Full-Graph Solution of Switched Capacitors Circuits by Means of Two-Graphs

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Abstract: - Circuits with switched capacitors are described by a capacitance matrix. As there are also graph methods of circuit analysis in addition to algebraic methods, it is clearly possible in theory to carry out an analysis of the whole switched circuit in two-phase switching exclusively by the graph method as well. The phase of switching are called even (E) and odd (O), not 1 and 2 to avoid confusion between the sign of the phases and of the nodes.

For this purpose it is possible to plot a Mason graph of a circuit, use two graphs to reduce oriented graphs for all the four phases of switching, and then plot a summary MC-graph from charge and voltage graphs. Summary MC-graph is now constructed by the incomplete common skeletons of the V-graph and the Q-graph in all four phases, branches obtained for EO and OE phase are drawn between these nodes, while their resulting transfer is multiplied by $z^{-\frac{1}{2}}$ to express the delay between the two phases.

This summary MC-graph can then be interpreted by the Mason's relation to provide transparent voltage transfers, and so it is possible to reach the final result a quite fully graphically. This method can be used for circuits containing operational amplifier with the break point frequency, too, but summa graph is rather complicated in this case, because this description is based on the modified nodal method, where the number of nodes is remains.

Key-Words: - Switched capacitors**,** two-graphs, Mason-Coates graph, summa graph, Mason´s rule, break point frequency.

1 Introduction

To symbolic analysis of electrical circuits is commonly used matrix calculus [1], [11], [12]. The modified nodal method can be used for description of the switched capacitor circuits, too. As stated in [8], [9] it is possible to describe switched capacitor circuits by modified nodal method by an equation system in a matrix form (1).

$$\begin{aligned} \mathbf{A_E}.\mathbf{V_E}(\mathbf{t}) &= \mathbf{B_E}.\mathbf{V_O} + \mathbf{g_E}.V \\ \mathbf{A_O}.\mathbf{V_O}(\mathbf{t}) &= \mathbf{B_O}.\mathbf{V_E} + \mathbf{g_O}.V \end{aligned} \qquad (1)$$

The matrices $\mathbf{A_E}$, $\mathbf{B_E}$, $\mathbf{A_O}$, $\mathbf{B_O}$, $\mathbf{g_O}$ and $\mathbf{g_E}$ have been obtained from two partial diagrams, one for the even phase and one for the odd phase by table description of graphs. In each of these phases, nodes are given a number in a square for charge graph (Q-graph) and in a triangle for the voltage graph (V-graph). The $\mathbf{A_E}$ matrix is obtained from the Q-graph and the V-graph for the even phase, the $\mathbf{A_O}$ matrix from the Q-graph and the V-graph for the odd phase, while the $\mathbf{B_E}$ matrix from the Q-graph for the even phase and the V-graph for the odd phase and the $\mathbf{B_O}$ matrix form the Q-graph for the odd phase and the V-graph for the even phase based on the expression of these graphs by a table. Thus obtained equation system (1) is consequently solved following [7], [8], [9], [10] by matrix calculus.

2 Using Two-Graphs for the Description of SC Circuits

If the difference between the even phase and the odd phase is expressed by means of the Z-transformation by multiplying by the $z^{-\frac{1}{2}}$ operator, it is then possible, supposed that $\mathbf{g_O} = \mathbf{g_E} = \mathbf{0}$, to rewrite the equation system (1) to the form (2):

$$\mathbf{A_E}.\mathbf{V_E} - z^{-\frac{1}{2}}\mathbf{B_E}.\mathbf{V_O} = \mathbf{0}$$
$$-z^{-\frac{1}{2}}\mathbf{B_O}.\mathbf{V_E} + \mathbf{A_O}.\mathbf{V_O} = \mathbf{0} \qquad (2)$$

3 Summary Graph Construction Based on Two-Graphs

Since any matrix can finally be demonstrated by a Mason-Coates graph (MC-graph), it is also possible to demonstrate the matrix (3) by such an MC-graph, which can be consequently evaluated by applying the Mason's rule. However, by means of this procedure there is a theoretical possibility to avoid the use of matrix calculus totally when solving circuits with switched capacitors and to solve a given circuit by the graph method only.

When using this graph-only solution we first make a summary MC-graph, which thus represents the matrix (3), by means of two-graphs and in the second step the transfer of the circuit is evaluated using Mason's rule.

4 The Principle of Summary Graph Construction

In this part is presented the general principles of the construction of the summary MC-graph by two-graphs in two main steps.

4.1 First step

The equation system (2) can be written e.g. in the following form (3).

$$C_{11}.V_{1E} + C_{12}.V_{2E} - z^{-\frac{1}{2}}C_{11}.V_{1O} - z^{-\frac{1}{2}}C_{12}.V_{2O} = 0$$
$$C_{21}.V_{1E} + C_{22}.V_{2E} - z^{-\frac{1}{2}}C_{21}.V_{1O} - z^{-\frac{1}{2}}C_{22}.V_{2O} = 0$$
$$-z^{-\frac{1}{2}}C_{11}.V_{1E} - z^{-\frac{1}{2}}C_{12}.V_{2E} + C_{11}.V_{1O} + C_{12}.V_{2O} = 0$$
$$-z^{-\frac{1}{2}}C_{21}.V_{1E} - z^{-\frac{1}{2}}C_{22}.V_{2E} + C_{21}.V_{1O} + C_{22}.V_{2O} = 0 \qquad (3)$$

