The Optimal Interval Partition and Second-Factor Fuzzy Set B_i on the Impacts of Fuzzy Time Series Forecasting

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Abstract: - This study uses two sets of Taiwanese data, the export values as the prediction varialbe and its foreign exchange spot rates as the auxiliary variable, to discuss two important issues of forecasting effects in the fuzzy time series analysis by using One- and Two-factor models. The first issue is the relation between the optimum number of partition equal intervals and forecasting error. The second issue is the setting of fuzzy matrix (B_i) in the model to compare its impacts on forecasting error when it is static or dynamic. The above two issues are investigated with the empirical results. First, the optimum number of partition equal intervals is to select 14 intervals for the information to have the smallest forecasting error in all models for one- or two-factor, or different number of window basis selected. However, if partitioning the information into more than 14 equal intervals, the forecasting error can not be reduced but presents a waving pattern. Second, when the information period is longer and if the selecting window basis is two, under any number of partition intervals, the forecasting error is always smaller for the dynamic B_i than for the static one. However, when the information period is shorter and the window basis is two or three, only partitioning into five or eight equal intervals, the forecasting error will also be smaller for the dynamic B_i.

Key-Words: Fuzzy tine series, Two-factor model, Interval partition, Fuzzy relationship matrix, Window base, Taiwan exports, MSE.

1 Introduction

Before fuzzifing time series data for fuzzy model prediction, one needs to determine the universe of discourse and to partition the set. The selected partition intervals will affect the forecasting accuracy of the fuzzy models [6]. Consequently, it is a vital research topic to explore how the selected number of partition intervals for the better results of fuzzy time series model. Previous studies have generally partitioned information into five or seven intervals. Song and Chissom [13-16] and Chen [1] have used seven intervals to fuzzify time series data to predict the number of students entering the University of Alabama. In addition, using the same partition technique on the same sample, Hwang et al. [7] proposed another one-factor model. Hsu et al. [4] applied the bivariate Markove model and partitioned five intervals to predict price limits and trading volume differences of weighted Taiwan stock index. Chen and Hwang [2] have used two-factor model to fuzzify seven intervals in a fuzzy set and engaged in temperature prediction. To avoid the operational

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complexity, these studies select only the simple random partition method (five or seven intervals) to their analysis among which the forecasting error (MSE) is not the smallest.

Huarng [5] has discussed the length of intervals used to segment the data and applied two methods (distribution-based and average-based) to find better intervals and improve forecasting results. Basing on the base-mapping table to determine the length of the interval, Huarng [6] did not explain the causes but only concluded that the more the partition intervals, the better the prediction accuracy. Huarng [6] has also suggested the multivariate guidance models to partition 16 intervals on a fuzzify time series data for the prediction of weighted index of Taiwan futures contracts. Li and Chen [9] applied the natural partitioning technique, which can recursively partition the universe of discourse level by leveling in a natural way for the purpose of replacing the base-mapping table. Both Huarng [6] and Li and Chen [9] have decided the length of intervals based on believes in experiences learned from domain experts. Lee et al. [8] had applied the two-factors high-order fuzzy time series model on the information of daily cloud density data and partitioned the universe of discourse (U) into nine intervals to predict changes in temperatures. Li and Cheng [11], based on the state-transition analysis, had overcome the hurdle of determining the 'k-order' in Chen's model. They quantified a deterministic maximum length of subsequence in the fuzzy time series which leaded to a certain state. They suggested that the length of interval will reflect the sensitivity of the invested information. The forecasting models must follow consistent principles under which the shorter the partition length, the more the intervals resulting in better forecasting precision.

Previous research indicates that the more of the partition sets, meaning the shorter the length of intervals, the higher the precision resulted. However, to provide every interval with a vocabulary and value or description, so called defining the fuzzy set, is an extremely difficult or even impossible task. Due to the increasing difficult ties and complexity during the process of empirical calculation and deduction, the pragmatic application of large number of partition sets is limited or restricted. Therefore, it is necessary to compromise between precision and complexity. No other related studies have discussed comprehensively to the subject but only considered differences in models and information periods, the resulted forecasting error from using different intervals to partition, in addition to their effects on future studies. The appropriateness of partition information into traditional five or seven intervals is worth of further verification.

