Probability Density Function of M-ary FSK Signal in the Presence of Noise, Interference and Fading

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Abstract: - In this paper the receiver for the demodulation of M-FSK signals in the presence of Gaussian noise, intersymbol interference, impulse interference and fading is considered. The probability density function of M-ary FSK signal in the presence of noise, interference and fading is derived. These interferences can seriously degrade the communication system performances.

Key-Words: - *M*-ary Frequency Shift Keying, Gaussian Noise, Intersymbol Interference, Impulse Interference, Rayleigh Fading, Nakagami Fading, Probability Density Function, Joint Probability Density Function

1 Introduction

In this paper we consider a system for coherent demodulation of *M*-ary FSK signals in the presence of Gaussian noise, impulse noise and variable signal amplitude. These disturbances can seriously degrade the performance of communication systems [1]-[3]. In the paper [4], the performance evaluation of several types of FSK and CPFSK receivers was investigated in detail using the modified moment's method. Also, the error probability of the cross-correlator receiver for binary digital frequency modulation detection is studied using theoretical analysis and computer simulations [5]. In [6] average bit-error probability performance for optimum diversity combining of noncoherent FSK over Rayleigh channels is determined. Performance analysis of wide-band *M*-ary FSK systems in Rayleigh fading channels is given in [7].

In order to view the influence of Gaussian noise, intersymbol interference and fading on the performances of *M*-ary FSK system we derived the probability density function of *M*-ary FSK receiver output signal, the joint probability density function of output signal and its derivative and the joint probability density function of output signal at two time instants.

The bit error probability, the signal error probability and the outage probability can be determined by the probability density function of an output signal. Also, the moment generating function, the cumulative distribution of an output signals and the moment and variance of output signals can be derived by probability density function of the output signals. The average level crossing rate and the average fade derivation of an output signal process can be calculated by the joint probability density of an output signals and its derivative. An expression for calculation of autocorrelation function can be derived by the joint probability density function of output signal at two time instants. Use of the Winner-Hinchine theorem gives us the spectral power density function of output *M*-ary FSK signal. Because of that the results obtained in this paper are significant. Based on this, the results obtained in this paper are of great significance.

This paper is organized as follows: first section is introduction. In second section the model of *M*-ary FSK system is defined. Signal characteristics, such as probability density function of output signal at one time instant, are given in third section. Signal characteristics at two time instants are derived in forth section.

In the next, fifth section, the numerical results in the case M=2 and various parameters values are given. The last section is the conclusion.

2 Model of the M-ary FSK System

The model of an M-ary FSK system, considered in this paper, is shown in Fig. 1. This system has *M* branches. Each branch consists of bandpass filter and correlator. Correlator is consisting of multiplier and lowpass filter.

The signal at the input of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, interference and variable signal amplitude.

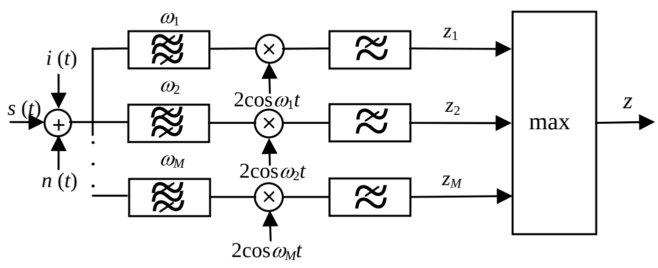

Fig.1. System model for coherent demodulation of M-ary FSK signal

Transmitted signal for the hypothesis H_i is:

$$s(t) = A\cos\omega_i t \tag{1}$$

where A denotes the amplitude of the modulated signal and can have Rayleigh or Nakagami-m distribution in our paper.

Gaussian noise at the input of the receiver is given with:

$$n(t) = \sum_{i=1}^{M} x_i \cos\omega_i t + y_i \sin\omega_i t, \quad i=1, 2,...M \tag{2}$$

where x_i and y_i are the components of Gaussian noise, with zero means and variances σ^2.

The interference $i(t)$ can be written as:

$$i(t) = \sum_{i=1}^{M} A_i \cos(\omega_i t + \theta_i) \tag{3}$$

If $i(t)$ is pulse interference it has distribution given by:

$$i(t) = \sum_{i=1}^{M} (A_i + cn)\cos(\omega_i t + \theta_i) \tag{4}$$

c is constant and n has Poisson's distribution:

$$p(n) = \frac{\lambda^n}{n!} e^{-\lambda} \tag{5}$$

λ is intensity of impulse process.

