Possible Solution of Decoupling and Invariance of Multi-variable Control Loop by Using Binding and Correction Members

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Abstract: - The paper describes one of possible methods how to control of multi-variable control loops. In this case the used method of control uses the so called main controllers, binding members and correction members. This control method combines classical approach for ensuring decoupling of multi-variable control loop by means of binding members and the use of the correction members for ensuring invariance of multi-variable control loop by means of two approaches. Main controllers can be proposed by arbitrary single-variable synthesis method. Simulation verification of the used control method is carried out for example of three-variable loop of a steam turbine.

Key-Words: - Decoupling of control loop, control, invariance of control loop, simulation.

1 Introduction

It is often required, at large numbers controlled plants, that their input and output variables have to be controlled simultaneously. The examples these controlled plants are e.g. aircraft autopilots, air-conditioning plants, chemical processes, distillation columns, steam boilers, spacecraft, steam turbines, etc. [1]. In these cases, it means that there is not only larger number of independent SISO (single-variable) control loop. These control loops are complex with several controlled variables where separate variables are not mutually independent. Mutual coupling of controlled variables is usually given by simultaneous action of each of input variables of controlled plant (manipulated variables and disturbance variables) to all controlled variables. These control loops are called MIMO (multi-variable) control loops and they are a complex of mutually influencing simpler control loops [1]. Special case of MIMO control loop is SISO control loop that have only one input signal (disturbance variable, manipulated variable) and one output signal (controlled variable) [2], [3].

One of possible examples of MIMO controlled plant is also an above mentioned steam turbine [4]–[6]. In the experimental part of the paper is considered three-variable controlled plant of steam turbine [4]. The selected method of control of the MIMO controlled plant uses the so called main controllers, binding members and correction members [1]. The main controllers can be designed via arbitrary SISO synthesis methods, e.g. [1], [7]–[12]. Binding members are determined from main controllers and from parameters of MIMO controlled plant and ensuring decoupling of MIMO control loop. Correction members are determined from parameters of MIMO controlled plant and ensure an elimination of influence of disturbance variables on MIMO control loop, i.e. the correction members ensure invariance of MIMO control loop. In the next part of the paper are described two approaches for ensuring invariance of MIMO control loop by means of correction members. The control method of MIMO control loop, described in the next part of this paper, is considered for MIMO controlled plant with same number input signals and output signals.

All simulation experiments were performed in the simulation mathematical education and research software MATLAB/SIMULINK [12]. MATLAB is a widely used tool not in education but also in research; in addition to that, many researchers have produced a wide variety of educational tools based on MATLAB [14], [15].

2 Multi-Variable Control Loop

2.1 Description of used multi-variable control loop

It will be considered multi-variable control loop with measurement of disturbance variables via the following figure (see Fig. 1) [1].

main controllers R_{11}, R_{22}, R_{33} are determined by means of arbitrary SISO synthesis method for modified diagonal transfer functions $S_{11,x}$, $S_{22,x}$ a $S_{33,x}$ (12). After that binding members are determined by using of (8). This relation serves to ensuring decoupling of control loop. Finally parameters of correction members, which ensured invariance of control loop, are calculated by using of (10) (absolute invariance) or (11) (approximate invariance).

To determination of modified transfer functions $S_{ii,x}$ (i = 1, 2, 3) was used (12), then

$$\begin{aligned}
S_{11,x}(s) &= S_{11}\frac{s_{11}}{s_{11}} + S_{12}\frac{s_{12}}{s_{11}} + S_{13}\frac{s_{13}}{s_{11}} = \\
&= S_{11} + S_{12}\frac{S_{23}S_{31}-S_{21}S_{33}}{S_{22}S_{33}-S_{23}S_{32}} + S_{13}\frac{S_{21}S_{32}-S_{22}S_{31}}{S_{22}S_{33}-S_{23}S_{32}} \\
S_{22,x}(s) &= S_{21}\frac{s_{21}}{s_{22}} + S_{22}\frac{s_{22}}{s_{22}} + S_{23}\frac{s_{23}}{s_{22}} = \\
&= S_{21}\frac{S_{13}S_{32}-S_{12}S_{33}}{S_{11}S_{33}-S_{13}S_{31}} + S_{22} + S_{23}\frac{S_{12}S_{31}-S_{11}S_{32}}{S_{11}S_{33}-S_{13}S_{31}} \\
S_{33,x}(s) &= S_{31}\frac{s_{31}}{s_{33}} + S_{32}\frac{s_{32}}{s_{33}} + S_{33}\frac{s_{33}}{s_{33}} = \\
&= S_{31}\frac{S_{12}S_{23}-S_{13}S_{22}}{S_{11}S_{22}-S_{12}S_{21}} + S_{32}\frac{S_{13}S_{21}-S_{11}S_{33}}{S_{11}S_{22}-S_{12}S_{21}} + S_{33}
\end{aligned} \qquad (25)$$

hence

$$S_{11,x} = \frac{1.294}{8.19s+1},\ S_{22,x} = \frac{-1.057}{0.479s+1},\ S_{33,x} = \frac{-1.02}{0.946s+1}. \qquad (26)$$

At design of parameters of main controllers R_{11}, R_{22}, and R_{33}, which are diagonal elements of transfer function matrix of controller $\boldsymbol{G}_R(s)$, the following SISO synthesis methods were used

a) Whiteley method [1], [12]
b) method of optimal module [1], [8]
c) method of desired model [8]
d) pole placement method [9], [11].

