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Capacity and Performance Analysis of Space-Time Block Codes in Rayleigh Fading Channels

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Abstract- Multi-Antenna systems are expected to play very important role in future multimedia wireless communication systems. Such systems are predicted to provide tremendous improvement in spectrum utilization. In this paper we consider the capacity analysis of Multiple-Input Multiple-Output (MIMO) systems. Space-Time coding schemes are the practical signal design techniques to realize the information theoretic capacity limits of MIMO systems. Here the orthogonal space-time block codes are considered for the capacity and error probability analysis of MIMO systems as a case study. The numerical and simulation results obtained using MATLAB are presented for the Multi-antenna system channel capacity and bit-error probability in Rayleigh fading channels.

Key-Words: space-time codes, transmit diversity, Rayleigh fading channels, Channel capacity, MIMO.

1 Introduction

Wireless communications has made a tremendous impact on the lifestyle of a human being. It is very difficult to survive without wireless in some form or the other. As compared to fixed wireless systems, today's wireless networks provide high-speed mobility (mobile users in fast vehicles) for voice as well as data traffic. The time-varying nature of wireless channels, such as fading, multipath makes it difficult for wireless system designers to satisfy the ever-increasing expectations of mobile users in terms of data rate and Quality of Service (QoS). The limited radio spectrum and limitation on the processing power availability in the portable handheld unit of mobile user are the other important constraints in designing wireless systems. Continuous exponential growth of Internet, Cellular Mobile and Multimedia Services in the near past has been the driving forces for the increased demand of data rates in Communication Networks. The integration of Internet and multimedia applications in wireless communications follows the quest for increased data rates and spectrally efficient signaling techniques.

3G cellular systems operating in 2 GHz band has promised data rates of at least 384 kbps for mobile and 2 Mbps for indoor applications. 4G systems are to yield about 20-40 Mbps. An IP based 4G systems which are considered to be an integration of 3G systems and wireless LAN (WLAN) systems, has promised more advanced services like enhanced multimedia, smooth video streaming, universal access and portability across all types of devices. Future wireless broadband applications like video conferencing and virtual reality would require data rates of hundreds of Mbps. The universal goal in all approaches towards 4G for achieving high data rates is increasing spectral efficiency using MIMO techniques.

Receiver diversity is used in present cellular mobile systems such as GSM, IS-136, etc. to gain certain benefits like improving quality and range of uplink. Though it is hard to locate more than two antennas in a small mobile handheld unit, it has been shown that transmit diversity can increase the channel capacity considerably. Error control coding can be combined with transmit diversity to achieve improved error performance of multiple antenna transmission systems and thus leads to coding gain advantage in addition to diversity benefit, at the cost of bandwidth expansion due to code redundancy. A joint design of error control coding, modulation and transmit diversity as a single block needs to use space-time codes, then it is possible to achieve coding gain as well as diversity benefit without bandwidth expansion. The combination of space-time codes with receive diversity can further enhance the performance of multi-antenna system by minimizing multipath fading effect and help achieve the capacity of MIMO systems. Recent research in MIMO systems has shown that large capacity gains over wireless channels are possible using multiple antennas at both ends of the wireless channel.

The work carried out by [8] and [9] on MIMO capacity limits shows that, capacity of MIMO channels increase approximately linearly with increased number of antennas. In 1998, Tarokh *et al* in [6] introduced the fundamentals of Space-Time coding utilizing multiple transmit antennas and optionally multiple receive antennas. Alamouti in [1] has proposed a simple 2 x 2 system achieving full diversity. Bell Laboratories Layered Space-Time (BLAST) coding technique has demonstrated spectral efficiencies of 42 b/s/Hz as compared to 2-3 b/s/Hz in cellular mobile and WLAN systems.

2 MIMO System Model

We consider a MIMO system as shown in Fig.1 with array of N_T transmit antennas and N_R receiving antennas. The transmitted signals in each symbol period are represented by an $N_T \times 1$ column matrix **x**.

