Analysis of Non-Linear Response in DS-CDMA Satellite Systems

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Abstract: - This paper shows an algorithm to calculate synchronous DS-CDMA satellite links with non-linear response amplifiers, where the probability error, active device parameters, harmonics, intermodulation products and back-off compensation, are used to evaluate the performance of the system. The analysis of carrier to interference ratio is carried out for N-carrier up-link, where a single carrier uses DS-CDMA.

Key-words: - Harmonics, Intermodulation, DS-CDMA, link budget, HPA, carrier to interference ratio.

1 Introduction

The nature of Frequency Division Multiple Access (FDMA) implies that the receiver channel has to amplify a set of carriers in a frequency intervals. Satellite systems with Code Division Multiple Access (CDMA) require also FDMA, because the bandwidth is very large. Then the non-linear transfer characteristic of the amplifiers used in earth and space stations generate a lot of spurious signal.

In this paper, we analyze a digital satellite communication system where Direct Sequence Code Division Multiple Access (DS-CDMA) is used to maximize the number of simultaneous users in the transmission media. In this technique, the satellite transponders identify the earth station (up-link), with a pseudorandom sequence (PN). The length and rate of the sequence determine the spread bandwidth and number of simultaneous user of the system. The architecture employed to obtain pseudo-random sequences is named PN sequence generator, and the features of the most important configurations are shown in [10].

In multi-carrier systems the non-linear effects are increased, intermodulation products generation reduces the power of the desired carriers [11], [13], [14], [15]. This behavior is caused by active devices, because their responses are non-linear.

There are two main techniques to achieve CDMA, Direct Sequence and Frequency Hopping (FH), but FH-CDMA is more sensible to Doppler effect, as a consequence of satellite velocities. Then, DS-CDMA is the only practical choice to use spread spectrum in satellite communication.

In section two, the carrier to interference ratio is presented. The back-off calculation for a single carrier is analyzed in section three. The case for an arbitrary number of carriers in the up-link is analyzed in detail in section four. The main results and conclusions are presented in section five and six.

2 C/I in Digital Satellite Systems

The total carrier to noise ratio is determined by (1).

$$\left(\frac{C}{N}\right)_T^{-1} = \left(\frac{C}{N}\right)_u^{-1} + \left(\frac{C}{N}\right)_d^{-1} + \left(\frac{C}{N}\right)_i^{-1} + \left(\frac{C}{I}\right)^{-1} \quad (1)$$

Where $(C/N)_T$ is total carrier to noise ratio, $(C/N)_u$ is the uplink carrier to noise ratio, $(C/N)_d$ is the downlink carrier to noise ratio, $(C/N)_i$ is the carrier to intermodulation noise ratio, and (C/I) is the carrier to interference ratio.

Harmonics generation for a single tone in the HPA can be calculated by equation (2), [13], [14], [15].

$$\begin{cases} x(t) = A\cos(\omega_c t) \\ f(t) \cong a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) \end{cases} \qquad (2)$$

Where $x(t)$ is the carrier, A is the amplitude of the carrier, ω_c is the angular frequency of the carrier, $f(t)$ is the non-linear magnitude response function, and finally a_0, a_1, a_2 and a_3 are constants.

The complete harmonics that are generated in the HPA is shown in the Table 1.

TABLE 1.
HARMONICS FOR A SINGLE CARRIER

Amplitude	**Frequency**
$\left(a_0 + \frac{a_2 A^2}{2}\right)$	D. C.
$\left(a_1 A + \frac{3a_3}{4} A^3\right)$	ω_c
$\left(\frac{a_2 A^2}{2}\right)$	$2\omega_c$
$\left(\frac{a_3 A^3}{4}\right)$	$3\omega_c$

Intermodulation can be determined by equations (3), (4), (5) y (6).

$$s_1(t) = A_1 \cos(\omega_1 t) \qquad (3)$$

$$s_2(t) = A_2 \cos(\omega_2 t) \qquad (4)$$

$$x(t) = s_1(t) + s_2(t) \qquad (5)$$

$$y(t) \cong a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \cdots \qquad (6)$$

Where $x(t)$ is the input signal; A_1 and A_2 are the amplitude of the carriers; ω_1 and ω_1 are the angular frequencies of the carriers; $y(t)$ is the non-linear magnitude response function; and a_0, a_1, a_2 and a_3 are constants.