Phases θ_i, $i=1,2,...,M$ have uniform probability density function.

These signals pass first through bandpass filters whose central frequencies ω_1, ω_2,..., ω_M correspond to hypotheses $H_1,H_2,...,H_M$.

After multiplying with signal from the local oscillator, they pass through lowpass filter. The filter cuts all spectral components which frequencies are greater than the border frequency of the filter.

If $z_1, z_2, \ldots z_M$ are the output signals of the branch of the receiver, then the M-FSK receiver output signal is:

$$z = \max\{z_1, z_2 ... z_M\} \tag{6}$$

The probability density of output signal is

$$p_z(z) = \sum_{i=1}^{M} p_{z_i}(z) \cdot \prod_{\substack{j=1 \\ j\neq i}}^{M} F_{z_j}(z) \tag{7}$$

The joint probability density function of the output signal z and its derivative is:

$$p_{z\dot{z}}(z,\dot{z}) = \sum_{i=1}^{M} p_{z_i\dot{z}_i}(z,\dot{z}) \cdot \prod_{\substack{j=1 \\ j\neq i}}^{M} F_{z_j}(z) \tag{8}$$

3 Signal Characteristics at One Time Instant

In the case of the hypothesis H_1, transmitted signal is:

$$s(t) = A\cos\omega_1 t \tag{9}$$

while the output branch signals of the receiver are:

$$z_1 = A + x_1 + A_1 \cos\theta_1 \tag{10}$$

$$z_k = x_k + A_k \cos\theta_k, \quad k=2,3,\ldots,M \tag{11}$$

It is necessary to define the probability density functions of the branches output signals and the cumulative density of these signals to obtain probability density function of M-ary FSK receiver output signal.

The conditional probability density functions for the signals z_1, z_2,..., z_M, in the presence of intersymbol interference and Rayleigh fading, are:

$$p_{z_1/A,\theta_1}(z_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 - A - A_1\cos\theta_1)^2}{2\sigma^2}} \tag{12}$$

$$p_{z_2/A,\theta_2}(z_2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_2 - A_2\cos\theta_2)^2}{2\sigma^2}} \tag{13}$$

….

$$p_{z_k / A,\theta_k}(z_k) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k - A_k \cos\theta_k)^2}{2\sigma^2}},$$

$$k=2,3,\ldots,M \tag{14}$$

By averaging (12) to (14) we obtain the probability density functions of the branches output signals as:

$$p_{z_1}(z_1) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 - A - A_1\cos\theta_1)^2}{2\sigma^2}} \cdot$$

$$\cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_1 \tag{15}$$

$$p_{z_2}(z_2) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_2 - A_2\cos\theta_2)^2}{2\sigma^2}} \cdot$$

$$\cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_2 \tag{16}$$

$$p_{z_k}(z_k) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k - A_k\cos\theta_k)^2}{2\sigma^2}} \cdot$$

$$\cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_k \tag{17}$$

The cumulative distributions of the signals z_1, z_2,..., z_M are:

$$F_{z_1}(z_1) = \int_{-\infty}^{z_1} \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 - A - A_1\cos\theta_1)^2}{2\sigma^2}} \cdot$$

$$\cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_1 dz_1 \tag{18}$$

$$F_{z_2}(z_2) = \int_{-\infty}^{z_k} \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_2 - A_2\cos\theta_2)^2}{2\sigma^2}} \cdot$$

$$\cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_2 dz_2 \tag{19}$$

…

$$F_{z_k}(z_k) = \int_{-\infty}^{z_k} \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k - A_k\cos\theta_k)^2}{2\sigma^2}} \cdot$$

$$\cdot \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} dA \frac{1}{2\pi} d\theta_k dz_k \tag{20}$$

The probability density function of the *M*-ary FSK receiver output signal in the case of the hypothesis H_1 can be obtained from:

$$p_{z_1}(z_1) = \sum_{i=1}^{M} p_{z_{1i}}(z_1) \cdot \prod_{\substack{j=1 \\ j\neq i}}^{M} F_{z_{1j}}(z_1) \tag{21}$$

The conditional probability density functions for the signals z_1, z_2,..., z_M, in the presence of impulse interference and Nakagami-*m* fading, are:

$$p_{z_1 / A,\theta_1}(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - A - (A_1 + cn)\cos\theta_1)^2}{2\sigma^2}} \tag{22}$$

σ denotes standard deviation.