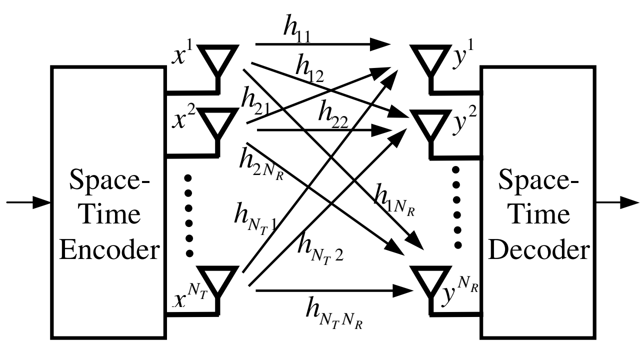

Fig.1: MIMO System model

Considering the Gaussian channel for which the optimum distribution of the transmitted signals is also Gaussian, the elements of **x** are considered to be zero-mean independent and identically distributed (i.i.d.) Gaussian variables. The wireless channel between transmitter and receiver is described by $N_R \times N_T$ complex matrix, denoted by **H**. The ijth entry of matrix **H** denoted by h_{ij} represents the channel fading coefficient from the ith transmit antenna to the jth receive antenna. Rayleigh distribution is the most representative of Non-Line of Sight (N-LOS) wireless radio propagation and hence the MIMO channel capacity has been investigated for Rayleigh fading channel model.

The received signals denoted by **y** are represented by $N_R \times 1$ column matrix. Similarly, the noise at the receiver is represented by $N_R \times 1$ column matrix, denoted by **n**. The elements of **n** are statistically independent complex zero-mean Gaussian random variables with equal variance. The received vector can be represented as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

3 Capacity Analysis of MIMO

The Shannon's capacity formula for capacity per channel use b/s/Hz is given by $C = \log_2(1 + \gamma H^2)$ where γ is received SNR and H^2 is channel transfer characteristic. As the Rayleigh channel model represented by channel matrix H has entries, which are independent and identically distributed (i.i.d.) complex zero mean Gaussian random variables with unit variance, H^2 is a chi-squared distributed variant.

The capacities for Single-Input Single-Output (SISO) i.e. no diversity, Single-Input Multiple-Output (SIMO) i.e. receive diversity, Multiple-Input Single-Output (MISO) i.e. transmit diversity, and MIMO i.e. combined transmit-receive diversity, for slow Rayleigh fading channel are given by [8]

SISO system (Single Antenna Link**):**

$$C = \log_2(1 + \gamma \chi_2^{\ 2}) \quad (2)$$

where $\chi_2^{\ 2}$ is a chi-squared random variable with 2 degrees of freedom.

SIMO system (Receive Diversity**):**

$$C = \log_2(1 + \gamma \chi^2_{\ 2N_R}) \quad (3)$$

MISO system (Transmit Diversity**):**

$$C = \log_2(1 + \frac{\gamma}{N_T} \chi^2_{\ 2N_T}) \quad (4)$$

MIMO system: For the case of $N_T \geq N_R$, lower bound on the capacity in terms of chi-squared random variable is given by

$$C \rangle \sum_{i=N_T-(N_R-1)}^{N_T} \log_2(1 + \frac{\gamma}{N_T} \chi^2_{2i}) \quad (5)$$

where χ_{2i}^{2} is a chi-squared random variable with $2i$ degrees of freedom.

For a special case of $N_T = N_R = N$, the lower bound in (5) reduces to

$$C = \sum_{i=1}^{N_T} \log_2 \left(1 + \frac{\gamma}{N_T} \chi^2_{2i}\right) \qquad (6)$$

Equation (6) shows that for large N, capacity increases at least linearly as a function of N.

4 The Orthogonal Space-Time Block Codes: A Case Study

Let us consider a STBC system as proposed in [2] with N_T transmitting and N_R receiving antennas and the channel is assumed to be quasi-static with flat fading, so that the path gains are constant over a frame period and varies independently from one frame to another. The STBC encoder takes a block of k-modulated signals as its input and generates N_T parallel signal sequences of length p according to transmission matrix $[X]_{N_T \times p}$, where p represents the number of time periods for transmission of one block of coded symbols. Each group of m bits selects a signal constellation consisting of 2^m points. The general form of the channel matrix **H** and the transmission matrix **X** for STBC system with N_T transmit antennas and N_R receive antennas is given by

$$H = \begin{pmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{pmatrix}$$

and

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \ddots & \vdots \\ x_{N_T 1} & \cdots & x_{N_T p} \end{pmatrix} \qquad (7)$$

