Nonlinear Integrator Backstepping for Traffic Flow Speed Control of Automated Freeway System

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Abstract: - In an automated freeway system environment, computers replace artificial driving and control vehicles accompany movement, which can greatly reduce randomness. In this paper, we propose a traffic flow speed controller with lower complexity which can send speed commands to regulate the speeds of vehicles in each section of a freeway. On the basis of the theory of nonlinear integrator backstepping, the controller can guarantee exponential convergence of the traffic flow speed at each section to the desired speed. Compared with former method, the establishment of this controller need not solve inverse matrix, and therefore, reduces algorithm's complexity, improves the algorithm efficiency and automatically satisfies the controllability condition. Simulations show that this controller can effectively reduce congestion and helps to achieve a smooth traffic flow on a congested freeway.

Key-Words: - Speed controller, automated freeway system, traffic control, nonlinear integrator backstepping, traffic flow model, exponential convergence.

1 Introduction

The freeway is not only an important routeway between cities, but also a key carrier of the urban traffic. It is a complex system which contains interaction between humans, vehicles and roads. By regarding this kind of system as a nonlinear system with some certain or random law, and applying the theory and technology in the fields of automatic control, computer and communication, we can transform the system into a controllable running system and maximize the system's performance.

On-ramp metering [1], [2] at freeway entrance ramps is a primary method to regulate the flow of incoming traffic to a freeway. But this approach provides very little feedback information and the traffic system is most of the time open-loop. Therefore, the freeway system is apt to present various types of instabilities.

Traffic flow instabilities such as traffic congestion are often caused by inappropriate speeds and headways that people choose when they operate their vehicles. In an automated highway system (AHS) environment, the driver's actions are replaced by those of a computer control system that is designed to optimize traffic flow, which will reduce people's subjective factors as well as greatly decrease the randomicity.

Current research [3]-[10] on automated freeway system is concerned with providing appropriate feedback control commands to the traffic system on the microscopic level. The goal of the macroscopic control approach is to prevent congestion, or at least avoid its amplification caused by traffic inhomogeneities.

Reference [3] proposes a traffic density controller for automated highway systems. The controller selects the traffic density as controlled variable and achieves tracking the desired density at each section. It drastically reduces congestion and helps to achieve a smooth traffic flow on a congested freeway. But, this controller need more computational efforts to solve the inverse matrix and probably gets a non-unique solution.

This paper proposes an improved controller design method for the controller proposed in [3]. This design selects the traffic flow speed as controlled variable, uses the theory of nonlinear integrator backstepping and can guarantee exponential convergence of the traffic flow speed at each section to the desired speed. It indirectly achieves correspondingly homogeneous traffic density, drastically reduces congestion and helps to achieve a smooth traffic flow on a congested freeway.

Compared with the method in [3], this controller need not solve inverse matrix, and therefore, reduces algorithm's complexity and automatically satisfies the controllability condition. Besides, since it improves the real-time performance, this controller will help a computer system to control more large-scale automated freeway system.

This paper is organized as follows. In sections 2 and 3, we give the problem statement and a discrete traffic flow model. In section 4, we present the detail of the controller design and analysis. In section 5, we provide the simulation results which show the benefits of our design. At last, the conclusion is given in section 6.

2 Problem Statement

Consider a freeway's a long segment which is divided into N sections. The length of each section is L_i, $i=1,2,...,N$. The initial traffic volume entering section 1 is assumed to be $q_0(n)$ vehicles per hour at sampling time nT ($n=1,2\cdots$) (nT is the time-discretization step size.). The desired space mean speed of traffic flow (traffic flow speed) in section i at sampling time nT is assumed to be $v_{di}(n)$.

The objective of the macroscopic control approach is to select an appropriate control law $u_i(n)$ which can guarantee exponential convergence of the traffic flow speed at each section to the desired speed, namely, $v_i(n)\to v_{di}(n)$ as $n\to\infty$.

