

Hierarchical Clustering in Power System Based on Fuzzy Transitive Closure

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Abstract: - This paper presents the applications of hierarchical clustering to the generators in a power system. A useful application of fuzzy mathematics is that the correction of clustering results and determination of whether it can obtain correct transitive closure. Thus, the fuzzy transitive closure plays an important role in hierarchical clustering. Based on the fuzzy relation matrix, the hierarchical cluster analysis can be achieved by using firstly computing a transitive closure matrix on which serial α -cut operations are to be performed. A specific feature of the proposed method is that the hierarchical clustering work can be performed in parallel with the algorithm. The proposed method retains the correctness of transitive closure by reducing the computation complexity. Results from applying the method to a power system are demonstrated to show the validity and effectiveness of the proposed method.

Key-Words: - Hierarchical Clustering, Power System, Fuzzy Transitive Closure, Fuzzy Similarity Relation, α -cut Operation, Complexity.

1 Introduction

The concept of similarity can be described in terms of regions in the feature space of a database. The fuzzy clustering analysis finds the fuzzy relation and then constructs the fuzzy model. The data clustering belongs to the area of fuzzy partitions, where similarity is measured by using membership values. A fuzzy similarity relation should satisfy some form of transitivity. The membership functions of clustering are defined based on a similarity relation; where the identification of clusters is based on transitive closure [1-5]. The cluster analysis refers to the partition of a data group into subgroups where the data points within a subgroup are more similar to one another than to any of the points belonging to other subgroups [6-13].

The cluster analysis of which the task is to classify non-processed data into certain categories depending on the features considered, is a fundamental tool commonly used in several scientific fields. Data in each category have the most resemblance while being very dissimilar with data from other categories. The cluster analysis divides highly complex systems into some smaller and relatively simpler systems. The traditional clustering

method strictly divides each studied object into a certain data cluster—the hard clustering method. In reality, however, a certain object may be recognized to partially belong to multiple clusters. The utilization of fuzzy concepts in the data clustering analysis can naturally describe the real classification problems. Such a clustering method is referred to as the fuzzy cluster analysis. The fuzzy cluster analysis produces a soft partition for a given set of data and provides more flexibility in describing the inherent structure within those data under study. There have been successful applications of fuzzy clustering in various engineering disciplines [14-16].

The methods of fuzzy clustering are divided into two main categories, nonhierarchical clustering and hierarchical clustering. The fuzzy c-means is the most known method of nonhierarchical fuzzy clustering. The transitive closure influences a hierarchical clustering. It is equivalent to the method of the single link. Thus the fuzzy hierarchical clustering provides a new perspective for agglomerative clustering. By using a fuzzy clustering technique to establish a fuzzy rule-based modeling, most methods map membership functions into input variables. Consequently, too many membership

functions may increase computational complexity. The similarity measure of fuzzy sets is merged in order to obtain a generalized concept represented by a new fuzzy set. This reduces the number of models used in clustering. The result will reduce computational complexity and simplify the rules. We can generate fuzzy sets from the fuzzy relation projection defined by the fuzzy cluster in each of the input space. We can then use a linguistic approximation technique to obtain a quantitative fuzzy model of the system.

In power system studies, creating a dynamic model is the very first step in system stability research, dynamic behaviors analysis, or other system functional tests. As systems become larger, their complexity likewise increases and the power system analysis has to tackle high-order models. However, computation on the high-order model is highly complex while the final analysis results may have unnecessary portions, leading to high investments but with no apparent advantages. If a group of generators can be aggregated into a single equivalent generator that constitutes a reduced-order equivalent group [16-18], then it simplify computational complexity and minimize time and costs.

This paper aims to discuss a fuzzy classification in terms of membership degrees and without the relational composition of the fuzzy transitive matrix. The correctness of transitive closure and clustering is still retained by the proposed method but will reduce computational complexity. A specific feature of the approach is that clustering can be performed in parallel when the algorithm is applied. The proposed method is simple and practical to use in clustering generator groups of a power system.

2 Fuzzy Relation

2.1 Similarity Relation

The fuzzy relation is a means of both mapping fuzziness from one universe to another and developing fuzzy functions through the Cartesian product of the two universes. Hence, the fuzzy relation R is a mapping from the Cartesian space $X \times Y$ to the interval $[0, 1]$, where the strength of the mapping is expressed by the membership function of the relation for ordered pairs from the two universes, or $\mu_R(x, y)$.

Assume that $R = [r_{ij}]$ is a binary fuzzy relation where $0 \leq r_{ij} \leq 1$. The relation R is a similarity relation if it has the following three properties.

(1) Reflexivity, i.e.

$$r_{ii} = 1, \text{ for every } i \in U \quad (1)$$

(2) Symmetry, i.e.

$$r_{ij} = r_{ji}, \text{ for every } i, j \in U \quad (2)$$

(3) Transitivity, i.e.

$$r_{ij}^n = \left[\bigvee_{k=1}^n (r_{ik} \wedge r_{kj}) \right], \text{ for every } i, j, k \in U \quad (3)$$

$$\text{or } R^n = R \circ R \circ \dots \circ R \subseteq R \text{ (or } r_{ij}^n \leq r_{ij} \text{)}$$

then we call R the fuzzy equivalence relation matrix.

The compatibility relation includes reflexive and symmetric. To obtain compatibility and transitivity, also called similarity relation, various forms of the composition operation can be described by the max-min transitive closure $R^n = [r_{ij}^n]$.

2.2 Transitive Closure

The fuzzy transitive closure plays an important role in hierarchical clustering. The algorithm for calculating transitive closure is presented in several works.

Transitive closure is essential to detect hidden relationships between two arbitrary transactions. The composition can be determined from the corresponding join. Transitive closure finds the minimal path of transitivity. It is done as follows: Derive a dependency Z between transactions x_i and x_j from the given dependencies X between x_i and x_k , and Y between x_k and x_j , as shown in Figure 1. The similarity relation can be considered to effectively group elements into crisp sets whose members are similar to each other in a specific degree.

Actually, most membership matrices are reflexive and symmetric, i.e. compatible relations. Therefore it is necessary, when computing transitive closure, to obtain a similar relation and proceed to clustering analysis. The correctness of hierarchical clustering depends on the computation of the fuzzy transitive closure. A novel approach proposed in Section 3 will rapidly obtain the transitive closure.

Zadeh advert the hierarchical classification by using the transitive closure, which is the fuzzy equivalence relation [3]. Dunn has provided that the transitive closure is equivalent to the single link method [6]. Miyamoto has shown the equivalence among the transitive closure, the single link, and the connected components of a fuzzy graph, and the clusters generated from the maximum spanning tree of a network [7].

