Advanced Modelling of Complex Processes by Fuzzy Networks

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Abstract: - This work presents an application of the novel theory of rule based networks for building models of processes characterised by uncertainty, non-linearity, modular structure and internal interactions. The application of the theory is demonstrated for a flotation process in the context of converting a multiple rule based system into an equivalent single rule based system by linguistic composition of the individual rule bases. During the conversion process, the transparency of the multiple rule based system is fully preserved while its accuracy is improved to a level comparable with the accuracy of the single rule based system.

Key-Words: - Hierarchical model, network model, data simulation, fuzzy logic, fuzzy systems, process model, input/output models, systems evaluation, knowledge base.

1 INTRODUCTION

A complex process is usually defined as a process composed of interconnected subprocesses that when taken together as a single entity exhibit some properties that are not to be seen otherwise. In particular, a complex process is often described by a number of features such as uncertainty, non-linearity, modular structure and internal interactions [1, 2]. These features undoubtedly present a serious challenge to the modelling of such a process.

In this context, fuzzy logic has already proved itself as a powerful tool for dealing with non-probabilistic uncertainty [9]. The most common cause for this type of uncertainty can be data that is in some way incomplete or ambiguous.

At the same time, the implementation of fuzzy logic by means of fuzzy systems helps with the tackling of non-linearity [3]. In this sense, the rule base of a fuzzy system is usually capable of representing quite well strongly non-linear functions in complex processes which usually can't be dealt with by other types of mathematical models.

However, in spite of the relative success of fuzzy systems in capturing uncertainty and non-linearity, there are other features of complexity, which can't be taken into account. For example, the interactions among subsystems and the high dimensionality in terms of large number of inputs may lead to the deterioration of fuzzy models. This deterioration can be attributed to the 'grey box' nature of fuzzy systems, which consider only the inputs to and the outputs from a complex process but not any interactions among separate subprocesses [6].

In contrast, a fuzzy network is a rule based network in the form of a 'white box' model that takes into some account the interactions among subprocesses [5]. This capability could bring considerable advantages in modelling complex processes in that all subprocesses and the associated interactions can be reflected in the model. In this context, the work presented here demonstrates the application of fuzzy networks for improving the model accuracy for a flotation process while preserving the model transparency.

2 BACKGROUND

2.1 Fuzzy Rule Based Systems

The most common type of fuzzy system consists of a single rule base, whereby the associated fuzzy model is described as a 'grey box' [3, 9]. A single rule based system (SRBS) deals with all process inputs simultaneously while not taking into account the interactions and the structure of the system. Such a system is shown in Figure 1, where RB is the rule base, $\{x_1,\ldots, x_m\}$ is the set of inputs and y is the output. In this case, the rules are derived from expert knowledge or data measurements about the whole process. The resulting SRBS model is usually quite accurate but its poor transparency may be an obstacle to the understanding of complex processes.

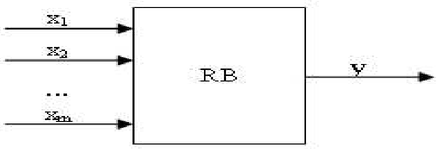

Fig.1 Single rule based system

Another quite common type of fuzzy system consists of multiple rule bases, whereby the associated fuzzy model is described as a 'white box' [11,12]. A multiple rule based system (MRBS) deals with process inputs sequentially while taking into account the interactions and the structure of the system. Such a system is shown in Figure 2, where $\{RB_1,\ldots, RB_{m-1}\}$ is the set of rule bases, $\{x_1,\ldots, x_m\}$ is the set of inputs, $\{z_1,\ldots, z_{m-2}\}$ is the set of interactions and y is the output. In this case, the rules are derived from expert knowledge or data measurements about the interacting subprocesses. In particular, a MRBS can be derived by functional decomposition of a SRBS such that all individual rule bases of the MRBS are subject to the stages of fuzzification, inference and defuzzification. The resulting MRBS model is transparent but its low accuracy may be a problem for the management of complex processes.

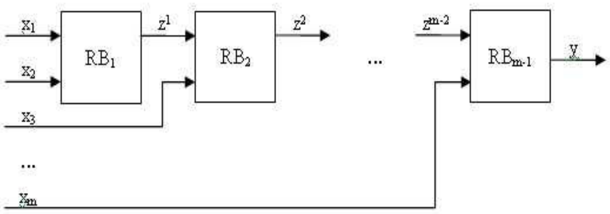

Fig.2 Multiple rule based system

A fairly novel type of fuzzy system consists of networked rule bases called fuzzy networks, whereby the associated fuzzy model is also described as a 'white box' [5]. A networked rule base system (NRBS) deals with process inputs sequentially while taking into account the interactions and the structure of the system. Such a system is shown in Figure 3, where $\{RB_1,\ldots, RB_{m-1}\}$ is the set of rule bases, $\{x_1,\ldots, x_m\}$ is the set of inputs, $\{z_1,\ldots, z_{m-2}\}$ is the set of interactions and y is the output. The identical mappings from the MRBS are represented by the sets of identity rule bases $\{I_{21}\},\ldots, \{I_{m-1,1}, I_{m-1,2}, \ldots\}$. In this case, the rules are also derived from expert knowledge or data measurements about the interacting subprocesses. In particular, a SRBS can be derived by linguistic composition of a NRBS such that the stages of fuzzification, inference and defuzzification are applied only once to the linguistically equivalent rule base of the derived SRBS. The resulting NRBS model is transparent and fairly accurate at the same time due to its hybrid nature, which facilitates the understanding and the management of complex processes.

