About the definition of parameters and regimes of active two-port networks with variable loads on the basis of projective geometry

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Abstract: - Disadvantages of some well- known methods of analysis of electric circuits with variable loads are analyzed. To interpret a mutual influence of the loads, some methods of the projective geometry are used. The application of the projective coordinates allows receiving the equation of the active two-port network in a normalized or relative form as well as defining the scales for the currents and conductivity of the loads. Such approach makes it possible to estimate the qualitative characteristics of the current regimes, to compare the regime efficiency of the different circuits. The formulas of the recalculation of the currents, which possess the group properties at change of conductivity of the loads, are obtained. It allows expressing the final values of the currents through the intermediate changes of the currents and conductivities. The generalized equivalent generator of the active two-port network in the form of the passive two-port network and a set of the sources of a current and voltage is proposed. The parameters of these sources do not depend on certain conductivities of the passive two-port network.

Key-Words: - Thevenin's theorem, load characteristics, projective geometry, two-port networks

1 Introduction

In the theory of electric circuits, an attention is given to the circuits with variable parameters of elements. In practice, it can be the power supply systems, for example, of the direct current, are containing the power supply of finite capacity and quantity (let it be two) of resistive loads with variable resistances. Therefore, a mutual influence of the loads takes place in the similar active two-port network. One of analysis problems is determination of dependence of the changes of characteristics or parameters of the circuit regime on the respective change of parameters of loads.

Also, it is important to present a circuit by the equivalent generator, to define normalized or relative values of the regime parameters with use, for example, of the characteristic or maximum values.Therefore, such definition of the regimes allows estimating the qualitative characteristics of the current regimes, and comparing regimes of different systems. Consequently, the problem of the representation of two-port network parameters in the normalized form results from, of the Y - parameters, for example. A range of properties, theorems and methods is well-known, and their use simplifies the decision of the problems of this kind. However, the known approaches do not completely disclose the properties of such circuits, which reduces the efficiency of the analysis.

For example, in the theorem of mutual changes of currents and resistance (the variation theorem), the changes of the load resistances are set in the form of increments [1]. Therefore, at circuit recalculation, in case of a number or group of changes of these resistances, these increments should be counted concerning the initial circuit. It shows that the group properties are not carried out, and the possibilities of this theorem are limited. The group property defines the resultant change through intermediate changes of the resistance and current (the group operation takes place in the form of addition or multiplication).

Thus, the obvious definition of the changes looks like formal and does not reflect the substantial side of the observed mutual influences: resistance – current.

The method of the equivalent generator or Thevenin's theorem represents an active two-pole as a source of the voltage with an internal resistance [1], [2], [3]. For analysis of the regime efficiency, it is convenient to present the external or load characteristic of an active two-pole through the normalized or relative expression. Scales for corresponding parameters of the circuit regime are the open circuit voltage, the internal resistance and the short circuit current. If the normalized load voltage is equal to 1\2 or normalized load resistance is equal to 1, it shows immediately that the load regime corresponds to the maximum power. However, if any resistance is changed in an active two-pole, the open circuit voltage is changed also. Therefore, it is not convenient to use this voltage as the parameter of the equivalent generator.

Similarly, the method of the equivalent generator represents the active two-port network as the passive two-port network and the separated sources of the voltage or current at the terminals of two loads [1]. The parameters of these sources correspond to the open circuit voltage or the short circuit current of both loads at the same time. But these parameters or quantities do not allow presenting directly the system of the equations of the active two-port network in the normalized kind. Moreover, if any resistance of the passive two-port network is changed also, the recalculation of the values of these sources of the voltage or the current is necessary.

The method of geometrical places or circular charts allows investigating the influence of various parameters only of the alternating current circuits on the regime characteristics [1]. But this method is not developed enough, since such basic geometrical concepts as the transformation or movement of point are not used. To a larger extent, this method represents the graphical constructions simply.

In a number of articles of the author, the approach is developed for interpretation of changes or "kinematics" of the circuit regimes on the basis of projective geometry [4], [5], [6]. It allows disclosing the invariant properties of circuits, i.e. such relationships which are identical to all parameters of the regime and sub circuits. Such invariant relationships are also the basis of well-founded definition of the relative regimes.

Also, the theorem of the generalized equivalent generator of an active two-pole, which develops the well-known Tevenin's and Norton's theorems, is formulated. It appears that the external characteristic is transformed into a bunch of straight lines for various values of the internal resistance of this two-pole. Since the coordinates of the center of a bunch do not depend on this changeable element, they can be accepted as the parameters of the generalized equivalent generator.

Proceeding from the obtained results, it is possible to draw a conclusion that such mathematical apparatus is adequate to the observed interrelations. In the present article, the obtained results are applied to the analysis of active two-port networks.

2 Projective coordinates of an active two-port network

Let us consider an active two-port network with changed conductivities of loads Y_{L1}, Y_{L2} in Fig.1. Taking into account the specified directions of the currents, a circuit is described by the following system of the equations:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_0 \end{bmatrix} = \begin{bmatrix} -Y_{11} & Y_{12} & Y_{10} \\ Y_{12} & -Y_{22} & Y_{20} \\ -Y_{10} & -Y_{20} & Y_{00} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} =$$

$$= \begin{bmatrix} -1.2 & 0.2 & 0.6 \\ 0.2 & -0.95 & 0.4 \\ -0.6 & -0.4 & 1.3 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ 5 \end{bmatrix} \qquad (1)$$

where Y - parameters are:

$$Y_{11} = Y_1 + Y_{13} - \frac{Y_{13}^2}{Y_\Sigma} = 1.2, \quad ,$$

$$Y_\Sigma = Y_{03} + Y_3 + Y_{13} + Y_{23} = 5.20833$$

$$Y_{12} = Y_{23}\frac{Y_{13}}{Y_\Sigma} = 0.2, \quad Y_{10} = Y_{03}\frac{Y_{13}}{Y_\Sigma} = 0.6,$$

$$Y_{22} = Y_2 + Y_{23} - \frac{Y_{23}^2}{Y_\Sigma} = 0.95,$$

$$Y_{20} = Y_{03}\frac{Y_{23}}{Y_\Sigma} = 0.4, \quad Y_{00} = Y_{03} - \frac{Y_{03}^2}{Y_\Sigma} = 1.3$$

Value dimensions are not indicated for simplifying of the record.

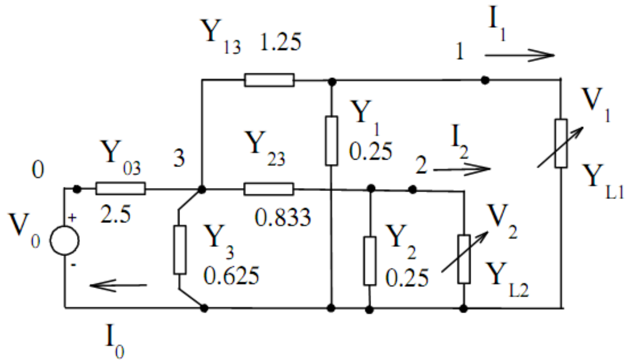


Fig.1. An active two-port network with changed conductivities of loads

Taking into account the voltages $V_1 = I_1 / Y_{L1}$, $V_2 = I_2 / Y_{L2}$, from the system of equations (1) the equation of two bunches of straight lines with parameters Y_{L1}, Y_{L2} are obtained:

$$V_0(Y_{20}Y_{12} + Y_{10}Y_{22}) - Y_{12}I_2 = I_1\left(Y_{22} + \frac{\Delta}{Y_{L1}}\right)$$

$$V_0(Y_{10}Y_{12} + Y_{20}Y_{11}) - Y_{12}I_1 = I_2\left(Y_{11} + \frac{\Delta}{Y_{L2}}\right) \qquad (2)$$

The bunches of these straight lines are presented in Fig. 2.