Since Song and Chissom's [13-16] fuzzy time series model, there are three main forecasting models been developed, including the two-factor, the heuristic model, and the Markov model. Future studies have based on one of those three to improve model construction to increase forecasting effects. Yu [18] proposed weighted and refined fuzzy time series models for TAIEX forecasting. Wang and Yang [17] introduced the application concept of entropy to measure the degrees of fuzziness when a time-invariant relation matrix is derived. Cheng et al [3] developed fuzzy time series models for forecasting IT costs applying two approaches: minimizing entropy and trapezoid fuzzification. Lee and Chen [9] constructed two-factor high-order fuzzy logical relationships based on the historical data to increase the forecasting accuracy rate. Singh [12] developed a simple computational method for fuzzy forecasting models based on different parameters. Li et al [10] used the fuzzy C mean method to cluster fuzzy data for forecasting TAIEX futures and enrollment. All of these papers modified the formulation of the model to minimize the forecasting error (MSE) against sample data. They adjusted parameters such as fuzzy data partition, the membership function, fuzzy relations and clustering. These improved models tried to compare forecasting effects with their corresponding models under the set condition of one information period. However, if the empirical information periods is prolonged or shorten, whether the forecasting effect of the model is subject to be influenced accordingly or if the model still behaves stably has never been investigated in the previous studies.

Hwang et al. [7] proposed a new fuzzy time series forecasting method. It follows the heuristic rules of one to three to confirm fuzzy relationships and to forecast the enrollment at year t; it must decide the number of years of enrollment information used. The number of years of the enrollment data we used is called the window basis which is set to *w* years. Using last year's variation as the criterion matrix and the variations of the past years' (*w*) as the operation matrix to perform the forecasting, called one-factor time-variant fuzzy time series model. Chen and Hwang [2] expanded Hwang's model and named the predicted variable the main-factor which introduced the additional informal as the second-factor to auxiliary the prediction of the main-factor. Chens' new model is called the two-factor model. In the two-factor model, the determination of the second-factor fuzzy set (B_i)

is based on the dynamic relationships with the main-factor. If the relationship is negative, then the second-factor fuzzy set (B_i) has an opposite directional setting for the main-factor fuzzy set (A_i). For example, in Chen et al. model to forecast temperature, the average temperature is the main-factor and the cloud density is the second-factor, and if partitioning into seven intervals, the main-factor fuzzy set (A_i) is $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ and $A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7$. Based on the negative relationship between the main- and second-factor, the second-factor fuzzy set could be formed accordingly and $B_1 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 1/u_5 + 1/u_6 + 1/u_7$. On the contrary, if the relationship is positive, $B_1 = 1/u_1 + 1/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 + 0/u_7$ however, there is an important assumption that the way of setting is irrelevant with the number of partition intervals. Setting the values of B_i not moving with the increasing number of partition intervals is called the static B_i . The ignored effect of different partition intervals should also be considered into the model. On the other hand, the values of B_i moving with the number of increasing partition intervals are called dynamic B_i. When the model has added the second-factor as the increment information to auxiliary the forecasting of main-factor, the differences in the setting of B_i could affect the forecasting error in the two-factor model which has never been mentioned in previous literatures. Therefore, this study focuses on the setting of the fuzzy set B_i and compares how the setting influences the forecasting error.

This study aims at the above mentioned two issues in the fuzzy time series forecasting model, using the one-factor and two-factor model as standards to measure and compare. Under different length of information periods, could these two fuzzy forecasting models maintain their stabilities in their forecasting effects? In the meantime, this study also partitions intervals into different numbers to verify the concept of “the shorter the interval, the smaller the forecasting error”. Further deduction on these two fuzzy time series models will be engaged to propose a valid number of partition intervals in order to reduce forecasting error. Finally, we will present an appropriate setting method for the second-factor fuzzy set (B_i) and discuss its effects on the forecasting error, when there are differences in the information periods, partition intervals, and retroacting window basis periods.

2 Fuzzy set theory and fuzzy time series

2.1 Defining the universe of discourse and the intervals.

Let U be the universe of discourse in which $U = \{u_1, u_2, ..., u_t\}$. As the problem domain, U can be defined properly. After the length of intervals is determined, U can be partitioned into equal length intervals $u_1, u_2, u_3, ..., u_t$. The midpoints of these intervals are $m_1, m_2, m_3, ..., m_t$ respectively.

2.2 Defining the fuzzy sets A_i and fuzzifying the data

Each fuzzy set A_i is assigned to a linguistic term, and can be defined by the intervals $\{u_1, u_2, u_3, ..., u_t\}$, $A_i = f_{Ai}(u_1)/u_1 + f_{Ai}(u_2)/u_2 + ... + f_{Ai}(u_t)/u_t$, where f_{Ai} is the membership function of fuzzy set A_i , $f_{Ai} : U \to [0,1]$. u_k is an element of the fuzzy set A_i , and $f_{Ai}(u_k)$ is the degree of belongingness of u_k to A_i , $f_{Ai}(u_k) \in [0,1]$ and $1 \le k \le n$.