3 Traffic Flow Model

Papageorgiou et al. [11]-[13] (1983, 1989, 1990a) proposed and improved a traffic flow model which has been tested and validated by real traffic data from the Boulevard Peripherique in Paris. However, Karaaslan et al. [3] (1990) demonstrated several shortcomings of Papageorgiou's model and proposed a more realistic one. This model's description is in the following form:

$$q_i(n)=\alpha k_i(n)v_i(n)+(1-\alpha)k_{i+1}(n)v_{i+1}(n) \tag{1}$$

$$k_i(n+1)=k_i(n)+\frac{T}{L_i}[q_{i-1}(n)-q_i(n)+r_i(n)-s_i(n)] \tag{2}$$

$$\begin{aligned} v_i(n+1)&=v_i(n)+\frac{T}{\tau}\{V_e[k_i(n)]-v_i(n)\}\\ &+\frac{T}{L_i}\frac{k_{i-1}(n)}{k_i(n)+\kappa^1}v_{i-1}(n)\\ &\times\left[\sqrt{v_{i-1}(n)v_i(n)}-v_i(n)\right]-\frac{\mu(n)T}{\tau L_i}w_i(n)\end{aligned} \tag{3}$$

where

$$V_e(k_i)=v_f[1-(\frac{k_i}{k_{jam}})^l]^m \tag{4}$$

$$\mu(n)=\begin{cases}\mu_1\dfrac{\rho}{k_{jam}-k_{i+1}(n)+\sigma} & if \quad k_{i+1}(n)>k_i(n)\\ \mu_2 & otherwise\end{cases} \tag{5}$$

$$w_i(n)=\frac{k_{i+1}(n)-k_i(n)}{k_i(n)+\kappa} \tag{6}$$

The parameters' meanings in above formulas are as follows.

$k_i(n)$ density in section i at time nT (in vehicles per kilometer per lane), where $n=1,2,\cdots$;

$v_i(n)$ space mean speed of vehicles in section i at time nT (in km h^{-1});

$q_i(n)$ traffic volume leaving section i ,entering section $i+1$ at time nT (in vehicles per hour);

$r_i(n)$ on-ramp traffic volume for section i (in vehicles per hour);

$s_i(n)$ off-ramp traffic volume for section i (in vehicles per hour);

L_i length of i th section(in km);

T time-discretization step size(in h).

Here α , ρ , σ , κ' , τ , μ_1 and μ_2 are positive constants with $0\le\alpha\le1$, and k_{jam} is the maximum possible density.

The variable $V_e[k_i(\bullet)]$ in (4) represents the density-dependent equilibrium spreed. $l>0$ and $m>1$ are real-valued parameters, and v_f is the free speed, which can be estimated from traffic data.

The term $w_i(n)$ in (3) under manual operation depends on the downstream density, and can be expressed as (6) where the positive constant κ is introduced to prevent abnormal growth of the velocity for section i when its density is very low.

In reference [3], for an AHS operating under homogeneous heavy traffic conditions, S_d which indicates the distance between two vehicles and V_e are defined according to the adopted safety policy.

$$S_d=\varphi(V_e)=hV_e+c \tag{7}$$

$$V_e(k_i)=\varphi^{-1}(\frac{1}{k_i})=\frac{1}{hk_i}-\frac{c}{h} \tag{8}$$

This method is somewhat reasonable. But the relationship between S_d and V_e , including time, parameters of brake system of vehicles, headway and so on, is very complex and not simple linear. At the same time, the S_d has close relationship with vehicles' type and the shortest distance between different types of vehicles has large difference. Furthermore, the so called "safety distance" is based on the car following model which belongs to microcosmic model and only concerns behaviours of single vehicle. Since the whole traffic flow is not a simple summation of every single vehicle's behaviour, the macroscopic model can not completely accord with the microcosmic model. It remains one of the difficulties not completely solved until now. Therefore, (8) is not completely correct.

This paper adopts (4) which was proposed by Papageorgiou [12], [13], [14] (1989, 1990a, 1990b) and commendably achieves the relationship between v_e and k_i . Namely, this paper use the modified freeway traffic flow model given by Karaaslan et al. [3] (1990).