Two parts of this equation (3), which are actually only sub-matrices $\mathbf{A_E}$, $\mathbf{A_O}$ of this equation can be demonstrated by MC-graphs. The construction of corresponding MC-graphs is following: If the first constituent is expressed from each first equation and the second constituent from the second ones, etc., the equations (3) will get the form (4)

$$C_{11}.V_{1E} = -C_{12}.V_{2E} \qquad C_{11}.V_{1O} = -C_{12}.V_{2O}$$
$$C_{22}.V_{2E} = -C_{21}.V_{1E}, \qquad C_{22}.V_{2O} = -C_{12}.V_{1O} \qquad (4)$$

Graph interpretation of equations (6) for e.g. the last equation is that the constituent $C_{22}.V_{2O}$, represented by the voltage of the V_{2O} node multiplied by the coefficient C_{22} (which means that the V_{2O} node's own loop has the transfer C_{22}), is given by the constituent $-C_{21}.V_{1O}$, i.e. by the addition to the V_{1O} node of $-C_{21}$ (therefore the branch going from the V_{1O} node to the V_{2O} node has the transfer $-C_{21}$). MC-graph of

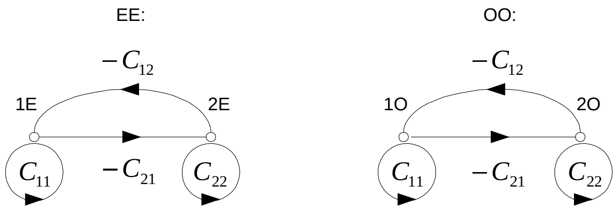


Fig.1. MC-graph of the equations (5)

4.2 Second step

Next, from e.g. the last equation in the system (3), the last constituent V_{2O} can be expressed (because it is the last equation), and this equation will thus obtain the form (5).

$$C_{22}.V_{2O} = z^{-\frac{1}{2}}C_{21}.V_{1E} + z^{-\frac{1}{2}}C_{22}.V_{2E} - C_{21}.V_{1O} =$$
$$= \left[z^{-\frac{1}{2}}C_{21}.V_{1E} + z^{-\frac{1}{2}}C_{22}.V_{2E} \right] + \left[-C_{21}.V_{1O} \right] \qquad (5)$$

However the second constituent $-C_{21}.V_{1O}$ of the equation (5) has already been demonstrated by the graph in Fig.1 as it is the last equation from the system (4). The principle of superposition implies that the total effect is given by the sum of effects; hence we have to add to the graph the remaining effect given by the relation (6)

$$z^{-\frac{1}{2}}C_{21}.V_{1E} + z^{-\frac{1}{2}}C_{22}.V_{2E} \qquad (6)$$

which indicates in general the addition of the elements of the $\mathbf{B}$ matrices (i.e. $\mathbf{B_E}$, $\mathbf{B_O}$) to the elements of $\mathbf{A}$ matrices (i.e. $\mathbf{A_E}$, $\mathbf{A_O}$.). By this, however, the equation (6) changes into the following form (7).

$$C_{22}.V_{2O} = z^{-\frac{1}{2}}C_{21}.V_{1E} + z^{-\frac{1}{2}}C_{22}.V_{2E} \tag{7}$$

The graph representation of the equation (7) is in this case then following:

The left side of the equation, i.e. the constituent $C_{22}.V_{2O}$, represents the voltage of node $2O$ (i.e. the voltage of the second node in the odd phase of V_{20}), with the own loop of the node $2O$ with the transfer C_{22}. This constituent $C_{22}.V_{2O}$ is given by the sum of constituents $z^{-\frac{1}{2}}C_{21}.V_{1E}$, representing the addition to the node V_{1E} of $z^{-\frac{1}{2}}C_{21}$, which means that the branch going from node 1E to the node 2O has the transition $z^{-\frac{1}{2}}C_{21}$. Further it is given by the constituent $z^{-\frac{1}{2}}C_{22}.V_{2E}$, representing the addition to the node V_{2E} of $z^{-\frac{1}{2}}C_{22}$, which means that the branch going from the node 2E to the node 2O has the transition $z^{-\frac{1}{2}}C_{22}$.

These described facts are illustrated by the graph in Fig.2.

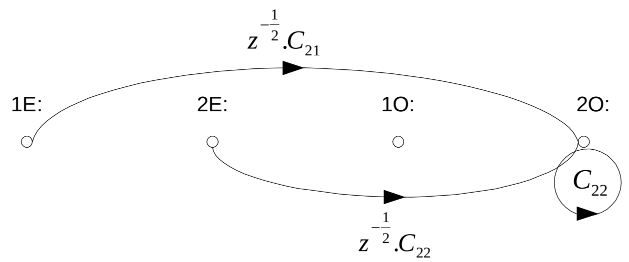


Fig.2. Graph of the equation (7)

4.3 General Rules of the Construction of the Summary Graph

Rules for summary graph construction can now be easily formulated by generalizing the construction described in the previous paragraph this way:

First we draw the nodes' own loops (and branches when applicable) as the results from the Q-graph and the V-graph for the even phase (EE) and the own loops (and branches when applicable) as the results from the Q-graph and the V-graph for the odd phase (OO).

Between thus obtained nodes and branches, we will consequently draw branches as the results from the Q-graph for one phase and the V-graph for the opposite phase, whose transfer obtained by evaluating the two-graph is multiplied by the coefficient $z^{-\frac{1}{2}}$.

These general rules will be now illustrated in following example.

5 Example

A solution of a switched capacitors circuit by the described method of a summary MC-graph constructed on the basis of two-graphs will be shown by solving a particular circuit with five nodes. This circuit consists of capacitor C_2, operational amplifier and two switched capacitors C_1 , C_3 , wiring diagram is shown in Fig.3.