$$p_{z_k / A,\theta_k}(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - (A_k + cn)\cos\theta_k)^2}{2\sigma^2}}, \quad k=2,3,\ldots,M \tag{23}$$

By averaging (22) and (23) we obtain the probability density functions of the branches output signals:

$$p_{z_1}(z_1) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - A - (A_1 + cn)\cos\theta_1)^2}{2\sigma^2}} \cdot$$

$$\cdot \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} \frac{2}{\Gamma(m_1)} \left(\frac{m_1}{\Omega_1}\right) A^{2m_1 - 1} e^{-\frac{m_1}{\Omega_1}A^2} dA \frac{1}{2\pi} d\theta_1 \tag{24}$$

$$p_{z_2}(z_2) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - (A_2 + cn)\cos\theta_2)^2}{2\sigma^2}} \cdot$$

$$\cdot \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} \frac{2}{\Gamma(m_2)} \left(\frac{m_2}{\Omega_2}\right) A^{2m_2 - 1} e^{-\frac{m_2}{\Omega_2}A^2} dA \frac{1}{2\pi} d\theta_2 \tag{25}$$

…

$$p_{z_k}(z_k)=\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z-(A_k+cn)\cos\theta_k)^2}{2\sigma^2}}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_k)}\left(\frac{m_k}{\Omega_k}\right)A^{2m_k-1}e^{-\frac{m_k}{\Omega_k}A^2}dA\frac{1}{2\pi}d\theta_k \qquad (26)$$

The cumulative distributions of the signals z_1, z_2,..., z_M are:

$$F_{z_1}(z_1)=\int_{-\infty}^{z}\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(r-A-(A_1+cn)\cos\theta_1)^2}{2\sigma^2}}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)A^{2m_1-1}e^{-\frac{m_1}{\Omega_1}A^2}dA\frac{1}{2\pi}d\theta_1 dr$$

(27)

$$F_{z_2}(z_2)=\int_{-\infty}^{z}\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(r-(A_2+cn)\cos\theta_2)^2}{2\sigma^2}}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)A^{2m_2-1}e^{-\frac{m_2}{\Omega_2}A^2}dA\frac{1}{2\pi}d\theta_2 dr$$

(28)

…

$$F_{z_k}(z_k)=\int_{-\infty}^{z}\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(r-(A_k+cn)\cos\theta_k)^2}{2\sigma^2}}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_k)}\left(\frac{m_k}{\Omega_k}\right)A^{2m_k-1}e^{-\frac{m_k}{\Omega_k}A^2}dA\frac{1}{2\pi}d\theta_k dr$$

(29)

The probability density function of the M-ary FSK receiver output signal in the case of the hypothesis H_1 can be obtained from (7):

$$p_z(z)=\sum_{i=1}^{M}p_i(z)\cdot\prod_{\substack{j=1\\ j\neq i}}^{M}F_j(z)$$

4 Signal Characteristics at Two Time Instants

The system model for coherent demodulation of M-ary FSK signal at two time instants, considered in this chapter, is shown in Fig. 2.

The M-ary FSK receiver output signals at instant t_1 are z_{11}, z_{21},..., z_{M1} and at the instant t_2, they are z_{12}, z_{22},..., z_{M2}. The indexes for the input signals are: first index is the number of the branch and the other signs time instant observed. For the output signal, the index represents the time instant observed.

The phases of the interference in each branch remain constant at both time instants.

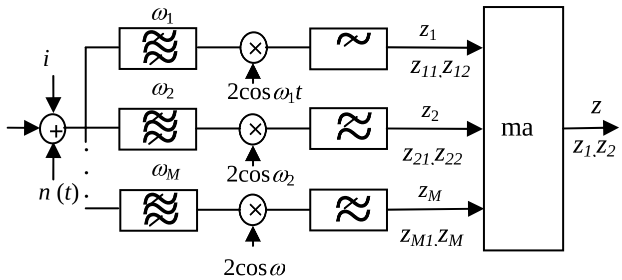

Fig.2. System model for coherent demodulation of M-ary FSK signal at two time instants

The joint probability density of the signals z_{11} and z_{12} is:

$$p_{z_{11}z_{12}}(z_{11},z_{12})=\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}}\cdot$$

$$\cdot e^{-\frac{(z_{11}-A-A_1\cos\theta_1)^2}{2\sigma^2(1-\gamma^2)}}\cdot$$

$$\cdot e^{-\frac{-2\gamma(z_{11}-A-A_1\cos\theta_1)(z_{12}-A-A_1\cos\theta_1)}{2\sigma^2(1-\gamma^2)}}\cdot$$

$$\cdot e^{-\frac{(z_{12}-A-A_1\cos\theta_1)^2}{2\sigma^2(1-\gamma^2)}}\cdot\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_1 \qquad (30)$$

where γ denotes the correlation coefficient of the noise.