2.3 Fuzzy time series

Suppose that $Y(t)(t = ...,0,1,2,.....)$is a subset of R. Let $Y(t)$ be the universe of discourse defined by the fuzzy set $f_i(t)$. If $F(t)$ consists of $f_i(t)(i = 1,2,......)$, then $F(t)$ is defined as a fuzzy time series on $Y(t)(t = ...,0,1,2,.....)$. Futhermore, we can also see that $F(t)$ is a function of time t, i.e., the values of $F(t)$ can be different at different times. According to Song and Chissom (1993), if $F(t)$ is caused by $F(t-1)$ only, then this relationship is represented by $F(t-1) \to F(t)$.

Let $F(t)$ be a fuzzy time series. If for any time t, $F(t-1) = F(t)$ and $F(t)$ only have finite elements, then $F(t)$ is called a time-invariant fuzzy times. Otherwise, it is called a time-variant fuzzy time series.

2.4 One-factors time-variant fuzzy time series model

2.4.1 One-factor criterion vector $C(t)$ and operation matrix $O^w(t)$ One-factor criterion vector is defined as following:

$$C(t) = f(t-1) = [C_1, C_2, \cdots, C_m], 0 \le C_j \le 1 \quad (1)$$

Operation matrix is defined as following:

$$O^w(t)=\begin{bmatrix} f(t-2) \\ f(t-3) \\ \vdots \\ f(t-w) \end{bmatrix}=\begin{bmatrix} O_{11} & O_{12} & \cdots & O_{1m} \\ O_{21} & O_{22} & \cdots & O_{2m} \\ \vdots & & & \vdots \\ O_{(w-1)1} & \cdots & \cdots & O_{(w-1)m} \end{bmatrix},$$

$$0 \le O_{ij} \le 1, 1 \le i \le w-1, 1 \le j \le m \quad (2)$$

Where $f(t-1)$ is the fuzzified variation between time ($t-1$) and ($t-2$) of the factor $F(t)$, m the number of interval in the universe of discourse, and w the window basis.

2.4.2 One-factor fuzzy relationship matrix $R(t)$

$$R(t)=O^w(t)\otimes C(t)$$

$$=\begin{bmatrix} O_{11}\times C_1 & O_{12}\times C_2 & \cdots & O_{1m}\times C_m \\ O_{21}\times C_1 & O_{22}\times C_2 & \cdots & O_{2m}\times C_m \\ \vdots & \vdots & \vdots & \vdots \\ O_{(w-1)1}\times C_1 & O_{(w-1)2}\times C_2 & \cdots & O_{(w-1)m}\times C_m \end{bmatrix}=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ R_{(w-1)1} & R_{(w-1)2} & \cdots & R_{(w-1)m} \end{bmatrix}$$

$$0 \le O_{ij} \le 1, 1 \le i \le w-1, 1 \le j \le m, 0 \le C_j \le 1 \quad (3)$$

Where $R_{ij}=O_{ij}\times C_j$, " $\times$ " is the multiplication operator. From the matrix model (3), the fuzzified variation $f(t)$ can be described as following:

$$f(t)=\lfloor \max(R_{11},R_{21},\ldots,R_{(w\ 1)1}), \max(R_{12},R_{22},\ldots,R_{(w\ 1)2}),\ldots \rfloor$$
$$,\max(R_{1m},R_{2m},\ldots,R_{(w\ 1)m})\ ]$$

(4)

The model algorithm of One-factor fuzzy time series can be presented as follows:

Step 1. Compute the variations of the enrollments between any two continuous years.

Step 2. Partition the universe of discourse and the length of intervals.

Step 3. Define fuzzy sets on universe of discourse U.

Step 4. Fuzzify the values of historical data.

Step 5. Choose a suitable window basis w, and calculate the output from operation matrix $O^w(t)$ and criterion matrix $C(t)$.

Step 6. Fuzzify the fuzzy forecasted variations derived in Step 5.

Step 7. Calculate the forecasted enrollments.

2.5 Two-factors time-variant fuzzy time series model

2.5.1 Two-factor criterion vector $C(t)$, $S(t)$ and operation matrix $O^w(t)$

$$C(t)=f(t-1)=[C_1,C_2,\cdots,C_m],\ 0 \le C_j \le 1 \quad (5)$$

$$S(t)=g(t-1)=[S_1,S_2,\cdots,S_m],\ 0 \le S_j \le 1 \quad (6)$$

Operation matrix is defined as following:

$$O^w(t)=\begin{bmatrix} f(t-2) \\ f(t-3) \\ \vdots \\ f(t-w) \end{bmatrix}=\begin{bmatrix} O_{11} & O_{12} & \cdots & O_{1m} \\ O_{21} & O_{22} & \cdots & O_{2m} \\ \vdots & & & \vdots \\ O_{(w-1)1} & \cdots & \cdots & O_{(w-1)m} \end{bmatrix},$$

$$0 \le O_{ij} \le 1, 1 \le i \le w-1, 1 \le j \le m \quad (7)$$

Where $f(t-1)$ is the fuzzified variation between time ($t-1$) and ($t-2$) of the first-factor $F(t)$, $s(t-1)$ the fuzzified variation at time ($t-1$) of the second-factor $S(t)$, m the number of interval in the universe of discourse, and w the window basis.