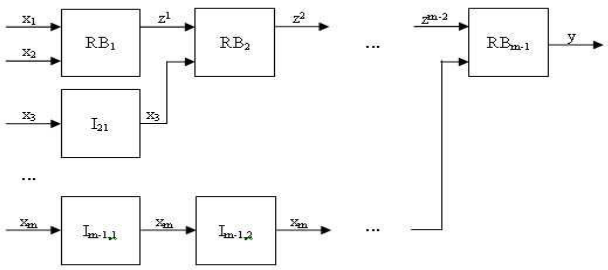

Fig.3 Networked rule based system

Overall, NRBSs represent a novel extension to SRBSs and MRBSs. As such, NRBSs have already been applied successfully for modelling the complex process of product pricing in the retail industry [4,8]. In particular, NRBSs provide a bridge between SRBSs and MRBSs that not only facilitates their use but may also improve some of their performance indicators such as accuracy and transparency. Also, linguistic composition of a NRBS into a SRBS is always based on physical considerations of complex processes. As opposed to this, functional decomposition of a SRBS into a MRBS is usually based on mathematical considerations of such processes.

2.2 Fuzzy Rule Based Networks

The novel theory of fuzzy networks introduces several formal presentation techniques for fuzzy systems such as Boolean matrices and binary relations [3]. It also presents techniques for formal manipulation of fuzzy rule bases based on several basic operations. Some of them are to be found in mathematics and are therefore well known, whereas others are quite novel in terms of the underlying theory and have been introduced only recently. The operations use Boolean matrices or binary relations for the presentation of the individual rule bases in order to facilitate the manipulation in the context of the linguistic composition approach.

2.2.1 Formal Presentation of Rule Bases

A typical single-input-single-output fuzzy rule base can be described by the three rules in Equations (1)-(3)

Rule1: If x is small, then y is low (1)

Rule 2: If x is medium, then y is high (2)

Rule 3: If x is big, then y is average (3)

where the input x takes the set of linguistic terms {small, medium, big} and the output y takes the set of linguistic terms {low, average, high}.
This rule base can be also described in a compressed form by the integer table shown in Table 1.

Table 1. Integer table for a rule base

Rule number	Input x	Output y
1	1 (small)	1 (low)
2	2 (medium)	3 (high)
3	3 (big)	2 (average)

where each linguistic term for the input and the output is presented as a positive integer in an increasing order from ‘small’ to ‘big’ and ‘low’ to ‘high’, respectively.

The rule base above can be formally presented by the Boolean matrix in Equation (4)

$$\begin{array}{c|ccc} x \backslash y & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 \end{array} \quad (4)$$

where the integer numbers for the linguistic terms of the input x are row labels, the integer numbers for the linguistic terms of the output y are column labels, a ‘1’ in the Boolean matrix corresponds to an existing rule and a ‘0’ corresponds to a missing rule.

2.2.2 Horizontal Merging of Rule Bases

Horizontal merging is a binary operation that can be applied to a pair of sequential rule bases, i.e. rule bases residing in different layers within the same level of a fuzzy network. This operation merges the operand rule bases into a single product rule base, as shown in Figure 4.

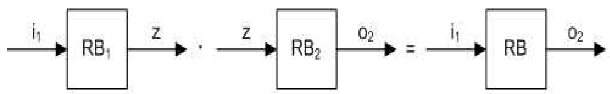

Fig.4 Horizontal merging of rule bases

When Boolean matrices are used as formal models for representing the rule bases of the operand rule bases, the horizontal merging operation is identical with Boolean matrix multiplication, as shown by Equations (5)-(7). The latter is similar to convex matrix multiplication, whereby each arithmetic multiplication is replaced by a ‘minimum’ operation and each arithmetic addition is replaced by a ‘maximum’ operation. Therefore, this operation can be applied only when all the outputs from the first rule base are fed forward as inputs to the second rule base in the form of an intermediate variable. In this case, the product rule base has the same inputs as the inputs to the first operand rule base and the same outputs as the outputs from the second operand rule base, whereas the intermediate variable does not appear in the product rule base.

$$RB_1: \quad \begin{array}{c|ccc} i_1 \backslash z & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 \end{array} \quad (5)$$

RB_2 : (6)

o_2	1	2	3
z			
1	0	1	0
2	0	0	1
3	1	0	0

RB : (7)

o_2	1	2	3
i_1			
1	0	1	0
2	1	0	0
3	0	0	1

2.2.3 Vertical Merging of Rule Bases

Vertical merging is a binary operation that can be applied to a pair of parallel rule bases, i.e. rule bases located in the same layer of a fuzzy network. This operation merges the operand rule bases from the pair into a single product rule base, as shown in Figure 5.

