A Transfer Method of Public Transport Network Based on Adjacency Matrix Multiplication searching algorithm

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Abstract: - In this paper, we model the public transport network (PTN) to an unweighted network by space L and space P method. The adjacency matrix is used to express this network, in which 1 denotes connection between two vertices and 0 denotes disconnection. According to the complex theory, the statistical characteristics (including average clustering coefficient, average shortest path length, and so on) of the real PTN of Hangzhou are analyzed. By the shortest path algorithm of matrix multiplication, all the least transfer routes between any two bus stations of Hangzhou are gotten. Then the PTN of Hangzhou is modeled to a weighted network using the straight-line distances between the bus stations which are computed by every station's longitudes and latitudes as the weights. To compare the weights (namely the straight-line distances) of all the least transfer routes, the transfer routes between any two bus stations which not only have the least transfer times but also the shortest straight-line distances are obtained. Finally, by some practical bus stations of Hangzhou, this transfer method is validated.

Key-Words: - Public transport network; Small-world; Weighted network; Transfer route; Matrix multiplication

1 Introduction

Many complex systems in the real world such as the social systems, the food chain systems and so on can be represented abstractly as complex networks [1-3]. The public transport network (PTN) is a typical complex network. The public transport network has an enormous impact on the metropolitan economy, and it is also responsible for the mobility of millions of passengers. Therefore studies of public transport network have drawn great researching enthusiasm from many scholars recently [4-6].

In China, bus is the main public transportation tool. So, public transport transfer is one of the most primary problems which bus passengers care about. The searching algorithms on the networks can solve the problem of public transport transfer. And Kleinberg has proved that small-world networks could be searched quickly [7, 8]. As a typical small-world network [9], PTN also can be searched quickly, namely it has the quick searching capability. It says that public transport transfer problem can be solved quickly and efficiently.

According to the statistical result about psychology inquisition of passengers' trip [10], the least transfer times is the most important factor when passages take a bus, then the traveling distance, time, expenses and so on. Based on the shortest path algorithms, many public transfer methods are presented, in which they only concerned the transfer times, no any other factors such as the traveling distance, expenses and so on[11-13]. In this paper, we model the PTN to an unweighted network firstly. Using the shortest path algorithm of matrix multiplication, all the least transfer routes between any two bus stations are gotten. Then we model the PTN to a weighted network by the straight-line distances between the bus stations which are computed by every station's longitudes and latitudes. To compare the weights, the transfer routes between any two bus stations which not only have the least transfer times but also the shortest straight-line distances are presented.

The paper is organized as follows. In the next section we present the statistical characteristics of PTNs of Hangzhou. In Section 3, we present our transfer method. In Section 4, this transfer method is applied to the real PTN of Hangzhou. Conclusion is given in the last Section.

2 PTN of Hangzhou's statistical characteristics

2.1 Modeling method of PTN

There are two main modeling methods of PTN so far: space P method and space L method [14-16]. In space L one vertex represents one bus station, and one edge represents one connection between two vertices if one bus station of two vertices is the successor of the other on one bus line. While in space P, one edge between two vertices indicates that there is at least one bus line that will halt at those two bus stations though vertex definition is the same. Fig. 1 and Fig. 2 show a PTN which include two bus lines and nine bus stations in space L and space P respectively.

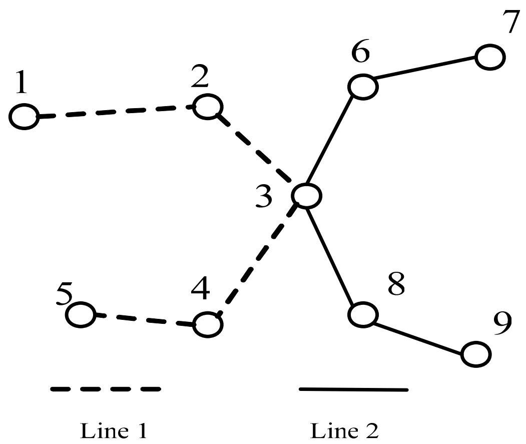

Fig. 1 A PTN in space L

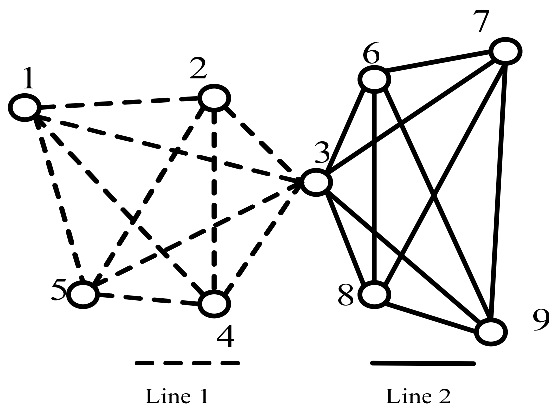

Fig. 2 A PTN in space P

To represent the PTN in computer, we use the adjacency matrix to denote the connections of the network, in which 1 denotes connection between two vertices and 0 denotes disconnection. So from Fig. 1, we can get the adjacency

$$\text{matrix } A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

And from Fig. 2, we can get the adjacency matrix

$$A_2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

2.2 Statistical characteristics

The practical data of PTN of Hangzhou are recorded from Internet [17]. By using space P and space L method [18, 19], we model PTN to two unweighted network. And adjacency matrix is used to denote the connections of the network, in which 1 denotes connection between two vertices and 0 denotes disconnection. To analyze the adjacency matrixes of PTN of Hangzhou, we obtain the average clustering coefficients, average shortest path lengths, total number of bus stations, total number of bus lines and degree distributions.