Similarly, the joint probability density of the signals z_{21} and z_{22} is:

$$p_{z_{21}z_{22}}(z_{21},z_{22})=\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}}\cdot$$

$$\cdot e^{-\frac{(z_{21}-A_2\cos\theta_2)^2}{2\sigma^2(1-\gamma^2)}}\cdot$$

$$\cdot e^{-\frac{-2\gamma\left(z_{21}-A_2\cos\theta_2\right)\left(z_{22}-A_2\cos\theta_2\right)}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{\left(z_{22}-A_2\cos\theta_2\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_2 \qquad (31)$$

…

$$p_{z_{k1}z_{k2}}\left(z_{k1},z_{k2}\right)=\int_0^\infty\int_{-\pi}^{\pi}\frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}}\cdot$$

$$\cdot e^{-\frac{\left(z_{k1}-A_k\cos\theta_k\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{-2\gamma\left(z_{k1}-A_k\cos\theta_k\right)\left(z_{k2}-A_k\cos\theta_k\right)}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{\left(z_{k2}-A_k\cos\theta_k\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_k \qquad (32)$$

The joint probability density of the signals z_{11} and z_{12}, when impulse noise and Nakagami-*m* fading are present at the system input is:

$$p_{z_{11}z_{12}}\left(z_{11},z_{12}\right)=\int_0^\infty\int_{-\pi}^{\pi}\frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}}\cdot$$

$$\cdot e^{-\frac{\left(z_{11}-A-(A_1+cn)\cos\theta_1\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{-2\gamma\left(z_{11}-A-(A_1+cn)\cos\theta_1\right)\left(z_{12}-A-(A_1+cn)\cos\theta_1\right)}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{\left(z_{12}-A-(A_1+cn)\cos\theta_1\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)A^{2m_1-1}e^{-\frac{m_1}{\Omega_1}A^2}\frac{1}{2\pi}d\theta_1 \qquad (33)$$

where γ denotes the correlation coefficient of the noise.

Similarly, the joint probability density of the signals z_{21} and z_{22} is:

$$p_{z_{21}z_{22}}\left(z_{21},z_{22}\right)=\int_0^\infty\int_{-\pi}^{\pi}\frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}}\cdot$$

$$\cdot e^{-\frac{\left(z_{21}-(A_2+cn)\cos\theta_2\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{-2\gamma\left(z_{21}-(A_2+cn)\cos\theta_2\right)\left(z_{22}-(A_2+cn)\cos\theta_2\right)}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{\left(z_{22}-(A_2+cn)\cos\theta_2\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}d\theta_2\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)A^{2m_2-1}e^{-\frac{m_2}{\Omega_2}A^2}\frac{1}{2\pi}d\theta_2 \qquad (34)$$

…

$$p_{z_{k1}z_{k2}}\left(z_{k1},z_{k2}\right)=\int_0^\infty\int_{-\pi}^{\pi}\frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}}\cdot$$

$$\cdot e^{-\frac{\left(z_{k1}-(A_k+cn)\cos\theta_k\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{-2\gamma\left(z_{k1}-(A_k+cn)\cos\theta_k\right)\left(z_{k2}-(A_k+cn)\cos\theta_k\right)}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{\left(z_{k2}-(A_k+cn)\cos\theta_k\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}$$

$$\cdot\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}\frac{2}{\Gamma(m_k)}\left(\frac{m_k}{\Omega_k}\right)A^{2m_k-1}e^{-\frac{m_k}{\Omega_k}A^2}\frac{1}{2\pi}d\theta_k \qquad (35)$$