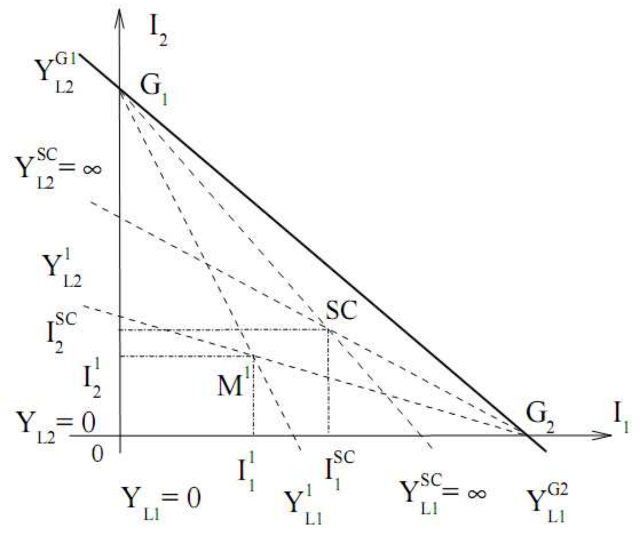


Fig.2. The bunches of load straight lines

A bunch center, a point G_1, corresponds to the bunch of the straight lines with the parameter Y_{L1}. Physically, the bunch center corresponds to such regime of the load Y_{L1} which does not depend on its values. It is carried out for the current and voltage $I_1 = 0, V_1 = 0$ at the expense of a choice of parameters of the second load Y_{L2}:

$$I_2^{G1} = V_0\left(Y_{20} + Y_{10}\frac{Y_{22}}{Y_{12}}\right) = V_0Y_{03}\left(1 + \frac{Y_2}{Y_{23}}\right) = 16.25$$

$$V_2^{G1} = -V_0\frac{Y_{10}}{Y_{12}} = -V_0\frac{Y_{03}}{Y_{23}} = -15. \qquad (3)$$

Negative value of conductivity of the load Y_{L2}, which already gives energy, corresponds to these parameters:

$$Y_{L2}^{G1} = I_2^{G1} / V_2^{G1} = -(Y_2 + Y_{23}) = -1.0833. \qquad (4)$$

The parameters of the center G_2 of the bunch Y_{L2} are expressed similary:

$$I_1^{G2} = V_0\left(Y_{10} + Y_{20}\frac{Y_{11}}{Y_{12}}\right) = \\ = V_0Y_{03}\left(1 + \frac{Y_1}{Y_{13}}\right) = 15 \qquad (5)$$

$$V_1^{G2} = -V_0\frac{Y_{20}}{Y_{12}} = -V_0\frac{Y_{03}}{Y_{13}} = -10,$$

$$Y_{L1}^{G2} = -(Y_1 + Y_{13}) = -1.5 \qquad (6)$$

Regimes, which are defined at qualitative level, we name as characteristic regimes. The regimes, which set the centers of bunches, are such characteristic regimes. One more form of such regimes is the short circuit regime on both loads that is presented by a point SC in Fig. 2. In this case, the currents are equal:

$$I_1^{SC} = Y_{10}V_0 = 3,\ I_2^{SC} = Y_{20}V_0 = 2. \qquad (7)$$

The open circuit regime on both loads is also characteristic and corresponds to the beginning of coordinates, a point 0.

Let the initial or current regime corresponds to a point M^1 which is set by the following parameters:

$$Y_{L1}^1 = 0.5,\ Y_{L2}^1 = 0.5,\ I_1^1 = 0.979,\ I_2^1 = 0.825$$

Let us use a definition of projective geometry coordinates [7], [8], [9]. Currents I_1, I_2 are the Cartesian non-uniform coordinates in the oordinate system $I_1\ 0\ I_2$. In turn, conductivities Y_{L1}, Y_{L2} define non-uniform projective coordinates m_1, m_2 which are set by a coordinate triangle $G_1\ 0\ G_2$ and a unit point SC. A point 0 is the beginning of coordinates and a straight line $G_1\ G_2$ is the infinitely remote straight line.

The non-uniform projective coordinate m_1^1 is set by a cross ratio of four points, three of these correspond to the points of the characteristic regimes, and the fourth corresponds to the point of the current regime:

$$m_1^1 = (0\ Y_{L1}^1\ \infty\ Y_{L1}^{G2}) = \frac{Y_{L1}^1 - 0}{Y_{L1}^1 - Y_{L1}^{G2}} : \frac{\infty - 0}{\infty - Y_{L1}^{G2}} = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G2}} = \frac{0.5}{0.5+1.5} = 0.25 \tag{8}$$

The conformity of the values Y_{L1} and m_1 is shown in Fig.3. The points $Y_{L1} = 0$, $Y_{L1}^1 = Y_{L1}^{G2}$ correspond to extreme or base values there. The point $Y_{L1} = \infty$ is the unit point.

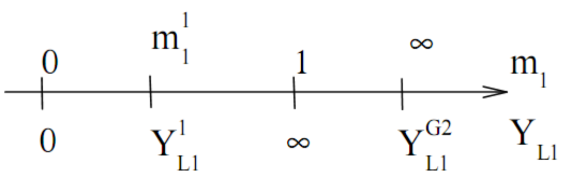


Fig.3. The conformity of a conductivity Y_{L1} and non-uniform projective coordinate m_1

The cross ratio for the projective coordinate m_2^1 is expressed similarly:

$$m_2^1 = (0\ Y_{L2}^1\ \infty\ Y_{L2}^{G1}) = \frac{Y_{L2}^1}{Y_{L2}^1 - Y_{L2}^{G1}} = \frac{Y_{L2}^1 / Y_{L2}^{G1}}{Y_{L2}^1 / Y_{L2}^{G1} - 1} = \frac{0.5}{0.5+1.0833} = 0.3158 \tag{9}$$

The conductivities Y_{L1}^{G2}, Y_{L2}^{G1} also are the scale values, as allow obtaining the normalized values of the load conductivities. Non-uniform coordinates of points of a straight line $G_1\ G_2$ have not final values. Therefore, homogeneous projective coordinates ξ_1, ξ_2, ξ_3 are entered which set non-uniform coordinates as follows:

$$m_1 = \frac{\xi_1}{\xi_3} = \frac{\rho\xi_1}{\rho\xi_3},\ m_2 = \frac{\xi_2}{\xi_3} = \frac{\rho\xi_2}{\rho\xi_3}, \tag{10}$$

where ρ is a coefficient of proportionality.

Homogeneous coordinates are defined by the ratio of the distances of points M^1, SC to parties of a coordinate triangle in Fig. 4:

$$\rho\xi_1^1 = \frac{\delta_1^1}{\delta_1^{SC}},\ \rho\xi_2^1 = \frac{\delta_2^1}{\delta_2^{SC}},\ \rho\xi_3^1 = \frac{\delta_3^1}{\delta_3^{SC}} \tag{11}$$

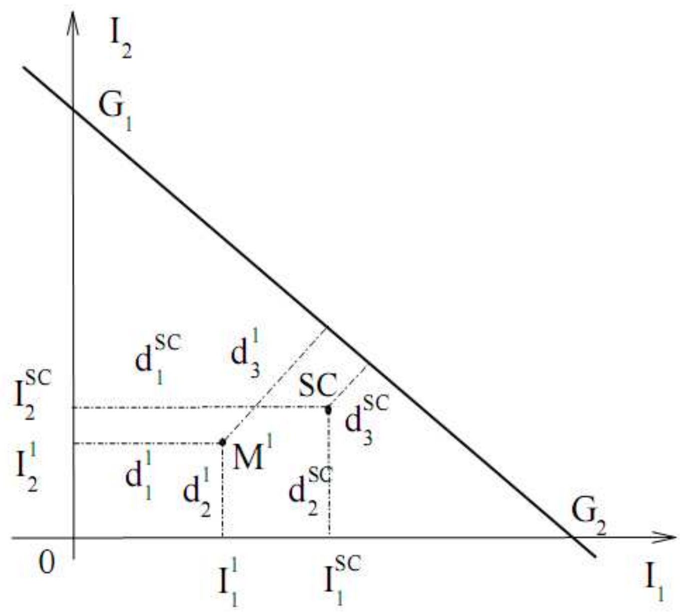


Fig.4. The distances of points M^1, SC to parties of a coordinate triangle

Taking into account a choice of the coordinate system, the distances are given:

$\delta_1^1 = I_1^1$, $\delta_2^1 = I_2^1$, $\delta_1^{SC} = I_1^{SC}$, $\delta_2^{SC} = I_2^{SC}$.