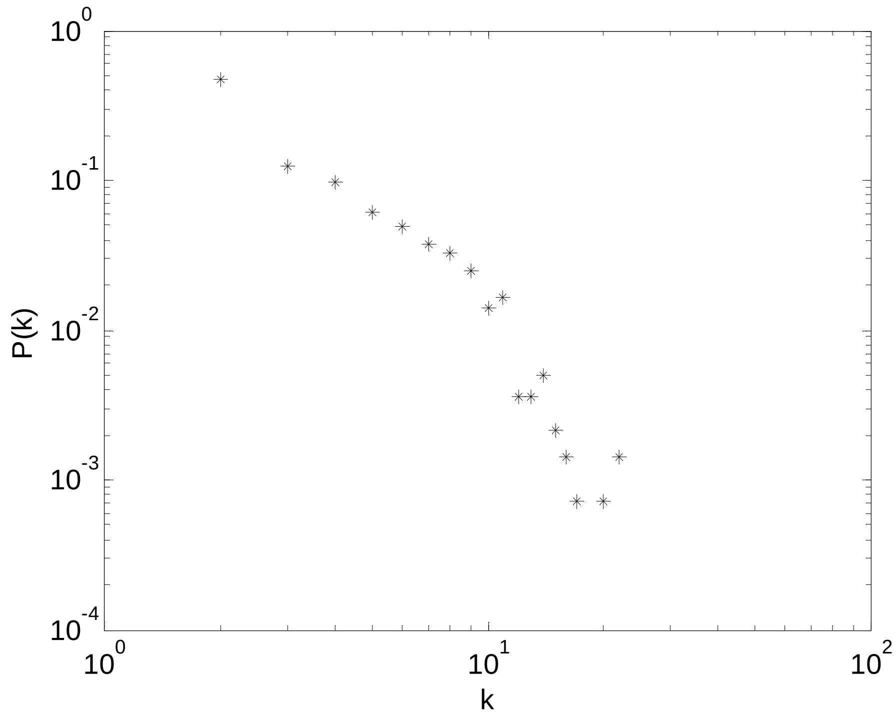

Fig. 3 Degree distribution of PTN of Hangzhou in space L

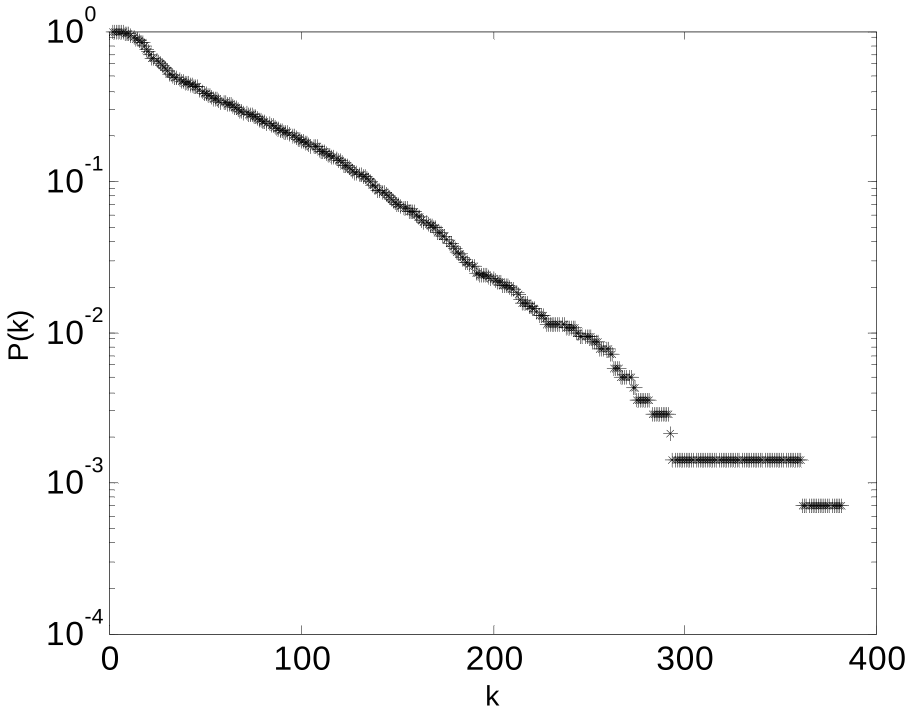

Fig. 4 Degree distribution of PTN of Hangzhou in space P

Table 1 shows the average clustering coefficients, average shortest path lengths, total number of bus stations, total number of bus lines of PTN of Hangzhou in space L. While Table 2 shows the same data of PTN of Hangzhou in space P. Fig. 3 shows the degree distribution of PTN of Hangzhou in space L, while Fig. 4 shows the one in space P.