In an automated freeway systems, the designed control law $u_i(n)$ replaces the last term $\frac{\mu(n)T}{\tau L_i}w_i(n)$ in (3), which indicates the controller sends the control command $u_i(n)$ to the vehicles in section i at sampling time nT and $u_i(n)$ will adjust the $v_{i+1}(n+1)$.

Let

$$f_i(n) = v_i(n) + \frac{T}{\tau}\{V_e[k_i(n)] - v_i(n)\} + \frac{T}{L_i}\frac{k_{i-1}(n)}{k_i(n)+\kappa^1}v_{i-1}(n) \times\left[\sqrt{v_{i-1}(n)v_i(n)} - v_i(n)\right] \quad (9)$$

On the basis of the above analysis, in this paper, the complete behavior of traffic flow in AHS is governed by the following equation:

$$q_i(n) = \alpha k_i(n)v_i(n) + (1-\alpha)k_{i+1}(n)v_{i+1}(n) \quad (10)$$

$$k_i(n+1) = k_i(n) + \frac{T}{L_i}[q_{i-1}(n) - q_i(n) + r_i(n) - s_i(n)] \quad (11)$$

$$v_i(n+1) = f_i(n) - u_i(n) \quad (12)$$

$$V_e(k_i) = v_f[1-(\frac{k_i}{k_{jam}})^l]^m \quad (13)$$

The boundary conditions are as same as [3]'s.

$$k_0(n) = \frac{q_0(n)/v_1(n) - (1-\alpha)k_1(n)}{\alpha} \quad (14)$$

$$v_0(n) = v_1(n) \quad (15)$$

$$k_{N+1}(n) = k_N(n) \quad (16)$$

$$v_{N+1}(n) = v_N(n) \quad \forall n. \quad (17)$$

4 Traffic Flow Speed Controller Design

We propose a macroscopic roadway traffic flow speed controller for AHS. The controller uses nonlinear integrator backstepping [15, 16, 17], which is a kind of systematic design method of controller and appropriate for uncertain system, to achieve the control law needed to track a desired speed profile. The following general lemma is used in the design and analysis of the proposed roadway controller.

4.1 Lemma

Consider the following discrete-time system:

$$z(n+1) = cz(n) + u(n)\ , \ z(0) = z_0 \quad (18)$$

Where c is a constant and $|c| < 1$. Then $u(n) \to 0$ exponentially implies $z(n) \to 0$ exponentially.

4.2 Controller Design

The main idea of the controller design is to apply backstepping and use Lemma twice. Selecting the traffic flow speed as controlled variable, designing the control input $u_i(n)$, we can achieve the desired $v_{di}(n)$ utilizing (12).

The controller design consists of two steps.

Step 1.

Define the track error for section as

$$\xi_i(n) = v_i(n) - v_{di}(n)\ , \ n = 1,2,... \quad (19)$$

Then, with (12), it follows that

$$\begin{aligned}\xi_i(n+1) &= v_i(n+1) - v_{di}(n+1) \\ &= f_i(n) - u_i(n) - v_{di}(n+1) \\ &= c_\varepsilon \xi_i(n) + \eta_i(n)\end{aligned} \quad (20)$$

where

$$\eta_i(n) = f_i(n) - u_i(n) - v_{di}(n+1) - c_\varepsilon \xi_i(n) \quad (21)$$

From Lemma, we have $\xi_i(n) \to 0$ as $n \to \infty$ if $|c_\varepsilon| < 1$ and $\eta_i(n) \to 0$ as $n \to \infty$. The goal of the next step is to choose the control input $u_i(n)$ that guarantees $\eta_i(n) \to 0$ as $n \to \infty$.

Step 2.