For a finding of distances to a straight line, the equation of this straight line is used:

$$\frac{I_1}{I_1^{G2}} + \frac{I_2}{I_2^{G1}} - 1 = 0. \quad (12)$$

Then the distance equal to

$$\delta_3^{SC} = \frac{1}{\mu_3}\left(\frac{I_1^{SC}}{I_1^{G2}} + \frac{I_2^{SC}}{I_2^{G1}} - 1\right), \text{ where}$$

$$\left(\frac{I_1^{SC}}{I_1^{G2}} + \frac{I_2^{SC}}{I_2^{G1}} - 1\right) = \frac{3}{15} + \frac{2}{16.25} - 1 = = -0.677 = \mu_3 \delta_3^{SC} \quad (13)$$

and μ_3 is a normalizing factor:

$$\mu_3 = \sqrt{\frac{1}{\left(I_1^{G2}\right)^2} + \frac{1}{\left(I_2^{G1}\right)^2}} = 0.0907.$$

Similarly:

$$\delta_3^1 = \frac{1}{\mu_3}\left(\frac{I_1^1}{I_1^{G2}} + \frac{I_2^1}{I_2^{G1}} - 1\right),$$

$$\left(\frac{I_1^1}{I_1^{G2}} + \frac{I_2^1}{I_2^{G1}} - 1\right) = -0.8839 = \mu_3 \delta_3^1 \quad (14)$$

Then, by expression (11), the homogeneous coordinates have a view:

$$\rho \xi_1^1 = \frac{I_1^1}{I_1^{SC}} = \frac{0.979}{3} = 0.3264,$$

$$\rho \xi_2^1 = \frac{I_2^1}{I_2^{SC}} = \frac{0.825}{2} = 0.4125, \quad (15)$$

$$\rho \xi_3^1 = \frac{I_1^1}{I_1^{G2}\mu_3\delta_3^{SC}} + \frac{I_2^1}{I_2^{SC}\mu_3\delta_3^{SC}} - \frac{1}{\mu_3\delta_3^{SC}} = = \frac{0.8839}{0.677} = 1.3057.$$

In turn, the non-uniform coordinates are equal to:

$$m_1^1 = \frac{\xi_1^1}{\xi_3^1} = \frac{0.3264}{1.3057} = 0.25,$$

$$m_2^1 = \frac{\xi_2^1}{\xi_3^1} = \frac{0.4125}{1.3057} = 0.3159,$$

that coincides with the values (8), (9).

Let us express the projective coordinates through the Cartesian coordinates I_1, I_2. We present the system of the equations (15) by a matrix form:

$$\begin{bmatrix} \rho\xi_1 \\ \rho\xi_2 \\ \rho\xi_3 \end{bmatrix} = [C] \cdot \begin{bmatrix} I_1 \\ I_2 \\ 1 \end{bmatrix} = [C] \cdot [I], \quad (16)$$

where matrix

$$[C] = \begin{bmatrix} \frac{1}{I_1^{SC}} & 0 & 0 \\ 0 & \frac{1}{I_2^{SC}} & 0 \\ \frac{1}{I_1^{G2}\mu_3\delta_3^{SC}} & \frac{1}{I_2^{G1}\mu_3\delta_3^{SC}} & \frac{-1}{\mu_3\delta_3^{SC}} \end{bmatrix}$$

Then the values ($I_1, I_2, 1$) are homogeneous Cartesian coordinates.

For our example, the matrix of transformation (16) is:

$$[C] = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ -\frac{1}{15\cdot 0.677} & -\frac{1}{16.25\cdot 0.677} & -\frac{1}{0.677} \end{bmatrix}$$

The reverse transformation to the transformation (16) is expressed as follows:

$$\begin{bmatrix} \rho I_1 \\ \rho I_2 \\ \rho I_3 \end{bmatrix} = \begin{bmatrix} I_1^{SC} & 0 & 0 \\ 0 & I_2^{SC} & 0 \\ \frac{I_1^{SC}}{I_1^{G2}} & \frac{I_2^{SC}}{I_2^{G1}} & -\mu_3\delta_3^{SC} \end{bmatrix} \cdot \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \quad (17)$$

From here, we pass to the non-uniform Cartesian coordinates:

$$I_1 = \frac{\rho I_1}{\rho 1} = \frac{I_1^{SC}\cdot\frac{\xi_1}{\xi_3}}{\frac{I_1^{SC}}{I_1^{G2}}\cdot\frac{\xi_1}{\xi_3} + \frac{I_2^{SC}}{I_2^{G1}}\cdot\frac{\xi_2}{\xi_3} - \mu_3\delta_3^{SC}} =$$

$$= \frac{I_1^{SC}\cdot m_1}{\frac{I_1^{SC}}{I_1^{G2}}\cdot m_1 + \frac{I_2^{SC}}{I_2^{G1}}\cdot m_2 - \mu_3\delta_3^{SC}} \quad (18)$$

$$I_2 = \frac{\rho I_2}{\rho 1} = \frac{I_2^{SC}\cdot\frac{\xi_2}{\xi_3}}{\frac{I_1^{SC}}{I_1^{G2}}\cdot\frac{\xi_1}{\xi_3} + \frac{I_2^{SC}}{I_2^{G1}}\cdot\frac{\xi_2}{\xi_3} - \mu_3\delta_3^{SC}} =$$

$$= \frac{I_2^{SC}\cdot m_2}{\frac{I_1^{SC}}{I_1^{G2}}\cdot m_1 + \frac{I_2^{SC}}{I_2^{G1}}\cdot m_2 - \mu_3\delta_3^{SC}}.$$

For our example, the transformation (18) leads to the same known values:

$$I_1^1 = \frac{3\cdot 0.25}{\frac{3}{15}\cdot 0.25 + \frac{2}{16.25}\cdot 0.3158 + 0.677} = 0.979$$

$$I_2^1 = \frac{2\cdot 0.3159}{0.7658} = 0.825$$

Thus, for the preset values of conductivities $Y_{L1},\ Y_{L2}$, we find the coordinates m_1, m_2, and the transformation (18) allows finding the currents I_1, I_2.

It is possible to present the system of the equations (18) by the normalized or relative form:

$$\frac{I_1}{I_1^{G2}} = \frac{\frac{I_1^{SC}}{I_1^{G2}}\cdot m_1}{\frac{I_1^{SC}}{I_1^{G2}}\cdot m_1 + \frac{I_2^{SC}}{I_2^{G1}}\cdot m_2 - \frac{I_1^{SC}}{I_1^{G2}} - \frac{I_2^{SC}}{I_2^{G1}} + 1} =$$

$$= \frac{\frac{I_1^{SC}}{I_1^{G2}}\cdot m_1}{\frac{I_1^{SC}}{I_1^{G2}}\cdot(m_1 - 1) + \frac{I_2^{SC}}{I_2^{G1}}\cdot(m_2 - 1) + 1},$$

$$\frac{I_2}{I_2^{G1}} = \frac{\frac{I_2^{SC}}{I_2^{G1}}\cdot m_2}{\frac{I_1^{SC}}{I_1^{G2}}\cdot(m_1 - 1) + \frac{I_2^{SC}}{I_2^{G1}}\cdot(m_2 - 1) + 1} \quad (19)$$

From this system of the equations, it is possible to obtain the equations of two bunches of the straight lines corresponding to expression (2):

$$1 - \frac{I_2}{I_2^{G1}} = \frac{I_1}{I_1^{G2}}\left[1 - \frac{1}{m_1}\left(1 - \frac{1 - I_2^{SC}/I_2^{G1}}{I_1^{SC}/I_1^{G2}}\right)\right]$$

$$1 - \frac{I_1}{I_1^{G2}} = \frac{I_2}{I_2^{G1}}\left[1 - \frac{1}{m_2}\left(1 - \frac{1 - I_1^{SC}/I_1^{G2}}{I_2^{SC}/I_2^{G1}}\right)\right] \quad (20)$$

