Simulation for Different Order Solitons in Optical Fibers and the Behaviors of Kink and Antikink Solitons

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Abstract:- Solitons are obtained by interaction effect of nonlinear effect and dispersion, so that nonlinear differential equation, KDV, is indicated to this matter. To understand how solitons are created and propagated, this equation has to be solved. The solution describes first, second and third order solitons family. Transmission relation is found by inverse scattering transform (IST) method. Also, the dynamic behavior of the Kink and Antikink solitons has been investigated by Sine- Gorden equation in physical systems. This equation has significant role in various physical branches. In optical fiber, Kink and Antikink solitons keep their forms when they propagate in fiber length. Because of indirect loss effect on nonlinear higher order, numerical simulation of those solitons is necessary. Therefore, Sine-Gorden equation has to be solved analytically.

Key-words:- solitons, optical fiber, nonlinear optics, KDV equation, Sine-Gorden equation, optical communication

1 Introduction

The generation of solitons in optical fibers, predicted by Hasegawa and Tappert [1] through the balance between the pulse broadening due to self-phase modulation and compression due to negative group velocity dispersion, has enabled the generation of stable picosecond and subpicosecond pulses in the near infrared. Verification of many of the predicted soliton pulse characteristics was carried out in a series of experiments by Mollenauer *et al* [2-4].

First of all, KDV equation can be solved for different order solitons.

In 1895, Kortweg De Vries (KDV) shows hydrodynamic wave equations, which move with velocity proportional to their amplitudes [5-8]. KDV equation is shown as:

$$u_t + 6uu_x + u_{xxx} = 0 \qquad (1)$$

u is the wave amplitude.

From physical point of view, solitons will be created from the interaction of self phase modulation (SPM) effect with group velocity dispersion (GVD) effect. If those effects are overlapped well, then the first order soliton will be created, otherwise higher order soliton may be created. In the case of higher order solitons, SPM dominates initially but GVD soon catches up and leads to pulse contraction. For first order soliton (N=1), there is a practical solution will be as follow:

$$u(x,t) = 2k^2 \sec h^2 k(x - ct - x_0) \qquad (2)$$

u (x, t) is a stationary single wave.

K is an arbitrary constant which shows height and the velocity of wave propagation is $c = 4k^2$. With increasing *K*, both parameters will be increased. x_0 is an arbitrary constant which can be stated the existence of single wave at t=0

2 Computer Simulation

With computer simulation, the three dimensional plot is shown in Fig. 1. As it is observed, there is an absolute equilibrium between the dispersion and nonlinear effect in KDV equation for first order soliton (N=1), and from Fig. 1b,

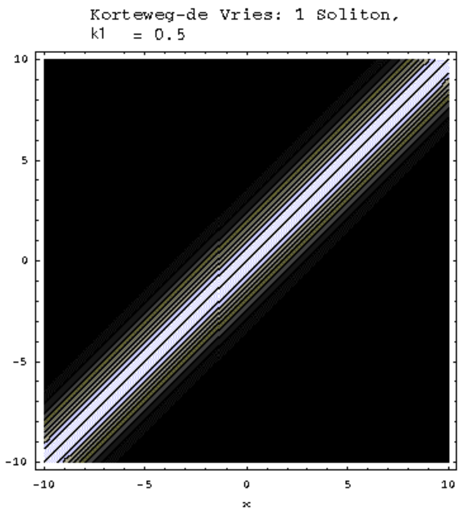

Fig. 1a: The two dimensional plot of the single soliton propagation, using KDV equation with k=0.5

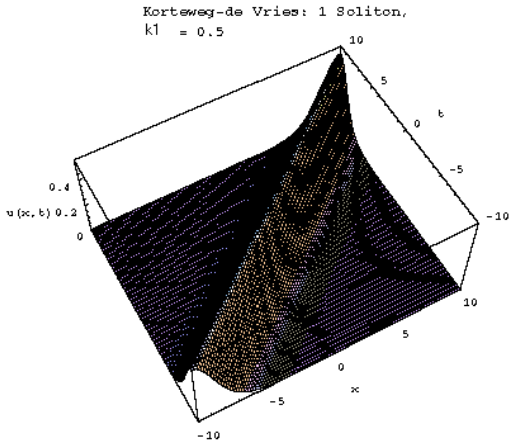

Fig. 1b: Three dimensional plot of a soliton propagation, using KDV equation with k=0.5

The spectral and temporal shapes of the soliton do not be changed in fiber length. Therefore, one can understand the meaning of the soliton by this equation. It is obvious in Fig. 1a that in long distance, solitons can keep their shaped without changing, because of the interaction of the nonlinear and dispersion effects. The two dimensional plot of the single soliton for arbitrary times is shown in Fig. 2. The significant of these solitons are their stability in long distances.