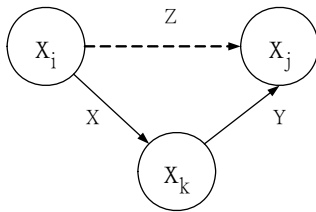


Fig. 1 Transitive relation

3 Cluster Analysis

The fuzzy clustering method is devised for defining linguistic labels on the input and output spaces. Clustering can be used to detect the possible groups that have similarity relations in their behaviors. It is also possible to make groups of related data to reduce computational complexity.

3.1 Classic Clustering

Clustering refers to identifying the number of subclasses of m clusters in a data universe X comprised of n data samples. For numerical data, one assumes that the members of each cluster bear more mathematical similarity to each other than to members of other clusters. Two important issues to consider in this regard are how to measure the similarity between pairs of observations and how to evaluate the partitions once they are formed.

This section summarizes two popular methods of clustering. The first is classification using similarity relations. This method makes use of certain special properties of similarity relations and the concept of defuzzification, known as an α -cut on the relations. The second clustering method is a very popular method known as fuzzy c-mean. This method uses concepts in n -dimensional Euclidean space to determine the geometric closeness of data points by assigning them to various clusters or classes and then determining the distance between the clusters. Clustering analysis can be divided into two categories:

(1) *Hierarchical*

Assume that n samples can be classified into m groups according to similar degree of some levels ($m \leq n$).

(2) *Partition*

Choose n initial prototypes from the input space and classify them as m groups using Euclidean distance. Every prototype belongs to a group ($m \leq n$).

By the use of fuzzy sets defined directly in the product space of the input variables, we can get the

clusters. The fuzzy clustering can be used as the main tool in obtaining rapid prototyping of a fuzzy model.

3.2 Proposed Method

The fuzzy relation can be used to analyze hierarchical clustering based on similar relation such as the clustering analysis method. The trait is marked with an α -value to show that a cluster belongs to another cluster in another level. The membership degrees of the feature vector in the clusters are firstly computed by using relational composition of the clustering algorithm. Later on, the output fuzzy set is obtained.

Here we present a new method to obtain the accurate hierarchical clustering by using fuzzy similarity based on α -cut. The proposed fuzzy clustering method is summarized as:

- (1) Define a fuzzy membership matrix R of the compatible relation.
- (2) Make an α -cut operation of R , according to similar properties of membership matrix R .
- (3) Find the defuzzified crisp similarity relations R' .

The result of the α -cut operation can be used for clustering while the transitive closure R^* can also be computed in parallel.

The hierarchical clustering based on fuzzy relation always finds similar relations—transitive closures and classifications. A specific feature of the proposed method is that the hierarchical clustering can be performed in parallel in the algorithm. The fuzzy hierarchical clustering analysis is shown in Figure 2.

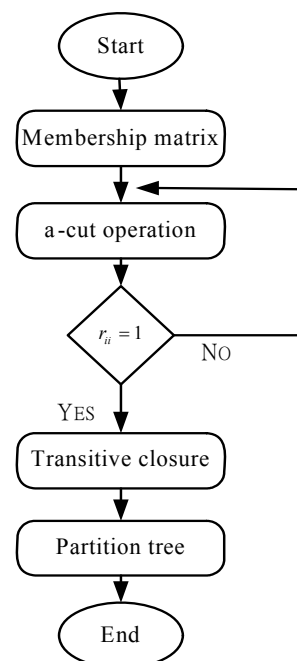


Fig. 2 Clustering procedure

The procedure for an α -cut algorithm is shown below:

Step 1 :

The binary fuzzy relation, R , is a mapping of $R : [0,1]^i \times [0,1]^j \rightarrow [0,1]$. It can be represented as an $i \times j$ matrix and is reflexive and symmetric.

$$R = [r_{ij}] = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1j} \\ r_{21} & r_{22} & \cdots & r_{2j} \\ \vdots & \vdots & \vdots & \vdots \\ r_{i1} & r_{i2} & \cdots & r_{ij} \end{bmatrix} \quad (4)$$

Step 2 :

α -cut operation

$$\alpha_k = \max_{2 \leq j \leq n} r_{kj} \quad (5)$$

We can get the level set of α -cut as:

$$\Lambda = \{\alpha_i, i = m, \dots, 1\}, \text{ where } \alpha_{i+1} > \alpha_i \quad (6)$$

$$R = [r_{ij}], r_{ij} \in [0,1], \alpha \in [0,1] \quad (7)$$

The crisp relation matrix R_α :

$$R_\alpha = [t_{ij}] \quad (8)$$

$$t_{ij} = \begin{cases} 1, & \text{if } r_{ij} \geq \alpha \\ 0, & \text{if } r_{ij} < \alpha \end{cases} \quad (9)$$

In this way it is possible to represent a fuzzy relation as the series of crisp relations comprising its α -cuts, each scaled by the value α .

Step 3 :

Find the set $J_{pq} = J_p \cup J_q$ such that the intersection operation is not a null of one column J_p and the others columns J_p , that is, the defuzzified crisp similarity relations R' . If the k th column is linearly dependent on the j th column, then there is same clustering. Otherwise, go to step 2.

Step 4 :

We can get the transitive closure R^* .

$$R^* = R_{\alpha 1} \vee R_{\alpha 2} \vee \cdots \vee R_{\alpha t} \quad (10)$$

where $1 \leq t \leq m$.

Step 5 :

Determine the number of clusters-the primary issue of this step. In hierarchical clustering methods, two fuzzy relations belonging to the same cluster at some level should remain together at all higher levels. It is needless to specify the number of clusters so that clustering proceeded autonomously. The complexity of the algorithm is determined from the number of

clusters. It can be reduced by the techniques described.

4 Complexity

In constructing a model, we always attempt to maximize its usefulness. This aim is closely connected with the relationship between three key characteristics of every system model: complexity, credibility, and uncertainty. This is important when a work is high complex and the transitive closure has to be calculated. How a model is constructed plays an important role in reducing complexity.

One of the main problems of applying fuzzy technique is the computational times and complexity. Engineers have to face this problem in complex system or especially in the field of information retrieval where the extremely large information maps of whole libraries or internet have to be processed at each user's request.

One of the important problems involved in the design of fuzzy clustering is their complexity. The complexity, which is defined as the size of the fuzzy transitive closures base of the fuzzy clustering, increases as the number of fuzzy relations increases; the number of fuzzy relations increases exponentially as the number of input variables increases. A value-based reduction technique is capable of reducing both the computational calculation and complexity.

A good algorithm retains the correctness of transitive closure by reducing the computational complexity. Thus, fuzzy transitive closure plays an important role in hierarchical clustering. The algorithm for calculating transitive closure is present in several works. Some methods in literatures are described as follows.