The system of the equations (19) allows obtaining also in a relative form the equations of load characteristics $I_1(V_1, V_2),\ I_2(V_1, V_2)$ which correspond to the two first equations of the system (1). For this purpose, we express the non-uniform coordinates m_1, m_2 through the currents and voltages:

$$m_1 = \frac{Y_{L1}}{Y_{L1} - Y_{L1}^{G2}} =$$
$$= \frac{I_1 / V_1}{(I_1 / V_1) - I_1^{G2} / V_1^{G2}} = \frac{I_1 / I_1^{G2}}{(I_1 / I_1^{G2}) - V_1 / V_1^{G2}}$$

$$m_2 = \frac{I_2 / I_2^{G1}}{(I_2 / I_2^{G1}) - V_2 / V_2^{G1}}.$$

Having substituted these values in system (19), we receive the required equations $I_1(V_1, V_2)$, $I_2(V_1, V_2)$, accordingly having excluded the currents I_2, I_1:

$$\frac{I_1}{I_1^{G2}} = \frac{V_1}{V_1^{G2}}\left(1 - \frac{I_1^{SC}}{I_1^{G2}}\right) - \frac{I_1^{SC}}{I_1^{G2}}\frac{V_2}{V_2^{G1}} + \frac{I_1^{SC}}{I_1^{G2}}$$

$$\frac{I_2}{I_2^{G1}} = -\frac{I_2^{SC}}{I_2^{G1}}\frac{V_1}{V_1^{G1}} + \frac{V_2}{V_2^{G1}}\left(1 - \frac{I_2^{SC}}{I_2^{G1}}\right) + \frac{I_2^{SC}}{I_2^{G1}} \qquad (21)$$

The obtained system of the equations represents the purely relative expressions. Therefore, such values, as

$$\left(1 - \frac{I_1^{SC}}{I_1^{G2}}\right), \ \frac{I_1^{SC}}{I_1^{G2}}, \ \frac{I_2^{SC}}{I_2^{G1}}, \left(1 - \frac{I_2^{SC}}{I_2^{G1}}\right),$$

represent the «normalized» Y^N-parameters:

$$Y_{11}^N = \left(1 - \frac{I_1^{SC}}{I_1^{G2}}\right) = 1 - Y_{12}^N,$$
$$Y_{12}^N = Y_{10}^N = \frac{1}{1 + Y_{20}Y_{11} / Y_{10}Y_{12}}$$

$$Y_{22}^N = \left(1 - \frac{I_2^{SC}}{I_2^{G1}}\right) = 1 - Y_{21}^N,$$
$$Y_{21}^N = Y_{20}^N = \frac{1}{1 + Y_{10}Y_{22} / Y_{20}Y_{12}}$$

Let us pass to absolute values of the regime parameters in the system (21):

$$I_1 = \frac{I_1^{G2}}{V_1^{G2}}\left(1 - \frac{I_1^{SC}}{I_1^{G2}}\right) \cdot V_1 - \frac{I_1^{SC}}{V_2^{G1}} V_2 + I_1^{SC}$$

$$I_2 = -\frac{I_2^{SC}}{V_1^{G1}} V_1 + \frac{I_2^{G1}}{V_2^{G1}}\left(1 - \frac{I_2^{SC}}{I_2^{G1}}\right) \cdot V_2 + I_2^{SC} \qquad (22)$$

From this, it follows that Y - parameters are expressed by the parameters of the characteristic regimes:

$$-Y_{11} = \frac{I_1^{G2}}{V_1^{G2}}\left(1 - \frac{I_1^{SC}}{I_1^{G2}}\right) = \frac{I_1^{G2} - I_1^{SC}}{V_1^{G2}} = -1.2$$

$$-Y_{22} = \frac{I_2^{G1}}{V_2^{G1}}\left(1 - \frac{I_2^{SC}}{I_2^{G1}}\right) = \frac{I_2^{G1} - I_2^{SC}}{V_2^{G1}} = -0.95$$

$$Y_{12} = -\frac{I_1^{SC}}{V_2^{G1}} = \frac{3}{15} = -\frac{I_2^{SC}}{V_1^{G2}} = 0.2 \qquad (23)$$

The systems of the equations (19), (20) are important relationships, as represent the purely relative expressions. It allows defining, at first, the coordinates m_1, m_2 (as relative values) for different load conductivities and, then, the values of the normalized currents.

Also as the equation (21), these expressions allow to estimate at once a qualitative condition of such circuit, how much the current regime is close to the characteristic values. Then, the currents I_1^{G2}, I_2^{G1} represent scales. In this sense, the initial systems of the equations (1), (2), containing the actual or absolute values of the Y- parameters, currents and voltages, are a little informative, because do not give of a direct representation about the qualitative characteristics of a circuit.

In addition, it is possible to notice that it is difficult to obtain the relative expressions of type (19 - 23) from the systems of the equations (1), (2) directly. Therefore, the application of the geometrical interpretation solves such problem by the easy and formalized method.

3 Variation of load conductivities

Let a initial regime corresponds to a point M^1, and a subsequent regime corresponds to a point M^2 with the concrete parameters of loads, $Y_{L1}^2 = 1, Y_{L2}^2 = 2$, $I_1^2 = 1.434$, $I_2^2 = 1.5504$.

An arrangement of points of the initial and subsequent regimes is shown in Fig. 5. Non-uniform coordinates are defined similarly to (8), (9):

$$m_1^2 = (0\ Y_{L1}^2\ \infty\ Y_{L1}^{G2}) = \frac{Y_{L1}^2}{Y_{L1}^2 - Y_{L1}^{G2}} = 0.4\ ,$$

$$m_2^2 = (0\ Y_{L2}^2\ \infty\ Y_{L2}^{G1}) = \frac{Y_{L2}^2}{Y_{L2}^2 - Y_{L2}^{G1}} = 0.6486\,.$$

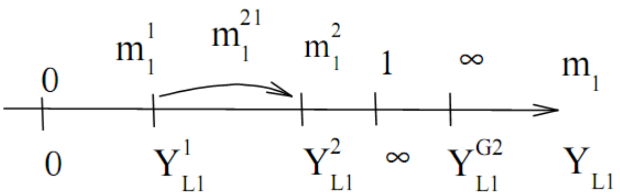


Fig.5. An arrangement of points of the initial and subsequent regimes

It is possible to define a valid change of a regime, using a group property of a cross ratio:

$$m_1^{21} = m_1^2 : m_1^1 = 0.4 : 0.25 = 1.6\,,$$

$$m_2^{21} = m_2^2 : m_2^1 = 0.6486 : 0.3158 = 2.0538$$

then:

$$m_1^2 = m_1^{21} \cdot m_1^1\,, m_2^2 = m_2^{21} \cdot m_2^1\,. \qquad (24)$$

Therefore, the change of the regime is expressed, naturally, through the cross ratio:

$$m_1^{21} = (0\ Y_{L1}^2\ Y_{L1}^1\ Y_{LH1}^{G2}) =$$
$$= \frac{Y_{L1}^2 - 0}{Y_{L1}^2 - Y_{L1}^{G2}} : \frac{Y_{L1}^1 - 0}{Y_{L1}^1 - Y_{L1}^{G2}} = m_1^2 : m_1^1,$$

$$m_2^{21} = (0\ Y_{L2}^2\ Y_{L2}^1\ Y_{L2}^{G1})\,. \qquad (25)$$

Let us define the homogeneous coordinates of a point M^2, using (11), (14), (15):

$$\delta_3^2 = \frac{1}{\mu_3}\left(\frac{I_1^2}{I_1^{G2}} + \frac{I_2^2}{I_2^{G1}} - 1\right),$$

$$\left(\frac{I_1^2}{I_1^{G2}} + \frac{I_2^2}{I_2^{G1}} - 1\right) = -0.809 = \mu_3 \delta_3^2\ .$$

then:

$$\rho\,\xi_1^2 = \frac{I_1^2}{I_1^{SC}} = \frac{1.434}{3} = 0.478,$$

$$\rho\,\xi_2^2 = \frac{I_2^2}{I_2^{SC}} = \frac{1.5504}{2} = 0.7752,$$

$$\rho\,\xi_3^1 = \frac{\delta_3^2}{\delta_3^{SC}} = \frac{0.809}{0.677} = 1.1949\,.$$