From the Lemma and (21), we have

$$\eta_i(n) = c_\eta \eta_i(n-1) + \beta_i(n-1) \tag{22}$$

$$\begin{aligned}\beta_i(n-1) = {} & f_i(n) - u_i(n) - v_{di}(n+1) \\ & - c_\varepsilon \xi_i(n) - c_\eta \eta_i(n-1)\end{aligned} \tag{23}$$

Let

$$\xi_i(0) = \xi_i(1) = v_i(1) - v_{di}(1) \tag{24}$$

then

$$\eta_i(0) = \xi_i(1) - c_\varepsilon \xi(0) \tag{25}$$

If

$$u_i(n) = f_i(n) - v_{di}(n+1) - c_\varepsilon \xi_i(n) - c_\eta \eta_i(n-1) \tag{26}$$

then

$$\beta_i(n-1) = 0 \tag{27}$$

From Lemma, $\eta_i(n) \to 0$ as $n \to \infty$.
Then, the control law is

$$u_i(n) = f_i(n) - v_{di}(n+1) - c_\varepsilon \xi_i(n) - c_\eta \eta_i(n-1) \tag{28}$$

Compared with the method in [3], the proposed controller need not solve inverse matrix, and therefore, reduces algorithm's complexity, achieves smaller computational efforts, can not get non-unique solution and automatically satisfies the controllability condition.

4.3 Proof

Formula (28) provides the solving method of control law $u_i(n)$. Application of Lemma along with this solution to (22) yields $\eta_i(n) \to 0$ as $n \to \infty$, $\forall i$. The use of the Lemma with (20) ensures that as $n \to \infty$, $\forall i$. Hence $\forall i$, we have $v_i(n) \to v_{di}(n)$ exponentially as $n \to \infty$.

5 Simulation Research

Consider a freeway's a long segment which is divided into 12 sections. The length of each section is 500m. The parameters of traffic flow model are as follows:

$v_f = 93.1 km \cdot h^{-1}$, $k_{jam} = 110 vehicles \cdot km^{-1} \cdot lane^{-1}$,
$T = 15/3600h$, $l = 1.86$, $m = 4.05$, $\alpha = 0.95$,
$\kappa = 50 vehicles \cdot km^{-1} \cdot lane^{-1}$, $\kappa^1 = 55 vehicles \cdot km^{-1} \cdot lane^{-1}$,
$\mu_1 = 12 km^2 h^{-1}$, $\mu_2 = 6 km^2 \cdot h^{-1}$, $\tau = 20.4/3600h$,
$\rho = 120 vehicles \cdot km^{-1} \cdot lane^{-1}$,

Controller's parameters: $c_\varepsilon = 0.9$, $c_\eta = 0.9$.

Five cases are considered.

In 1,2,3,4 cases, the initial traffic volume entering section 1 is assumed to be 1500 vehicles per hour. The initial density and mean speed of each section are as shown in Table 1.

In the first case, as shown in Figs 1 and 2, no feedback from the roadway is applied, and therefore $u_i(n)$ is replaced with the corresponding term in (3). From these figures, we see that the initial traffic congestion in sections 6-8 causes a traffic jam.

In the second and third cases, we use the proposed controller to achieve desired traffic flow speeds of 45km/hour and 60km/hour respectively. The simulation results shown in Figs 3-8 demonstrate that the initial congested conditions are quickly dampened out by the proposed controller and the traffic flow is regulated to achieve the desired traffic flow speeds. At the same time, traffic densities are indirectly influenced and get to a correspondingly homogeneous state. For example, let us compare Fig.1 and Fig.2 with Fig.3 and Fig.4. In Fig.1, the velocities of traffic flow in sections 6-8 are very low, which denotes a traffic jam there. In Fig.3, at initial time, velocities of traffic flow in sections 6-8 are very low. But the velocities quickly increase with time and finally stabilize at the desired speed. Considering there is not controller in Fig.1 and there is the proposed controller in Fig.3, we can see the controller can rapidly banish congestion and achieve desired traffic flow speed. In Fig.2, the densities of traffic flow in sections 6-8 are very high, which also denotes a traffic jam there. In Fig.4, at initial time, densities of traffic flow in sections 6-8 are very high. But the densities quickly decrease with time and finally stabilize. Considering there is not controller in Fig.2 and there is the proposed controller in Fig.4, we can see the controller can rapidly banish congestion and achieve a smooth traffic flow density.

In the fourth case, we set the desired traffic flow speeds to 50km/h and assume that there exists interference as follows:

(a) The input traffic flow rate in section 1 increases exponentially from 1500 to 2000 vehicles per hour as shown in Fig. 9.

Its formula is $q_0 = 2000 - 500e^{-22(n-1)T}$.