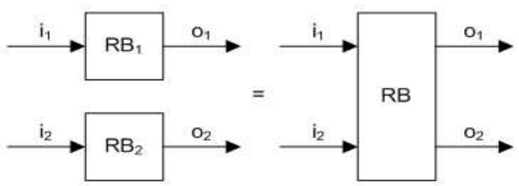

Fig.5 Vertical merging of rule bases

When Boolean matrixes are used as formal models for representing the rule bases of the operand rule bases, the vertical merging operation is like an expansion of the first operand matrix along its rows and columns, as shown by Equations (8)-(10). In particular, the product matrix is obtained by expanding each non-zero element from the first operand matrix to a block that is the same as the second operand matrix and by expanding each zero element from the first operand matrix to a zero block of the same dimension as the second operand matrix. In this case, the inputs to the product rule base represent the union of the inputs to the operand rule bases, whereas the outputs from the product rule base represent the union of the outputs from the operand rule bases. This operation can always be applied due to the ability to concatenate the inputs and the outputs of any two parallel rule bases.

RB_1 : (8)

o_1	1	2	3
i_1			
1	1	0	0
2	0	0	1
3	0	1	0

RB2 : (9)

o_2	1	2	3
i_2			
1	0	1	0
2	1	0	0
3	0	0	1

RB : (10)

o_1, o_2	11	12	13	21	22	23	31	32	33
i_1, i_2									
11	0	1	0	0	0	0	0	0	0
12	1	0	0	0	0	0	0	0	0
13	0	0	1	0	0	0	0	0	0
21	0	0	0	0	0	0	0	1	0
22	0	0	0	0	0	0	1	0	0
23	0	0	0	0	0	0	0	0	1
31	0	0	0	0	1	0	0	0	0
32	0	0	0	1	0	0	0	0	0
33	0	0	0	0	0	1	0	0	0

3 ISSUES AND PROBLEMS

3.1 Flotation Process Description

The flotation process from the mining industry is a typical complex process that deals with the enrichment of raw ore [10]. It usually consists of a certain number of interacting subprocesses, which

are characterised by non-probabilistic uncertainty. The subprocesses represent different parts of the flotation process, whereby the associated uncertainty is related to the incomplete and ambiguous information available about the factors affecting the ore enrichment at each stage. The paragraphs below describe the modelling of the first stage of a flotation process using the three types of fuzzy systems from Section 2.
The purpose of flotation is the improvement of the characteristics of multi-component poly-metal ores. In particular, flotation is implemented by processing a mixture of finely ground ore, water and reagents called pulp, through a sequence of two stages, as shown in Figure 6.

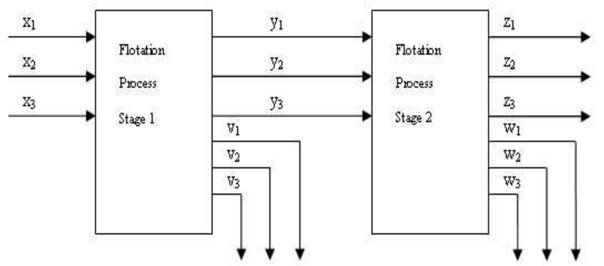

Fig. 6 General block scheme of the flotation process

At the first stage of the flotation process, there are only three measurable inputs: x_1 – the concentration of copper in the pulp given in [%] , x_2 – the concentration of iron in the pulp given in [%], x_3 – the pulp debit given in [l/min]. However, there are a number of unknown input factors characterising the pulp such as acidity, density and temperature, which can't be quantified in a mathematical model. The same measurable inputs appear also as outputs y_1, y_2, y_3 from the first stage of the process and as outputs z_1, z_2, z_3 from the second stage of the process. At the output of the first and the second stage, the concentration of iron and copper in the pulp is usually increased, whereas the pulp debit may either increase or decrease depending on the quantity of the water and reagents added to and removed from the pulp. The variables v_1, v_2, v_3 and w_1, w_2, w_3 at the output of the two stages of the flotation process represent waste quantities of the corresponding outputs, which are removed from the pulp for further treatment or disposal. For the purpose of initial prototyping, only the concentration of copper in the pulp is considered as an output of the first stage for the three types of fuzzy system models from Section 2.