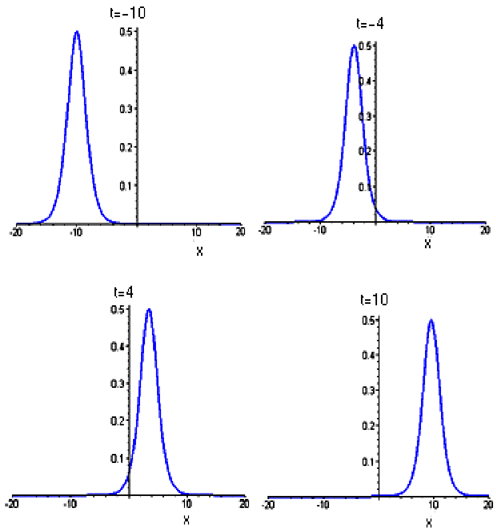

Fig. 2: The soliton propagation using KDV equation

For two solitonic response, if two single waves with different K exist, because of different velocity, interact with each other. Before discussing the two solitonic response, it is better that we write the solitonic response in different form. With substituting $\delta = -x_0$ in (2), one can write as:

$$u(x,t) = 2\frac{\partial^2}{\partial x^2}\log\phi(x,t) \qquad (3)$$

OR

$$\phi(x,t) = 1 + A_1e^{2\eta_1} + A_2e^{2\eta_2} + A_3e^{2(\eta_1+\eta_2)} \qquad (4)$$

With substituting (3), (4) in (2), one can confirm that equation (2) with $c=4k^2$ can be described.

Also, one can assume that:

$$\eta_1 = k_1x - \beta_1t + \delta_1$$
$$\eta_2 = k_2x - \beta_2t + \delta_2 \qquad (5)$$

Where δ_1 and δ_2 are arbitrary constants and و β_1 β_2 A_3 و A_2 و A_1 و may be determined in a way that the equation (3) can describe the equation (1).

If we assume $A_1 = \frac{1}{2k_1}, A_2 = \frac{1}{2k_2}$ one can write:

$$\phi(x,t) = \det B(x,t) \qquad (6)$$

$$B(x,t)=\begin{bmatrix} 1+\dfrac{1}{2k_1}e^{2\eta_1} & \dfrac{1}{k_1+k_2}e^{(\eta_1+\eta_2)} \\ \dfrac{1}{k_1+k_2}e^{(\eta_1+\eta_2)} & 1+\dfrac{1}{2k_2}e^{2\eta_2} \end{bmatrix} \quad (7)$$

Now, one can obtain two solitonic responses as:

$$u(x,t)=\frac{2(k_1-k_2)\left(\sec h\left[\frac{\sqrt{k_1}(x-2tk_1)}{\sqrt{2}}\right]^2 k_1+\csc h\left[\frac{\sqrt{k_2}(x-2tk_2)}{\sqrt{2}}\right]^2 k_2\right)}{-\sqrt{2}\sqrt{k_2}\coth\left[\frac{\sqrt{k_2}(x-2tk_2)}{\sqrt{2}}\right]+\sqrt{2}\sqrt{k_1}\tanh\left[\frac{\sqrt{k_1}(x-2tk_1)}{\sqrt{2}}\right]^2} \quad (8)$$

And now, Sine-Gorden equation can be investigated for Kink and Antikink solitons in optical fibers [11-13].

This equation is a relativity nonlinear equation in space-time without dimension 1+1. Before this equation, there was only Korteweg de vries (KDV) in nineteen century [9, 10]. Not only one can use Sine-Gorden equation in relativity field phenomena, but also it can be used in solid state physics, nonlinear optics, and so on. Solitonic solutions of Sine-Gorden are better than KDV and modified KDV solutions [7-9].