(b) The on-ramp traffic volume for section 2(in vehicles per hour) is shown in Fig. 10.

Its formula is $r2 = 120 + 20\sin(15nT)$.

(c) The off-ramp traffic volume for section 10(in vehicles per hour) is shown in Fig. 11.

Its formula is $s10 = 150 + 30\sin(32nT + 50)$.

The simulation result shown in Fig.12 shows that the desired traffic flow speed is achieved exponentially with the proposed roadway controller.

In the fifth case, assuming the initial density and mean speed of each section are as shown in Table 2, setting the desired traffic flow speeds to 75km/h, we achieve the simulation under strong interferences which are as follows:

(a) The initial traffic volume entering section 1 is $q_0 = 1500 + 150\sin(18nT)$,

(b) The on-ramp traffic volume for section 2(in vehicles per hour) is $r2 = 220 + 35\sin[15(n-1)T]$,

(c) The off-ramp traffic volume for section 5(in vehicles per hour) $s5 = 150 + 37\sin[35(n-1)T + 45]$,

(d) The on-ramp traffic volume for section 7(in vehicles per hour) $r7 = 180 + 50\sin(18nT)$,

(e) The off-ramp traffic volume for section 10(in vehicles per hour) $s10 = 120 + 46\sin(50nT + 50)$.

The simulation result shown in Fig.13 and Fig.14 shows that, even under the condition with strong interferences, the traffic flow speeds can exponentially converge to the desired values at each section and the traffic densities at each section only fluctuate in a very small range.

We give a true comparison between the controller proposed in this paper and the controller in reference [3] in term of working time charge. We assume that two controller respectively control two same freeways' segments under the same initial conditions described in this section. After controlling, both the stationary velocities of traffic flow in all sections of the two freeways are 60km/hour. We make the simulation under the MATLAB software entironment. We observed that working time charge of the controller proposed in this paper and the controller in reference [3] are 0.013164 seconds and 0.032354 seconds respectively. The revised algorithm of this paper improved 59.3% working time charge. So, we can see that the revised algorithm highly improves the algorithm efficiency.

Table 1. Initial densities and velocities of a single-lane freeway

Section	1	2	3	4	5	6	7	8	9	10	11	12
Initial density (vehicles km^{-1} $lane^{-1}$)	18	18	18	18	18	52	52	52	18	18	18	18
Initial velocity (km h^{-1})	81	81	81	81	81	29	29	29	81	81	81	81

Table 2. Initial densities and velocities of a single-lane freeway

Section	1	2	3	4	5	6	7	8	9	10	11	12
Initial density (vehicles km^{-1} $lane^{-1}$)	18	25	18	25	18	25	25	18	25	18	25	25
Initial velocity (km h^{-1})	81	71	81	71	81	71	71	81	71	81	71	71