3.2 Flotation Process Modelling

The SRBS model is not capable of capturing the influence of the inputs on the outputs of the flotation process. Moreover, this model is not capable of modelling the interactions between the inputs and the outputs. As opposed to this, the MRBS and the NRBS models are usually better in capturing the influence of the inputs on the outputs as well as in modelling the interactions between them. This is why MRBS and NRMS models are the main focus of this work, whereas the SRBS model is used only for comparison purposes.
The first model represents a MRBS. It has very good transparency and can deal with process inputs sequentially. However, this model is expected to be worst in terms of accuracy because of the fuzzification – defuzzification error accumulated for every individual rule base [9].
The second model is obtained from the first model and represents a NRBS. It also has very good transparency and can deal with process inputs sequentially. This model is expected to have better accuracy than the first model because of the single fuzzification - defuzzification sequence applied to its rule base [3].
The third model represents a SRBS. It is expected to be the best in terms of accuracy [9]. However, this model is not transparent and can deal with process inputs only simultaneously. This model is considered here for comparison purposes.
All the models are based on the data from the flotation process studied in [10] and part of the data used is shown in Table 2. The data includes about 75 measurements for the three inputs and the output introduced at the start of the current section.

Table2. Partial data sets for the flotation process

	Initial Copper [%]	Initial Iron [%]	Debit [l/min]	Enriched Copper [%]
Data Point 1	0.407	2.927	16.14	3.737
Data Point 2	0.406	2.891	16.02	3.963
Data Point 3	0.391	2.890	18.28	4.827
…	…	…	…	…
Data Point 74	0.384	2.932	18.59	4.276
Data Point 75	0.383	2.925	18.05	4.037
Data Point 76	0.385	2.909	17.83	4.017

4 SOLUTIONS AND RECOMMENDATIONS

4.1 Implementation of Rule Based System Models

The two merging operations from Sections 2 have been implemented in MATLAB® environment. In order to work with Boolean matrices as formal models of rule bases, two additional MATLAB® functions have been implemented [7]. These functions convert the integer table representation of a rule base into a Boolean matrix form and vice versa, i.e. the Boolean matrix representation of a rule base into an integer table form.

The result of the first additional function is a Boolean matrix with rows that represent all possible permutations of the linguistic terms of the inputs from the integer table sorted in ascending order. The columns of that matrix represent all possible permutations of the linguistic terms of the outputs from the integer table sorted in ascending order. An element of the product Boolean matrix is set to 1, if it reflects an existing mapping from an input / output permutation from the operand integer table, or to 0 otherwise.

The result of the second additional function is a rule base represented in the form of an integer table. For a fuzzy system with m inputs and n outputs, the first m columns of the table represent the linguistic terms of the inputs to the system and the next n columns represent the linguistic terms of the outputs from the system. This product integer table is used for creating a fuzzy system in the Fuzzy Logic Toolbox™ for MATLAB®.

4.2 Multiple Rule Based System Model

The first model represents a MRBS structured as a hierarchical fuzzy system. It is based on inferential composition of two interacting rule bases. In this case, the composition is applied to the rule bases such that each of them is subject to fuzzification, inference and defuzzification. The MRBS model is illustrated in Figure 7.

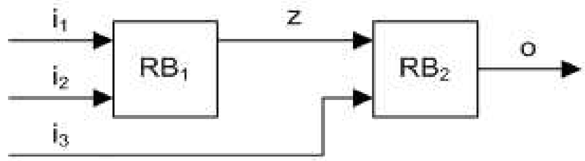

Fig.7 Block scheme of the MRBS model

The first rule base RB_1 in the MRBS model has two inputs i_1 and i_2 - the initial concentration of copper in the pulp and the initial concentration of iron in the pulp. These two inputs are presented by eleven linguistic terms each, as shown in Figures 8-9. The output z from the first rule base RB_1 is the intermediate concentration of copper in the pulp and it is presented by eleven linguistic terms, as shown in Figure 10. The first and the last several rules from the rule base RB_1 are presented as an integer table in Table 3.

