

Consider again (1), with $n = 8$, $y = \frac{1}{1 + \Omega^{16}}$

Under the transformation $\Omega = \sin^8(\frac{\omega_1}{2}) + \sin^8(\frac{\omega_2}{2})$, i.e. $p = 4$, one gets the zero-phase filter with magnitude response in Fig.3

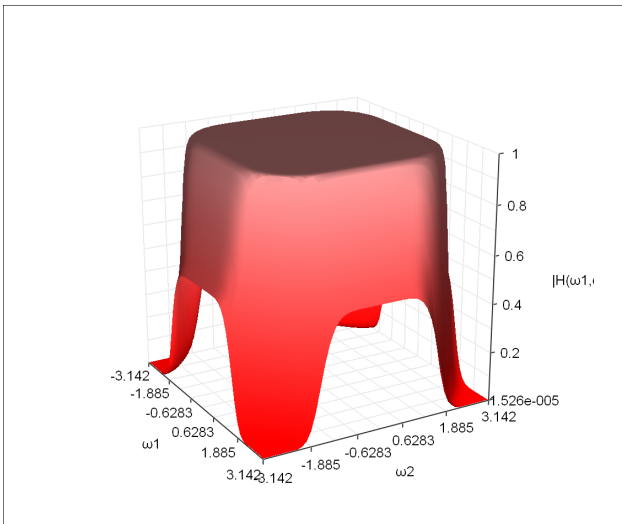


Fig.3. Plot $H(j\omega_1, j\omega_2) = \frac{1}{1 + (\sin^8(\frac{\omega_1}{2}) + \sin^8(\frac{\omega_2}{2}))^{16}}$

By considering $\varepsilon = 2$, we have $y = \frac{1}{1 + \varepsilon^{16}\Omega^{16}}$

Now the transformation $\Omega = \sin^8(\frac{\omega_1}{2}) + \sin^8(\frac{\omega_2}{2})$

$$y = \frac{1}{1 + \varepsilon^{16}(\sin^8(\frac{\omega_1}{2}) + \sin^8(\frac{\omega_2}{2}))^{16}} \text{ yields}$$

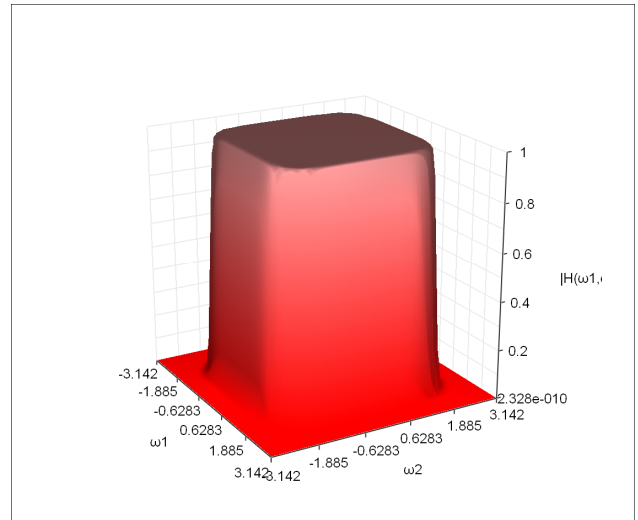


Fig.4.

$$\text{Plot } H(j\omega_1, j\omega_2) = \frac{1}{1 + 2^{16}(\sin^8(\frac{\omega_1}{2}) + \sin^8(\frac{\omega_2}{2}))^{16}}$$

with cut-off frequencies to be given by

$$\varepsilon^{16}(\sin^8(\frac{\omega_1}{2}) + \sin^8(\frac{\omega_2}{2}))^{16} = 1$$

or equivalently

$$\sin^8(\frac{\omega_1}{2}) + \sin^8(\frac{\omega_2}{2}) = \frac{1}{2}$$

$$\text{or } \left\| (\sin(\frac{\omega_1}{2}), \sin(\frac{\omega_2}{2})) \right\|_8 = \sqrt[8]{.5} \approx 0.917$$

On the other hand

$$\left\| (\sin(\frac{\omega_1}{2}), \sin(\frac{\omega_2}{2})) \right\|_8 \approx \max \left(\left| \sin(\frac{\omega_1}{2}) \right|, \left| \sin(\frac{\omega_2}{2}) \right| \right)$$

i.e. the cut-off frequencies can be defined by

$$\omega_1, \omega_2 = 2 \cdot \arcsin(0.917) = 2.3209$$

The previous filters are symmetrical in ω_1, ω_2 thanks to $\Omega = \sin^{2p}(\frac{\omega_1}{2}) + \sin^{2p}(\frac{\omega_2}{2})$. Introducing now the real parameters a, b , one takes the transformation

$\Omega = a^{2p} \sin^{2p}(\frac{\omega_1}{2}) + b^{2p} \sin^{2p}(\frac{\omega_2}{2})$ that provide the (non-causal) filter

$$H(z_1^{-1}, z_2^{-1}) = \frac{1}{1 + \varepsilon^{2n} \left(a^{2p} \left(\frac{2 - z_1^{-1} - (z_1^{-1})^{-1}}{4} \right)^p + b^{2p} \left(\frac{2 - z_2^{-1} - (z_2^{-1})^{-1}}{4} \right)^p \right)^{2n}}$$

(5)

Example III.3

Starting from $y = \frac{1}{1 + \varepsilon^{16} \Omega^{16}}$ ($n = 8$) and using

$\Omega = a^{2p} \sin^{2p}(\frac{\omega_1}{2}) + b^{2p} \sin^{2p}(\frac{\omega_2}{2})$, with $p = 4$, compute the parameters ε, a , and b , in (5), such that cut-off frequencies to be $\omega_{10} = \pm\pi/2$, $\omega_{20} = \pm\pi/3$.