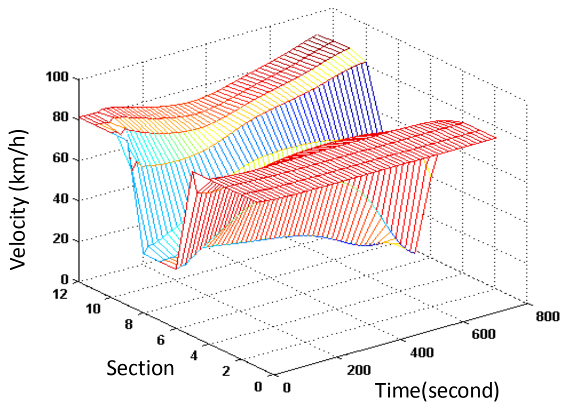

Fig.1 Velocity profile without control

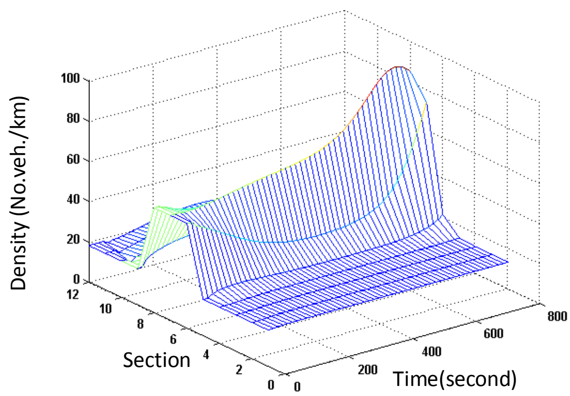

Fig.2 Density profile without control

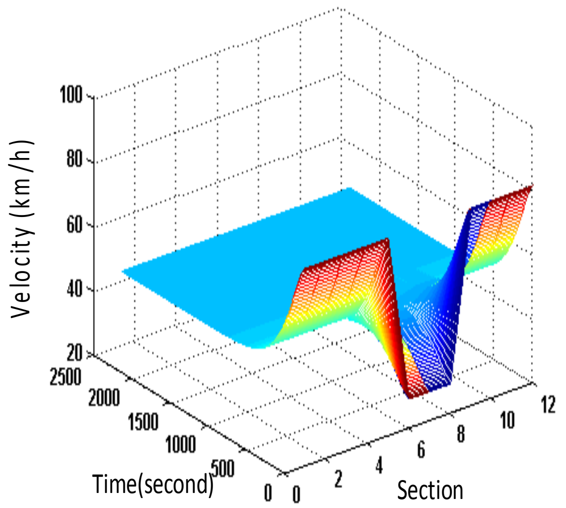

Fig.3 Velocity profile: desired velocity is 45 km/h

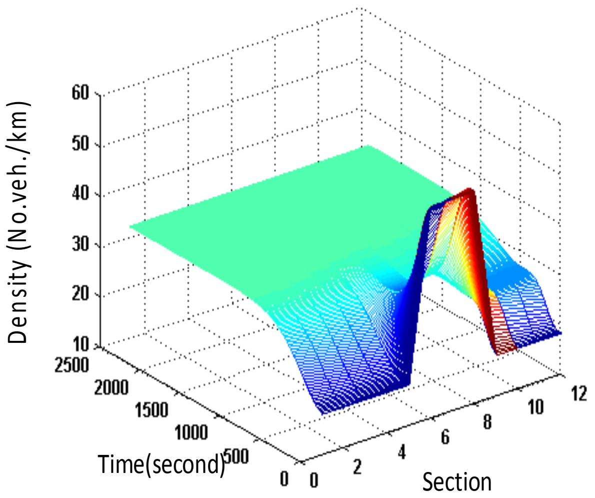

Fig.4 Density profile: desired velocity is 45 km/h

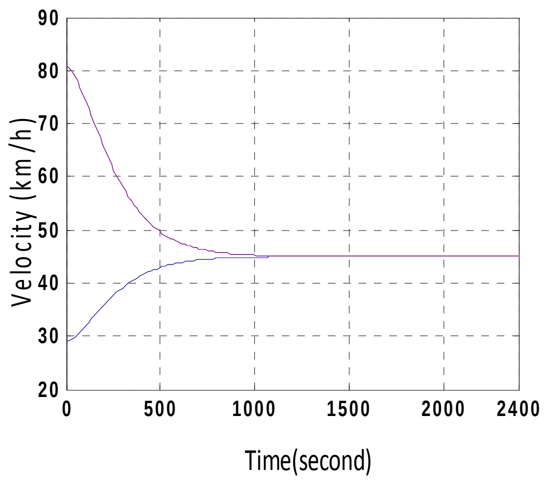

Fig.5 Velocity in each section: desired velocity is 45km/h

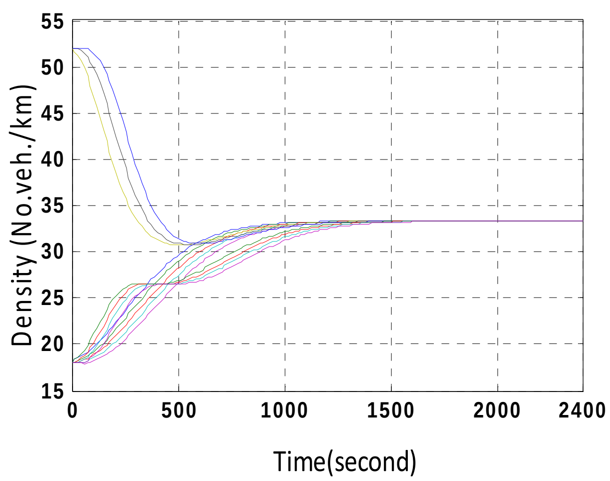

Fig.6 Density in each section: desired velocity is 45km/h

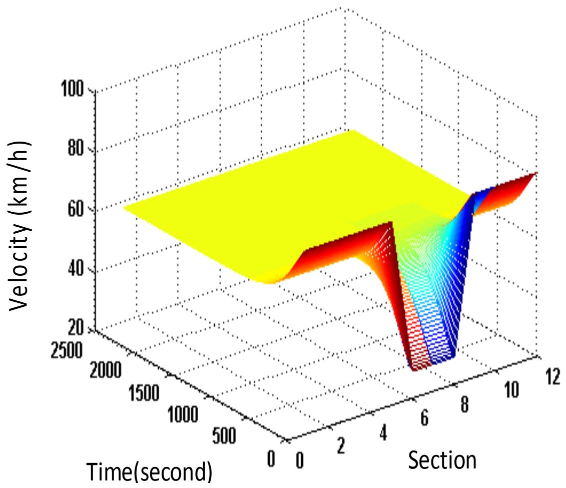

Fig.7 Velocity profile: desired velocity is 60 km/h

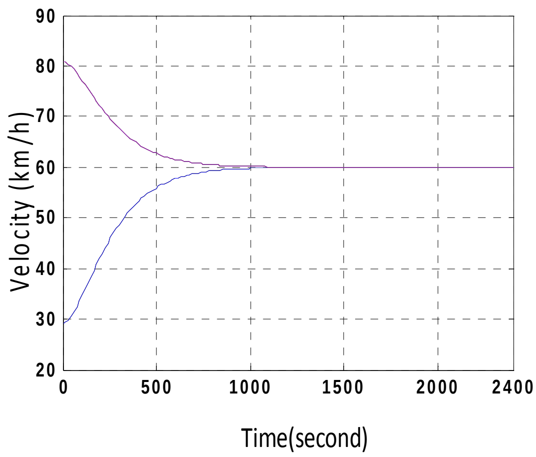

Fig.8 Velocity in each section: desired velocity is 60km/h