Let us check up the values of the non-uniform coordinates:

$$m_1^2 = \frac{\xi_1^2}{\xi_3^2} = \frac{0.478}{1.194} = 0.4,$$

$$m_2^2 = \frac{\xi_2^2}{\xi_3^2} = \frac{0.7752}{1.194} = 0.6487\,. \qquad (26)$$

Using the transformations (18), we receive the current

$$I_1^2 = \frac{I_1^{SC} \cdot m_1^2}{\dfrac{I_1^{SC}}{I_1^{G2}} \cdot m_1^2 + \dfrac{I_2^{SC}}{I_2^{G1}} \cdot m_2^2 - \mu_3 \delta_3^{SC}}\,.$$

Taking into account (21), we present the non-uniform coordinates, m_1^2 and m_2^2, in a form:

$$m_1^2 = m_1^{21}\frac{\xi_1^1}{\xi_3^1},\quad m_2^2 = m_2^{21}\frac{\xi_2^1}{\xi_3^1}\,.$$

Therefore, the current

$$I_1^2 = \frac{(I_1^{SC} \cdot m_1^{21}) \cdot \xi_1^1 / \xi_3^1}{\left(\dfrac{I_1^{SC}}{I_1^{G2}} \cdot m_1^{21}\right) \cdot \dfrac{\xi_1^1}{\xi_3^1} + \left(\dfrac{I_2^{SC}}{I_2^{G1}} \cdot m_2^{21}\right) \dfrac{\xi_2^1}{\xi_3^1} - \mu_3 \delta_3^{SC}}$$

Then, taking into account the expression (17), we receive the transformation:

$$\begin{bmatrix}\rho I_1^2\\ \rho I_2^2\\ \rho 1\end{bmatrix}=\begin{bmatrix}I_1^{SC}\cdot m_1^{21} & 0 & 0\\ 0 & I_2^{SC}\cdot m_2^{21} & 0\\ \frac{I_1^{SC}m_1^{21}}{I_1^{G2}} & \frac{I_2^{SC}m_2^{21}}{I_2^{G1}} & -\mu_3\delta_3^{SC}\end{bmatrix}\cdot\begin{bmatrix}\xi_1^1\\ \xi_2^1\\ \xi_3^1\end{bmatrix} \quad (27)$$

Using (16), we receive the resultant transformation as a product of the two matrixes:

$$\begin{bmatrix}\rho I_1^2\\ \rho I_2^2\\ \rho 1\end{bmatrix}=\begin{bmatrix}I_1^{SC}\cdot m_1^{21} & 0 & 0\\ 0 & I_2^{SC}\cdot m_2^{21} & 0\\ \frac{I_1^{SC}}{I_1^{G2}}\cdot m_1^{21} & \frac{I_2^{SC}}{I_2^{G1}}\cdot m_2^{21} & -\mu_3\delta_3^{SC}\end{bmatrix}\times$$

$$\times\begin{bmatrix}\frac{1}{I_1^{SC}} & 0 & 0\\ 0 & \frac{1}{I_2^{SC}} & 0\\ \frac{1}{I_1^{G2}\mu_3\delta_3^{SC}} & \frac{1}{I_2^{G1}\mu_3\delta_3^{SC}} & -\frac{1}{\mu_3\delta_3^{SC}}\end{bmatrix}\cdot\begin{bmatrix}I_1^1\\ I_2^1\\ 1\end{bmatrix}$$

or in a definitive form:

$$\begin{bmatrix}\rho I_1^2\\ \rho I_2^2\\ \rho 1\end{bmatrix}=\begin{bmatrix}m_1^{21} & 0 & 0\\ 0 & m_2^{21} & 0\\ \frac{m_1^{21}-1}{I_1^{G2}} & \frac{m_2^{21}-1}{I_2^{G1}} & 1\end{bmatrix}\cdot\begin{bmatrix}I_1^1\\ I_2^1\\ 1\end{bmatrix} \quad (28)$$

For our example, the transformation (28) has a view:

$$\begin{bmatrix}\rho I_1^2\\ \rho I_2^2\\ \rho 1\end{bmatrix}=\begin{bmatrix}1.6 & 0 & 0\\ 0 & 2.0538 & 0\\ 0.04 & 0.0648 & 1\end{bmatrix}\cdot\begin{bmatrix}I_1^1\\ I_2^1\\ 1\end{bmatrix}.$$

Let us check up the values of the currents of the subsequent regime:

$$I_1^2=\frac{\rho I_1^2}{\rho 1}=\frac{1.6\cdot 0.979}{0.04\cdot 0.979+0.0648\cdot 0.825+1}=1.4335$$

$$I_2^2=\frac{\rho I_2^2}{\rho 1}=\frac{2.0538\cdot 0.825}{1.0926}=1.55072.$$

Let us notice that a group property of the transformation (28) is carried out owing to group properties of the expressions (24). Let a regime once again change, i.e. we receive the values of the changes m_1^{32}, m_2^{32}.

Then the final value of the regime is expressed as:

$$m_1^3=m_1^{32}\cdot m_1^2=m_1^{32}\cdot m_1^{21}\cdot m_1^1=m_1^{31}\cdot m_1^1,$$

$$m_2^3=m_2^{31}\cdot m_2^1. \quad (29)$$

Using (28), we receive the resultant transformation as a product of the matrixes:

$$\begin{bmatrix}\rho I_1^3\\ \rho I_2^3\\ \rho 1\end{bmatrix}=\begin{bmatrix}m_1^{32} & 0 & 0\\ 0 & m_2^{32} & 0\\ \frac{m_1^{32}-1}{I_1^{G2}} & \frac{m_2^{32}-1}{I_2^{G1}} & 1\end{bmatrix}\times$$

$$\times\begin{bmatrix}m_1^{21} & 0 & 0\\ 0 & m_2^{21} & 0\\ \frac{m_1^{21}-1}{I_1^{G2}} & \frac{m_2^{21}-1}{I_2^{G1}} & 1\end{bmatrix}\cdot\begin{bmatrix}I_1^1\\ I_2^1\\ 1\end{bmatrix}.$$

or in a definitive form:

$$\begin{bmatrix} \rho I_1^3 \\ \rho I_2^3 \\ \rho 1 \end{bmatrix} = \begin{bmatrix} m_1^{31} & 0 & 0 \\ 0 & m_2^{31} & 0 \\ \frac{m_1^{31}-1}{I_1^{G2}} & \frac{m_2^{31}-1}{I_2^{G1}} & 1 \end{bmatrix} \cdot \begin{bmatrix} I_1^1 \\ I_2^1 \\ 1 \end{bmatrix} \quad (30)$$

3.1. Special case of the transformation (28)

Let us take interest only by a change of the current I_1. It is necessary to receive a transformation which defines the subsequent current I_1^2 only through the initial current I_1^1. For the explanatory, we consider Fig. 6. Let the initial regime corresponds to a point B^1 with the concrete parameters of loads Y_{L1}^1, Y_{L2}^1 and current I_1^{B1}. Then the regime passes into a point B^2 at the expence of a change $Y_{L1}^1 \to Y_{L1}^2$. We name such case as own change of a regime.

Further, the regime changes at the expence of a change $Y_{L2}^1 \to Y_{L2}^2$, and passes into a point C^2. Such case is mutual change of a regime.

The specified points B^1, C^2 correspond to above considered points M^1, M^2. Further, we use the results [5].