(1) Square algorithm [19]

This popular method is known as the square algorithm. It can be found through two compositions of R^2 that produce a transitive matrix R^* . It means that the max-min composition of R with itself results in a similarity relation. The theorem suggests calculating the transitive closure, $R^* = R^{(n-1)}$, by calculating the sequence of relations

$$R \rightarrow R^2 \rightarrow R^4 \rightarrow \cdots \rightarrow R^{2^k} = R^* \quad (11)$$

until no new relation is produced or $2^k \geq n-1$.

The max-min composition is defined by the set and membership function. The physical analogy behind transitivity is that the shorter the chain, the stronger the relation. In particular, the strength of the link between two elements must be greater than or

equal to the strength of any indirect chain involving other element.

The relation must satisfy the following properties:

The square operation may accelerate computing the transitive matrix by, at most, $(\log_2 n)+1$ compositions.

(2) Ascending value method [20]

Assume a fuzzy matrix R_{i-j-k} . If R_{i-j-k} possesses similarity relation and it must satisfy the following two properties:

Find the maximum r_l of all r , where $r_1 > r_2 > \dots > r_k$. The transitive closure will be formed by updating all r elements. R is thus symmetric. An advantage of this method is that the right upper matrix just has to be computed, reducing composition time.

(3) Left-upper n-order sub-matrix algorithm [21]

By looking for the next maximum of each left-upper matrix A_k , transitive closure is formed and its procedure is as follows.

- (a) Assume an $n \times n$ fuzzy relation matrix A_n . We get the first left-upper matrix A_2 . The transitive closure B_2 of A_2 equals to A_2 .
- (b) From the other $(n-2)$ columns of A_n , find the next maximum x_2^* , suppose $a_{13} = x_2^*$, then a_3 is replaced by b_3 and the transitive closure B_3 of A_3 formed.
- (c) Judging from this point, look for the maximum x_k^* of the other $(n-k)$ columns in the k -th row.
- (d) When $k=n-1$, we can obtain transitive closure B_n of the fuzzy relation matrix A_n .

(4) S-K-Q-Δ method [22]

In this method, any two elements must be less than or equal to the third element between the three elements r_{ij}, r_{ik}, r_{jk} of the upper triangle matrix. The matrix $R^{(s)}$ is replaced by $R^{*(s)}$. Afterwards, the transitive closure is obtained through the following three steps below.

- (a) Assume S-matrix $R^{(s)}$,
- (b) Take the fuzzy band,
- (c) The fuzzy values $r_{ij}^f, r_{ik}^f, r_{ij}^f$ of any fuzzy band $Q_{i-k}^{(f)}$ in relation matrix $R_{(k)}^{(s)}$ must satisfy the properties below.

Finally, transitive closure can be formed.

Table 1 Comparison of Complexity

Algorithm	Complexity	Computational times
Square algorithm	$n^3 \log n$	$n^3 \log(n-1)$
Ascending value method	n^3	$n^2(n-1)/2$
Left-upper n-order sub-matrix algorithm	n^3	$n^3/6 - n/6 - (n-1)$
S-K-Q-Δ method	n^3	$n(n-1)(n-2)/2$
Proposed method	n^2	$n(n-1)/2$

The result of our method compared with others is given on Table 1. Table 1 show the proposed method simplifies the computation time.

5 Example

5.1 Example 1: The New England System

In order to show the proposed fuzzy clustering method, the 10-machine New England power system is used here to demonstrate the effectiveness. The test system comprises 10 generators, 39 buses and 46 lines, as given in Figure 3. The detailed system data and initial operating conditions can be found in [18].

The swing curves of all generators in the external system are shown in Figure 4. Note that generator 10 is equivalent for representing a large system, and generators 6 and 7 are belonging to the study area; therefore, they will not belong in the external system.

In this example we will directly characterize the membership matrix R , which satisfies the compatibility relation. It will later be formed into a fuzzy similarity relation matrix R^* . This algorithm does not need composition of any fuzzy relation to complete the clustering.

$$R = \begin{bmatrix} 1.00 & 0.92 & 0.57 & 0.56 & 0.61 & 0.56 & 0.60 & 0.58 & 0.15 & 0.64 \\ 0.92 & 1.00 & 0.58 & 0.59 & 0.64 & 0.57 & 0.60 & 0.59 & 0.15 & 0.64 \\ 0.57 & 0.58 & 1.00 & 0.57 & 0.72 & 0.79 & 0.64 & 0.43 & 0.11 & 0.67 \\ 0.56 & 0.59 & 0.57 & 1.00 & 0.54 & 0.48 & 0.42 & 0.59 & 0.19 & 0.44 \\ 0.61 & 0.64 & 0.72 & 0.54 & 1.00 & 0.87 & 0.58 & 0.48 & 0.12 & 0.62 \\ 0.56 & 0.57 & 0.79 & 0.48 & 0.87 & 1.00 & 0.62 & 0.42 & 0.11 & 0.65 \\ 0.60 & 0.60 & 0.64 & 0.42 & 0.58 & 0.62 & 1.00 & 0.44 & 0.10 & 0.85 \\ 0.58 & 0.59 & 0.43 & 0.59 & 0.48 & 0.42 & 0.44 & 1.00 & 0.23 & 0.46 \\ 0.15 & 0.15 & 0.11 & 0.19 & 0.12 & 0.11 & 0.10 & 0.23 & 1.00 & 0.11 \\ 0.64 & 0.64 & 0.67 & 0.44 & 0.62 & 0.65 & 0.85 & 0.46 & 0.11 & 1.00 \end{bmatrix} \rightarrow$$

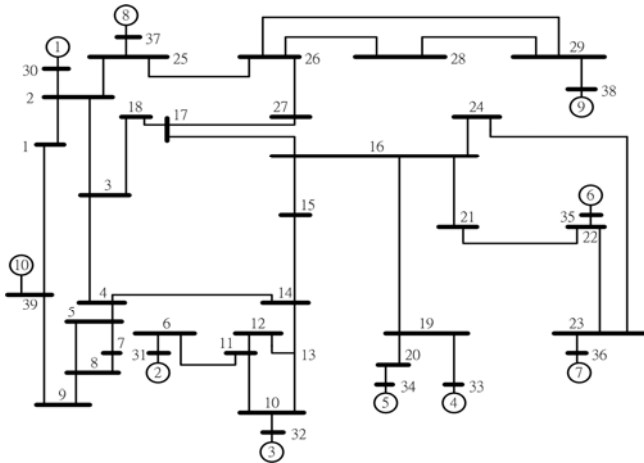


Fig. 3 One-line diagram of the New England system

$$R_{\alpha=0.92} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In this case one can see in the relation, $R_{\alpha=0.92}$, that there are four classes. Column 1 and 2 are identical while column 3, 4, 5, 6, 7, 8, 9 and 10 are each unique. The data points can be classified into four classes, as described below:

$\{x_1, x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}, \{x_9\}, \{x_{10}\}$

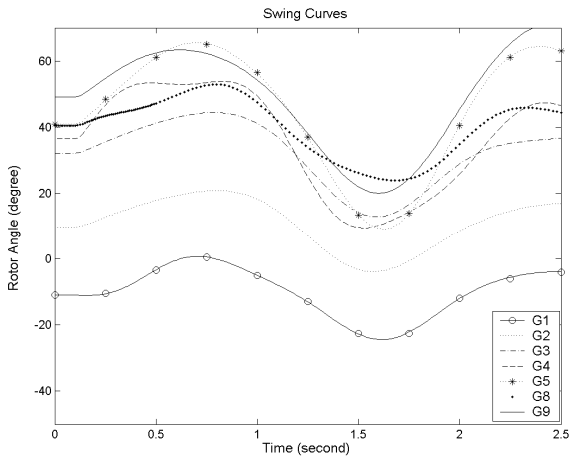


Fig. 4 Swing curves of all generators in the external system

$$R_{\alpha=0.87} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R^* = \begin{bmatrix} 1.00 & 0.92 & 0.64 & 0.59 & 0.64 & 0.64 & 0.64 & 0.59 & 0.23 & 0.64 \\ 0.92 & 1.00 & 0.64 & 0.59 & 0.64 & 0.64 & 0.64 & 0.59 & 0.23 & 0.64 \\ 0.64 & 0.64 & 1.00 & 0.59 & 0.79 & 0.79 & 0.67 & 0.59 & 0.23 & 0.67 \\ 0.59 & 0.59 & 0.59 & 1.00 & 0.59 & 0.59 & 0.59 & 0.59 & 0.23 & 0.59 \\ 0.64 & 0.64 & 0.79 & 0.59 & 1.00 & 0.87 & 0.67 & 0.59 & 0.23 & 0.67 \\ 0.64 & 0.64 & 0.79 & 0.59 & 0.87 & 1.00 & 0.67 & 0.59 & 0.23 & 0.67 \\ 0.64 & 0.64 & 0.67 & 0.59 & 0.67 & 0.64 & 1.00 & 0.59 & 0.23 & 0.85 \\ 0.59 & 0.59 & 0.59 & 0.59 & 0.59 & 0.59 & 0.59 & 1.00 & 0.23 & 0.23 \\ 0.23 & 0.23 & 0.23 & 0.23 & 0.23 & 0.23 & 0.23 & 0.23 & 1.00 & 0.23 \\ 0.64 & 0.64 & 0.67 & 0.59 & 0.67 & 0.67 & 0.85 & 0.59 & 0.23 & 1.00 \end{bmatrix}$$

$$\rightarrow R_{\alpha=0.85} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

By taking α -cuts of fuzzy relation R at $\alpha=1, 0.92, 0.87, 0.85, 0.79, 0.67, 0.64, 0.59$ and 0.23 , we get the following:

$$R_{\alpha=1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Choosing $\alpha= 0.87$ and 0.85 , the following defuzzified crisp similarity relations and their associated classes are derived: $\{x_1, x_2\}, \{x_5, x_6\}, \{x_3\}, \{x_4\}, \{x_7\}, \{x_8\}, \{x_9\}, \{x_{10}\}$ and $\{x_1, x_2\}, \{x_5, x_6\}, \{x_7, x_{10}\}, \{x_3\}, \{x_4\}, \{x_8\}, \{x_9\}$.

When we take an α -cut of this fuzzy similarity relation at $\alpha=0.79$, if the element of the similarity relation R is greater than α , it is set as 1, otherwise as 0. We then get the defuzzified relation matrix $R'_{\alpha=0.79}$. Then observing column 6, $\{x_5, x_6\}$ and $\{x_3\}$ are similar to each other to a degree of $\alpha=0.79$. Hence, the classes are formed by the refinement levels of a similarity relation $R'_{\alpha=0.79}$. This step can be interpreted as grouping elements that are similar to each other and only to each other to a degree not less

than α . Six distinct classes are identified: $\{x_1, x_2\}$, $\{x_3, x_5, x_6\}$, $\{x_7, x_{10}\}$, $\{x_4\}$, $\{x_8\}$, $\{x_9\}$. The results of clustering for different values of α are shown in Table 2.

Finally, we can express the partition tree of 10-generator as shown in Figure 5.

$$R_{\alpha=0.79} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow R'_{\alpha=0.79} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & I & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow R'_{\alpha=0.67} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & I & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & I & 0 & 0 & I \\ 0 & 0 & 1 & 0 & 1 & 1 & I & 0 & 0 & I \\ 0 & 0 & I & 0 & I & I & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & I & I & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow R'_{\alpha=0.64} = \begin{bmatrix} 1 & 1 & I & 0 & I & I & I & 0 & 0 & 1 \\ 1 & 1 & I & 0 & 1 & I & I & 0 & 0 & 1 \\ I & I & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ I & I & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ I & I & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow R'_{\alpha=0.59} = \begin{bmatrix} 1 & 1 & 1 & I & 1 & 1 & 1 & I & 0 & 1 \\ 1 & 1 & 1 & I & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & I & 1 & 1 & 1 & I & 0 & 1 \\ I & 1 & I & 1 & I & I & I & 1 & 0 & I \\ 1 & 1 & 1 & I & 1 & 1 & 1 & I & 0 & 1 \\ 1 & 1 & 1 & I & 1 & 1 & 1 & I & 0 & 1 \\ 1 & 1 & 1 & I & 1 & 1 & 1 & I & 0 & 1 \\ I & 1 & I & 1 & I & I & I & 1 & 0 & I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & I & 1 & 1 & 1 & I & 0 & 1 \end{bmatrix}$$

$$\rightarrow R'_{\alpha=0.23} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & I & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & I & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & I & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & I & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & I & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & I & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & I & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & I & 1 \\ I & I & I & I & I & I & I & 1 & 1 & I \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & I & 1 \end{bmatrix}$$