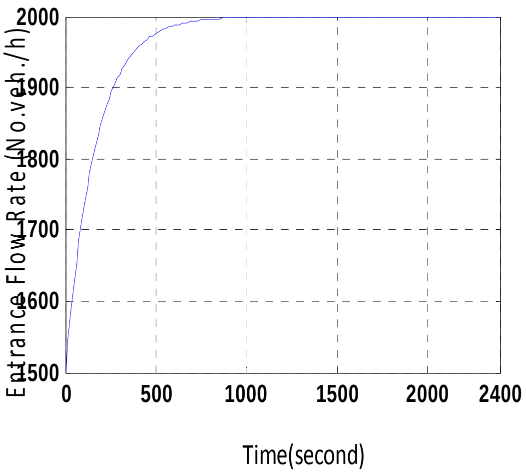

Fig.9 Increasing entrance flow rate

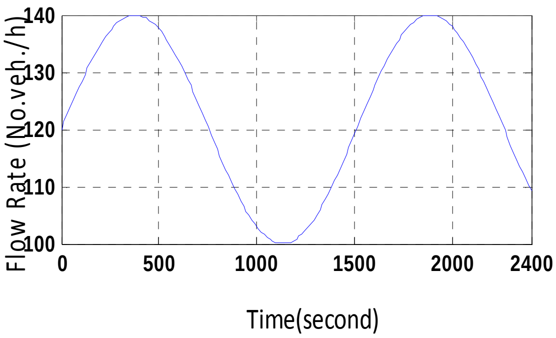

Fig.10 on-ramp traffic flow rate for section 2

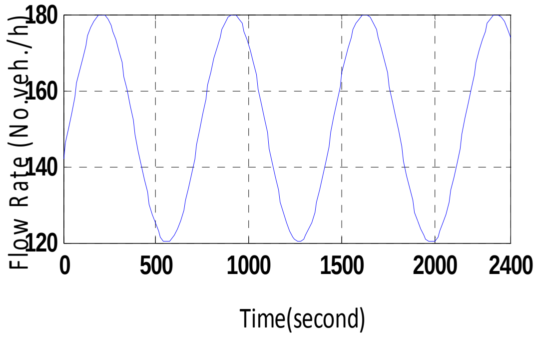

Fig.11 off-ramp traffic flow rate for section 11

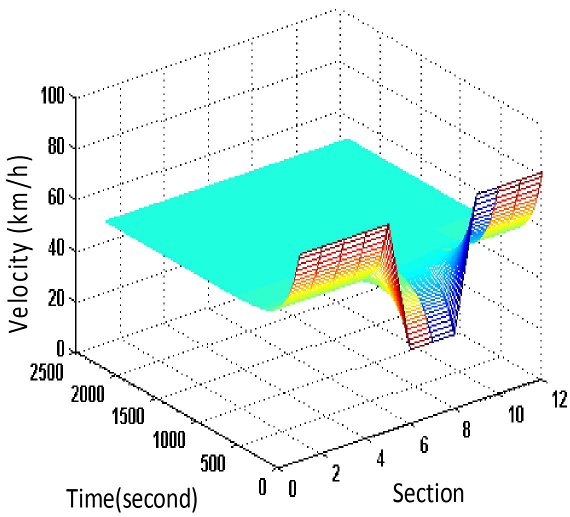

Fig.12 Velocity profile with interference:

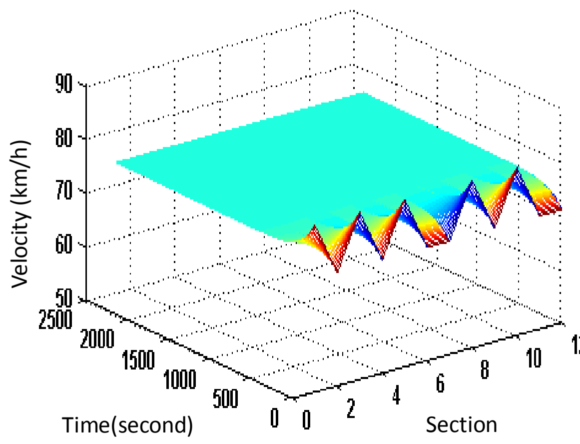

Fig.13 Velocity profile with strong interference: desired velocity is 75km/h

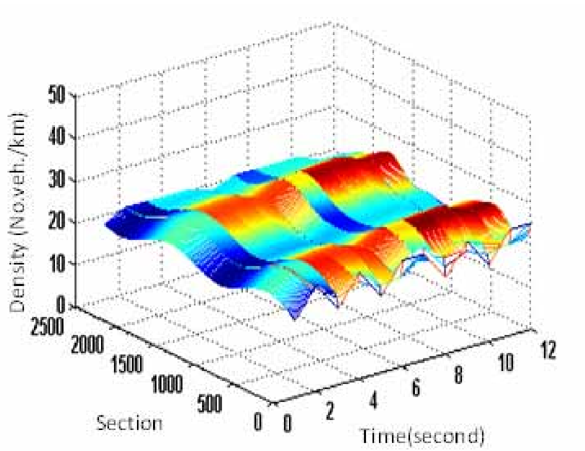

Fig.14 Density profile with strong interference: desired velocity is 75km/h

6 Conclusion

This paper proposes a design method of traffic flow speed controller which can achieve homogeneous traffic flow speed in an automated freeway system environment. Compared with the former method, the proposed controller need not solve inverse matrix, reduces algorithm's complexity and automatically satisfies the controllability condition. Improving real-time performance, this controller will help a computer system to control more large-scale automated highway system. Simulations show that the controller can guarantee exponential convergence of the traffic flow speed at each section to the desired speed and can effectively reduce traffic flow instabilities. It releases the pressure of congestion and helps to achieve a smooth traffic flow on a congested highway.

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