Let us receive expressions of a change of the regime, $B^1 \to B^2$. For this purpose, we map the points B, B^1, B^2, G^2 of a straight line with the parameter Y_{L2}^1 on the axis I_1 and we introduce the corresponding values of currents. Using (22), we make the cross ratio for this own change of the regime:

$$m_1^{21} = (B, B^2, B^1, G^2) = (0\, Y_{L1}^2\, Y_{L1}^1\, Y_{L1}^{G2}) = 1.6 \quad (31)$$

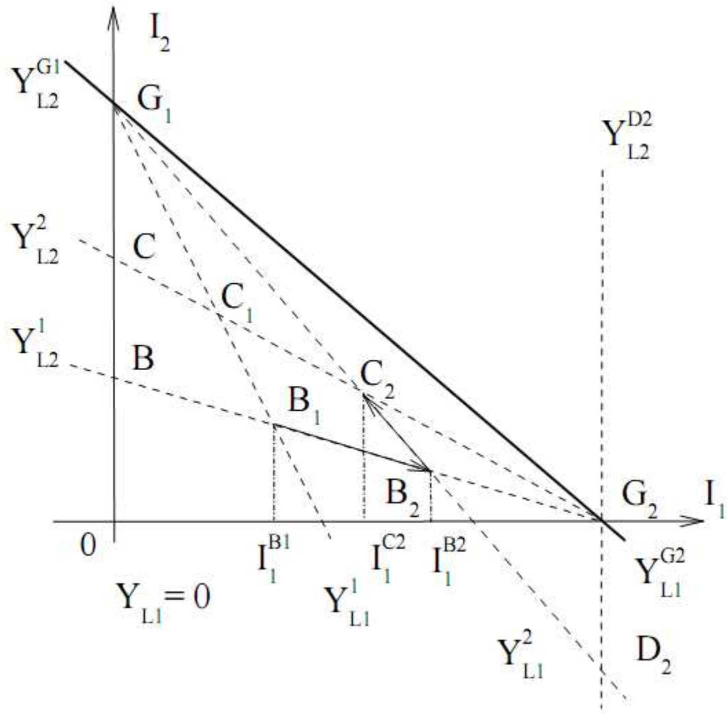


Fig.6. Changes of the current I_1

Also, this cross ratio is expressed through the introduced currents:

$$m_1^{21} = (0\ I_1^{B2}\ I_1^{B1}\ I_1^{G2}) = \frac{I_1^{B2}-0}{I_1^{B2}-I_1^{G2}} : \frac{I_1^{B1}-0}{I_1^{B1}-I_1^{G2}} =$$
$$= \frac{1.5073}{1.5073-15} : \frac{0.979}{0.979-1.5} = 1.6$$

Let us make the cross ratio for the mutual change of the regime. For this purpose, it is necessary to map the points G^1, C^2, B^2, D^2 of a straight line with parameter Y_{L1}^2 on the axis I_1 also. Then we receive the expression:

$$m_1^{22} = (G^1\ C^2\ B^2\ D^2) = (0\ I_1^{C2}\ I_1^{B2}\ I_1^{G2}) = \\ = (Y_{L2}^{G1}\ Y_{L2}^2\ Y_{L2}^1\ Y_{L2}^{D2}) \quad (32)$$

The choice of points G^1, D^2, as the base points, leads to the same base points, as well as for (31).

The given circuit, concerning of the first load, is a generator of the current I_1^{G2} at the expence of the second load conductivity equal to Y_{L2}^{D2}. The physical sense of the value Y_{L2}^{D2} consists in this fact. Let us define the value of conductivity Y_{L2}^{D2}. For this purpose, we present expression (2) in such form:

$$(I_1 - I_1^{G2}) = -\frac{I_2}{Y_{12}}(Y_{11} + \Delta / Y_{L2}\ ),$$

Let the current $I_1 = I_1^{G2}$. Then the conductivity

$$Y_{L2} = Y_{L2}^{D2} = -\frac{\Delta}{Y_{11}} = -0.9166 .$$

Also, the same value is obtained from the second expression (20). For this purpose, we find the corresponding value of the cross ratio:

$$m_2^{D2} = -\frac{1 - I_1^{SC} / I_1^{G2}}{I_2^{SC} / I_2^{G1}} + 1 = \frac{\mu_3 \delta_3^{SC}}{I_2^{SC} / I_2^{G1}} = -5.5$$

Then the value of the conductivity Y_{L2}^{D2} is found from expression (9).

Now we calculate m_1^{22}, using the expression (32):

$$m_1^{22} = \frac{I_1^{C2} - 0}{I_1^{C2} - I_1^{G2}} : \frac{I_1^{B2} - 0}{I_1^{B2} - I_1^{G2}} = \\ = \frac{1.434}{1.434 - 15} : \frac{1.5073}{1.5073 - 15} = 0.946 \quad (33)$$

$$m_1^{22} = \frac{Y_{L2}^2 - Y_{L2}^{G1}}{Y_{L2}^2 - Y_{L2}^{D2}} : \frac{Y_{L2}^1 - Y_{L2}^{G1}}{Y_{L2}^1 - Y_{L2}^{D2}} = \\ = \frac{2 + 1.0833}{2 + 0.9166} : \frac{0.5 + 1.0833}{0.5 + 0.9166} = 0.946 \quad (34)$$

As common base points are used for the own and mutual change, the group properties are carried out. Therefore, the resultant change is expressed by their product:

$$m_1(C2, B1) = m_1^{22} \cdot m_1^{21} = 0.16 \cdot 0.946 \quad (35)$$

$$m_1(C2, B1) = \frac{I_1^{C2} - 0}{I_1^{C2} - I_1^{G2}} : \frac{I_1^{B1} - 0}{I_1^{B1} - I_1^{G2}} = \\ = (0\, I_1^{C2}\ I_1^{B1}\ I_1^{G2}) \quad (36)$$

It is apparently, the intermediate value of the current I_1^{B2} is excluded. Thus, knowing changes of load conductivities, we find the values of changes m_1^{22}, m_1^{21} and resultant change (35) by (31), (32). A final value of a current I_1^{C2} is obtained from (36).

3.2. Comparative analysis with a traditional recalculation of a circuit.

Let us lead the basic statements of a traditional approach to recalculation of currents of an active two-port network. Expressions for the currents I_1, I_2 are obtained from the equation (1):

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -Y_{11} & Y_{12} & Y_{10} \\ Y_{12} & -Y_{22} & Y_{20} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} =$$

$$= \begin{cases} -Y_{11}V_1 + Y_{12}V_2 + I_1^{SC} \\ \ \ Y_{12}V_1 - Y_{22}V_2 + I_2^{SC} \end{cases} \quad (37)$$

The short circuit currents I_1^{SC}, I_2^{SC} are considered as initial values of the load currents I_1^0, I_2^0. Now, we introduce the load resistors as increments ΔR_{L1}^1, ΔR_{L2}^1 concerning initial zero values in Fig. 7,a. Therefore, the relationships take place for the subsequent values of the currents and voltages:

$$V_1^1 = \Delta R_{L1}^1 \cdot I_1^1, V_2^2 = \Delta R_{L2}^1 \cdot I_2^1 .$$

We transform the expression (37) as follows:

$$\begin{cases} I_1^1 = -Y_{11} \cdot \Delta R_{L1}^1 \cdot I_1^1 + Y_{12} \cdot \Delta R_{L2}^1 \cdot I_2^1 + I_1^0 \\ I_2^1 = \ \ Y_{12} \cdot \Delta R_{L1}^1 \cdot I_1^1 - Y_{22} \cdot \Delta R_{L2}^1 \cdot I_2^1 + I_2^0 \end{cases},$$

or:

$$\begin{cases} \ \ I_1^1(1 + Y_{11} \cdot \Delta R_{L1}^1) - Y_{12} \cdot \Delta R_{L2}^1 \cdot I_2^1 = I_1^0 \\ -Y_{12} \cdot \Delta R_{L1}^1 \cdot I_1^1 + I_2^1(1 + Y_{22} \cdot \Delta R_{L2}^1) = I_2^0 \end{cases}$$