2.5.2 Two-factor fuzzy relationship matrix $R(t)$

$$R(t)=O^w(t)\otimes S(t)\otimes C(t)$$

$$=\begin{bmatrix} O_{11}\times S_1\times C_1 & O_{12}\times S_2\times C_2 & \cdots & O_{1m}\times S_m\times C_m \\ O_{21}\times S_1\times C_1 & O_{22}\times S_2\times C_2 & \cdots & O_{2m}\times S_m\times C_m \\ \vdots & \vdots & \vdots & \vdots \\ O_{(w-1)1}\times S_1\times C_1 & O_{(w-1)2}\times S_2\times C_2 & \cdots & O_{(w-1)m}\times S_m\times C_m \end{bmatrix}=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ R_{(w-1)1} & R_{(w-1)2} & \cdots & R_{(w-1)m} \end{bmatrix}$$

$$0 \le O_{ij} \le 1, 1 \le i \le w-1, 1 \le j \le m, 0 \le S_j \le 1, 0 \le C_j \le 1 \quad (8)$$

Where $R_{ij}=O_{ij}\times S_{ij}\times C_j$, " $\times$ " is the multiplication operator. From the matrix model (8), the fuzzified variation $f(t)$ can be described as following:

Where $R_{ij}=O_{ij}\times S_{ij}\times C_j$, " $\times$ " is the multiplication operator. From the matrix model (8), the fuzzified variation $f(t)$ can be described as following:

$$f(t)=\lfloor \max(R_{11},R_{21},\ldots,R_{(w\ 1)1}), \max(R_{12},R_{22},\ldots,R_{(w\ 1)2}),\ldots \rfloor$$
$$,\max(R_{1m},R_{2m},\ldots,R_{(w\ 1)m})\ ]$$

(9)

The model algorithm of Two-factor fuzzy time series can be presented as follows:

Step 1. Decide the first factor and calculate its variations.

Step 2. Decide universe of discourse and the length of intervals.

Step 3. Determine the second factor $S(t)$ as presented in step (1) and (2).

Step 4. Decide the window basis for fuzzy relationship matrix.

Step 5. Compute the fuzzified variation and defuzzify.

This study uses mean square error (MSE) to measure forecasting accuracy. The MSE value can be represented by:

$$MSE = \frac{1}{n}\sum_{k=1}^{n}(x^{(0)}(k) - \hat{x}^{(0)}(k))^2 \qquad (10)$$

where $X^{(0)}(K)$ is the actual value; $\hat{X}^{(0)}(K)$ the predicted value; n denotes the number of data. The smaller MSE value is, the closer is predicted value of the model to the historical data, meaning the high prediction capability of the model.

3 Data information and results

3.1 Data information

The data are obtained from AREMOS economic database, which includes the amount of Taiwan exports and the foreign exchange spot rate from January 1995 to March 2002, with a total of 87 data points. According to the international finance theory, amount of exports and foreign exchange spot rate have a close relationship. Therefore, in our analysis, the amount of exports is the predicted variable and the foreign exchange spot rate is the auxiliary variable as the increment information for forecasting. The correlation between the two variables is calculated to understand their moving direction. The correlation is found to be 0.80331 (p-value=0.0001), meaning that there is a significant positive relation between the amount of exports and foreign exchange spot rate. The Mean Square Error (MSE) is the forecasting error to evaluate model's forecasting accuracy.

Due to the possible influence of information period on forecasting power, the length for information period is controlled for in this study and further separated into three sample sets, including a short period of a total 27 data points (January 2000 to March 2002), a middle term of a total 51 data points (January 1998 to March 2002) and a long period of a total 87 data points (January 1995 to March 2002). Under different length of information periods, how prior information, interval partition, incremenet information and the setting of second-factor fuzzy set affect model forecasting power are discussed.

3.2 Empirical result

This study includes the window basis with a range of two to seven ($w = 2 \sim w = 7$) and selects five to 16 equal partition intervals to discuss the variation of forecasting error in the two-factor model when under either one- or two-factor condition.