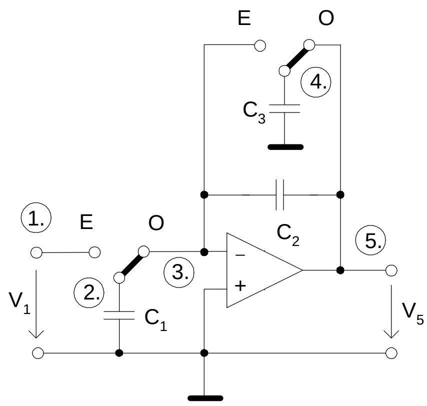


Fig.3. Circuit diagram for the example case

Full-graph solution by described method is following:

First we draw two circuit diagrams: a partial circuits diagram for the even phase and a partial circuits diagram for the odd phase separately by the algorithm described in [8], [9].

These diagrams are shown in Fig.4, where the node numbers in the squares are the numbers of the nodes of the charge (i.e. Q-) graph and the node numbers in the triangles are the numbers of the nodes of voltage (i.e. V-) graph after renumbering the nodes, caused according to the rules stated in [8] by the operational amplifier.

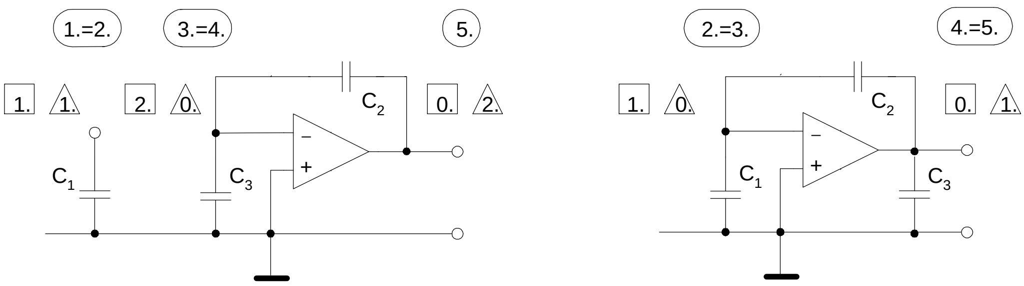


Fig.4. Diagrams of circuits for even and odd phases

To the diagrams for individual phases, we can assign directed graphs drawn in Fig.5. For orientation there are the original numbers of nodes from the diagram in Fig.4 and Fig.5 in the circles.

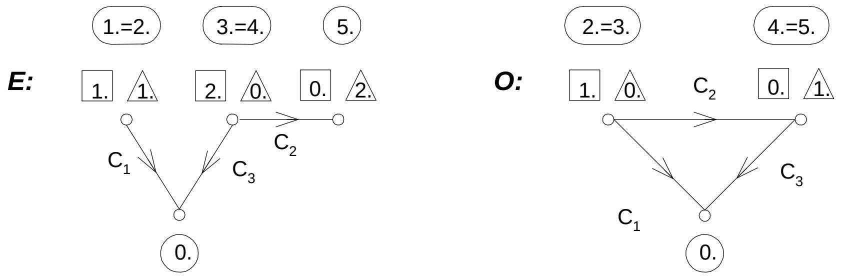
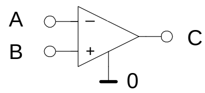


Fig.5. Graphs for even and odd phases

An ideal operational amplifier then causes merging of the input nodes 3 and 0 in the voltage V-graph and merging of the output nodes 5 and 0 in the charge Q-graph, as is shown in Table 1. [8], [9]. For both even and odd phases it is necessary to draw a special voltage and charge graphs.

These graphs are shown in Fig.6.

Table 1. Q- and V- Graphs of the Ideal Operational Amplifier

Element	Q-graph	V-graph
	A ○ ○ C=0 B ○	A=B ○ ○ C ○ 0

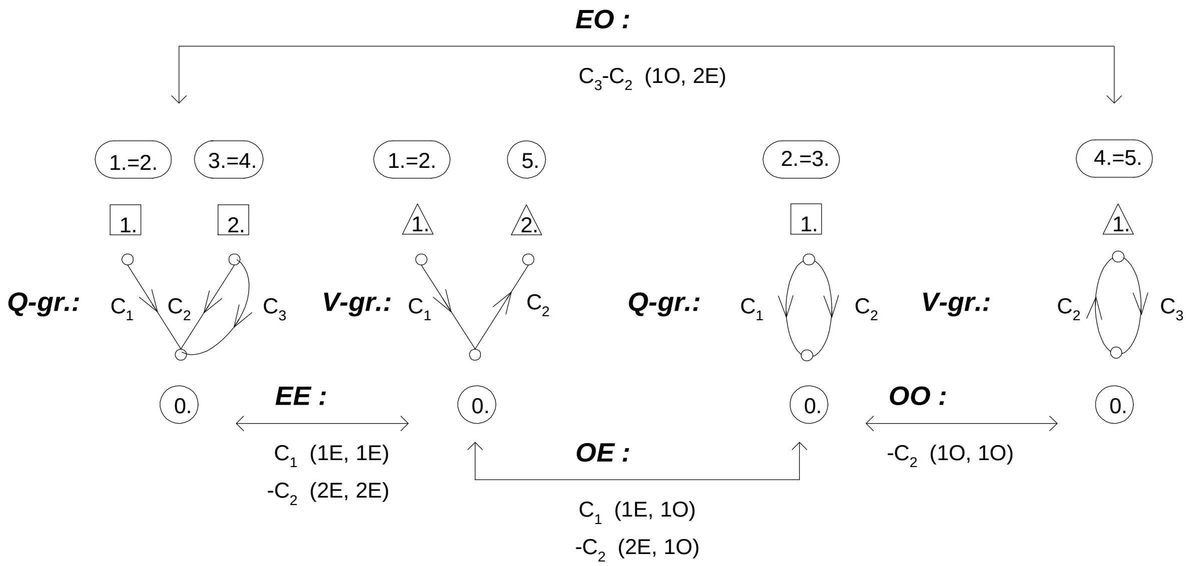


Fig.6. Two-graphs for even and odd phases

As is described in [2], [5], [6] determinant Δ of the matrix $\mathbf{Y}$ (or the matrix $\mathbf{C}$ in this case switched capacitors circuits) of the circuit is given by (8)

$$\Delta = \sum_{W} \pm (product \quad of \quad capacitors) \tag{8}$$

where

$$W = \begin{Bmatrix} set \quad of \quad spanning \\ trees \quad of \quad V-graph \end{Bmatrix} \cap \\ \cap \begin{Bmatrix} set \quad of \quad spanning \\ trees \quad of \quad Q-graph \end{Bmatrix} \qquad (9)$$