The code rate R and the spectral efficiency η of STBC is given by

$$R = k/p, \text{ and } \eta = km/p \text{ b/s/Hz} \qquad (8)$$

It can be been shown that the rate of STBC with full transmission diversity has $R \leq 1$. The code with full rate $R = 1$ requires no bandwidth (B) expansion whereas the code with $R < 1$ requires bandwidth expansion of $1/R$. The minimum value of p to achieve full rate is given by [2]

$$p = \min(2^{4c+d}) \qquad (9)$$

where minimization is taken over the set

$$c, d \mid 0 \leq c, 0 \leq d \leq 4, \text{ and } 8c+2^d \geq N_T. \qquad (10)$$

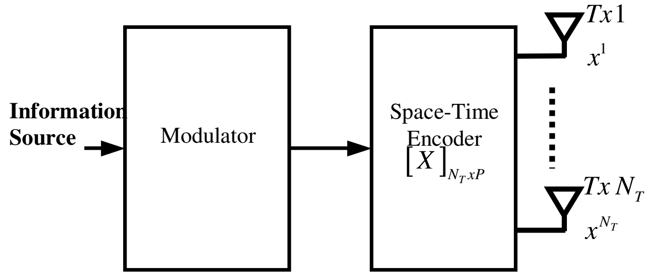

Fig.2: STBC Encoder

The generalized STBC design requires orthogonal designs, meaning that the symbols transmitted from each transmit antenna are mutually orthogonal. When the rows of the transmission matrix $\mathbf{X}_{NT}$ of a STBC of size $N_T \times p$ are orthogonal to each other, the signal sequences from any two transmit antennas in each block will be orthogonal. The orthogonality enables achieve full transmit diversity and allows receiver to easily decouble the signals transmitted from different antennas and a simple maximum likelihood (ML) receiver which performs decoding based on linear processing of received signals can be used. It can be shown that a real orthogonal STBC design exists only for N_T=2, 4, 8 whereas complex orthogonal STBC design exists only for N_T= 2 [2], [12].

At a particular time t, the received signal y_t corresponding to the t_{th} input block spanning over p time slots is given by

$$\mathbf{Y_t} = \mathbf{H_t} \mathbf{X_t} + \boldsymbol{N_t} \qquad (11)$$

where $\mathbf{Y_t}$ is $N_R \times p$ matrix, $\mathbf{H_t}$ is $N_R \times N_T$ fading channel coefficient matrix with i.i.d. entries of complex Gaussian random variables, $\mathbf{X_t}$ are $N_T \times p$ matrix and $\boldsymbol{N_t}$ is $N_R \times p$ receiver noise matrix with i.i.d. complex Gaussian random variables with zero mean and variance $No/2$.

The received signal by j^{th} antenna at time t denoted by y_t^j is given by

$$y_t^{\,j} = \sum_{i=1}^{N_T} h_{ij} x_t^{\,i} + n_t^{\,j}\,; 1 \leq i \leq N_T \text{ and } 1 \leq j \leq N_R \qquad (12)$$

where y_t^j is a $N_R \times 1$ matrix, H is $N_R \times N_T$ matrix, x_t^i is $N_T \times 1$ and n_t^j is $N_R \times 1$ matrix.

The ML decoding scheme at the receiver with perfect CSI is given by

$$\sum_{t=1}^{N_T} \sum_{j=1}^{N_R} \left| y_t^j - \sum_{i=1}^{N_T} h_{ij}^t x_t^i \right|^2 \qquad (13)$$

for all codewords and decides in favour of the codeword that minimizes the sum.

5 Capacity Analysis of MIMO-STBC over Rayleigh Fading Channels

Considering that the receiver knows the channel whereas the transmitter does not know the channel, the general expression for channel capacity of a random MIMO channel is given in [8] as

$$C = E\left[B \log_2 \det\left(I + \frac{E_s}{N_T N_o} Q \right) \right] \text{ b/s/Hz} \qquad (14)$$

where E[.] is the expectation operator, B is the channel bandwidth, det (**x**) denotes the determinant of matrix **x**, I denotes the identity matrix of dimension N_R, *Es* is the transmitted energy on N_T transmit antennas per symbol and ***Q*** is the Wishart matrix where $\mathbf{Q} = \mathbf{H}\mathbf{H}^{\mathbf{H}}$ for $N_R < N_T$ and $\mathbf{Q} = \mathbf{H}^{\mathbf{H}}\mathbf{H}$ when $N_R \geq N_T$ and $\mathbf{H}^{\mathbf{H}}$ is the transpose conjugate of the channel matrix **H**.