3.2.1 The comparison of forecasting error for different partition intervals

In the search of the best partition intervals for a smaller forecasting error model to verify the existence of the most appropriate length of intervals, this study adopts a generally used range of five to 16 equal partition intervals in our three separate information data sets. Through the application of the fuzzy time series two-factor model, comparison results for changes in the forecasting error (MSE) are summarized in the following.

Based on table 3-1, 3-2 & 3-3, figure 3-1, 3-2 and 3-3 are presented to compare the partition intervals with the frequency accumulated for four smaller forecasting errors (MSE) obtained from the three separate data information sets. The frequency in each figure represents the accumulated occurrence of the four smallest forecasting errors for each partition intervals, from one- and two-variable, and under the window basis of two to seven. These four smallest forecasting errors are regarded as the most appropriate equal partition intervals with relatively smaller forecasting errors. This study discovers that under the two-factor model, the selection of 14 equal partition intervals has the smallest forecasting error, no matter the information periods in the three data set, one- or two-variable type, or the window basis. After the partition intervals has reached to 14, forecasting errors show a waving condition and even adding more partition intervals will not further reduce the forecasting error.

3.2.2 The setting of second-factor fuzzy B_i on model forecasting error

In order to investigate whether the setting of second-factor fuzzy B_i would influence model forecasting error, this study sets B_i as static or dynamic. The method of setting is in the appendix 1 & 2. Under three different (short or long) information periods including January 1995 to March 2002, January 1998 to March 2002, and January 2000 to March 2002 as well as different retroacting window basis periods including the range of $w = 2 \sim w = 7$, changes in forecasting errors are compared. The empirical results are included in table 3-4-1 ~ 3-4-6, 3-5-1 ~ 3-5-6 and 3-6-1 ~3-6-6.

Table 3-1 Forecasting error (MSE) comparison for one-and two variables under two factor model (Taiwan export amounts, January 1995~March 2002, in million NT dollars)

partition interval	Amount	model	Two-factor model											
		type	One variable						Two variables					
		window	w=2	w=3	w=4	w=5	w=6	w=7	w=2	w=3	w=4	w=5	w=6	w=7
5 intervals	41747.2		***2.73×10^9	3.57×10^9	3.83×10^9	3.96×10^9	4.04×10^9	4.07×10^9	****2.66×10^9	**3.22×10^9	*3.42×10^9	3.47×10^9	3.52×10^9	3.55×10^9
6	34789.4		***2.55×10^9	*3.5×10^9	3.88×10^9	4.14×10^9	4.37×10^9	4.45×10^9	****2.53×10^9	**3.44×10^9	3.75×10^9	4.02×10^9	4.22×10^9	4.29×10^9
7	29819.5		***2.25×10^9	*3.31×10^9	3.59×10^9	3.6×10^9	3.75×10^9	3.87×10^9	****2.18×10^9	**3.14×10^9	3.42×109	3.42×10^9	3.52×10^9	3.65×10^9
8	26092		****2.15×10^9	**3.31×10^9	3.89×10^9	4.03×10^9	4.16×10^9	4.22×10^9	****2.15×10^9	***3.23×10^9	*3.73×10^9	3.88×10^9	3.99×10^9	4.05×10^9
9	23192.9		****2.19×10^9	**2.85×10^9	3.08×10^9	3.16×10^9	3.71×10^9	3.73×10^9	****2.19×10^9	***2.76×10^9	*3.03×10^9	3.11×10^9	3.53×10^9	3.55×10^9
10	20873.6		****2.14×10^9	**2.75×10^9	3.1×10^9	3.02×10^9	3.38×10^9	3.48×10^9	****2.14×10^9	***2.71×10^9	3.08×10^9	*3×10^9	3.34×10^9	3.44×10^9
11	18976		****2.11×10^9	**2.87×10^9	3.25×10^9	3.23×10^9	3.69×10^9	3.74×10^9	****2.11×10^9	***2.81×10^9	*3.21×10^9	3.19×10^9	3.65×10^9	3.7×10^9
12	17394.67		****2×10^9	**2.63×10^9	3.2×10^9	3.2×10^9	3.56×10^9	3.64×10^9	****2×10^9	***2.55×10^9	*3.15×10^9	3.16×10^9	3.51×10^9	3.59×10^9
13	16056.62		****2.06×10^9	**2.72×10^9	3.16×10^9	3.15×10^9	3.32×10^9	3.44×10^9	****2.06×10^9	***2.63×10^9	*3.09×10^9	3.1×10^9	3.27×10^9	3.38×10^9
14	14909.72		****1.97×10^9	**2.53×10^9	3.02×10^9	3.02×10^9	3.23×10^9	3.3×10^9	****1.97×10^9	***2.47×10^9	*2.98×10^9	2.99×10^9	3.19×10^9	3.26×10^9
15	13915.74		****1.93×10^9	**2.53×10^9	3.07×10^9	3.12×10^9	3.24×10^9	3.38×10^9	****1.93×10^9	***2.48×10^9	*3.04×10^9	3.1×10^9	3.22×10^9	3.36×10^9
16	13046		****1.9×10^9	**2.46×10^9	3.04×10^9	3×10^9	3.2×10^9	3.33×10^9	****1.9×10^9	***2.4×10^9	*3×10^9	2.98×10^9	3.17×10^9	3.3×10^9

****: the smallest forecasting error (MSE), ***: the second smallest, **: the third smallest, *: fourth smallest MSE compared horizontally.