The *M*-ary FSK receiver output signals at two time instants, z_1 and z_2, are equal:

$$z_1=\max\left\{z_{11},z_{21},...,z_{M1}\right\} \qquad (36)$$

$$z_2=\max\left\{z_{12},z_{22},...,z_{M2}\right\} \qquad (37)$$

The joint probability density of the output signals z_1 and z_2 is:

$$p_{z_1z_2}\left(z_1,z_2\right)=\sum_{i=1}^{M}\sum_{j=1}^{M}p_{z_{i1}z_{j2}}\left(z_1,z_2\right)\cdot\prod_{\substack{k=1\\k\neq i}}^{M}\prod_{\substack{l=1\\l\neq i}}^{M}F_{z_{k1}z_{l2}}\left(z_1,z_2\right)$$

$$(38)$$

The joint probability density of signals z_{11} and z_{12} and their derivatives $\dot{z}_{11}$ and $\dot{z}_{12}$ is:

$$p_{z_{11}z_{12}\dot{z}_{11}\dot{z}_{12}}\left(z_{11},z_{12},\dot{z}_{11},\dot{z}_{12}\right)=\frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}}\cdot$$

$$\int_{0}^{\infty}\int_{-\pi}^{\pi} e^{-\frac{\left(z_{11}-A-A_1\cos\theta_1\right)^2-2\gamma\left(z_{11}-A-A_1\cos\theta_1\right)\left(z_{12}-A-A_1\cos\theta_1\right)}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{\left(z_{12}-A-A_1\cos\theta_1\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{\dot{z}_{11}^2+\dot{z}_{12}^2}{2\sigma_1^2}}\cdot$$

$$\cdot\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_1 \qquad (39)$$

Similarly,

$$p_{z_{21}z_{22}\dot{z}_{21}\dot{z}_{22}}\left(z_{21},z_{22},\dot{z}_{21},\dot{z}_{22}\right)=\frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}}\cdot$$

$$\cdot\int_{0}^{\infty}\int_{-\pi}^{\pi} e^{-\frac{\left(z_{21}-A_2\cos\theta_2\right)^2-2\gamma\left(z_{21}-A_2\cos\theta_2\right)\left(z_{22}-A_2\cos\theta_2\right)}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{\left(z_{22}-A_2\cos\theta_2\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{\dot{z}_{21}^2+\dot{z}_{22}^2}{2\sigma_{21}^2}}\cdot$$

$$\cdot\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_2 \qquad (40)$$

and:

$$p_{z_{k1}z_{k2}\dot{z}_{k1}\dot{z}_{k2}}\left(z_{k1},z_{k2},\dot{z}_{k1},\dot{z}_{k2}\right)=\frac{1}{2\pi\sigma^2\sqrt{1-\gamma^2}}\cdot$$

$$\cdot\int_{0}^{\infty}\int_{-\pi}^{\pi} e^{-\frac{\left(z_{k1}-A_2\cos\theta_k\right)^2-2\gamma\left(z_{k1}-A_2\cos\theta_k\right)\left(z_{k2}-A_2\cos\theta_k\right)}{2\sigma^2\left(1-\gamma^2\right)}}\cdot$$

$$\cdot e^{-\frac{\left(z_{k2}-A_2\cos\theta_k\right)^2}{2\sigma^2\left(1-\gamma^2\right)}}\cdot\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{\dot{z}_{k1}^2+\dot{z}_{k2}^2}{2\sigma_{21}^2}}\cdot$$

$$\cdot\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}}dA\frac{1}{2\pi}d\theta_k \qquad (41)$$

The joint probability density of the signals z_1 and z_2 and their derivatives $\dot{z}_1$ and $\dot{z}_2$ is:

$$p_{z_1z_2\dot{z}_1\dot{z}_2}\left(z_1,z_2,\dot{z}_1,\dot{z}_2\right)=\sum_{i=1}^{M}\sum_{j=1}^{M}p_{z_{i1}z_{j2}\dot{z}_{11}\dot{z}_{12}}\left(z_1,z_2,\dot{z}_1,\dot{z}_2\right)\cdot$$

$$\cdot\prod_{\substack{k=1\\k\neq i}}^{M}\prod_{\substack{l=1\\l\neq i}}^{M}F_{z_{k1}z_{l2}}\left(z_1,z_2\right) \qquad (42)$$

Similar situation is when Nakagami-*m* fading is present at the system input.

5 The Numerical Results

We now consider the dual branch FSK receiver because of its easy implementation and very good performances. It is employed in many practical telecommunication systems.

The probability density function in this case has the form:

$$p_z\left(z_1\right)=p_{z_{11}}\left(z_1\right)\cdot F_{z_{12}}\left(z_1\right)+p_{z_{12}}\left(z_1\right)\cdot F_{z_{11}}\left(z_1\right)$$

(43)

Fig. 3. shows the influence of the intersymbol interference on the probability density function at the output of the dual branch FSK receiver.