In other words, there is a term in the expression for Δ corresponding to each spanning tree that is common to the charge (Q-gr.) and voltage graphs (V-gr.).

This principle is used to construction summary MC-graph.

Summary MC-graph is now constructed by first finding the incomplete common skeletons of the V-graph and the Q-graph in the event phase and in the odd one: in the even phase there is one incomplete common skeleton formed by the C_1 element connected to the node 1 of both graphs, another incomplete skeleton is formed by the C_2 element connected to the node 2 of both graphs. But as the arrows in the Q-graph and the V-graph aim in the opposite direction, the C_2 element has a negative sign, i.e. there will be $-C_2$. In the odd phase there is an incomplete skeleton formed by C_2 connected to the 1st node, the arrows in the Q-graph and the V-graph aim in the opposite directions, so C_2 has a negative sign $-C_2$. Thus obtained loops of summary Mason-Coates graph are in Fig.7.

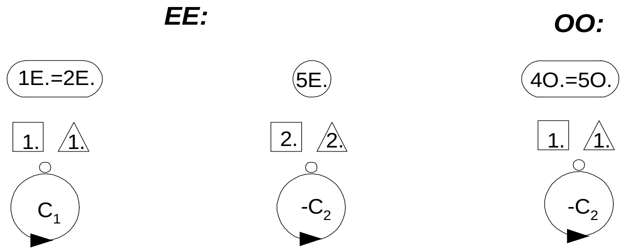


Fig.7. The nodes' own loops of the summary MC-graph

The common skeleton of the V-graph in the E phase and of the Q-graph in the O phase is formed by the C_1 element, whose graph goes from the node 1 in the even phase and from the node 1 in the odd phase, and by the C_2 element, but as the arrows at C_2 in the Q-graph and V-graph go against each other, the C_2 element has got a negative sign, i.e. there is $-C_2$. Thus the graph for the OE phase is given. In the EO phase we look for common incomplete skeletons from the Q-graph in the E phase and from the V-graph for the O phase. There are two branches going parallel with transfers C_2 and C_3, while the arrows at C_2 go in opposite directions so in the sum of these parallel branches C_2 will have a negative sign: C_3-C_2.

After completing the graph will get the form shown in Fig.8.

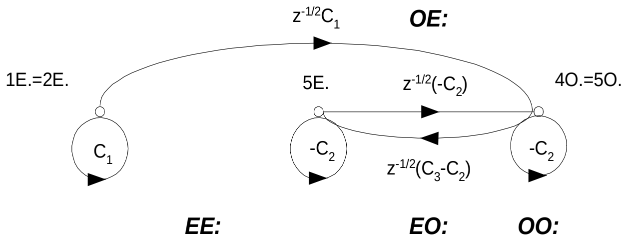


Fig.8 Summary MC-graph

The voltage transfers (10) and (11) will now be obtained from a shortened graph, i.e. a graph in which there will not be the entry node's own loop, by means of the Mason rule [1]:

$$\frac{V_{5E}}{V_{1E}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{V - \sum S^{(K)}.V^{(K)}} =$$

$$= \frac{z^{-\frac{1}{2}}.C_1.z^{-\frac{1}{2}}.(C_3 - C_2)}{(-C_2).(-C_2) - z^{-\frac{1}{2}}.(C_3 - C_2).z^{-\frac{1}{2}}.(-C_2)} =$$

$$= -\frac{z^{-1}.C_1.(C_2 - C_3)}{C_2^2 - z^{-1}.C_2.(C_2 - C_3)} \qquad (10)$$

$$\frac{V_{5O}}{V_{1E}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{V - \sum S^{(K)}.V^{(K)}} =$$

$$= \frac{z^{-\frac{1}{2}}.C_1.(-C_2)}{(-C_2).(-C_2) - z^{-\frac{1}{2}}.(C_3 - C_2).z^{-\frac{1}{2}}.(-C_2)} =$$

$$= -\frac{z^{-\frac{1}{2}}.C_1}{C_2 - z^{-1}.(C_2 - C_3)} \qquad (11)$$

The voltage transfers $\frac{V_{5O}}{V_{1O}}$ and $\frac{V_{5E}}{V_{1O}}$ between nodes 1O and 5O, 1O and 5E can not be calculated, because the node 1O is not in the graph.