The complete intermodulation products that are generated by the non-linear response of the HPA is shown in the Table 2.

TABLE 2.
INTERMODULATION PRODUCTS FOR TWO CARRIERS

Amplitude	**Frequency**
$a_0 + \frac{1}{2} a_2 \left(A_1^2 + A_2^2\right)$	D. C.
$a_1 A_1 + \frac{3}{4} a_3 A_1 \left(A_1^2 + 2A_2^2\right)$	ω_1
$a_1 A_2 + \frac{3}{4} a_3 A_2 \left(A_2^2 + 2A_1^2\right)$	ω_2
$\frac{1}{2} a_2 A_1^2$	$2\omega_1$
$\frac{1}{2} a_2 A_2^2$	$2\omega_2$
$a_2 A_1 A_2$	$\omega_1 \pm \omega_2$
$\frac{1}{4} a_3 A_1^3$	$3\omega_1$
$\frac{1}{4} a_3 A_2^3$	$3\omega_2$
$\frac{3}{4} a_3 A_2 A_1^2$	$2\omega_1 \pm \omega_2$
$\frac{3}{4} a_3 A_1 A_2^2$	$2\omega_2 \pm \omega_1$

The total carrier to noise ratio is very important to determine the ratio of energy per bit to noise density and error probability as well. The error probability for non-linear magnitude response for DS-CDMA is illustrated in [11], where the analysis assume a single carrier in the up-link.

3 Back-Off for a single carrier

The amplifiers have a non-linear response if the input power level is high and it is possible to employ a polynomial model, in the output level prediction. The input back-off is calculated by (7).

$$[BO_i] = [EIRP_R] - [EIRP_S] \qquad (7)$$

Where: $[BOi]$ is the input back-off, dBW; $[EIRP_R]$ is the real up-link EIRP, dBW; and $[EIRP_S]$ is the EIRP for HPA saturation, dBW. The input back-off is the uplink compensation needed to saturate satellite

HPA. In order to use a normalized curve of TWT non-linear response, it is necessary to define R_{TWTA}.

$$R_{TWTA} = [EIRP_R]/[EIRP_S] \qquad (8)$$

Where: R_{TWTA} is the real to saturation EIRP Ratio. It is obvious that response of a particular TWTA may be modified, if its TWT is changed, because each tube has a particular response. The following equations offer a way to obtain a normalized prediction for TWTA non-linear input-output ratio [2].

$$P = \frac{R_{TWTA}\alpha_p}{\left(1+\beta_p R_{TWTA}{}^2\right)} \qquad (9)$$

$$Q = \frac{R_{TWTA}{}^3\alpha_q}{\left(1+\beta_q R_{TWTA}{}^2\right)^2} \qquad (10)$$

$$Z = \sqrt{P^2 + Q^2} \qquad (11)$$

Where: α_P, β_P, α_q and β_q are TWT coefficients; P is the in-phase normalized non-linear response, and; Q is the in-quadrature normalized non-linear response.

$$\lfloor EIRP_{R,S} \rfloor = \lfloor EIRP_{S,S} \rfloor Z \qquad (12)$$

Where: Z is the magnitude of normalized non-linear response. It is possible to back with the real non-normalized parameters. Finally, we can obtain the output back-off in the real amplifier, as a consequence of compensating the input power level.

$$[BO_O] = \lfloor EIRP_{S,S} \rfloor - \lfloor EIRP_{R,S} \rfloor \qquad (13)$$

Where: [*BOo*] is the output back-off, dBW; [$EIRP_{R,S}$] Is the real output EIRP, dBW; and, [$EIRP_{S,S}$] is the output EIRP in saturation, dBW. In this paper, we use three TWT devices to compare the non-linear response in satellite HPA, see Table 3.