It is used a polynomial approach at design of parameters of main controllers by means of pole placement method. Further it is considered that roots of the closed control loop (poles), which influence quality and stability of the closed control loop, are selected like multiple roots.

It is possible to use also other methods of parameters design of main (diagonal) controllers of transfer function matrix of controller $\boldsymbol{G}_R(s)$, e.g. Ziegler-Nichols methods, Naslin method, the symmetrical optimum method, SIMC method, etc. [1], [7]–[12].

To calculation of binding members R_{ij} ($i \neq j$), which ensuring decoupling of control loop, was used (8). It means these binding members were gained from the following relations

$$\begin{aligned}
R_{12}(s) &= \frac{s_{21}}{s_{22}}R_{22} = \frac{S_{13}S_{32}-S_{12}S_{33}}{S_{11}S_{33}-S_{13}S_{31}}R_{22} \\
R_{13}(s) &= \frac{s_{31}}{s_{33}}R_{33} = \frac{S_{12}S_{23}-S_{13}S_{22}}{S_{11}S_{22}-S_{12}S_{21}}R_{33} \\
R_{21}(s) &= \frac{s_{12}}{s_{11}}R_{11} = \frac{S_{23}S_{31}-S_{21}S_{33}}{S_{22}S_{33}-S_{23}S_{32}}R_{11} \\
R_{23}(s) &= \frac{s_{32}}{s_{33}}R_{33} = \frac{S_{13}S_{21}-S_{11}S_{33}}{S_{11}S_{22}-S_{12}S_{21}}R_{33} \\
R_{31}(s) &= \frac{s_{13}}{s_{11}}R_{11} = \frac{S_{21}S_{32}-S_{22}S_{31}}{S_{22}S_{33}-S_{23}S_{32}}R_{11} \\
R_{32}(s) &= \frac{s_{23}}{s_{22}}R_{22} = \frac{S_{12}S_{31}-S_{11}S_{32}}{S_{11}S_{33}-S_{13}S_{31}}R_{22}
\end{aligned} \qquad (27)$$

To determine of correction members KC, which ensuring invariance of control loop, was used (10) and (11). Relation (10) ensures absolute invariance of control loop, e.g. the first four correction members are following

$$\begin{aligned}
KC_{11}(s) &= \frac{1}{\det\boldsymbol{G}_S}\left(s_{11}S_{V,11} + s_{21}S_{V,21} + s_{31}S_{V,31}\right) \\
KC_{12}(s) &= \frac{1}{\det\boldsymbol{G}_S}\left(s_{11}S_{V,12} + s_{21}S_{V,22} + s_{31}S_{V,32}\right) \\
KC_{13}(s) &= \frac{1}{\det\boldsymbol{G}_S}\left(s_{11}S_{V,13} + s_{21}S_{V,23} + s_{31}S_{V,33}\right) \\
KC_{21}(s) &= \frac{1}{\det\boldsymbol{G}_S}\left(s_{12}S_{V,11} + s_{22}S_{V,21} + s_{32}S_{V,31}\right) \\
&\vdots
\end{aligned} \qquad (28)$$

where $\det\boldsymbol{G}_S$ is a determinant of transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$ and s_{ki} (k, i = 1, 2, 3) are algebraic supplements of separate elements of a transfer function matrix of controlled plant $\boldsymbol{G}_S(s)$ (see (25) or (27)).

Relation (11) ensures that control loop is approximately invariant, hence

$$\begin{aligned}
&KC_{11} = \frac{S_{V11}}{S_{11}},\ KC_{22} = \frac{S_{V22}}{S_{22}},\ KC_{33} = \frac{S_{V33}}{S_{33}} \\
&KC_{ij} = 0 \qquad i \neq j,\ \ i, j = <1, 2, 3>.
\end{aligned} \qquad (29)$$

Transfer function matrix of controllers $\boldsymbol{G}_R(s)$ with utilization of four chosen SISO synthesis methods for design of parameters of main controllers is following

Table 2 - Quality of control for simulation example of three-variable control loop by ensuring invariance of control loop via (35)

Synthesis method *)	J_{K1}		J_{K2}		J_{K3}	
	ISE	ITAE	ISE	ITAE	ISE	ITAE
1	7.853	1471	0.669	296.0	1.499	1059
2	7.959	1493	0.679	299.5	1.524	1073
3	3.243	786.8	2.488	1332	4.328	3607
4	1.215	252.6	1.396	1093	0.598	845.1

*) Numbers in the previous tables (Table 1 and Table 2) in the column “Synthesis method” represent the used SISO synthesis method of at design of parameters of main controllers, i.e.: 1 - Whiteley method, 2 - method of optimal module, 3 - method of desired model, 4 - pole placement method.