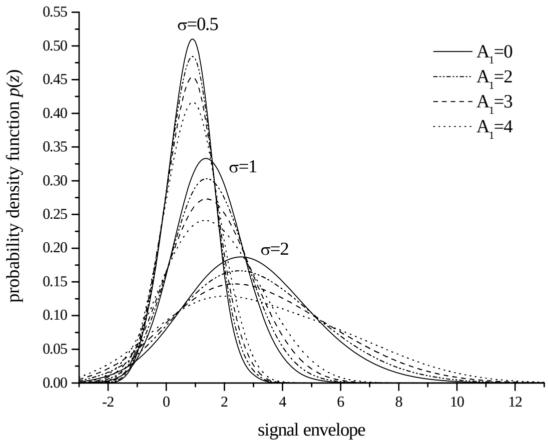

Fig. 3. Probability density function p(z) of dual branch FSK receiver output signal for different values of signal-to-interference power

We note that with the decrement of the signal-to-interference power ratio (i.e. with increasing of the intersymbol interference amplitude), the curve has lower maximum and becomes wider. Also, we can see that with increase of parameter σ, maximum decrease and translatory shift along signal envelope axis.

Fig. 4. shows that similar transformations of curve happen when signal-to-noise power ratio varies while interference remain constant.

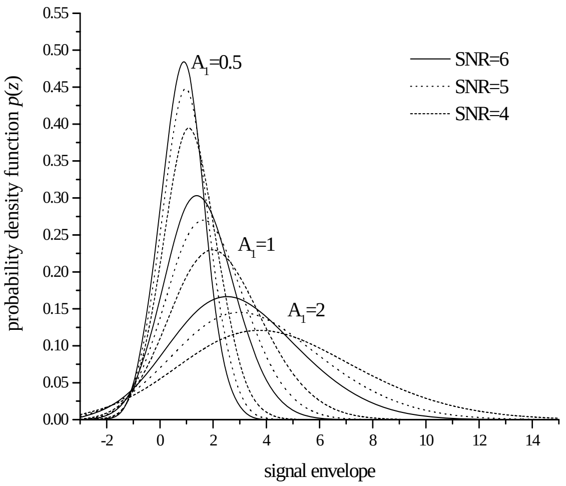

Fig. 4. The probability density function p(z) on the dual branch FSK receiver output signal for different values of signal-to-noise power ratio

The probability density functions $p(z)$ versus output signal z, with and without interference and impulse noise, for various values of fading severity parameter m and average power Ω are shown in Figs. 5 and 6, respectively.

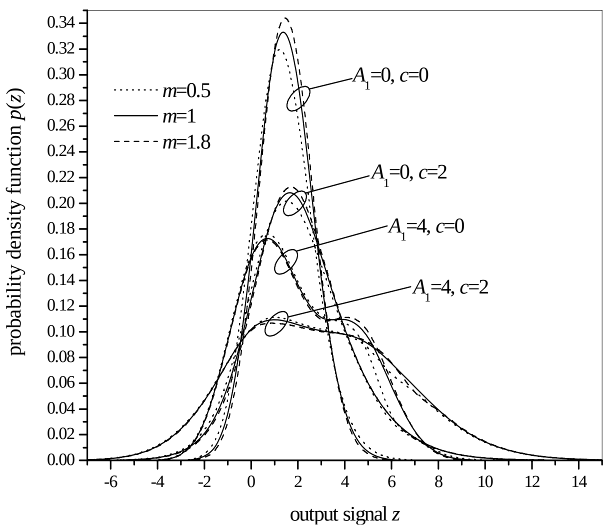

Fig. 5. The probability density functions p(z) for the parameters σ=1, λ=1 and Ω=2

In Figs. 7. to 9. probability density functions $p(z)$ versus output signal z, are given depends on parameter λ.

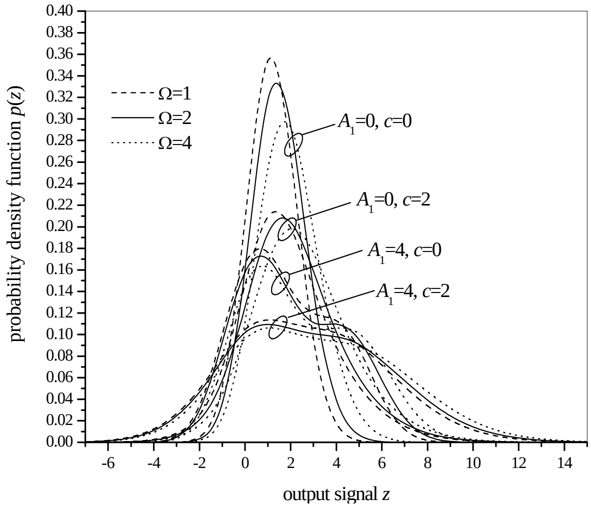

Fig. 6. The probability density functions p(z) for the parameters σ=1, λ=1 and m=1

The probability density functions $p(z)$ versus output signal z, when dependence is on standard deviation σ, are given in Figs. 10. to 12.