Table 3-2 Forecasting error (MSE) comparison for one-and two variables under two factor model (Taiwan export amounts, January 1998~March 2002, in million NT dollars)

partition interval	Amount	model	Two-factor model											
		type	One variable						Two variables					
		window	w=2	w=3	w=4	w=5	w=6	w=7	w=2	w=3	w=4	w=5	w=6	w=7
5 intervals	41747.2		***3.17×10^9	3.99×10^9	4.13×10^9	4.44×10^9	4.63×10^9	4.74×10^9	****2.86×10^9	**3.46×10^9	*3.52×10^9	3.79×10^9	3.92×10^9	4.01×10^9
6	34790		***2.89×10^9	4.18×10^9	4.67×10^9	5.06×10^9	5.44×10^9	5.62×10^9	****2.85×10^9	**3.82×10^9	*4.16×10^9	4.39×10^9	4.66×10^9	4.81×10^9
7	29820		***2.56×10^9	3.86×10^9	4.18×10^9	4.3×10^9	4.64×10^9	4.88×10^9	****2.47×10^9	**3.41×10^9	*3.59×10^9	3.69×10^9	3.91×10^9	4.12×10^9
8	26092		***2.51×10^9	4.22×10^9	4.73×10^9	4.95×10^9	5.32×10^9	5.53×10^9	****2.41×10^9	**3.76×10^9	*4.17×10^9	4.34×10^9	4.59×10^9	4.79×10^9
9	23193		***2.64×10^9	3.44×10^9	3.71×10^9	3.9×10^9	4.58×10^9	4.64×10^9	****2.61×10^9	**2.96×10^9	*3.1×10^9	3.16×10^9	3.82×10^9	3.93×10^9
10	20873.6		***2.52×10^9	3.39×10^9	3.74×10^9	3.69×10^9	4.41×10^9	4.62×10^9	****2.51×10^9	**3.02×10^9	3.29×10^9	*3.22×10^9	3.91×10^9	4.12×10^9
11	18976		***2.52×10^9	*3.64×10^9	4.01×10^9	4.01×10^9	4.82×10^9	5.02×10^9	****2.52×10^9	***3.48×10^9	**3.52×10^9	3.49×10^9	3.89×10^9	4.07×10^9
12	17395		***2.32×10^9	3.05×10^9	3.82×10^9	3.86×10^9	4.58×10^9	4.77×10^9	****2.29×10^9	**2.66×10^9	*3.37×10^9	3.4×10^9	4.09×10^9	4.27×10^9
13	16057		***2.45×10^9	3.36×10^9	3.75×10^9	3.82×10^9	4.13×10^9	4.34×10^9	****2.43×10^9	**2.97×10^9	*3.35×10^9	3.41×10^9	3.71×10^9	3.91×10^9
14	14910		***2.31×10^9	*3.02×10^9	3.74×10^9	3.73×10^9	4.11×10^9	4.28×10^9	****2.29×10^9	*2.61×10^9	3.27×10^9	3.25×10^9	3.6×10^9	3.76×10^9
15	13916		***2.24×10^9	*3.02×10^9	3.77×10^9	3.83×10^9	4.07×10^9	4.36×10^9	****2.2×10^9	**2.62×10^9	3.31×10^9	3.36×10^9	3.6×10^9	3.88×10^9
16	13046		***2.27×10^9	*2.98×10^9	3.75×10^9	3.72×10^9	4.05×10^9	4.28×10^9	****2.25×10^9	**2.59×10^9	3.31×10^9	3.27×10^9	3.59×10^9	3.81×10^9

****: the smallest forecasting error (MSE), ***: the second smallest, **: the third smallest, *: fourth smallest MSE compared horizontally.