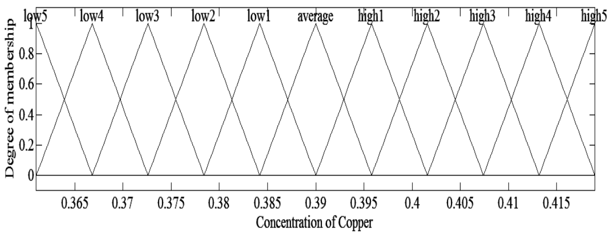

Fig. 8 Linguistic terms for the initial concentration of copper

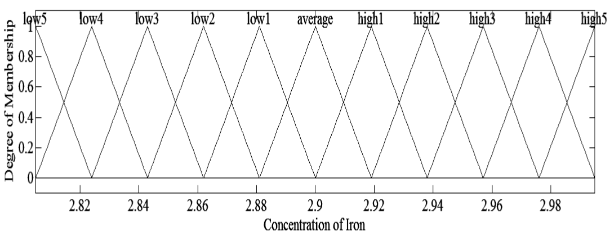

Fig.9 Linguistic terms for the initial concentration of iron

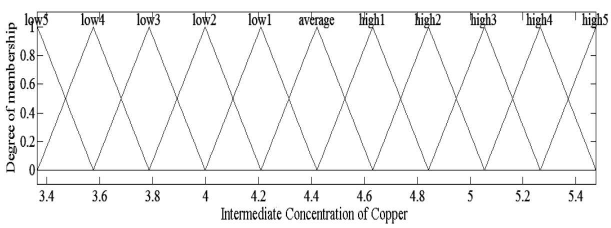

Fig. 10 Linguistic terms for the intermediate concentration of copper

Table 3 Partial presentation of the first rule base for the MRBS model

	Rule Antecedent	Rule Consequent
Rule 1	[1 2]	7
Rule 2	[1 4]	6
Rule 3	[1 5]	10
…	…	…
Rule 42	[10 8]	4
Rule 43	[11 10]	3
Rule 44	[11 11]	8

The second rule base RB_2 in the MRBS model has two inputs - the intermediate concentration of copper in the pulp z and the pulp debit i_3. The first input to RB_2 is the same as the output from RB_1 and it is already presented by the eleven linguistic terms in Figure 10. The second input to RB_2 is also presented by eleven linguistic terms, as shown in Figure 11. The output o from the second rule base RB_2 is the new concentration of copper in the pulp and it is presented by the eleven linguistic terms for the first input z in Figure 10, because z has the same physical meaning and variation range as o. The first and the last several rules from the rule base RB_2 are presented in Table 4.

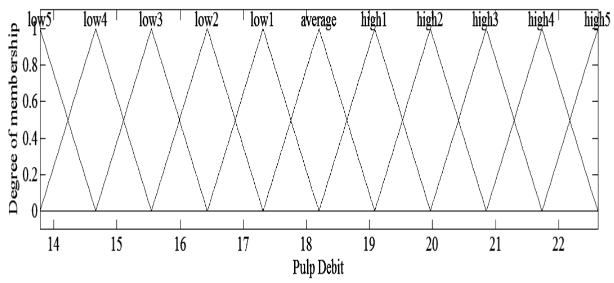

Fig. 11 Linguistic terms for the pulp debit

Table 4 Partial presentation of the second rule base for the MRBS model

	Rule Antecedent	Rule Consequent
Rule 1	[1 3]	1
Rule 2	[1 4]	1
Rule 3	[2 3]	2
…	…	…
Rule 44	[11 3]	11
Rule 45	[11 8]	11
Rule 46	[11 10]	11

The first rule base in the MRBS model has 44 rules, whereas the second rule base has 46 rules. The rules in these rule bases are derived using simple data clustering of the input and output data sets [7]. All inputs and outputs in these two rule bases are considered in their variation ranges.

The MRBS model is simulated for all available measurements. The results from the model simulation are shown in Figure 12, where the data output is given by the ´o´ marker and the model output is given by the ´x´ marker.

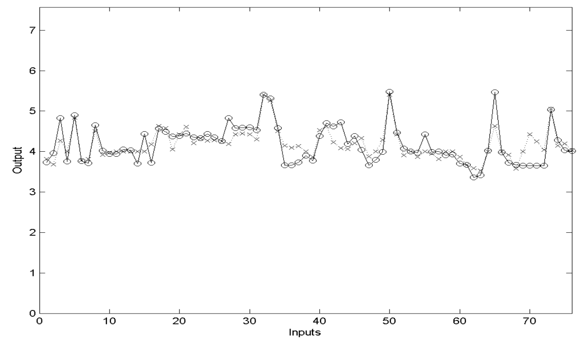

Fig. 12 Simulation results for the MRBS model

The two output surfaces of the model are presented in Figures 13-14, from where it can be seen that they are strongly non-linear due to the non-linearity of the process. In this case, the first surface represents the first rule base, whereas the second surface represents the second rule base.