Table 2 Clustering Groups for Different α

α	Clustering groups
1	$\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}, \{x_9\}, \{x_{10}\}$
0.92	$\{x_1, x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}, \{x_7\}, \{x_8\}, \{x_9\}, \{x_{10}\}$
0.87	$\{x_1, x_2\}, \{x_5, x_6\}, \{x_3\}, \{x_4\}, \{x_7\}, \{x_8\}, \{x_9\}, \{x_{10}\}$
0.85	$\{x_1, x_2\}, \{x_5, x_6\}, \{x_7, x_{10}\}, \{x_3\}, \{x_4\}, \{x_8\}, \{x_9\}$
0.79	$\{x_1, x_2\}, \{x_3, x_5, x_6\}, \{x_7, x_{10}\}, \{x_4\}, \{x_8\}, \{x_9\}$
0.67	$\{x_1, x_2\}, \{x_3, x_5, x_6, x_7, x_{10}\}, \{x_4\}, \{x_8\}, \{x_9\}$
0.64	$\{x_1, x_2, x_3, x_5, x_6, x_7, x_{10}\}, \{x_4\}, \{x_8\}, \{x_9\}$
0.59	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_{10}\}, \{x_9\}$
0.23	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$

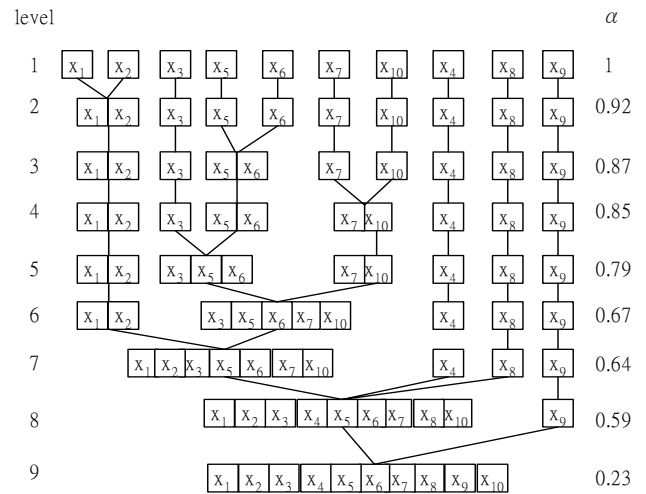


Fig. 5 Partition tree of 10-generator

5.2 Example 2: A 16-generator System

In this subsection, the proposed hierarchical clustering method is applied to a 16-generator and 68-bus test system as shown in Figure 6. The detailed system data and initial operating conditions can be found in [23]. Note that generator 15 is a large equivalent and thus not considered in clustering analysis because of its high inertia constant $H = 300$ sec. Swing curves of system generators are to be shown for the purpose of comparison.

The simulation process includes first applying a three-phase short circuit fault at bus 29 and then clearing the fault after three cycles by opening the line connecting bus 28 and bus 29. The study period is from 0 sec to 5 sec. Table 3 tabulates the computed normalized fuzzy similarity relation matrix.

The hierarchical clustering method previously discussed is then applied to process the fuzzy similarity relation matrix in Table 3. Since there are 15 generators considered in clustering study, the similarity relation matrix for this system will be of dimension 15×15 . After performing hierarchical clustering on the computed similarity relation matrix, the partition tree of 16-generator constructed is shown in Figure 7.

The partition tree in Figure 7 gives the detailed description of generator clustering at different levels of α -cut. As seen in Figure 7, the generator pairs (2, 3) and (6, 7) have comparatively high degree of similarity relation and units 4, 5, 6 and 7 as a whole can also serve as a similarity group. However, generators 12 and 13 are similarity to a less extent. For example, in level 8, five similarity groups can be obtained:

- Group 1: (2, 3)
- Group 2: (4, 5, 6, 7)
- Group 3: (10, 11)
- Group 4: (12, 13)
- Group 5: (14, 16)

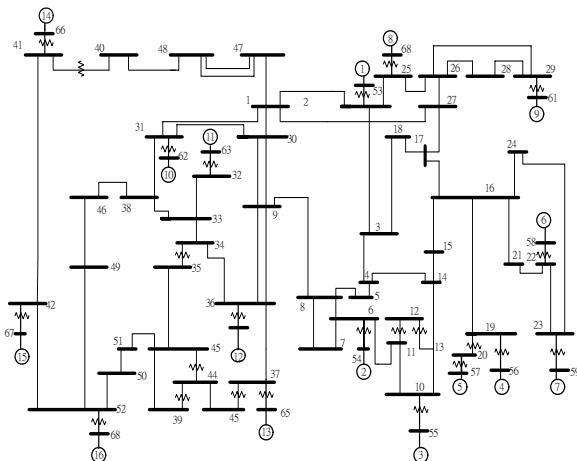


Fig. 6 One-line diagram of 16-generator system

This observation is consistent with those made in [23].

For the purpose of comparison, the swing curves of system generators following the disturbance are shown in Figure 8, which depicts the swing curves of all generators under study, and Figure 9, which are resolved from Figure 8 with a view to revealing fuzzy similarity relation generators more clearly. From Figure 7 and Figure 9 it is found that the similarity relation groups obtained by examining the swing curves are the same as those identified by using the proposed transitive closure and hierarchical clustering method. Moreover, the partition tree in Figure 7 reveals more detailed information as to the relationship among the identified c fuzzy similarity relation by its specific feature of creating generator clusters at different levels of α -cut. For example, the group (2, 3) and (4, 6, 7) are recognized to have stronger association with respect to the group (10, 11).

The results presented above reveal that the main advantage of the proposed data analysis technique lies in its capability of creating objective clustering of generators at different levels of α -cut. An appropriate number of fuzzy similarity relation groups can be determined according to the need for reduction in model complexity when this method is to be utilized for transitive closure based dynamic equivalence.

Table 3 Fuzzy Similarity Relation Matrix

$i \backslash j$	1	2	3	4	5	6	7	8
1	1.0000	0.8484	0.8420	0.8417	0.8403	0.8490	0.8490	0.7619
2	0.8484	1.0000	0.9770	0.8892	0.8574	0.8498	0.8410	0.7606
3	0.8420	0.9770	1.0000	0.8890	0.8622	0.8588	0.8425	0.7932
4	0.8417	0.8924	0.8890	1.0000	0.9609	0.9746	0.9765	0.8268
5	0.8403	0.8574	0.8622	0.9609	1.0000	0.9628	0.9612	0.7880
6	0.8490	0.8498	0.8588	0.9746	0.9628	1.0000	0.9891	0.8016
7	0.8490	0.8410	0.8425	0.9765	0.9612	0.9891	1.0000	0.7978
8	0.7619	0.7606	0.7932	0.8268	0.7880	0.8016	0.7978	1.0000
9	0.5623	0.7465	0.5360	0.4982	0.5726	0.6049	0.8003	0.7619
10	0.4367	0.7618	0.6258	0.1952	0.7788	0.7854	0.4055	0.5614
11	0.4056	0.1773	0.4981	0.6357	0.6269	0.6542	0.2710	0.7606
12	0.5625	0.4549	0.3371	0.6538	0.5893	0.2994	0.4652	0.4367
13	0.7453	0.6381	0.7492	0.4437	0.3362	0.6345	0.5862	0.5624
14	0.8670	0.7642	0.5861	0.8391	0.7748	0.2995	0.6619	0.7616
16	0.9263	0.9380	0.7739	0.3721	0.5526	0.5741	0.7492	0.2664