The capacity of equivalent STBC channel with code rate *R* will then be given by

$$\overline{C}_{STBC} = E\left[B\,R \log_2 \det\left(I_r + \frac{E_s}{R N_T N_o} Q \right) \right] \text{ b/s/Hz} \qquad (15)$$
$$= E\left[B\,R \log_2 (1+\gamma_s) \right]$$

with the bound for channel capacity at the output of STBC system expressed as

$$\overline{C}_{STBC} \leq E\left[B\,R \log_2 (1+\overline{\gamma}_{STBC}) \right] \qquad (16)$$

where γ_s is the effective instantaneous SNR per symbol at the receiver given by [13]

$$\gamma_s = \frac{E_s}{N_T R N_o} \|H\|_F^2 \; and \; \|H\|_F^2 = \sum_{i=1}^{N_T} \sum_{j=1}^{N_R} \|h_{ij}\|^2 \qquad (17)$$

and $\overline{\gamma}_{STBC}$ is average SNR per symbol in STBC channel.

The difference between (14) and (15) reduces with increasing SNR. It can be shown that the capacity of a MIMO-Rayleigh fading channel with *M*-ary modulation scheme and N_T transmit antennas approaches $N_T \log_2 M$ b/s/Hz while the capacity with orthogonal STBC approaches $R\log_2 M$ b/s/Hz for large values of SNR. The capacity loss with the use of STBC can be expressed as $(N_T - R) \log_2 M$ bits/channel use.

6 Error Probability Analysis over Rayleigh Fading Channels

The well-known Bit-Error Probability (BEP) equations as given in [17], for Binary Phase Shift Keying (BPSK) and QPSK over an AWGN channel are given as $P_{Mb}(\gamma_s) = Q(\sqrt{2\gamma_s})$ and $P_{Mb}(\gamma_s) = Q(\sqrt{\gamma_s})$ respectively. The complex exact equation for Symbol Error Probability (SEP) of *M*-ary PSK in AWGN channel can be approximated under the assumptions of large values of SNR and large values of M. The approximated SEP expression can be given as

$$P_M(\gamma_s) \approx 2Q\left(\sqrt{2\gamma_s} \sin\frac{\pi}{M} \right) \qquad (18)$$

and the approximated BEP is given by

$$P_{Mb} \approx \frac{1}{\log_2 M} P_M \qquad (19)$$

Defining the error probability of *M*-ary signal constellation with STBC in AWGN channel as $P_{STBC,M}$, the error probability with Rayleigh fading channel can be obtained by averaging $P_M(\gamma_s)$ over the PDF of γ_s and can be given as in [13]

$$P_{STBC,M} = \int_0^\infty P_M(\gamma_s)\, p_{Rayleigh}(\gamma_s)\, d\gamma_s \qquad (20)$$

By making use of SEP, the exact BEP for BPSK and QPSK can be derived using approach in [17] and is given by

$$P_{STBC,PSK,Mb} = \frac{1}{2}\left(1 - \sum_{k=0}^{N_T N_R - 1} \mu \left(\frac{1-\mu^2}{4} \right)^k \binom{2k}{k} \right) \qquad (21)$$

and the approximations for the BEPs of BPSK and QPSK can be obtained as

$$P_{STBC,2b} \approx 1 - \sum_{k=0}^{N_T N_R - 1} \mu \left(\frac{1-\mu^2}{4} \right)^k \binom{2k}{k} \qquad (22)$$

and

$$P_{STBC,4b} \approx \frac{1}{2}\left(1 - \sum_{k=0}^{N_T N_R - 1} \mu \left(\frac{1-\mu^2}{4} \right)^k \binom{2k}{k} \right) \qquad (23)$$

where $\mu = \sqrt{\overline{\gamma}_c / (1+\overline{\gamma}_c)}$ and $\mu = \sqrt{\overline{\gamma}_c / (2+\overline{\gamma}_c)}$ for BPSK and QPSK, respectively.