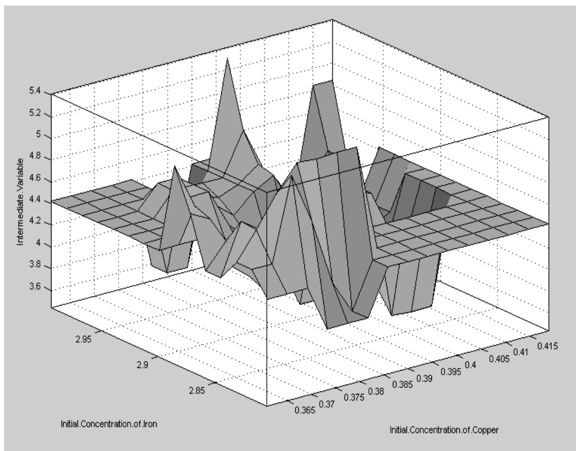

Fig.13 First output surface for the MRBS model

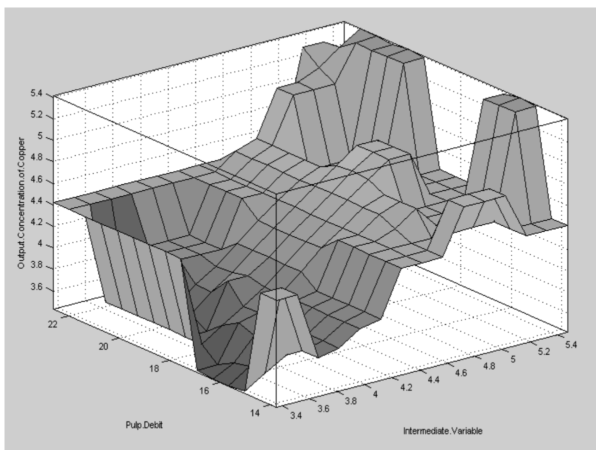

Fig.14 Second output surface for the MRBS model

4.3 Networked Rule Based System Model

The second model represents a NRBS and it can be obtained from the first model. In this case, the two rule bases are the same as the ones used for the MRBS model and they are composed linguistically into an equivalent single rule base. An identity rule base (IRB) is added to the NRBS model for the purpose of making the operations of vertical and horizontal merging of rule bases compatible. The first and the last several rules from the rule base IRB are presented in Table 5, from where it can be seen that output is identical to the input. The equivalent single rule base for the model is then subject to a single fuzzification - defuzzification sequence. The NRBS model is illustrated in Figure 15.

Table 5 Partial presentation of the identity rule base for the NRBS model

	Rule Antecedent	Rule Consequent
Rule 1	[1]	1
Rule 2	[2]	2
Rule 3	[3]	3
...	...	...
Rule 9	[9]	9
Rule 10	[10]	10
Rule 11	[11]	11

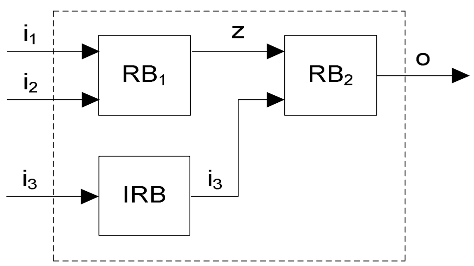

Fig. 15 Block scheme of the NRBS model

The linguistic composition of the individual rule bases is applied, as shown in Equation (11)

$$RB = (RB_1 + IRB) * RB_2 \tag{11}$$

where RB_1 is the first rule base, RB2 is the second rule base, IRB is the identity rule base, RB is the equivalent single rule base of the NRBS model. The symbols ‘*’ and ‘+’ denote horizontal and vertical merging operations of rule bases, respectively. The first and the last several rules from the rule base RB are presented in Table 6.

Table 6 Partial presentation of the rule base for the NRBS model

	Rule Antecedent	Rule Consequent
Rule 1	[1 2 4]	7
Rule 2	[1 2 5]	7
Rule 3	[1 2 6]	7
...	...	...
Rule 249	[10 8 1]	4
Rule 250	[11 10 1]	3
Rule 251	[11 11 1]	8

The NRBS model has three inputs and one output. The inputs and the output are presented by the same eleven linguistic terms as the ones used for the MRBS model. The overall number of rules for the equivalent single rule base of the model is 251. The rules in these rule bases are derived using simple data clustering of the input and output data sets, as the one used for the MRBS model. All inputs and outputs in the equivalent single rule base are considered in their variation ranges.

The NRBS model is simulated for all available measurements. The results from the model simulation are shown in Figure 16, where the data output is given by the ´o´ marker and the model output is given by the ´x´ marker.

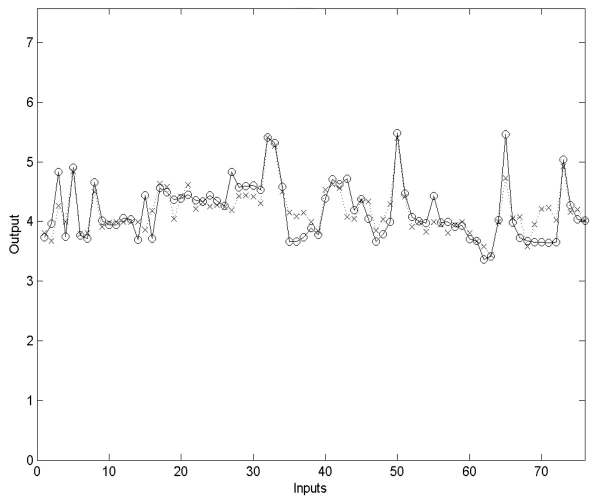

Fig 16 Simulation results for the NRBS model

The output surface of the model is presented in Figure 17, from where it can be seen that it is strongly non-linear due to the non-linearity of the process. The peaks in the surface correspond to strong variations of the output for small variations of the inputs, which are due to the non-measurable input factors that are not included in the model.