Table 3-3 Forecasting error (MSE) comparison for one-and two variable under two factor model (Taiwan export amounts, January 2000~March 2002, in million NT dollars)

partition interval	Amount	model	Two-factor model											
		type	One variable						Two variables					
		windo w	w=2	w=3	w=4	w=5	w=6	w=7	w=2	w=3	w=4	w=5	w=6	w=7
5	41747.2		*3.05×10^9	*3.05×10^9	3.22×10^9	3.78×10^9	3.77×10^9	3.64×10^9	***2.68×10^9	****2.66×10^9	**2.91×10^9	3.22×10^9	3.18×10	3.18×10^9
6	34790		***3.07×10^9	*3.33×10^9	3.8×10^9	4.56×10^9	4.72×10^9	4.81×10^9	****2.96×10^9	**3.22×10^9	3.62×10^9	4.36×10^9	4.42×10	4.5×10^9
7	29820		***2.46×10^9	*2.65×10^9	2.97×10^9	3.26×10^9	3.53×10^9	3.33×10^9	****2.44×10^9	**2.63×10^9	2.98×10^9	3.27×10^9	3.37×10	3.34×10^9
8	26092		***2.42×10^9	*3.02×10^9	3.52×10^9	3.87×10^9	4.02×10^9	4.1×10^9	****2.4×10^9	**3×10^9	3.44×10^9	3.79×10^9	3.94×10	4.01×10^9
9	23193		***2.56×10^9	*2.92×10^9	3.4×10^9	3.65×10^9	3.78×10^9	3.79×10^9	****2.49×10^9	**2.84×10^9	3.37×10^9	3.62×10^9	3.75×10	3.75×10^9
10	20873.6		****2.44×10	**2.7×10^9	3.1×10^9	3.06×10^9	3.18×10^9	3.18×10^9	***2.47×10^9	*2.73×10^9	3.15×10^9	3.11×10^9	3.18×10	3.18×10^9
11	18976		****2.48×10	**2.83×10^9	3.23×10^9	3.27×10^9	3.5×10^9	3.53×10^9	***2.52×10^9	*2.87×10^9	3.23×10^9	3.27×10^9	3.5×10^9	3.53×10^9
12	17395		****2×10^9	**2.31×10^9	3.37×10^9	3.37×10^9	3.56×10^9	3.61×10^9	***2.03×10^9	*2.34×10^9	3.41×10^9	3.41×10^9	3.56×10	3.61×10^9
13	16057		****2.37×10	**2.72×10^9	**3.17×1	*3.19×10	3.36×10^9	3.4×10^9	****2.37×10^9	***2.72×10^9	**3.17×10^9	**3.19×1	3.36×10	3.4×10^9
14	14910		****1.93×10	**2.15×10^9	3.09×10^9	3.09×10^9	3.26×10^9	3.3×10^9	***1.96×10^9	*2.18×10^9	3.12×10^9	3.12×10^9	3.26×10	3.3×10^9
15	13916		****1.98×10	***2.34×1	**3.26×1	*3.34×10	3.45×10^9	3.46×10^9	****1.98×10^9	***2.34×10^9	**3.26×10^9	*3.34×10	3.45×10	3.46×10^9
16	13046		****1.91×10	**2.13×10^9	3.13×10^9	3.09×10^9	3.25×10^9	3.27×10^9	***1.93×10^9	*2.15×10^9	3.15×10^9	3.12×10^9	3.28×10	3.3×10^9

****: the smallest forecasting error (MSE), ***: the second smallest, **: the third smallest, *: fourth smallest MSE compared horizontally.

Appendix 2

Multivariate fuzzy time series two-factor model: setting of dynamic B_i

5 intervals

1	1	1	0.5	0	0
2	1	1	1	0.5	0
3	0.5	1	1	1	0.5
4	0	0.5	1	1	1
5	0	0	0.5	1	1

6 intervals

1	1	1	1	0.5	0	0
2	1	1	1	1	0.5	0
3	1	1	1	1	1	0.5
4	0.5	1	1	1	1	1
5	0	0.5	1	1	1	1
6	0	0	0.5	1	1	1

7 intervals

1	1	1	1	0.5	0	0	0
2	1	1	1	1	0.5	0	0
3	1	1	1	1	1	0.5	0
4	0.5	1	1	1	1	1	0.5
5	0	0.5	1	1	1	1	1
6	0	0	0.5	1	1	1	1
7	0	0	0	0.5	1	1	1

8 intervals

1	1	1	1	0.5	0	0	0	0
2	1	1	1	1	0.5	0	0	0
3	1	1	1	1	1	0.5	0	0
4	1	1	1	1	1	1	0.5	0
5	0.5	1	1	1	1	1	1	0.5
6	0	0.5	1	1	1	1	1	1
7	0	0	0.5	1	1	1	1	1
8	0	0	0	0.5	1	1	1	1