TABLE 3.
TWT DEVICES USED IN SIMULATION

	TWT$_1$	**TWT$_2$**	**TWT$_3$**
α_p	1.90947	2.11075	2
β_p	1.07469	2.22764	1
α_q	4.35023	7.33959	0
β_q	2.33525	2.11475	0

The *TWT$_1$* and *TWT$_2$* are commercial tubes, and *TWT$_3$* is an ideal tube, where there is not phase distortion.

4 Back-Off for N-Carrier

In the previous section, the amplitude non-linearities have been analyzed and modeled. But, in many digital satellite communications systems, the modulation techniques employed to transmit are very sensible to Doppler effect. The velocity of the LEO satellites in some PCS satellite systems causes a large Doppler shift in the carrier frequency. Then the analysis of phase non-linearities are quite important in satellite applications specially in non-GEO orbits. Consider the multi-carrier input signal in equation (14).

$$v = \sum_{i=1}^{n} \left\{ A_i \cos\left(\omega_i t + \theta_i(t)\right) \right\} \qquad (14)$$

Where $\theta_i(t)$ is the modulated phase, and n is the number of carriers in the system. It is possible to express this set as a carrier having a mean angular frequency, equations (14) and (15).

$$v = R(t)\cos\left(\omega_0 t + \theta(t)\right) \qquad (15)$$

$$R^2(t) = \left\{ \sum_{i=1}^{n} \left[A_i \cos\left[(\omega_i - \omega_0)t + \theta_i(t)\right]\right] \right\}^2 + \left\{ \sum_{i=1}^{n} \left[A_i \mathrm{sen}\left[(\omega_i - \omega_0)t + \theta_i(t)\right]\right] \right\}^2 \qquad (16)$$

$$\theta(t) = \tan^{-1}\left\{ \frac{\left\{ \sum_{i=1}^{n} \left[A_i \mathrm{sen}\left[(\omega_i - \omega_0)t + \theta_i(t)\right]\right] \right\}^2}{\left\{ \sum_{i=1}^{n} \left[A_i \cos\left[(\omega_i - \omega_0)t + \theta_i(t)\right]\right] \right\}^2} \right\} \qquad (17)$$

$$\omega_0 = \frac{\omega_1 + \omega_n}{2} \qquad (18)$$

Where ω_1 and ω_n are the angular frequencies of the first and last carriers in the system, respectively.

In the amplitude-phase non-linear behavior in the TWTA, the corresponding output signal $V(t)$ is determined by equation (19).

$$V(t) = G[R(t)]\cos\{\omega_0 t + \theta(t) + F[R(t)]\} \qquad (19)$$

Where G[R(t)] and F[R(t)] represent the amplitude and phase transfer characteristics of the TWTA, respectively. The amplitude of the individual output carrier is determined by (20) [12].

$$B_n = a_1\sqrt{\frac{2P_{ti}}{n}}\left\{1 + 3\frac{a_3}{a_1}\left(\frac{P_{ti}}{n}\right)\left(n - \frac{1}{2}\right) + \ldots\right\} \qquad (20)$$

Where P_{ti} is the total input power, W. The third-order intermodulation products are the most important, because their amplitudes are frequently the highest. The amplitudes of these intermodulation products can be obtained using (21) and (22) [5].

$$V_{2,1} = \frac{3}{4}a_3\left(\frac{2P_{ti}}{n}\right)^{3/2}\left\{1 + \frac{2a_5}{3a_3}\left(\frac{P_{ti}}{n}\right)[12.5 + 15(n-2)] + \ldots\right\} \qquad (21)$$

$$V_{1,1,1} = \frac{3}{2}a_3\left(\frac{2P_{ti}}{n}\right)^{3/2}\left\{1 + 10\frac{a_5}{a_3}\left(\frac{P_{ti}}{n}\right)\left[\frac{3}{2} + (n-3)\right] + \ldots\right\} \qquad (22)$$