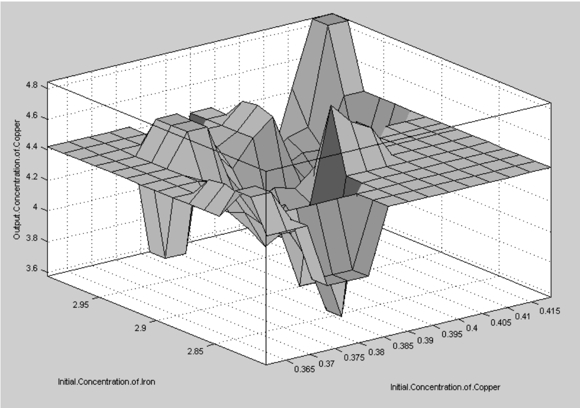

Fig. 17 Output surface for the NRBS model

4.4 Single Rule Based System Model

The third model represents a SRBS. This model is very similar to the NRBS in that it also has a single rule base RB with the same three inputs and one output as the NRBS model. However, this single rule base is derived in advance and not from other rule bases. The first and the last several rules from the rule base RB are presented in Table 7. The SRBS model is illustrated in Figure 18.

Table 7 Partial presentation of the rule base for the SRBS model

	Rule Antecedent	Rule Consequent
Rule 1	[1 2 10]	7
Rule 2	[1 4 10]	6
Rule 3	[1 5 11]	10
...	...	...
Rule 44	[11 10 7]	3
Rule 45	[11 11 1]	8
Rule 46	[11 11 6]	8

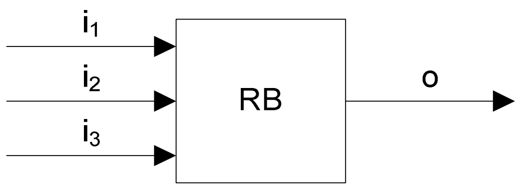

Fig 18 Block scheme of the SRBS model

The inputs and the output are presented by the same eleven linguistic terms as the ones used for the MRBS model. The overall number of rules for the rule base of the model is 58. The rules in these rule bases are derived using simple data clustering of the input and output data sets, as the one used for the MRBS model. All inputs and outputs in the single rule base are considered in their variation ranges.

The SRBS model is simulated for all available measurements. The results from the model simulation are shown in Figure 19, where the data output is given by the ‘o´ marker and the model output is given by the ´x´ marker.

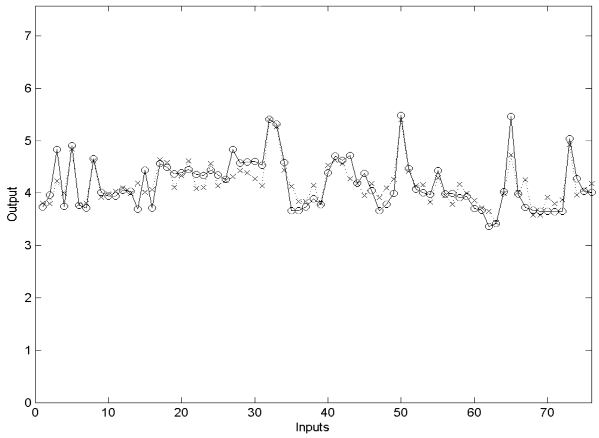

Fig19 Simulation results for the SRBS model

The output surface of the model is presented in Figures 20. It can be seen that the surface is strongly non-linear due to the non-linearity of the process.

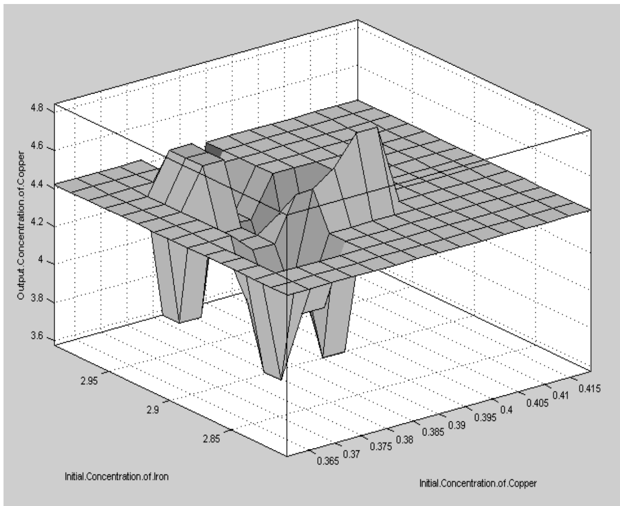

Fig. 20 Output surface for the SRBS model

4.5 Evaluation of Models Performance

The three rule based models are evaluated by quantitative metrics based on two indicators – mean absolute percentage error (MAPE) and overall transparency index (OTI).