5 Solving the Circuit by Matrix Calculus

For comparison we can mention a solution of the same circuit from the Fig.3 by matrix calculus using transformations of coordinates and reduction of the number of variables [1], [5] [7]. The circuit has five nodes and so its capacitance matrix will have them too (12).

$$\tilde{\mathbf{C}}_{\mathbf{O}} = \begin{array}{c} \\ 1.: \\ 2.: \\ 3.: \\ 4.: \\ 5.: \end{array} \begin{array}{c} \begin{matrix} 1.: & 2.: & 3.: & 4.: & 5.: \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 \\ 0 & 0 & C_2 & 0 & -C_2 \\ 0 & 0 & 0 & C_3 & 0 \\ 0 & 0 & -C_2 & 0 & C_2 \end{bmatrix} \end{array} \qquad (12)$$

An ideal operational amplifier will cause leaving out the row with the index of the outgoing node of the operational amplifier and the column with the index of entry node of the operational amplifier, i.e. the 5th row and the 3rd column, by which the matrix will be reduced to the form (13).

$$\mathbf{C}_{\mathbf{O}} = \begin{array}{c} \\ 1.: \\ 2.: \\ 3.: \\ 4.: \end{array} \begin{array}{c} \begin{matrix} 1.: & 2.: & 4.: & 5.: \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & 0 & -C_2 \\ 0 & 0 & C_3 & 0 \end{bmatrix} \end{array} \qquad (13)$$

The resulting capacitance matrix containing all phases of switching will then have the form (14).

$$\mathbf{C} = \begin{bmatrix} \mathbf{C_{EE}} & -z^{-\frac{1}{2}}.\mathbf{C_{EO}} \\ -z^{-\frac{1}{2}}.\mathbf{C_{OE}} & \mathbf{C_{OO}} \end{bmatrix} =$$

$$= \begin{array}{r} \\ 1E.=2E.: \\ 3E.=4E.: \\ 2O.=3O.: \end{array} \begin{array}{c} \begin{matrix} 1E.=2E.: & 5E.: & 4O.=5O.: \end{matrix} \\ \left[\begin{array}{cc|c} C_1 & 0 & 0 \\ 0 & -C_2 & z^{-\frac{1}{2}}(C_2 - C_3) \\ \hline -z^{-\frac{1}{2}}C_1 & z^{-\frac{1}{2}}C_2 & -C_2 \end{array}\right] \end{array} \qquad (14)$$

Hence the desired transfers will be following:

$$\frac{V_{5O}}{V_{1E}} = \frac{\Delta_{1:3}}{\Delta_{1:1}} = \frac{-(-z^{-\frac{1}{2}}C_1).(-C_2).(-1)^4}{C_2^2 - z^{-\frac{1}{2}}C_2.z^{-\frac{1}{2}}(C_2 - C_3)} =$$

$$= -\frac{z^{-\frac{1}{2}}C_1}{C_2 - z^{-1}.(C_2 - C_3)} \qquad (15)$$

and

$$\frac{V_{5E}}{V_{1E}} = \frac{\Delta_{1:2}}{\Delta_{1:1}} = \frac{-(-z^{-\frac{1}{2}}C_1).z^{-\frac{1}{2}}(C_2 - C_3).(-1)^3}{C_2^2 - z^{-\frac{1}{2}}C_2.z^{-\frac{1}{2}}(C_2 - C_3)} =$$

$$= -\frac{z^{-1}.C_1.(C_2 - C_3)}{C_2 - z^{-1}.(C_2 - C_3)} \qquad (16)$$

By comparing relations (10), (11) and (15), (16) it is obvious that the results are identical.

6 Solving Circuits Considering Operational Amplifier with Diffraction of its Frequency Characteristics of Amplification

6.1 The Break Point Frequency

Nevertheless, the voltage amplification decrease itself is caused by the input resistances and the capacity between the base and the collector of transistors which the operational amplifier is made up. Since an operational amplifier in a switched circuit processes high frequencies, the frequency characteristics of its amplification is taken into account by considering either one frequency ω_T

$$A = \frac{A_0.\omega_T}{\omega_T + sA} \qquad (17)$$

or both frequencies ω_T, ω_2

$$A = \frac{A_0.\omega_T}{\omega_T + sA}.\frac{\omega_2}{\omega_2 + s} \qquad (18)$$

of its diffraction, where A_0 is the maximum value of amplification in an open loop of feedback.

6.2 MC-Graph of the Voltage Control Voltage Source Break Point Frequency

The differential operational amplifier with a break point frequencies on its amplification characteristics can be considered as the Voltage Control Voltage

Source (VCVS).

In [1], [3] is described construction of the graph VCVS, i.e. graph of the equation (22).

$$-A.V_A + A.V_B = V_C \qquad (19)$$

The equation (20) can be after rewritten into (20)

$$-A.V_A + A.V_B = 1.V_C \qquad (20)$$

interpreted so that the addition to the variable V_C multiplied by the coefficient 1 from the variable V_A has the value of $-A$, and the addition from the variable V_B has the value of A. Therefore the very loop at the node V_C has the transfer 1, the branch from the node V_A to the node V_C has the transfer $-A$ and finally, the branch from the node V_B to the node V_C has the transfer A. The above mentioned construction is shown by the graph in Fig.9.

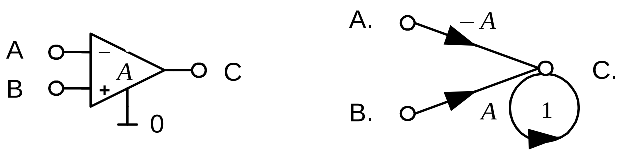


Fig.9 MC-graph of the VCVS

Because the output current of the VCVS (and thus the output charge of the VCVS) is not defined at the source voltage VCVS, the last equation of system of charge node equations (21)

$$\begin{aligned} C_{11}.V_A + C_{12}.V_B + C_{13}.V_C &= 0 \\ C_{21}.V_A + C_{22}.V_B + C_{23}.V_C &= 0 \\ C_{31}.V_A + C_{32}.V_B + C_{33}.V_C &= 0 \end{aligned} \qquad (21)$$

(ie the equation for the output charge) is replaced by the (20), thus the set of charge equations (21) is in form (22).

$$\begin{aligned} C_{11}.V_A + C_{12}.V_B + C_{13}.V_C &= 0 \\ C_{21}.V_A + C_{22}.V_B + C_{23}.V_C &= 0 \\ -A.V_A + A.\ V_B + (-1).V_C &= 0 \end{aligned} \qquad (22)$$

Because the omitted equation $C_{31}.V_A + C_{32}.V_B + C_{33}.V_C = 0$, after rewritten $C_{33}.V_C = -C_{31}.V_A - C_{32}.V_B$ indicates that the transfer of branches from node A to node C is C_{31} and the transfer of branches from node B to node C is C_{21}, these branches entering node C in the resulting graph will be not.