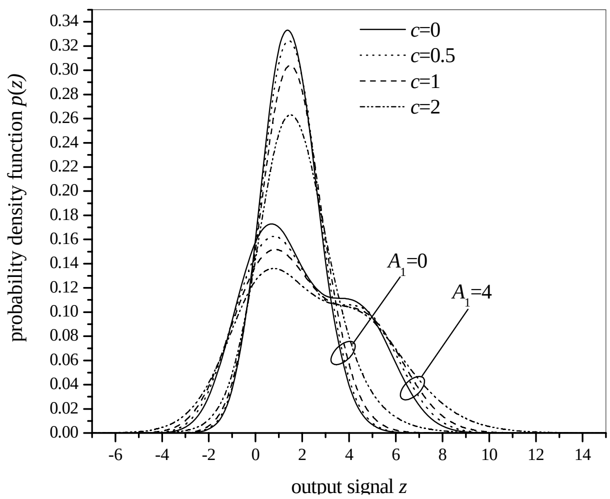

Fig. 7. The probability density functions p(z) for the parameters σ=1, λ=0.5, Ω=2 and m=1

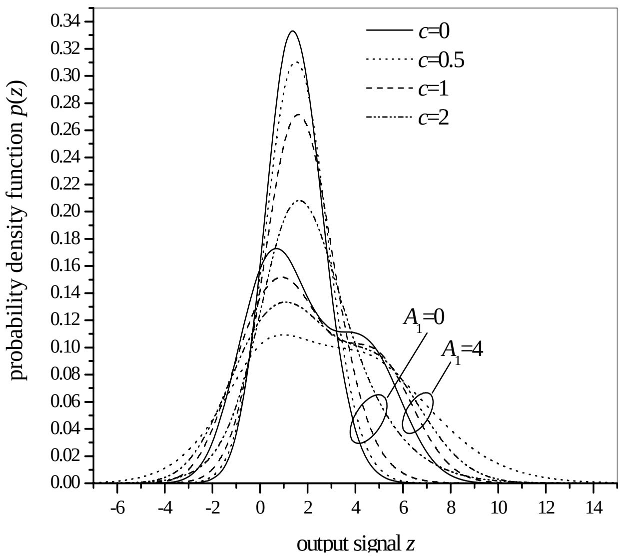

Fig. 8. The probability density functions p(z) for the parameters σ=1, λ=1, Ω=2 and m=1

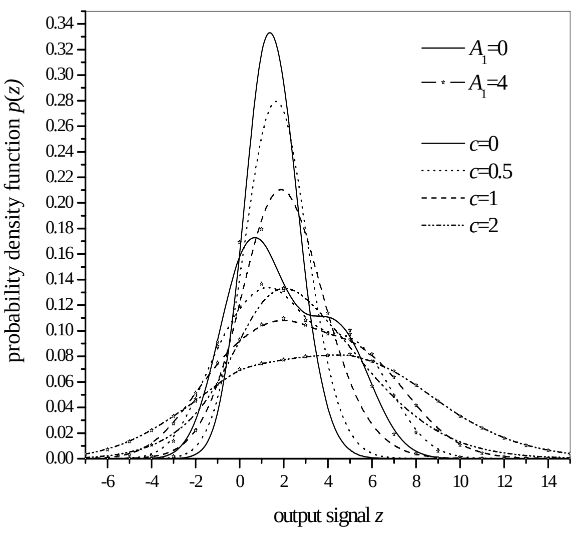

Fig. 9. The probability density functions p(z) for the parameters σ=1, λ=2, Ω=2 and m=1