$i \backslash j$	9	10	11	12	13	14	16
1	0.5623	0.4367	0.4056	0.5625	0.7453	0.8670	0.9263
2	0.7465	0.7618	0.1773	0.4549	0.6381	0.7642	0.9380
3	0.5360	0.6258	0.4981	0.3371	0.7492	0.5861	0.7739
4	0.4982	0.1952	0.6357	0.6538	0.4437	0.8391	0.3721
5	0.5726	0.7788	0.6269	0.5893	0.3362	0.7748	0.5526
6	0.6049	0.7854	0.6542	0.2994	0.6345	0.2995	0.5741
7	0.8003	0.4055	0.2710	0.4652	0.5862	0.6619	0.7492
8	0.7619	0.5614	0.7606	0.4367	0.5624	0.7616	0.2664
9	1.0000	0.2936	0.5622	0.4367	0.6755	0.8342	0.5644
10	0.2936	1.0000	0.9590	0.6653	0.1994	0.3555	0.1996
11	0.5622	0.9590	1.0000	0.8390	0.8328	0.6532	0.5626
12	0.4367	0.6653	0.8390	1.0000	0.9547	0.5614	0.6049
13	0.6755	0.1994	0.8328	0.9547	1.0000	0.4055	0.5938
14	0.8342	0.3555	0.6532	0.5614	0.4055	1.0000	0.9380
16	0.5644	0.1996	0.5626	0.6049	0.5938	0.9380	1.0000

6 Conclusion

A useful application of fuzzy mathematics is the hierarchical clustering whose correctness depends on the computation of fuzzy transitive closure. It does not have to specify the number of clusters so that clustering can proceed autonomously. Our method can easily retrieve the membership functions for input variables and can generate a small number of fuzzy subsets for each fuzzy variable. The α -cut algorithm proposed here is essentially a method of selecting the smallest clustering, which accurately represents the system. The proposed method is simple and practical to use in clustering generator groups of a power system. Its validity has been verified through computer simulations. Moreover, this new algorithm describes a given system by using fuzzy rules quite well and is easy to implement in a computer. In addition, the transitive closure computation complexity is reduced while the hierarchical clustering can be performed in parallel.

References:

- [1] S. Janssens, B. Baets, and H. Meyer, Transitivity of comparison measures, *Proceedings of IEEE International Conference on Fuzzy Systems*, 2002, pp. 1369-1374.
- [2] D. Baets and H. Meyer, T-transitive closures, openings and approximations of similarity relations, *Proceedings of IEEE International Conference on Fuzzy Systems*, 2002, pp. 1375-1380.
- [3] L. Zadeh, Similarity relations and fuzzy orderings, *Information Sciences*, Vol. 3, 1971, pp. 177-200.
- [4] M. Wallace and S. Kollias, Computationally efficient incremental transitive closure of sparse fuzzy binary relations, *Proceedings of IEEE International Conference on Fuzzy Systems*, 2004.
- [5] S. Tamura, S. Higuchi, and K. Tanaka, Pattern classification based on fuzzy relations, *IEEE Trans. on System, Man, and Cybernetics*, Vol. SMC-1, 1971, pp. 61-66.
- [6] J. Dunn, Well-separated clusters and optimal fuzzy partitions, *Journal of Cybernetics*, vol. 4, 1974, pp. 95-104.
- [7] S. Miyamoto, An overview and new methods in fuzzy clustering, *Proceedings of KES International Conference on Knowledge-Based Intelligent Electronic Systems*, 1998, pp. 33-40.
- [8] R. Hathaway, J. Bezdek, and Y. Hu, Generalized fuzzy c-means clustering strategies using L_p norm distances, *IEEE Trans. on Fuzzy Systems*, Vol. 8, No. 5, 2000, pp. 576-582.
- [9] N. Pal and J. Bezdek, On cluster validity for the fuzzy c-means model, *IEEE Trans. on Fuzzy Systems*, Vol. 3, No. 3, 1995, pp. 370-379.
- [10] J. Bezdek, D. Dubois, and H. Prade, *Fuzzy sets in approximate reasoning and information systems*, Kluwer Academic, Boston, 1999.
- [11] M. Setnes and R. Babuska, Fuzzy relational classifier trained by fuzzy clustering, *IEEE Trans. on Systems, Man and Cybernetics*, Vol. 29, No. 5, 1999, pp. 619-625.
- [12] X. Chang and W. Li, A c-means clustering based fuzzy modeling method, *Proceedings of IEEE International Conference on Fuzzy Systems*, 2000, pp. 937-940.
- [13] J. Engelfriet, and J. Treur, Multi-interpretation operators and approximate classification, *International Journal Approximate Reasoning*, Vol. 32, 2003, pp. 43-61.
- [14] A. Nuhanovic, M. Glavic, and N. Prljaca, Validation of a clustering algorithm for voltage stability analysis on the Bosnian electric power system, *IEE Proceedings-Generation, Transmission and Distribution*, Vol. 145, No. 1, 1998, pp. 21-26.
- [15] S. K. Joo, C. C. Liu, L. E. Jones, and J. W. Choe, Coherency and aggregation techniques