The MAPE is a widely used indicator for model validation [9]. It is calculated as the absolute value of the difference between each data point in the data sets from the flotation process and the output of the corresponding rule based model. This difference is then divided by each data point and summed over all points from the simulations of the three models. Finally, the sum is divided by the overall number of simulated points, as shown in Equation (12)

$$MAPE = \left(\sum_{i=1}^{n} | d_i - m_i | / d_i \right) / n \qquad (12)$$

where d_i and m_i, $i=1,n$ denote the data output and the model output for the i-th simulated point, respectively.

The OTI is a novel indicator for model validation [5]. It is estimated by first subtracting the sum of identity rule bases and identity intermediate variables from the overall sum of rule bases and intermediate variables. The result of this subtraction is then divided by the sum of inputs and output. The assumption here is that each rule base or intermediate variable improves the transparency by taking into account the modular structure and the interactions of the flotation process. However, this is not the case for identity rule bases and identity intermediate variables in the NRBS model, which represent mathematical constructs for the linguistic composition of the rule bases from the MRBS, but don't have any physical meaning. In particular, the OTI is obtained, as shown in Equation (13)

$$OTI = (N_n + N_z - N_{idn} - N_{idz}) / (N_i + N_o) \qquad (13)$$

where N_n is the number of rule bases, N_z is the number of intermediate variables, N_{idn} is the number of identity rule bases, N_{idz} is the number of identity intermediate variables, N_i is the number of inputs and N_o is the number of outputs.

The comparative evaluation of the three rule based models of the flotation process with respect to accuracy and transparency is summarised in Table 8.

Table 8 Comparative evaluation of models

Indicator / Model	MRBS	NRBS	SRBS
MAPE	4.76 %	4.60 %	4.35 %
OTI	0.75	0.75	0.25

The simulation results show that in terms of accuracy the SRBS model is the best, the NRBS model is slightly worse, whereas the MRBS is the worst of all. The superiority of the SRBS model in terms of accuracy can be attributed to the presence of approximation errors as a result of multiple fuzzification-inference-defuzzification applied to the rule bases in the MRBS model or multiple linguistic composition applied to the rule bases in the NRBS model. As far as transparency is concerned, the SRBS model is the worst, whereas the MRBS model and the NRBS are better and equal to each other.

4.6 Improvement of Models Performance

The accuracy of the NRBS model could be further improved due to the Boolean matrix multiplication nature of the horizontal merging operation. This potential improvement is a subject of on-going research and it follows from the fact that the Boolean matrix multiplication nature of the horizontal merging operation allows the number of linguistic terms for the intermediate variables connecting any pair of rule bases that are horizontally merged to be increased, while preserving the overall number of rules in the NRBS model equal to the number of rules in the linguistically equivalent SRBS model.

In this context, the MAPE of a NRBS model is expected to decrease with the increase of the number of linguistic terms for the intermediate variables connecting the individual rule bases. The latter is due to the decreased approximation error during the linguistic composition of the rule bases in the MRBS.

A two-step algorithm for improving the accuracy of a NRBS model is proposed as follows:

1. A given MRBS model is converted to a NRBS model, whereby IRBs are introduced for the presentation of any identity intermediate variables.
2. The number of linguistic terms for all other intermediate variables is increased until the

error of the NRBS model becomes close enough to the error of the SRBS model.

5 FUTURE RESEARCH DIRECTIONS

The theory of rule based networks presented in this work can be further extended in the context of complex networks. In particular, a number of network complexity indicators could be considered which would also provide details about the complexity of the process that is modelled by the rule based network. Some of these indicators could be the following:

- in-degree and out-degree for a node in the rule based network, i.e. the number of inputs to and outputs from an individual rule base in the network,
- overall in-degree and out-degree for a level, i.e. the number of inputs to and outputs from the rule bases in a particular horizontal level of the rule based network,
- overall in-degree and out-degree for a layer, i.e. the number of inputs to and outputs from the rule bases in a particular vertical layer of the rule based network,
- degree of completeness for a level, i.e. the number of occupied layer positions in a particular level of the rule based network as a proportion of the overall number of layer positions in this level,
- degree of completeness for a layer, i.e. the number of occupied level positions in a particular layer of the rule based network as a proportion of the overall number of level positions in this layer,
- overall degree of completeness for a rule based network, i.e. the number of occupied positions as a proportion of the overall number of positions in the underlying grid structure of the network.

In terms of applications, the theory of rule based networks could be further validated on other complex processes in areas such as manufacturing, business, finance, transport and the environment. This is because many processes in these areas are characterised by uncertainty, non-linearity, modular structure and interactions. Therefore, these processes could be easily handled by the theory of rule based networks that is aimed at dealing with all these aspects of complexity.

6 CONCLUSION

This work illustrates the application of the novel theory of rule based networks for building and improving a fuzzy model for a complex flotation process from the mining industry. The theoretical concepts introduced in this work can be applied to any rule based models and not only to fuzzy models. As a whole, the theory of rule based networks facilitates the building of rule based models for complex industrial and other processes, characterised by uncertainty, non-linearity, modular structure and interactions.

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