9 intervals

1	1	1	1	1	0.5	0	0	0	0
2	1	1	1	1	1	0.5	0	0	0
3	1	1	1	1	1	1	0.5	0	0
4	1	1	1	1	1	1	1	0.5	0
5	0.5	1	1	1	1	1	1	1	0.5
6	0	0.5	1	1	1	1	1	1	1
7	0	0	0.5	1	1	1	1	1	1
8	0	0	0	0.5	1	1	1	1	1
9	0	0	0	0	0.5	1	1	1	1

10 intervals

1	1	1	1	1	1	0.5	0	0	0	0
2	1	1	1	1	1	1	0.5	0	0	0
3	1	1	1	1	1	1	1	0.5	0	0
4	1	1	1	1	1	1	1	1	0.5	0
5	1	1	1	1	1	1	1	1	1	0.5
6	0.5	1	1	1	1	1	1	1	1	1
7	0	0.5	1	1	1	1	1	1	1	1
8	0	0	0.5	1	1	1	1	1	1	1
9	0	0	0	0.5	1	1	1	1	1	1
10	0	0	0	0	0.5	1	1	1	1	1

11 intervals

1	1	1	1	1	1	0.5	0	0	0	0	0
2	1	1	1	1	1	1	0.5	0	0	0	0
3	1	1	1	1	1	1	1	0.5	0	0	0
4	1	1	1	1	1	1	1	1	0.5	0	0
5	1	1	1	1	1	1	1	1	1	0.5	0
6	0.5	1	1	1	1	1	1	1	1	1	0.5
7	0	0.5	1	1	1	1	1	1	1	1	1
8	0	0	0.5	1	1	1	1	1	1	1	1
9	0	0	0	0.5	1	1	1	1	1	1	1
10	0	0	0	0	0.5	1	1	1	1	1	1
11	0	0	0	0	0	0.5	1	1	1	1	1

12 intervals

1	1	1	1	1	1	1	0.5	0	0	0	0	0
2	1	1	1	1	1	1	1	0.5	0	0	0	0
3	1	1	1	1	1	1	1	1	0.5	0	0	0
4	1	1	1	1	1	1	1	1	1	0.5	0	0
5	1	1	1	1	1	1	1	1	1	1	0.5	0
6	1	1	1	1	1	1	1	1	1	1	1	0.5
7	0.5	1	1	1	1	1	1	1	1	1	1	1
8	0	0.5	1	1	1	1	1	1	1	1	1	1
9	0	0	0.5	1	1	1	1	1	1	1	1	1
10	0	0	0	0.5	1	1	1	1	1	1	1	1
11	0	0	0	0	0.5	1	1	1	1	1	1	1
12	0	0	0	0	0	0.5	1	1	1	1	1	1

13 intervals

1	1	1	1	1	1	1	1	0.5	0	0	0	0	0
2	1	1	1	1	1	1	1	1	0.5	0	0	0	0
3	1	1	1	1	1	1	1	1	1	0.5	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0.5	0	0
5	1	1	1	1	1	1	1	1	1	1	1	0.5	0
6	1	1	1	1	1	1	1	1	1	1	1	1	0.5
7	1	1	1	1	1	1	1	1	1	1	1	1	1
8	0.5	1	1	1	1	1	1	1	1	1	1	1	1
9	0	0.5	1	1	1	1	1	1	1	1	1	1	1
10	0	0	0.5	1	1	1	1	1	1	1	1	1	1
11	0	0	0	0.5	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0.5	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0.5	1	1	1	1	1	1	1

14 intervals

1	1	1	1	1	1	1	1	0.5	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	0.5	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	0.5	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0.5	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	0.5	0	0
6	1	1	1	1	1	1	1	1	1	1	1	1	0.5	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5
8	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1
9	0	0.5	1	1	1	1	1	1	1	1	1	1	1	1
10	0	0	0.5	1	1	1	1	1	1	1	1	1	1	1
11	0	0	0	0.5	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0.5	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0.5	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0.5	1	1	1	1	1	1	1

15 interva 閲 s

1	1	1	1	1	1	1	1	0.5	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	0.5	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	0.5	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0.5	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	0.5	0	0	0
6	1	1	1	1	1	1	1	1	1	1	1	1	0.5	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5	0
8	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5
9	0	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1
10	0	0	0.5	1	1	1	1	1	1	1	1	1	1	1	1
11	0	0	0	0.5	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0.5	1	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0.5	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0.5	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0.5	1	1	1	1	1	1	1

16 intervals

1	1	1	1	1	1	1	1	1	0.5	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	0.5	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0.5	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	0.5	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	1	0.5	0	0	0
6	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5	0
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5
9	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	0	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	0	0	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0.5	1	1	1	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0.5	1	1	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0.5	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0.5	1	1	1	1	1	1	1	1	1
16	0	0	0	0	0	0	0	0.5	1	1	1	1	1	1	1	1