- incorporating rotor and voltage dynamics, *IEEE Trans. on Power Systems*, Vol. 19, No. 2, 2004, pp. 1068-1075.
- [16] M. H. Haque, Identification of coherent generators for power system dynamic equivalents using unstable equilibrium point, *IEE Proceedings*, Vol. 138, Pt. C, 1991, pp. 546-552.
- [17] E. Pires de Souza and A. Leiti da Silva, An efficient methodology for coherency-based dynamic equivalents, *IEE Proceedings*, Vol. 139, Pt. C, 1992, pp. 371-382.
- [18] S. Zhao, X. Chang, Y. Pan and R. He, A reduced order method for swing mode eigenvalue calculating based on fuzzy coherency recognition, *Proceedings of the IEEE Conference on Power System Technology, 1998*, pp. 1402-1405.
- [19] M. Mishref and E. Emam, Transitivity and sub-inverses in fuzzy matrices, *Fuzzy Sets and Systems*, Vol. 52, 1992, pp. 337-343.
- [20] S. Li, The simplest method of ascending value to find fuzzy transitive closure, *Fuzzy Sets and Systems*, Vol. 38, 1990, pp. 91-96.
- [21] G. Fu, An algorithm for computing the transitive closure of a fuzzy similarity matrix, *Fuzzy Sets and Systems*, Vol. 51, 1992, pp. 189-194.
- [22] K. Shi and D. Ng, Composition of fuzzy transitive closure by S-K-Q- Δ method, *The Journal of Fuzzy Mathematics*, Vol. 1, No. 1, 1993, pp. 25-32.
- [23] G. Rogers, *Power System Oscillations*, Kluwer Academic, 2000.
- [24] S. C. Wang and P. H. Huang, Fuzzy c-means clustering applied to direct power system coherency identification, *WSEAS Trans. on Power Systems*, Vol. 1, No. 9, 2006, pp. 1634-1640.
- [25] Y. Hu, Y. Cai, and S. Rodier, Satellite Data Analysis with FPGA Reconfigurable Computation, *WSEAS Trans. on Systems*, Vol. 3, No. 5, 2004, pp. 2142-2147.
- [26] S. C. Wang, C. J. Wu, and S. W. Yang, Applying FPGA-based Chip to Apparent Power and Power Factor Measurement Considering Nonsinusoidal and Unbalanced Conditions, *WSEAS Trans. on Circuits and Systems*, Vol. 8, No. 7, 2009, pp. 559-568.
- [27] S. C. Wang and P. H. Huang, Description of Wind Turbine Power Curve via Fuzzy Modeling, *WSEAS Trans. on Power Systems*, Vol. 1, No. 5, 2006, pp. 786-792.
- [28] S. C. Wang and C. J. Wu, Using Fuzzy Modeling to Describe Power System QV Curve, *Proceedings of the 9th WSEAS International Conference on Instrumentation, Measurement, Circuits and Systems (IMCAS '10)*, 2010, pp. 137-140.
- [29] S. C. Wang and C. J. Wu, Analysis of Extensive Power Factor by FPGA under Power Quality Disturbance, *Proceedings of the 9th WSEAS International Conference on Instrumentation, Measurement, Circuits and Systems (IMCAS '10)*, 2010, pp. 148-152.
- [30] Y. H. Chan, C. J. Wu, and S. C. Wang, Power Quality Assessment of Specially Connected Transformers, *Proceedings of the 9th WSEAS International Conference on Instrumentation, Measurement, Circuits and Systems (IMCAS '10)*, 2010, pp. 157-161.
- [31] C. W. Tao and J. S. Taur, Flexible Complexity Reduced PID-Like Fuzzy Controllers, *IEEE Trans. on Systems, Man and Cybernetics*, Vol. 30, Vol. 4, 2000, pp. 510-516.