Solution: Our filter must be

$$y = \frac{1}{1 + \varepsilon^{16} \left(a^8 \sin^8(\frac{\omega_1}{2}) + b^8 \sin^8(\frac{\omega_2}{2}) \right)^{16}}$$

Without loss of generality the constant ε is incorporated to a and b . So, one can set $\varepsilon = 1$. The cut-off frequencies are found from the equation

$$\left(a^8 \sin^8(\frac{\omega_1}{2}) + b^8 \sin^8(\frac{\omega_2}{2}) \right)^{16} = 1$$

Or equivalently

$$a^8 \sin^8(\frac{\omega_1}{2}) + b^8 \sin^8(\frac{\omega_2}{2}) = 1$$

Since,

$$\left\| \left(a \sin(\frac{\omega_1}{2}), b \sin(\frac{\omega_2}{2}) \right) \right\|_8 \approx \max \left(\left| a \sin(\frac{\omega_1}{2}) \right|, \left| b \sin(\frac{\omega_2}{2}) \right| \right)$$

Using $\omega_{10} = \pm\pi/2$, $\omega_{20} = \pm\pi/3$ from $\max \left(\left| a \sin(\frac{\omega_{10}}{2}) \right|, \left| b \sin(\frac{\omega_{20}}{2}) \right| \right) = 1$ we take

$$a = \sqrt{2}, b = 2$$

Hence, our filter is

$$H(z_1^{-1}, z_2^{-1}) = \frac{1}{1 + \left(2^4 \left(\frac{2 - z_1^{-1} - (z_1^{-1})^{-1}}{4} \right)^4 + 2^8 \left(\frac{2 - z_2^{-1} - (z_2^{-1})^{-1}}{4} \right)^4 \right)^{16}}$$

and can be depicted easily in Fig.5 taking

$$\left| H(\omega_1, \omega_2) \right| = \frac{1}{1 + \left(2^4 \sin^8(\frac{\omega_1}{2}) + 2^8 \sin^8(\frac{\omega_2}{2}) \right)^{16}}$$

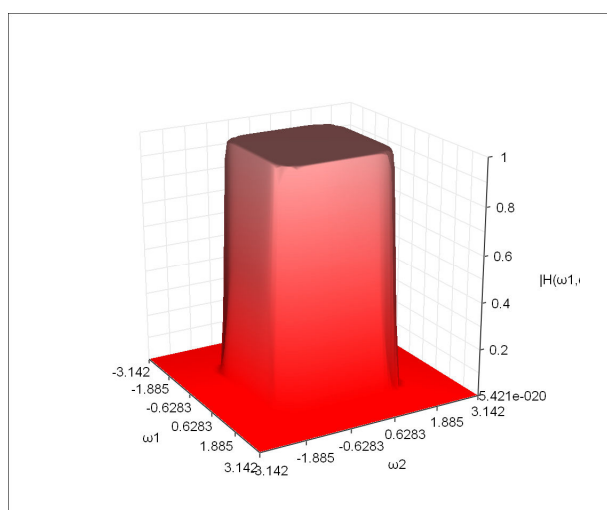


Fig.5: Magnitude response of the filter of Example III.3

In Fig.5, the magnitude response is illustrated if $\omega_2 = 0$

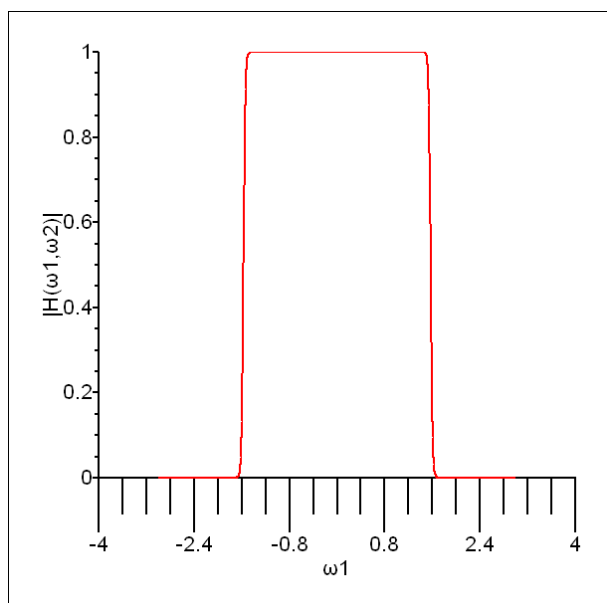


Fig.6: Magnitude response of the filter of Example III.3 if $\omega_2 = 0$

4 Conclusion

In this brief, new transformations for designing 2-D (Two-Dimensional) IIR filters are introduced. The resulting non-causal IIR filter is proven to be BIBO stable. Some numerical examples illustrate the validity and usefulness of the proposed transformation. Analogous transformations can be

derived for highpass, bandstop and bandpass non-causal 2-D IIR filters.

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