Single solitonic solution is two various cases of Kink and Antikink solutions [10,16]. But a Kink solution is a solution with boundary conditions; in left infinity is zero and in right infinity is 2π. Boundary conditions for an Antikink solution in left infinity are zero and in right infinity is -2π. The Sine-Gorden equation is as follow [9, 10,14]:

$$\phi_{xx}-\phi_{tt}=\sin\phi \quad (9)$$

To solve this equation, first of all it is assumed that:

$$\phi(x,t)=u(x-vt)=u(\xi)$$

Where v is velocity of soliton. But *u*, *v* are two functions which can be determined. We have:

$$\frac{d^2u}{d\xi^2}-v^2\frac{d^2u}{d\xi^2}=(1-v^2)\frac{d^2u}{d\xi^2}=\sin u \quad (10)$$

With dividing both sides of the equation (10) by $\mathbf{1-v^2}$ and multiplying both sides by $\frac{\boldsymbol{du}}{\boldsymbol{d\xi}}$, therefore we have:

$$\frac{d^2u}{d\xi^2}\frac{du}{d\xi}=\frac{\sin u}{(1-v^2)}\frac{du}{d\xi} \quad (11)$$

With multiplying both sides of equation (12) by $\frac{\boldsymbol{du}}{\boldsymbol{d\xi}}$, one can obtain:

$$\frac{du}{d\xi}\left[\frac{1}{2}\left(\frac{du}{d\xi}\right)^2+\frac{\cos u}{(1-v^2)}\right]=0 \quad (13)$$

One can define A as follow:

$$\frac{1}{2}\left(\frac{du}{d\xi}\right)^2+\frac{\cos u}{(1-v^2)}=A \quad (14)$$

First of all, the ordinary differential equation for u is obtained as follow:

$$\frac{du}{d\xi}=\left(2A-\frac{2\cos u}{(1-v^2)}\right)^{1/2} \quad (15)$$

With assumption $B=\frac{A}{(1-v^2)}$ one can write the variable in (15) as follow:

$$\int_{u_0}^{u}\frac{du'}{\sqrt{B-\cos u'}}=\left(\frac{2}{(1-v^2)}\right)^{1/2}\int_{\xi_0}^{\xi}d\xi' \quad (16)$$

The integral in above equation is a variable function of v and B, where v is soliton velocity, and B is integral constant. With selection of B=1, a single wave with velocity $\mathbf{0<|v|<1}$ is obtained. In this case, the left side of equation (16) with assumption $1-\cos u=2\sin^2\left(\frac{1}{2}u\right)$ may be suggested as follow:

$$\frac{d}{du}\ln\tan\left(\frac{1}{4}u\right)=\frac{1}{2\sin\left(\frac{1}{2}u\right)} \quad (17)$$

The both sides of equation (16) can be written as follow:

$$u(\xi) = 4\tan^{-1}\left\{\tan^{-1}\left(\frac{1}{4}u\right)\exp\left[\frac{\xi-\xi}{\sqrt{1-v^2}}\right]\right\} \tag{18}$$

With solving equation for *u*:

$$\phi(x,t) = 4\tan^{-1}\left\{\pm\exp\left[\frac{x-vt}{\sqrt{1-v^2}}\right]\right\} \tag{19}$$

The kink and antikink solitons can be determined by this solution [10-12].
In equation (19), plus sign is for kink soliton and minus sign is for antikink soliton.
The other solution which can be obtained from equation (16) as follow:

$$\phi(x,t) = 4\cot^{-1}\left\{\mp\exp\left[\frac{x-vt}{\sqrt{1-v^2}}\right]\right\} \tag{20}$$

Again the plus sign is for antikink soliton and the minus sign is for kink soliton.
Here, we investigate the solution of single soliton.

3 Simulation of solitons

a: Kink soliton:

In computer program, the kink velocity *v*=0.1 is considered and transmission parameter a, is defined as follow: [9, 10]

$$a = \sqrt{\frac{1-v_k}{1+v_k}} \tag{12}$$

The simulation result for a single kink soliton is shown in Fig. (3):

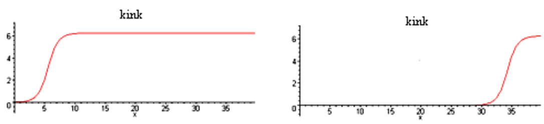

Fig. 3: Two dimensional plot of Sine-Gorden equation for Kink single soliton with *v*=0.1

b: Antikink soliton

If the transmission parameter in kink soliton is negative, that is:

$$a = -\sqrt{\frac{1-v_k}{1+v_k}} \tag{21}$$

We have the antikink soliton with the same velocity. The simulation result is shown in Fig. (4). For antikink soliton.

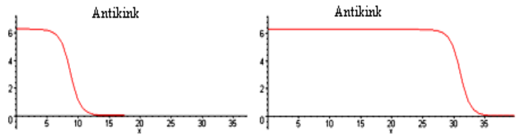

Fig. 4: Two dimensional plot of Sine-Gorden equation for Antikink single soliton with *v*=0.1

The two solitonic solution is divided in some cases [14-16]. The interaction of kink with kink soliton, the interaction of kink with antikink solitons, the interaction of kink with antikink soliton and so on which can be shown here.