6.3 Example

A solution of a circuit by the described method will be shown by solving a particular circuit with capacitor C_2, two switched capacitors C_1, C_3 and amplifier with the voltage amplification A, whose wiring diagram is in Fig.10.

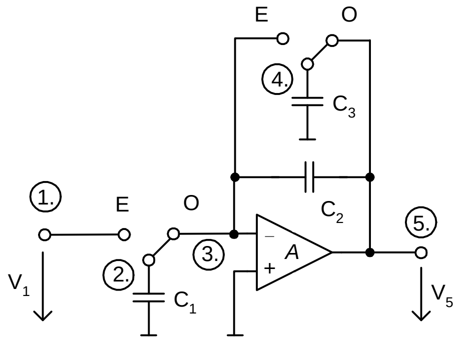


Fig.10. Diagram for the example case

First we draw two circuit diagrams: a partial diagram for the even phase and for the odd phase separately, these diagrams are shown in following Fig.11.

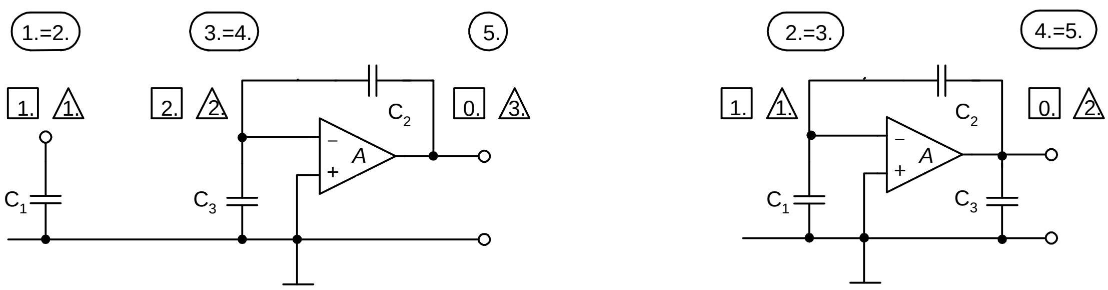


Fig.11. Diagrams of circuits for even and odd phases

For both even and odd phases it is necessary to draw a special voltage (V-gr.) and charge (Q-gr.) graphs. These graphs are in Fig.14 includes capacitors only.

Summary MC-graph is now constructed by first finding the incomplete common skeletons of the V-graph and the Q-graph in the EE, OO, EO and OE phases.

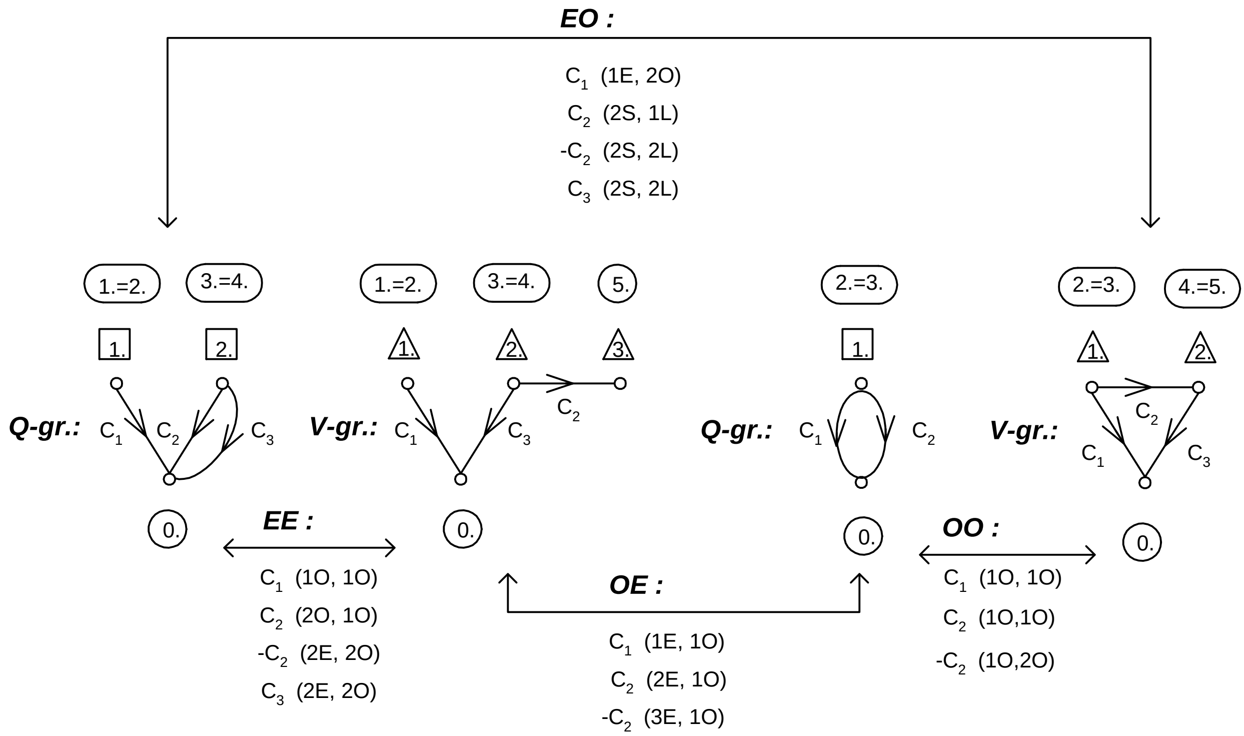


Fig.12. Two-graphs for even and odd phases

First we draw the nodes' in both phases. Because graph in Fig.12 has three nodes in even and two nodes in odd phases, summary MC-graph in Fig.13 has three nodes in even and two nodes in odd phases, too.

