK signals in the presence of Gaussian noise, impulse noise and variable signal amplitude. These disturbances can seriously degrade the performance of communication systems [1]-[3]. In the paper [4], the performance evaluation of several types of FSK and CPFSK receivers was investigated in detail using the modified moment's method. Also, the error probability of the cross-correlator receiver for binary digital frequency modulation detection is studied using theoretical analysis and computer simulations [5]. In [6] average bit-error probability performance for optimum diversity combining of noncoherent FSK over Rayleigh channels is determined. Performance analysis of wide-band M -ary FSK systems in Rayleigh fading channels is given in [7].

In order to view the influence of Gaussian noise, intersymbol interference and fading on the performances of M -ary FSK system we derived the probability density function of M -ary FSK receiver output signal, the joint probability density function of output signal and its derivative and the joint probability density function of output signal at two time instants.

The bit error probability, the signal error probability and the outage probability can be determined by the probability density function of an output signal. Also, the moment generating function, the cumulative distribution of an output signals and the moment and variance of output signals can be

derived by probability density function of the output signals. The average level crossing rate and the average fade derivation of an output signal process can be calculated by the joint probability density of an output signals and its derivative. An expression for calculation of autocorrelation function can be derived by the joint probability density function of output signal at two time instants. Use of the Winner-Hinchine theorem gives us the spectral power density function of output M -ary FSK signal. Because of that the results obtained in this paper are significant. Based on this, the results obtained in this paper are of great significance.

This paper is organized as follows: first section is introduction. In second section the model of M -ary FSK system is defined. Signal characteristics, such as probability density function of output signal at one time instant, are given in third section. Signal characteristics at two time instants are derived in forth section.

In the next, fifth section, the numerical results in the case $M=2$ and various parameters values are given. The last section is the conclusion.

2 Model of the M-ary FSK System

The model of an M -ary FSK system, considered in this paper, is shown in Fig. 1. This system has M branches. Each branch consists of bandpass filter and correlator. Correlator is consisting of multiplier and lowpass filter.

The signal at the input of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, interference and variable signal amplitude.

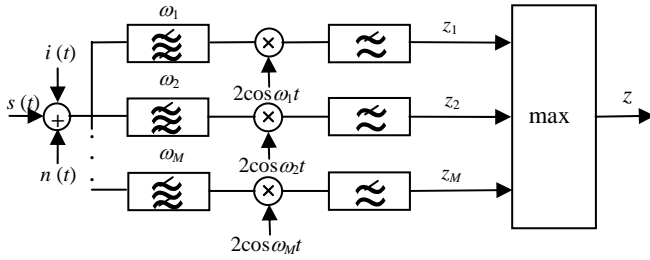


Fig.1. System model for coherent demodulation of M-ary FSK signal

Transmitted signal for the hypothesis H_i is:

$$s(t) = A \cos \omega_i t \quad (1)$$

where A denotes the amplitude of the modulated signal and can have Rayleigh or Nakagami- m distribution in our paper.

Gaussian noise at the input of the receiver is given with:

$$n(t) = \sum_{i=1}^M x_i \cos \omega_i t + y_i \sin \omega_i t, \quad i=1, 2, \dots, M \quad (2)$$

where x_i and y_i are the components of Gaussian noise, with zero means and variances σ^2 .

The interference $i(t)$ can be written as:

$$i(t) = \sum_{i=1}^M A_i \cos(\omega_i t + \theta_i) \quad (3)$$

If $i(t)$ is pulse interference it has distribution given by:

$$i(t) = \sum_{i=1}^M (A_i + cn) \cos(\omega_i t + \theta_i) \quad (4)$$

c is constant and n has Poisson's distribution:

$$p(n) = \frac{\lambda^n}{n!} e^{-\lambda} \quad (5)$$

λ is intensity of impulse process.

Phases $\theta_i, i=1, 2, \dots, M$ have uniform probability density function.

These signals pass first through bandpass filters whose central frequencies $\omega_1, \omega_2, \dots, \omega_M$ correspond to hypotheses H_1, H_2, \dots, H_M .

After multiplying with signal from the local oscillator, they pass through lowpass filter. The filter cuts all spectral components which frequencies are greater than the border frequency of the filter.