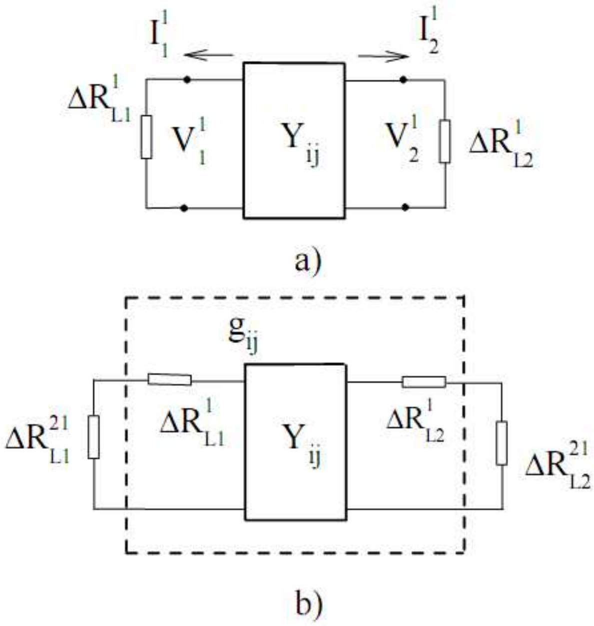


Fig.7. A traditional approach to recalculation of currents: a)- load resistors ΔR_{L1}^1, ΔR_{L2}^1 are as increments concerning initial zero values; b)- the changes ΔR_{L1}^{21}, ΔR_{L2}^{21} are concerning values ΔR_{L1}^1, ΔR_{L2}^1

The decision of this system of the equations gives calculated relationships:

$$I_1^1 = \frac{I_1^0 + \Delta R_{L2}^1 (Y_{22} I_1^0 + Y_{12} I_2^0)}{\Delta}$$

$$I_2^1 = \frac{I_2^0 + \Delta R_{L1}^1 (Y_{11} I_2^0 + Y_{12} I_1^0)}{\Delta}, \qquad (38)$$

where

$$\frac{1}{\Delta} = \frac{1}{(1 + Y_{11} \Delta R_{L1}^1)(1 + Y \Delta R_{L2}^1) - (Y_{12})^2 \Delta R_{L1}^1 \Delta R_{L2}^1}$$

Let us carry out the analysis of these relationshirs. We introduce intermediate changes ΔR_{L1}^{21}, ΔR_{L2}^{21} thus:

$$\Delta R_{L1}^2 = \Delta R_{L1}^{21} + \Delta R_{L1}^1, \; \Delta R_{L2}^2 = \Delta R_{L2}^{21} + \Delta R_{L2}^1 .$$

Therefore, the denominator will contain, both changes ΔR_{L1}^{21}, ΔR_{L2}^{21}, and initial values of resistance ΔR_{L1}^1, ΔR_{L2}^1. Thus, the structure of a denominator of expression (38) shows that group properties are not carried out for the intermediate changes of the load resistances. Therefore, the subsequent changes must be counted concerning initial zero values.

However, if we count off the changes ΔR_{L1}^{21}, ΔR_{L2}^{21} concerning values ΔR_{L1}^1, ΔR_{L2}^1, the scheme of other two-port network with conductivity parameters g_{ij} is obtained in Fig. 7,b. Thus, recalculation of parameters of new two-port network is required.

The offered approach on the basis of the projective geometry does not require recalculation of parameters and, naturally, possesses greater flexibility and convenience.

4 Generalized equivalent generator of an active two-port network

The expression (37) shows that an active two-port network represents a passive part which is set by parameters of conductivity Y_{ij}, and two generators of a current I_1^{SC}, I_2^{SC} that is shown in Fig.8.

In turn, the passive two-port network can be defined by various equivalent schemes. We notice that the currents I_1^{SC}, I_2^{SC} are set by parameters Y_{10}, Y_{20} which depend practically on all elements of the two-port network, except Y_1, Y_2 .

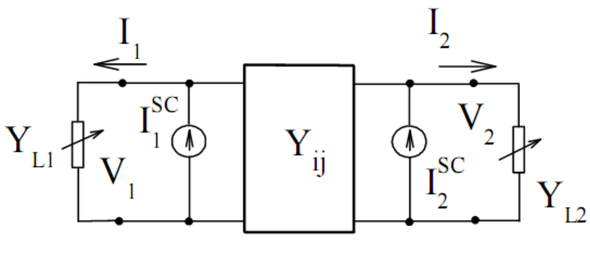


Fig.8. A traditional equivalent generator of an active two-port network

Therefore, at possible changes, for example, of a conductivity Y_3, it is necessary to make recalculation of values of these generators of a current that is inconvenient. The conductivity Y_3 can be a part of a possible third load. In this sense, the parameters of generalized equivalent generator, offered for an active two-pole, do not depend on such element of a circuit [4], [5].

Let us introduce, similarly, the generalized equivalent generator for the active two-port network on the basis of coordinates of the centers of bunches of straight lines and the relationships (3-6). We consider, at first, the generalized equivalent generator of this two-port network concerning load Y_{L1} which is the active two-pole with a changeable element Y_{L2} in Fig. 9,a. The family of external or load characteristics $I_1(V_1)$ with a changeable parameter Y_{L2} is presented in Fig. 9,b. This family represents the bunch of the straight lines with familiar center G_2 and center's coordinates correspond to expressions (5), (6).

Accordingly, at change of the load Y_{L1}, the bunch of the straight lines is formed with the center G_1, coinciding with the beginning of the coordinates. The equation of this bunch is obvious:

$$I_1 = Y_{L1} V_1 .$$

The equation of the bunch with the parameter Y_{L2} is obtained from the expressions (1), (2) and accepts a view:

$$(I_1 - I_1^{G2}) = -Y_{i1}(V_1 + V_1^{G2}), \quad (39)$$

where a internal conductivity of the active two-pole is:

$$Y_{i1} = Y_{11} - \frac{(Y_{12})^2}{Y_{L2} + Y_{22}} . \quad (40)$$

Let us transform the expression (39) to a relative form:

$$\frac{I_1}{I_1^{G2}} - 1 = -\frac{Y_{i1}}{Y_{L1}^{G2}} \left(\frac{V_1}{V_1^{G2}} + 1 \right) \quad (41)$$

Thus, the value Y_{L1}^{G2} is a scale for the internal conductivity. The expressions (39), (41) allow making directly the scheme of the generalized equivalent generator in Fig. 9,c.

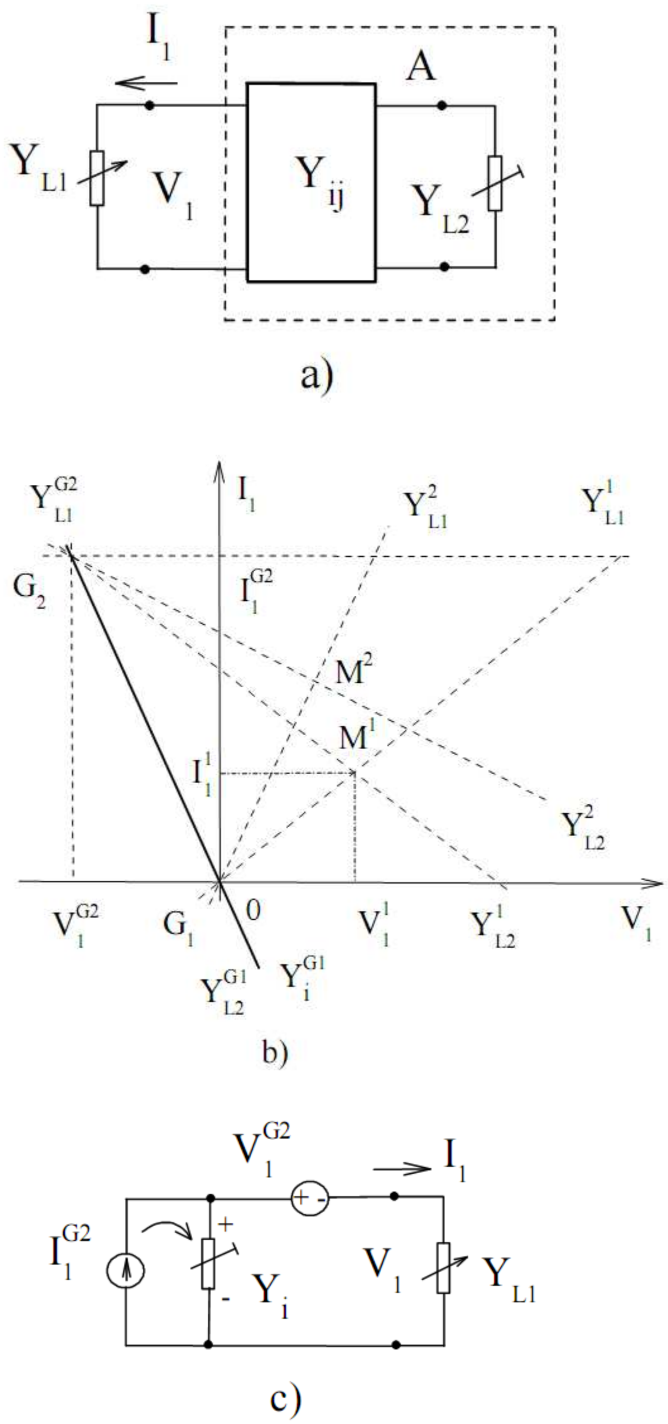


Fig.9. An active two-pole with a changeable element Y_{L2} : a)- scheme of this active two-pole; b)- family of load characteristics; c)- generalized equivalent generator

The value $Y_{i1}^{G1} = -Y_{L1}^{G2} = 1.5$ is the characteristic value for the conductivity Y_i. In this case, a voltage of the conductivity Y_i is equal and opposite in a direction to the voltage of the source V_1^{G2}. Therefore, $I_1 = 0, V_1 = 0$ for all values of conductivity Y_{L1}.