Finally, the carrier to interference ratio can be computed by (23).

$$\left(\frac{C}{I}\right)_r = \frac{B_n^2}{D_{2,1}(r,n)V_{2,1}^2 + D_{1,1,1}(r,n)V_{1,1,1}^2} \qquad (23)$$

Where $D_{2,1}(r,n)$ and $D_{1,1,1}(r,n)$ are the number of third-order intermodulation products that fall right on the r-th carrier.

The active device selected to obtain the results presented in the following section was the $TWTA_2$.

This device has a moderate non-linear response. AM-PM conversion is one of the most important problem to solve in digital satellite communications, because phase and frequency modulations are almost always chosen as a consequence of noise level and very high media attenuations.

5 Results

The main results obtained in this work, were the output back-off and carrier to interference ratio in the DS-CDMA multi-carrier satellite system. The output back-off calculated for a practical system is illustrated in Table 4.

TABLE 4.
OUTPUT BACK-OFF

P_{ti}	$[BO_i]$	A_0	B_1	B_n	$[BO_o]$
0.02	13.97940	0.20000	0.38557	0.02655	8.50973
0.04	10.96910	0.28284	0.53046	0.03561	5.95787
0.06	9.20819	0.34641	0.63201	0.04144	4.64165
0.08	7.95880	0.40000	0.70993	0.04554	3.82223
0.1	6.98970	0.44721	0.77214	0.04854	3.26758
0.12	6.19789	0.48990	0.82284	0.05079	2.87412
0.14	5.52842	0.52915	0.86464	0.05250	2.58659
0.16	4.94850	0.56569	0.89928	0.05381	2.37216
0.18	4.43697	0.60000	0.92802	0.05483	2.20997
0.2	3.97940	0.63246	0.95181	0.05561	2.08623
0.22	3.56547	0.66332	0.97139	0.05622	1.99168
0.24	3.18759	0.69282	0.98735	0.05669	1.92014
0.26	2.83997	0.72111	1.00020	0.05703	1.86766
0.28	2.51812	0.74833	1.01033	0.05726	1.83203
0.3	2.21849	0.77460	1.01809	0.05739	1.81235
0.32	1.93820	0.80000	1.02379	0.05742	1.80891
0.34	1.67491	0.82462	1.02768	0.05732	1.82294
0.36	1.42668	0.84853	1.02998	0.05710	1.85659
0.38	1.19186	0.87178	1.03091	0.05673	1.91289
0.4	0.96910	0.89443	1.03063	0.05620	1.99572
0.42	0.75721	0.91652	1.02931	0.05546	2.10995
0.44	0.55517	0.93808	1.02708	0.05450	2.26157
0.46	0.36212	0.95917	1.02408	0.05328	2.45792
0.48	0.17729	0.97980	1.02042	0.05177	2.70806
0.5	0.00000	1.00000	1.01620	0.04992	3.02342

In Figure 1, the output back-off corresponding to Table 2 is shown.

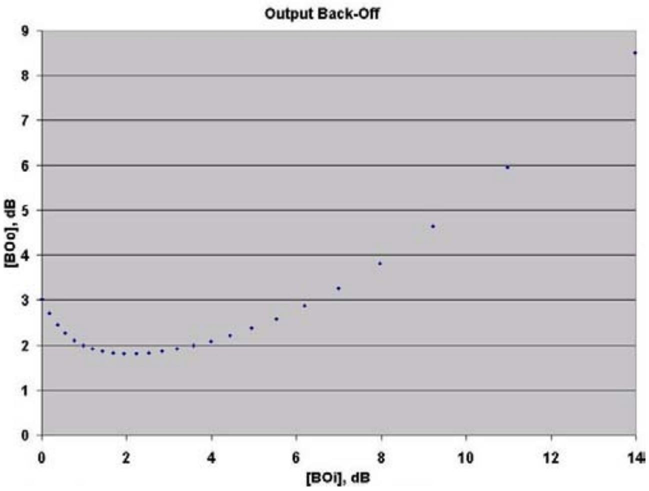

Fig. 1 Output Back-Off.