Optimal adjustment of control loop is considered here from the point of view of minimal size of ISE criterion or ITAE criterion (Table 1 and Table 2). However quite different point of view can be rally considered for optimal adjustment. Namely requirements for the smallest overshooting and for the shortest time of control are generally valid for optimal adjustment. However these requirements are antagonistic and therefore the optimal adjustment of controller is always a compromise between them.

Degree of internal coupling of MIMO controlled plant is often evaluated via RGA (Relative Gain Array) [18], [20], [21]. RGA values are depended on frequency. These values are usually determined for frequency equal to zero, i.e. for steady state. From the point of view of control, it is ideal state when values of diagonal elements of RGA matrix are approaching to the value of one and aside-from-diagonal elements of RGA matrix approaching to the value of zero. The RGA matrix ($\boldsymbol{\Lambda}$) can be calculated from the following equation

$$\boldsymbol{\Lambda} = \boldsymbol{G}(\mathrm{j}\omega) \otimes (\boldsymbol{G}^{-1}(\mathrm{j}\omega))^T = \boldsymbol{G}(0) \otimes (\boldsymbol{G}^{-1}(0))^T \quad (38)$$

where $\boldsymbol{G}(s)$ is a transfer function matrix of examined object ($s = \mathrm{j}\omega$), e.g. MIMO controlled plant, MIMO closed loop, $\otimes$ operator implies an element by element multiplication (Schur product).

It is possible to use RGA values to compare properties of original MIMO controlled plant and MIMO control loop from the point of view degree internal coupling.

RGA matrix of three-variable controlled plant is following

$$\boldsymbol{\Lambda}(\boldsymbol{G}_S(0)) = \begin{bmatrix} 0.8548 & 0.0912 & 0.0540 \\ 0.0942 & 0.9068 & -0.0010 \\ 0.0510 & 0.0020 & 0.9470 \end{bmatrix} \quad (39)$$

and RGA matrix of closed loop transfer function matrix $\boldsymbol{G}_{W/Y}(s)$ of three-variable control loop, for all MIMO controller (30) - (33), is in the form

$$\boldsymbol{\Lambda}(\boldsymbol{G}_{W/Y}(0)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (40)$$

It is obvious from the simulations of control loop shown above (see Fig. 10 - Fig. 17) that the condition of decoupling of control loop was fulfilled. Fulfilment of this condition was ensured via binding members, which are aside-from-diagonal elements of transfer function matrix of controller $\boldsymbol{G}_R(s)$ (30) - (33). Binding members were determined from so called main controllers, which are main diagonal elements of the transfer function matrix of controller. The determination of main controllers is carried out by any SISO synthesis method for modified diagonal elements of the transfer function matrix of controlled plant.

From the simulation of control loop is also obvious that the control loop is absolute invariant (see Fig. 10 - Fig. 13 and (34)), let us say, approximate invariant (see Fig. 14 - Fig. 17 and (35)). In this second case influence of disturbance variables is eliminated only at steady state. Fulfilment of this condition was ensured via separate elements of transfer function matrix of correction members $\boldsymbol{G}_{KC}(s)$ (34) - (35), i.e. via correction members *KC*. Correction members were determined from transfer function matrix of the controlled plant and disturbance variables.

4 Conclusion

In this paper was to described and shown the one of possible approaches to control of MIMO control loops. Advantage of described and used control method is its simplicity. This control method enables to use any known SISO synthesis method to design of main controllers. The control method combines classical way of ensuring decoupling of control loop via binding members, which are aside-from-diagonal elements of transfer function matrix of controller $\boldsymbol{G}_R(s)$, and the use of the correction members for ensuring absolute invariance or approximate invariance of MIMO control loop. Simulation verification of proposed control method was presented on three-variable control loop of steam turbine.

Designed parameters of matrix controllers and correction members have good results of the control and fulfilled basic control requirements such as the stability, the reference signal tracking and disturbance attenuation.

The described and used control method is valid under the following condition, i.e. this method can be used only for MIMO controlled system with same number of input and output signals. MIMO controlled plant containing transport delay, non-minimal phase or having high order dynamics can be also cause of certain limitations of the control method.

The future work will be focused on the reduction of limitations of proposed control method, verification of alternative approach to ensuring decoupling of control loop [19] and also simulation verification of proposed, let us say, modified version of control method for other MIMO controlled plants, e.g. model of balance platform system [22], model of heating system [23].

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