Similarly, relationships of type of the expressions (39), (40) are obtained for a second load Y_{L2}. Therefore, the whole scheme of the equivalent generator of the active two-port network can be constituted in Fig.10,a. This scheme gives a evident representation about mutual influence of loads and allows to carry out calculations.

Also, it is possible to obtain one more scheme of the equivalent generator in Fig. 10,b. For this purpose, we consider the expression (23) for parameter Y_{11}:

$$Y_{11} = \frac{I_1^{G2} - I_1^{SC}}{U_1^{G2}} = \frac{J_1}{V_1}.$$

This expression defines input conductivity of the passive two-port network at short circuit of the first load and short circuit of the second pair terminals of the two-port network in Fig.11,a. The similar relationship is obtained for parameter Y_{22} in Fig. 11,b. The association of these two schemes leads to the scheme in Fig.10,b.

Taking into account the superposition theorem, the values of all entering sources of a current and voltage are decreased twice in the scheme in Fig. 10,b. Calculations prove such scheme of the equivalent generator.

We will obtain a system of equations, which describes the given scheme of the equivalent generator. Using the specified designations in Fig. 10,b, it is possible to write down:

$$U_1 = \frac{V_1^{G2}}{2} + V_1, \quad J_1 = \frac{I_1^{G2}}{2} - I_1,$$

$$U_2 = \frac{V_2^{G1}}{2} + V_2, \quad J_2 = \frac{I_2^{G1}}{2} - I_2. \qquad (42)$$

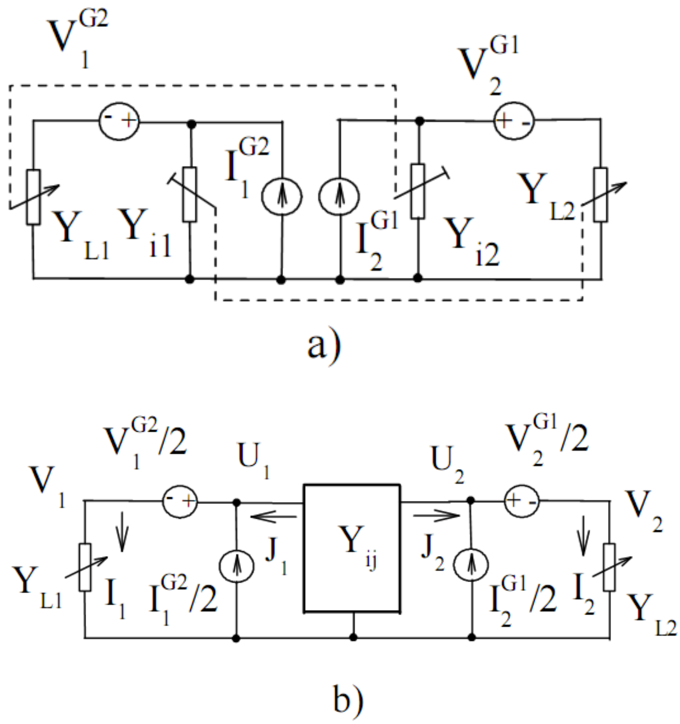


Fig.10. The proposed equivalent generator of the active two-port network: a)- is as an association of two active two-poles: b)-is as generalization of the equivalent generator of active two-port network

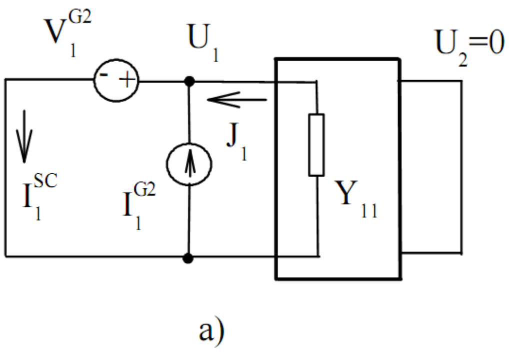
Following relationships take place for the passive two-port network:

$$J_1 = Y_{11}U_1 - Y_{12}U_2$$

$$J_2 = -Y_{12}U_1 + Y_{22}U_2.$$

Then, taking into account (42), after a number of transformations, the expression is obtained:

$$I_1 = Y_{11}V_1 - Y_{12}V_2 + \\ + \left(-Y_{11}\frac{V_1^{G2}}{2} + Y_{12}\frac{V_2^{G2}}{2} + \frac{I_1^{G2}}{2} \right) \qquad (43)$$



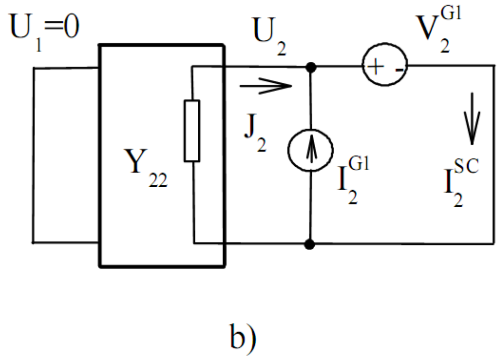


Fig.11. The definition of input conductivity of the passive two-port network: a)- at short circuit of the first load and short circuit of the second pair terminals; b)- at short circuit of the second load and short circuit of the first pair terminals

Let us compare this form to the expression (37). Then the expression in brackets should be equal to the short circuit current I_1^{SC}. We can check up its value:

$$I_1^{SC} = \left(-1.2\frac{10}{2} + 0.2\frac{15}{2} + \frac{15}{2}\right) = 3.$$

In addition, the structure of the short circuit current expression shows components of this current. A component $Y_{11}V_1^{G2}/2$ corresponds to own current of the two-port network and depends on its parameters. A component $I_1^{G2}/2$ is defined by a current source. A component $Y_{12}V_2^{G1}/2$ corresponds to a mutual current on account of a voltage source on the second pair of terminals. An expression for a current I_2 is obtained similarly to the equation (43).

The offered scheme of the equivalent generator is convenient at modeling of an active two-port network.

Parameters of voltage and current sources of the generalized equivalent generator of the active two-port network in Fig.10 do not depend on the conductivity Y_3. It represents a practical interest. This conductivity Y_3 can be a part of third changeable load. Therefore, only Y- parameters of the passive two-port network are recalculated.

5 Conclusions

1. The projective geometry adequately interprets "kinematics" of a circuit with changeable parameters of elements, allows to carry out deeper analysis and to receive relationships useful in practice. The given approach is applicable to the analysis of «flow » kind processes of the various physical natures.

2. The application of the projective coordinates allows receiving the equation of the active two-port network in a normalized or relative form, to define scales for the currents and conductivities of loads. Such approach allows estimating qualitative characteristics of current regimes, to compare the regime efficiency of different circuits.

3. The formulas of the recalculation of the currents, which possess the group properties at change of conductivity of the loads, are obtained. It allows expressing the final values of the currents through intermediate changes of the currents and conductivities.

4. The generalized equivalent generator of the active two-port network in the form of the passive two-port network and a set of the sources of a current and voltage is proposed. The parameters of these sources do not depend on conductivity which represents a possible third load.

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