The carrier to interference ratio calculated is illustrated in Table 5 and Figure 2.

TABLE 5.
CARRIER TO INTERFERENCE RATIO

P_{ti}	$[BO_i]$	$V_{2,1}$	$V_{1,1,1}$	[C/I r]
0.02	13.97940	-2.708E-06	-5.414E-06	32.11441
0.04	10.96910	-7.160E-06	-1.431E-05	26.22266
0.06	9.20819	-1.228E-05	-2.455E-05	22.85224
0.08	7.95880	-1.765E-05	-3.526E-05	20.52729
0.1	6.98970	-2.300E-05	-4.593E-05	18.78468
0.12	6.19789	-2.819E-05	-5.625E-05	17.41710
0.14	5.52842	-3.311E-05	-6.603E-05	16.31261
0.16	4.94850	-3.772E-05	-7.516E-05	15.40265
0.18	4.43697	-4.198E-05	-8.359E-05	14.64094
0.2	3.97940	-4.593E-05	-9.135E-05	13.99348
0.22	3.56547	-4.958E-05	-9.850E-05	13.43333
0.24	3.18759	-5.299E-05	-1.052E-04	12.93764
0.26	2.83997	-5.623E-05	-1.114E-04	12.48591
0.28	2.51812	-5.939E-05	-1.175E-04	12.05896
0.3	2.21849	-6.258E-05	-1.236E-04	11.63840
0.32	1.93820	-6.590E-05	-1.300E-04	11.20641
0.34	1.67491	-6.950E-05	-1.369E-04	10.74577
0.36	1.42668	-7.351E-05	-1.445E-04	10.24016
0.38	1.19186	-7.808E-05	-1.532E-04	9.67445
0.4	0.96910	-8.337E-05	-1.634E-04	9.03513
0.42	0.75721	-8.957E-05	-1.753E-04	8.31051
0.44	0.55517	-9.685E-05	-1.893E-04	7.49083
0.46	0.36212	-1.054E-04	-2.058E-04	6.56796
0.48	0.17729	-1.154E-04	-2.252E-04	5.53478
0.5	0.00000	-1.272E-04	-2.479E-04	4.38416

In Figure 2 is possible to identify the minimum of output back-off. This parameter should be short in order to maximize the total carrier to noise ratio.

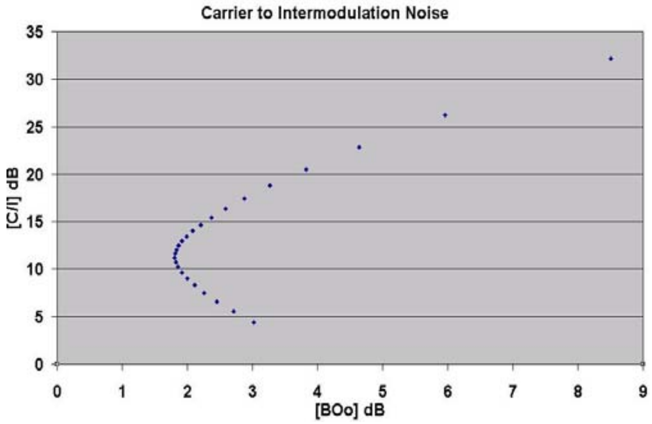

Fig. 2 Carrier to interference ratio.

The multi-carrier case for a back-off calculation is illustrated in Figure 3.