7 Numerical and Simulation Results

The numerical and simulation results are presented to illustrate and verify the information theoretic capacity of MIMO systems and to observe the effect of several STBCs on MIMO channel capacity. Fig.3 below shows the graph for the ergodic capacity plotted against SNR for different MIMO systems assuming a Rayleigh fading model. The graph shows that increasing the number of antennas increases the ergodic capacity. The capacity using different STBCs over a Rayleigh fading channel is given Fig. 4.

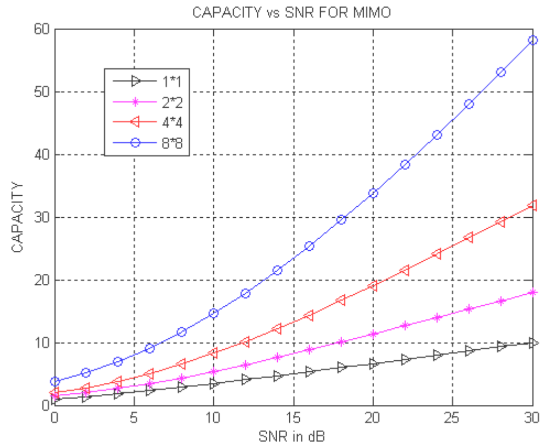

Fig.3: MIMO Ergodic Capacity

The results of error probability based on the analysis given in Section 6 and simulation results are given in Fig. 5 and Fig. 6 respectively.

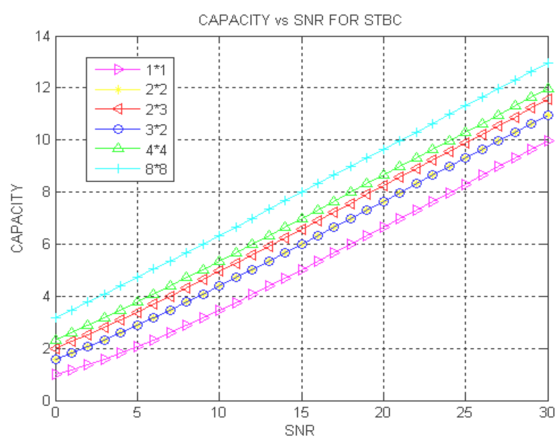

Fig.4: MIMO-STBC Capacity

8 Conclusion

In this paper, the capacity and BER of various MIMO systems in Rayleigh fading channels has been examined. It has been seen that the use of multiple antennas increases the capacity although significant improvement can be achieved using equal or higher number of receive antennas compared to transmit antennas.

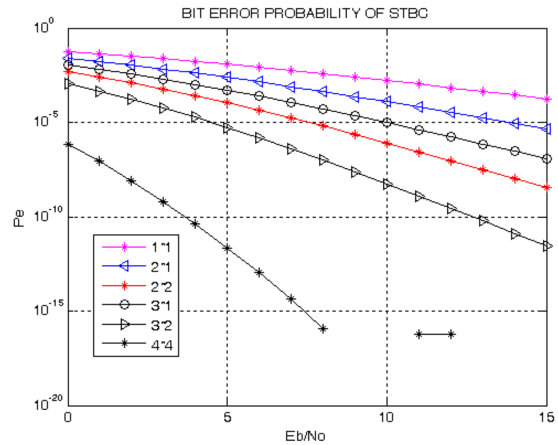

Fig. 5: BEP of MIMO-STBC

Fig. 5 shows the analytical results of Bit-Error Probability of BPSK with various STBCs based on (21) and the simulation results and the comparison is shown in Fig. 6.

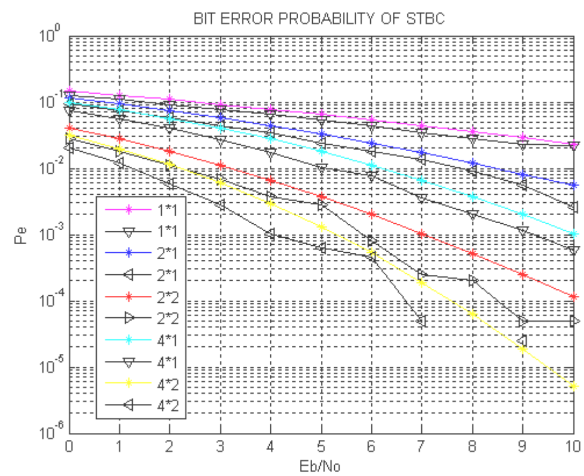

Fig. 6: Comparison of BEP Results

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