(a) Interaction of kink with kink soliton.

Here the interaction of two kink soliton with v=0.2 is simulated. The interaction of two single kink-kink soliton which have the same velocity and amplitude, is shown in Fig. 5. It is observed that each soliton keep its shape after the collision.

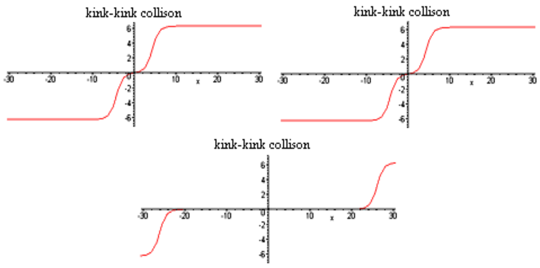

Fig. 5: Interaction of two Kink-Kink solitons with *v*=0.2

(b) Interaction of kink–antikink soliton:

Here the interaction of kink and antikink soliton with *v*=0.2 is simulated,which is shown in two dimensions (Fig. 6) and in three dimensions (Fig. 7). The solution for Sine-Golden equation for the kink and antikink solitons and their interaction behavior are observed. In Fig. 6, one can see that after the interaction, each of them are continued to move without changing in their shapes. Also, for better understand of the solitonic interactions, the three dimensional plot is shown in Fig. 7.

4 Conclusion

Solitons can be described as ideal pulses and it is very important in optical telecommunication systems. Solitons technology is very rapidly developing area of science and engineering, which promises a big change in the functionality and capacity of optical data transmission and networking. In this paper, with using KDV equation for numerical simulation, the first and second order soliton can be shown. Also, one can continue this method for behavior of higher order solitons.

Moreover, the propagation of kink and antikink solitons and the interaction between them are investigated by Sine-Gorden equation.

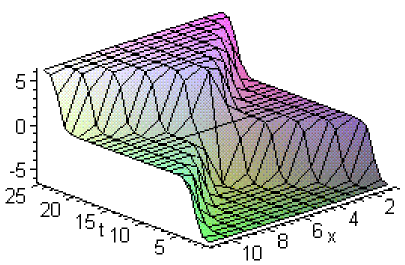

Fig. 7: Three dimensional plot of collision of Kink-Antikink solitons with velocity v=0.2

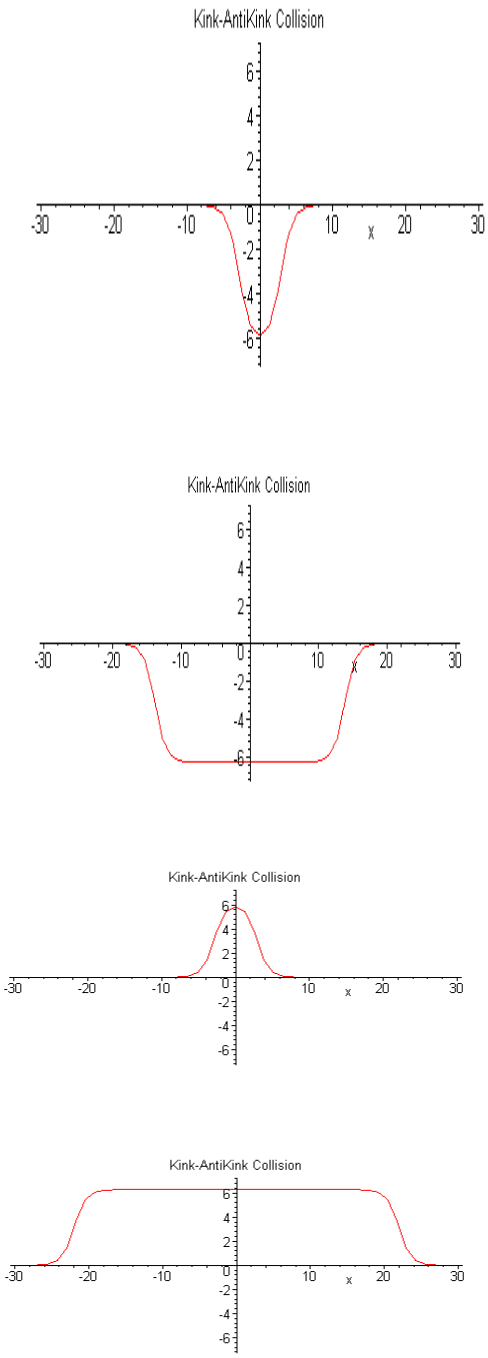

Fig. 6: Collision of two solitons Kink-Antikink with together with velocity v=0.2.

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