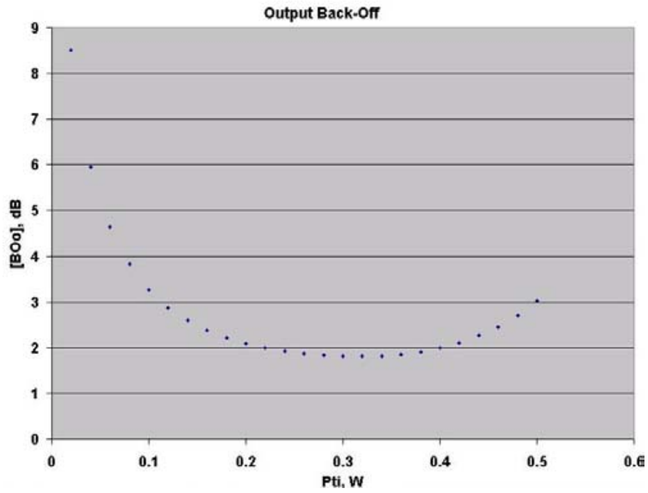

Fig. 3 Output Back-Off versus carrier average power.

In the Figure 4 the output back-off is shown for a large interval of carrier amplitude. It is possible to adjust the link in order to reduce the output back-off.

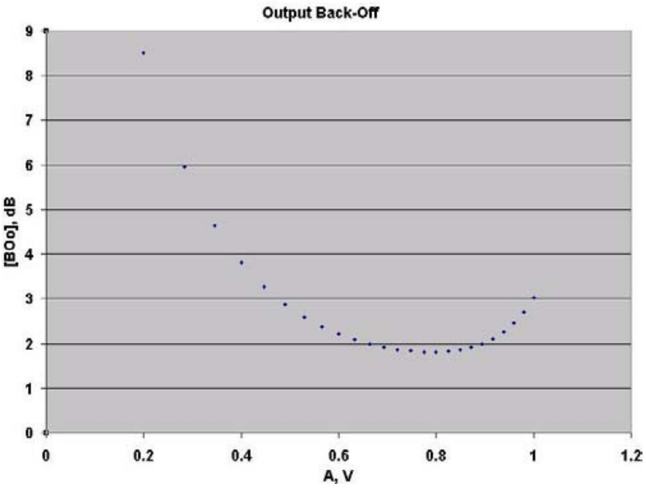

Fig. 4 Output Back-Off versus carrier amplitude.

The Figure 5 shows that the coefficient B_n does not vary significantly, if the operation point is near the best case for output back-off generation.

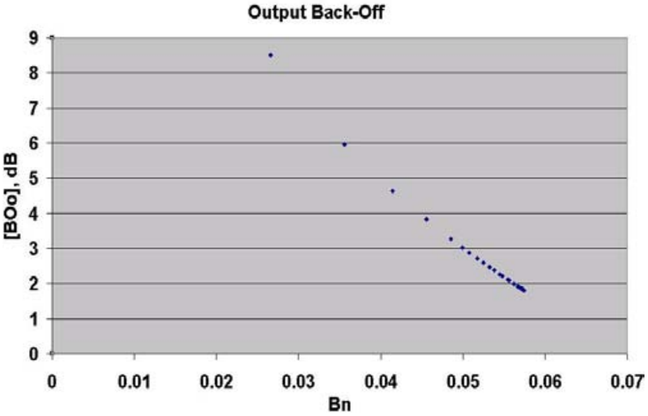

Fig. 5 Output Back-Off versus B_n.

6 Conclusions

The non-linear behavior of the devices used to amplify the signal in the transponders, causes several effects that can reduce the performance of the system: harmonics, intermodulation products, AM-AM and AM-PM, are some of the most important aspect to take in consideration.

In this paper the carrier to interference ratio has been computed for DS-CDMA satellite systems, where non-linear responses (phase and magnitude) are considered, besides some results has been plotted in order to compare the back-off compensation for an arbitrary number of carriers.

Link budget was calculated by ViaSat software. ViaSat is a preliminary software to teach digital satellite systems in the Metropolitan Autonomous University, where we enhance its features constantly.

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