In the second step, between thus obtained nodes, we will consequently draw branches and own loops of the VCVS from Fig.11. This step is illustrated in Fig.13.

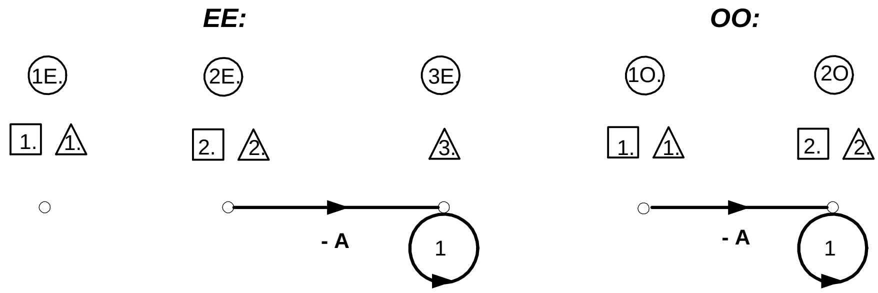


Fig.13 Graph with VCVS after second step

Between thus obtained nodes and branches, we will consequently draw branches and own loops as the results finding the incomplete common skeletons of the V-graph and the Q-graph in the event phase and in the odd one, as is described in part 4.

After completing the summary MC-graph will get the form shown in Fig.14.

The voltage transfer for example $\frac{V_{5O}}{V_{1E}}$ (23) will now be obtained from a shortened graph, i.e. a graph in which there will not be the entry node's own loop and branches going into entry node, by means of the Mason rule [1]:

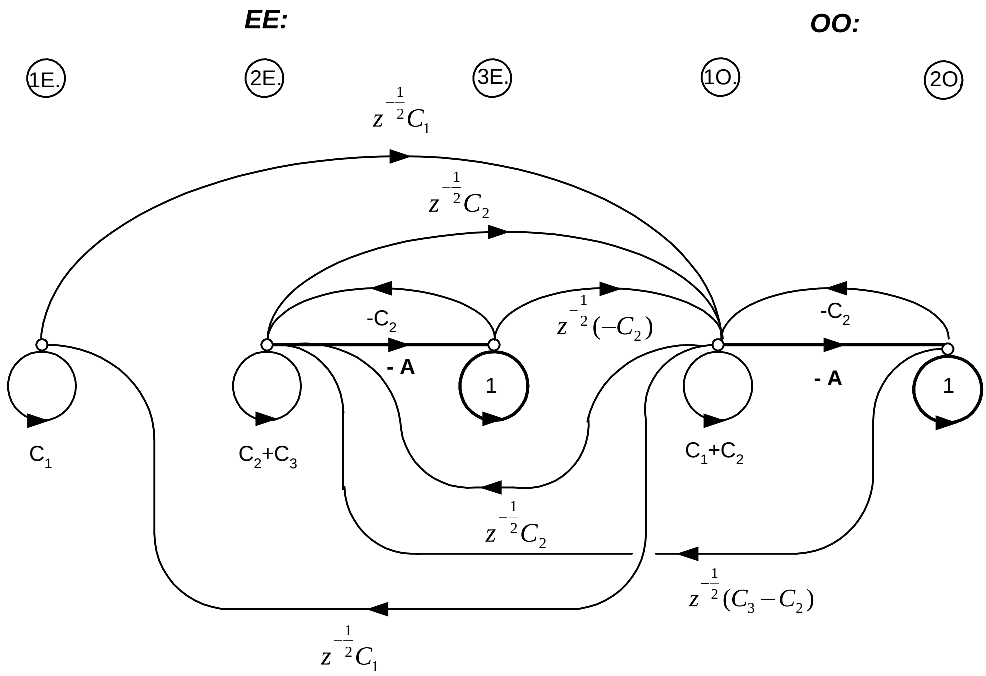


Fig.14 Resulting summary MC-graph

$$\frac{V_{5O}}{V_{1E}}=\frac{\sum p_{(i)}.\Delta_{(i)}}{V-\sum S^{(K)}.V^{(K)}}=$$

$$=\frac{z^{-\frac{1}{2}}.C_1.(-A).\{(C_2+C_3).1-(-A).(-C_2)\}}{\begin{aligned}&(C_2+C_3).(C_1+C_2)-(-A).(-C_2).\{(C_1+C_2).1\}-(-A).(-C_2).\{(C_2+C_3).1\}-z^{-\frac{1}{2}}.C_2.z^{-\frac{1}{2}}.C_2.\{1.1\}\\&+z^{-\frac{1}{2}}.C_2.(-A).z^{-\frac{1}{2}}.(C_3-C_2).\{1\}+(-A).z^{-\frac{1}{2}}.(-C_2).z^{-\frac{1}{2}}.C_2-(-A).z^{-\frac{1}{2}}.(-C_2).(-A).z^{-\frac{1}{2}}.(C_3-C_2)\\&+(-A).(-C_2).(-A).(-C_2)\end{aligned}} \tag{23}$$

7 Solving the Circuit by Matrix Calculus

For comparison we can mention a solution of the same circuit from the Fig.10 by matrix calculus. The circuit has five nodes and so its capacitance matrix $\mathbf{C_O}$ (12) will have them, too.