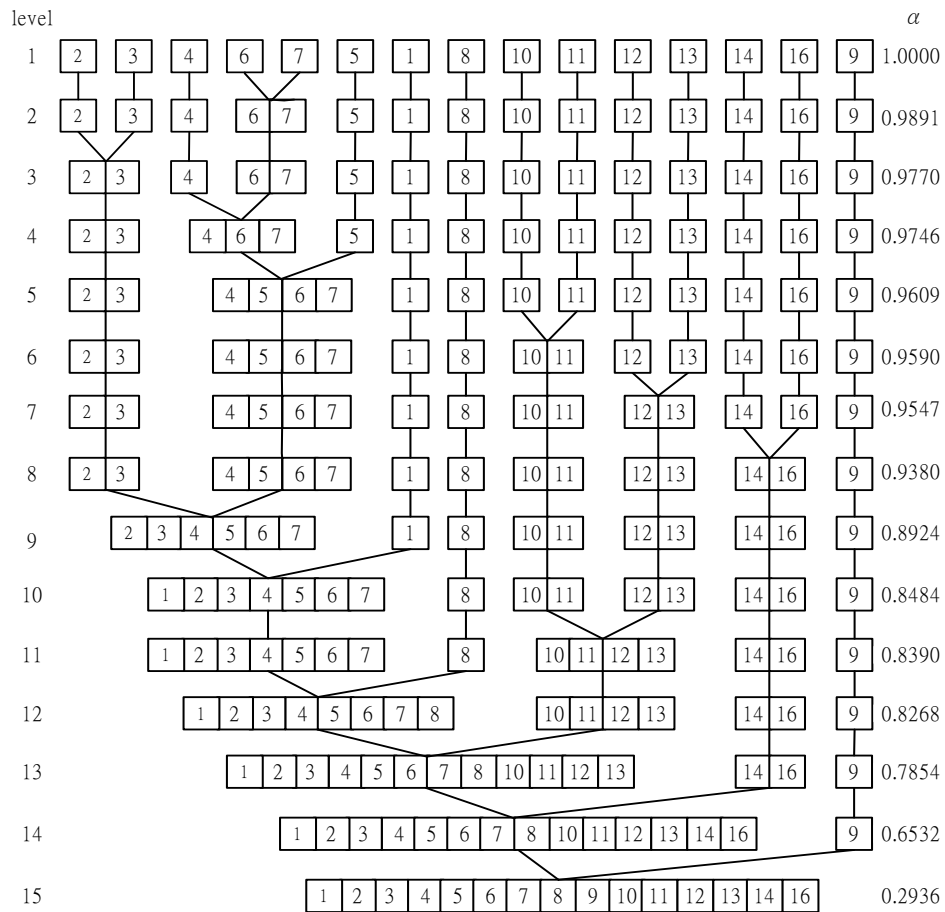


Fig. 7 Partition tree of 16-generator

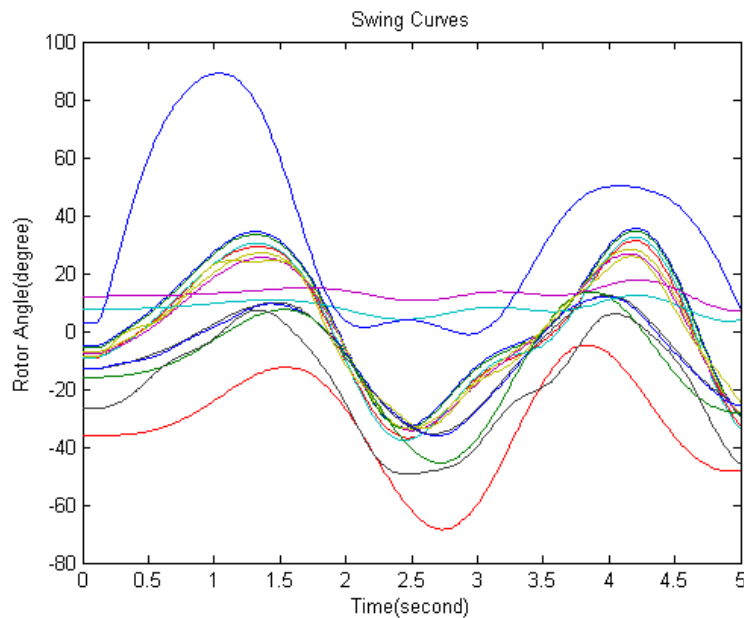
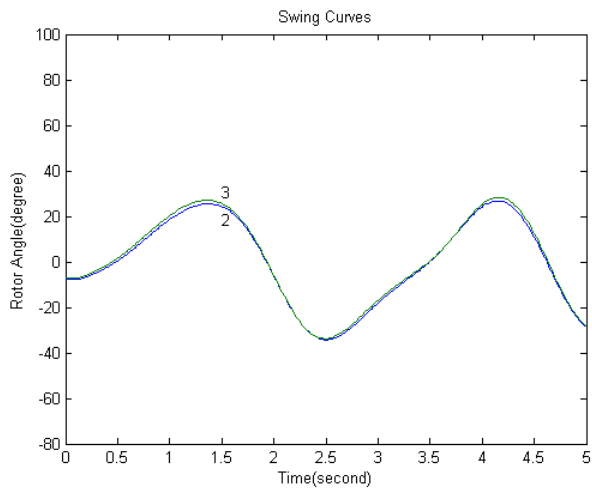
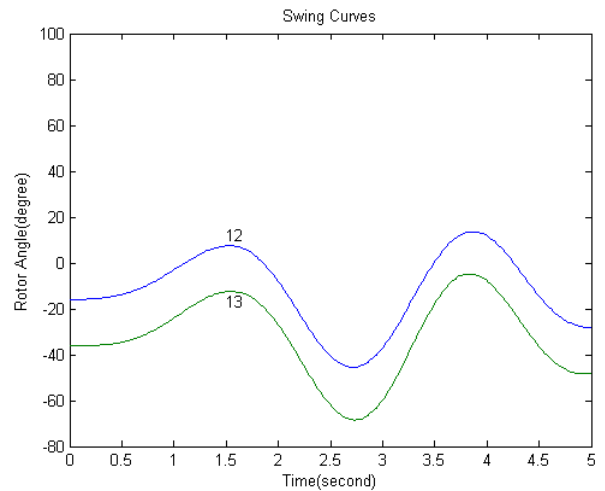


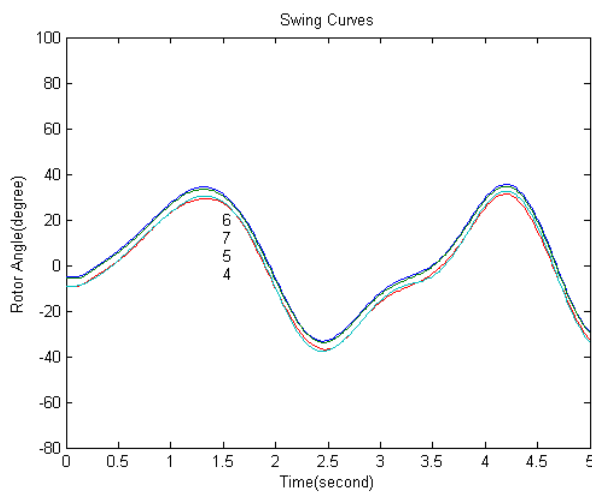
Fig. 8 Swing curves of all generators under study (The generators, in descending order of the rotor angle values at the time of zero, are: 16, 14, 9, 6, 7, 5, 4, 3, 2, 1, 10, 11, 12, 8, and 13.)



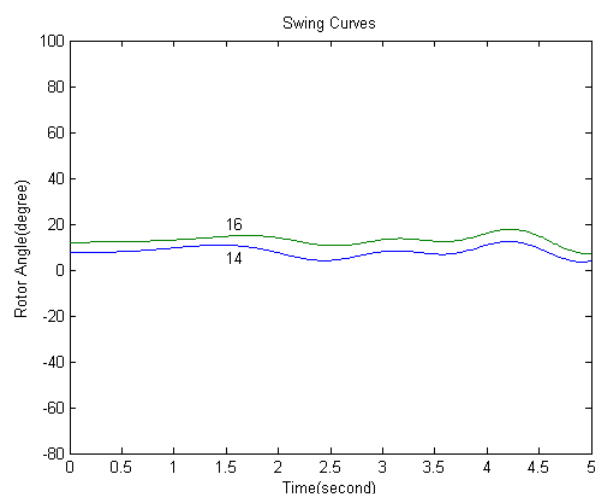
(a)



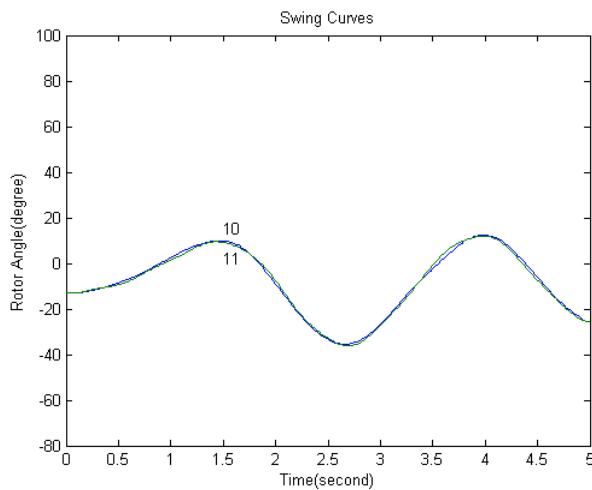
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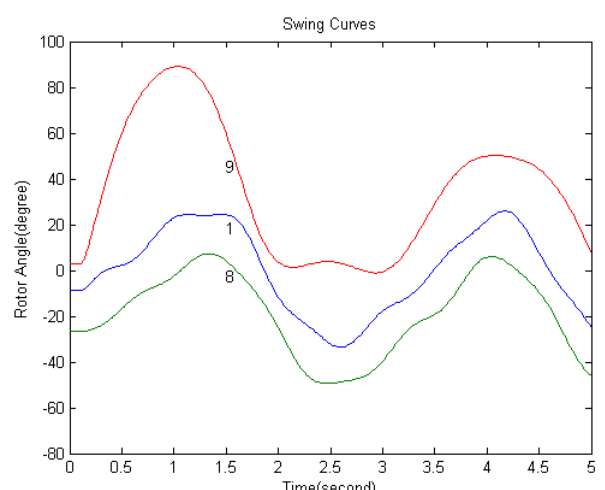
(b)



(e)



(c)



(f)

Fig. 9 Swing curves of (a) generators (2, 3) (b) generators (4, 5, 6, 7) (c) generators (10, 11) (d) generators (12, 13) (e) generators (14, 16) (f) generators (1, 8, 9)