If z_1, z_2, \dots, z_M are the output signals of the branch of the receiver, then the M -FSK receiver output signal is:

$$z = \max \{ z_1, z_2, \dots, z_M \} \quad (6)$$

The probability density of output signal is

$$p_z(z) = \sum_{i=1}^M p_{z_i}(z) \cdot \prod_{\substack{j=1 \\ j \neq i}}^M F_{z_j}(z) \quad (7)$$

The joint probability density function of the output signal z and its derivative is:

$$p_{z\dot{z}}(z, \dot{z}) = \sum_{i=1}^M p_{z_i \dot{z}_i}(z, \dot{z}) \cdot \prod_{\substack{j=1 \\ j \neq i}}^M F_{z_j}(z) \quad (8)$$

3 Signal Characteristics at One Time Instant

In the case of the hypothesis H_1 , transmitted signal is:

$$s(t) = A \cos \omega_1 t \quad (9)$$

while the output branch signals of the receiver are:

$$z_1 = A + x_1 + A_1 \cos \theta_1 \quad (10)$$

$$z_k = x_k + A_k \cos \theta_k, \quad k=2, 3, \dots, M \quad (11)$$

It is necessary to define the probability density functions of the branches output signals and the cumulative density of these signals to obtain probability density function of M -ary FSK receiver output signal.

The conditional probability density functions for the signals z_1, z_2, \dots, z_M , in the presence of intersymbol interference and Rayleigh fading, are:

$$p_{z_1/A, \theta_1}(z_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 - A - A_1 \cos \theta_1)^2}{2\sigma^2}} \quad (12)$$

$$p_{z_2/A, \theta_2}(z_2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_2 - A_2 \cos \theta_2)^2}{2\sigma^2}} \quad (13)$$

....

$$p_{z_k/A, \theta_k}(z_k) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k - A_k \cos \theta_k)^2}{2\sigma^2}},$$

$$k=2,3,\dots,M \quad (14)$$

By averaging (12) to (14) we obtain the probability density functions of the branches output signals as:

$$p_{z_1}(z_1) = \int_0^\infty \int_{-\pi}^\pi \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 - A - A_1 \cos \theta_1)^2}{2\sigma^2}} \cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_1 \quad (15)$$

$$p_{z_2}(z_2) = \int_0^\infty \int_{-\pi}^\pi \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_2 - A_2 \cos \theta_2)^2}{2\sigma^2}} \cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_2 \quad (16)$$

$$p_{z_k/A, \theta_k}(z_k) = \int_0^\infty \int_{-\pi}^\pi \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k - A_k \cos \theta_k)^2}{2\sigma^2}} \cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_k \quad (17)$$

The cumulative distributions of the signals z_1, z_2, \dots, z_M are:

$$F_{z_1}(z_1) = \int_{-\infty}^{z_1} \int_0^\infty \int_{-\pi}^\pi \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 - A - A_1 \cos \theta_1)^2}{2\sigma^2}} \cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_1 dz_1 \quad (18)$$

$$F_{z_2}(z_2) = \int_{-\infty}^{z_2} \int_0^\infty \int_{-\pi}^\pi \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_2 - A_2 \cos \theta_2)^2}{2\sigma^2}} \cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_2 dz_2 \quad (19)$$

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$$F_{z_k}(z_k) = \int_{-\infty}^{z_k} \int_0^\infty \int_{-\pi}^\pi \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k - A_k \cos \theta_k)^2}{2\sigma^2}} \cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_k dz_k \quad (20)$$

The probability density function of the M -ary FSK receiver output signal in the case of the hypothesis H_1 can be obtained from:

$$p_{z_1}(z_1) = \sum_{i=1}^M p_{z_{1i}}(z_1) \cdot \prod_{\substack{j=1 \\ j \neq i}}^M F_{z_{1j}}(z_1) \quad (21)$$

The conditional probability density functions for the signals z_1, z_2, \dots, z_M , in the presence of impulse interference and Nakagami- m fading, are:

$$p_{z_1/A, \theta_1}(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - A - (A_1 + cn) \cos \theta_1)^2}{2\sigma^2}} \quad (22)$$

σ denotes standard deviation.

$$p_{z_k/A, \theta_k}(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - (A_k + cn) \cos \theta_k)^2}{2\sigma^2}}, \quad k=2,3,\dots,M \quad (23)$$

By averaging (22) and (23) we obtain the probability density functions of the branches output signals:

$$p_{z_1}(z_1) = \int_0^\infty \int_{-\pi}^\pi \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - A - (A_1 + cn) \cos \theta_1)^2}{2\sigma^2}} \cdot \sum_{n=0}^\infty \frac{\lambda^n}{n!} e^{-\lambda} \frac{2}{\Gamma(m_1)} \left(\frac{m_1}{\Omega_1}\right) A^{2m_1-1} e^{-\frac{m_1 A^2}{\Omega_1}} dA \frac{1}{2\pi} d\theta_1 \quad (24)$$

$$p_{z_2}(z_2) = \int_0^\infty \int_{-\pi}^\pi \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - (A_2 + cn) \cos \theta_2)^2}{2\sigma^2}} \cdot \sum_{n=0}^\infty \frac{\lambda^n}{n!} e^{-\lambda} \frac{2}{\Gamma(m_2)} \left(\frac{m_2}{\Omega_2}\right) A^{2m_2-1} e^{-\frac{m_2 A^2}{\Omega_2}} dA \frac{1}{2\pi} d\theta_2 \quad (25)$$

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