But in matrix (12) last row is replaced by equation of the VCVS, i.e. (20), as is described in part 6.2, because modified nodal method is used for describe. Thus the capacitance matrix $\mathbf{C_O}$ with 5 rows and 5 columns will be in following form (24).

$$\begin{array}{cc} & \begin{array}{ccccc}1.: & 2.: & 3.: & 4.: & 5.:\end{array} \\ \mathbf{C_O}= \begin{array}{c}1.:\\2..\\3.:\\4.:\\5.:\end{array} & \begin{bmatrix}0 & 0 & 0 & 0 & 0\\0 & C_1 & 0 & 0 & 0\\0 & 0 & C_2 & 0 & -C_2\\0 & 0 & 0 & C_3 & 0\\0 & 0 & -A & 0 & -1\end{bmatrix}\end{array} \tag{24}$$

The resulting capacitance matrix **C** containing all four phases of switching consists of four matrix (24) and will then have the following form (25).

$$\mathbf{C}=\begin{bmatrix}\mathbf{C_{EE}} & -z^{-\frac{1}{2}}.\mathbf{C_{EO}}\\ -z^{-\frac{1}{2}}.\mathbf{C_{OE}} & \mathbf{C_{OO}}\end{bmatrix}=$$

$$= \left[\begin{array}{ccccc|ccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 & 0 & -z^{-\frac{1}{2}}C_1 & 0 & 0 & 0 \\ 0 & 0 & C_2 & 0 & -C_2 & 0 & 0 & -z^{-\frac{1}{2}}C_2 & 0 & z^{-\frac{1}{2}}C_2 \\ 0 & 0 & 0 & C_3 & 0 & 0 & 0 & 0 & -z^{-\frac{1}{2}}C_3 & 0 \\ 0 & 0 & -A & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -z^{-\frac{1}{2}}C_1 & 0 & 0 & 0 & 0 & C_1 & 0 & 0 & 0 \\ 0 & 0 & -z^{-\frac{1}{2}}C_2 & 0 & z^{-\frac{1}{2}}C_2 & 0 & 0 & C_2 & 0 & -C_2 \\ 0 & 0 & 0 & -z^{-\frac{1}{2}}C_3 & 0 & 0 & 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -A & 0 & -1 \end{array}\right] \quad (25)$$

The last row in $\mathbf{C_{OE}}$ and $\mathbf{C_{EO}}$ matrix consists of zeros, because an operational amplifier has not memory (i.e. transfer between phases E, O is equal zero).

This matrix (25) is in next step reduced by closing the switch into form (26). Hence the desired transfer (for example $\frac{V_{5O}}{V_{1E}}$) can be expressed by applying the algebraic complements theory in following form (27).

But computing of the complement is rather difficult.

$$\left[\begin{array}{ccc|cc} C_1 & 0 & 0 & -z^{-\frac{1}{2}}C_1 & 0 \\ 0 & C_2+C_3 & -C_2 & -z^{-\frac{1}{2}}C_2 & z^{-\frac{1}{2}}C_2 - z^{-\frac{1}{2}}C_3 \\ 0 & -A & -1 & 0 & 0 \\ \hline -z^{-\frac{1}{2}}C_1 & -z^{-\frac{1}{2}}C_2 & -z^{-\frac{1}{2}}C_2 & C_1+C_2 & -C_2 \\ 0 & 0 & 0 & -A & -1 \end{array}\right] \quad (26)$$

$$\frac{V_{5O}}{V_{1E}} = \frac{\Delta_{1:5}}{\Delta_{1:1}} =$$

$$= \frac{\left|\begin{array}{ccc|c} 0 & C_2+C_3 & -C_2 & -z^{-\frac{1}{2}}C_2 \\ 0 & -A & -1 & 0 \\ \hline -z^{-\frac{1}{2}}C_1 & -z^{-\frac{1}{2}}C_2 & z^{-\frac{1}{2}}C_2 & C_1+C_2 \\ 0 & 0 & 0 & -A \end{array}\right|}{\left|\begin{array}{cc|cc} C_2+C_3 & -C_2 & -z^{-\frac{1}{2}}C_2 & z^{-\frac{1}{2}}(C_2-C_3) \\ -A & -1 & 0 & 0 \\ \hline -z^{-\frac{1}{2}}C_2 & z^{-\frac{1}{2}}C_2 & C_1+C_2 & -C_2 \\ 0 & 0 & -A & -1 \end{array}\right|} \quad (27)$$

It is necessary an expansion for the numeration (along the item of the last row in this case), because algebraic complements $\Delta_{1:5}$, $\Delta_{1:1}$ were of higher grade than 3.

Conclusion

While in case of using the graph method a graph was indicated, a voltage and charge graphs are plotted and from this two-graphs a summary graph was drawn and evaluated by the Mason´s rule, after which the result was obtained, in case of solving by the matrix calculus the procedure was much more complicated. First a partial capacitance matrix had to be composed, in the next step it was modified by an operational amplifier. From four matrices obtained by this capacitance matrix was constructed and was reduced by the activity of switches.

In literature [3], [4] etc. we can find description of a solution procedure by a method of two graphs, which leads to construction of a matrix, from which the desired voltage transfers for corresponding phases are calculated by the method of algebraic complements.

However, the described method enables us to carry out the whole solution procedure in the graphic form. With regard to the evaluation of the resulting summary MC graph it is suitable for manual solution of rather simple circuits. Its certain disadvantage is the renumbering of node in different ways for the charge graph and for the voltage graph, which may seem complicated.

In case of an ideal operational amplifier is solving by this method very simply, because the number of the nodes of the graph is decreased, but if the break point frequencies of the operational amplifier are considered, the number of the nodes stay unchanged, as we can see in comparsion of Fig.8 and Fig.14.

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