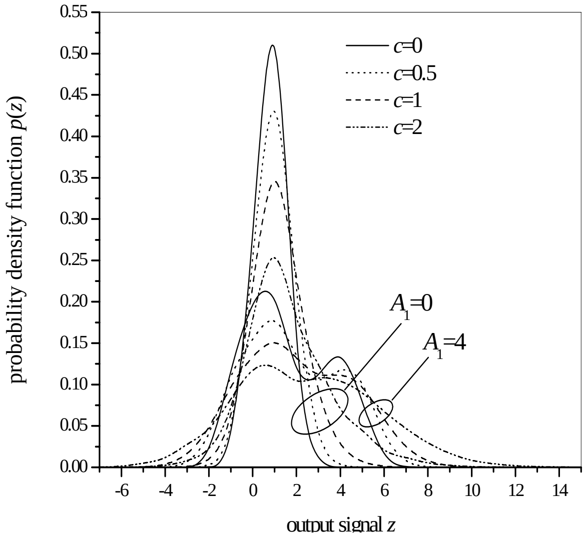

Fig. 10. The probability density functions p(z) for the parameters σ=0.5, λ=1,Ω=2 and m=1

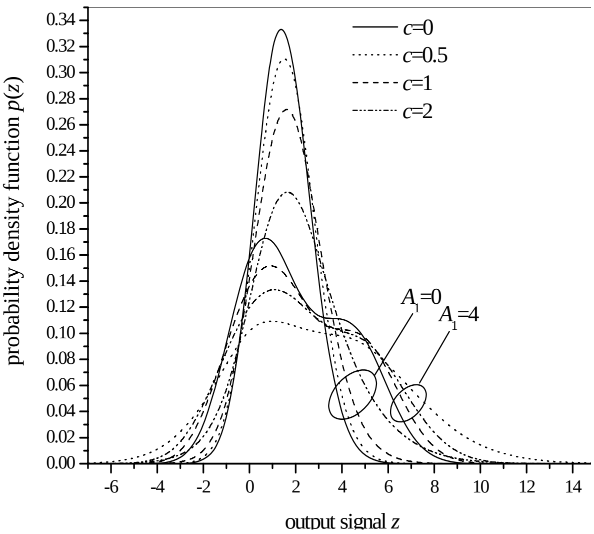

Fig. 11. The probability density functions p(z) for the parameters σ=1, λ=1, Ω=2 and m=1

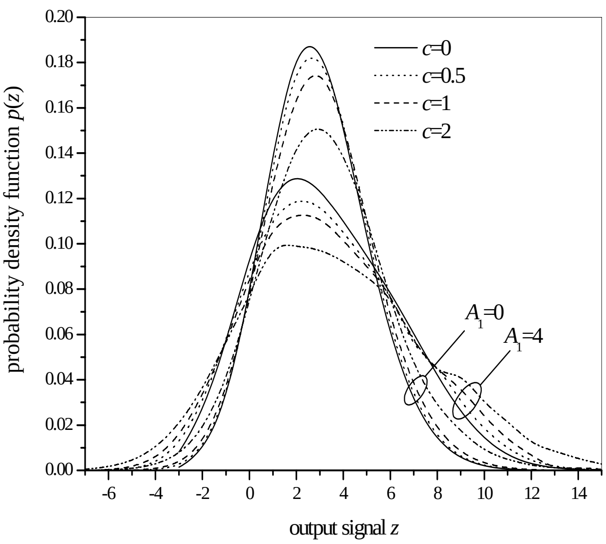

Fig. 12. The probability density functions p(z) for the parameters σ=2, λ=1, Ω=2 and m=1

6 Conclusion

In this paper, the statistical characteristics of the signal at the output of the receiver for coherent FSK demodulation are derived. The input signal of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, intersymbol interference and variable signal amplitude. The interference appears in each receiver branch. In this paper the probability density function of an *M*-ary FSK receiver output signal, the joint probability density function of output signal and its derivative, and the joint probability density function of output signal of two time instants are derived.

The joint probability density function of the output signal at two times instants is important for the case when the noise is correlated. The bit error probability and the outage probability can be determined by the probability density function of an output signal. The level crossing rate and average fade duration of an output signal process can be calculated by the joint probability density function of an output signal and its derivative. An expression for calculation autocorrelation function of the output signal can be obtained by a joint probability density function of an output signals at two time instants. Also, by this function a likelihood function of an *M*-FSK system when the decision is done by two samples can be calculated.

Many of the wireless communication systems use some form of diversity combining to reduce multupath fading appeared in the channel. Among the simpler diversity combining schemes, the two most popular are selection combining (SC) and switch and stay combining (SSC). In the paper [6] Alouini and Simon develop, analyze and optimize a simple form of dual-branch switch and stay combining (SSC).

Our future papers will be developed to diversity combining and reducing fading in the diversity systems.

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