In this paper, we study on the transfer method of the public transport networks mainly. As we know, if passengers want to travel from one bus station to another, firstly they must find if these two stations are at least on one same bus line. If yes, they do not need to any transfer. Else, they need to find a transfer station to transfer to a new bus line. This definition is the same of the space P method. Therefore we focus on the statistical characteristics of the PTN which is modeled by space P method.

Table 1 Empirical statistical data of Hangzhou in space L

Number of bus lines	Number of bus stations	Average shortest path length	average clustering coefficient
328	1404	10.97	0.16

Table 2 Empirical statistical data of Hangzhou in space P

Number of bus lines	Number of bus stations	Average shortest path length	average clustering coefficient
328	1404	2.65	0.72

The data in table 2 and Fig. 4 show that PTN modeled by space P method has small average shortest path length, big average clustering coefficient and random degree distribution, so it is a typical small-world network. And easily we can get that the average number of least transfer times is equal to the average shortest path length of PTN in space P minus one. So, the average number of least transfer times is very small (between one and two). It means that passengers can travel from one bus station to another conveniently. It also shows that our method is feasible.

3 Presentation of the transfer method

3.1 Obtain the least transfer routes

Dijkstra algorithm is applied to obtain the shortest path length matrix D of every two bus stations from the adjacency matrix A of PTN. This matrix D is equal to the searching steps matrix T (namely the matrix multiplication times matrix) which denotes the number of searching steps (namely the number of matrix multiplication times) of every two stations. For example, Fig. 5 presents a simple PTN of 3 lines. Line 1 is consisted of station 1, 2 and 3; line 2 is consisted of station 2, 4 and 6; line 3 is consisted of station 5, 6, 7 and 8. Fig. 6 presents this PTN modeled by space P method.

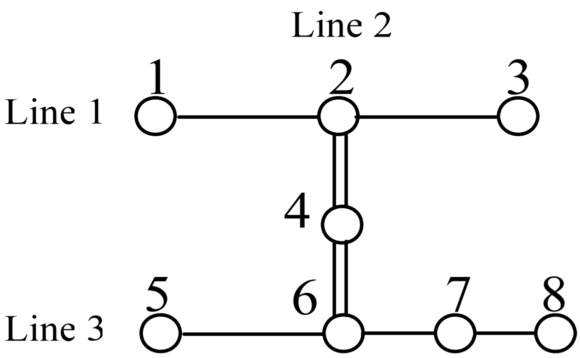

Fig. 5 A simple PTN

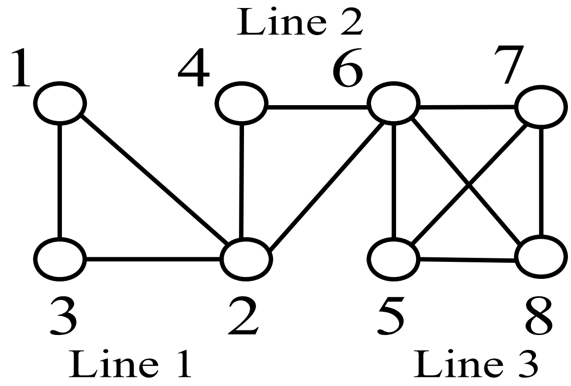

Fig. 6 The simple PTN modeled by space P method

Space P method is employed to model the PTN presented by Fig. 5 to an unweighted network. Then we can obtain the adjacency matrix A and the shortest path length matrix D as follow.

$$A=\begin{bmatrix}0&1&1&0&0&0&0&0\\1&0&1&1&0&1&0&0\\1&1&0&0&0&0&0&0\\0&1&0&0&0&1&0&0\\0&0&0&0&0&1&1&1\\0&1&0&1&1&0&1&1\\0&0&0&0&1&1&0&1\\0&0&0&0&1&1&1&0\end{bmatrix}$$

$$D=\begin{bmatrix}0&1&1&2&3&2&3&3\\1&0&1&1&2&1&2&2\\1&1&0&2&3&2&3&3\\2&1&2&0&2&1&2&2\\3&2&3&2&0&1&1&1\\2&1&2&1&1&0&1&1\\3&2&3&2&1&1&0&1\\3&2&3&2&1&1&1&0\end{bmatrix}$$

The value of $D_{i,j}$ denotes the shortest path length between station i and j. It also denotes that the least number of transfers between station

i and j is $D_{i,j}-1$ (the same station need not to be considered transfer). Besides, for the matrix multiplication algorithm, the number of matrix multiplication times $T_{i,j}$ between station i and j is denoted by the value of $D_{i,j}$, namely $T_{i,j}=D_{i,j}$.

When we use the matrix multiplication searching algorithm to search the target vertex d from the start vertex s, firstly we judge that if $T_{s,d}=0$ or $T_{s,d}=1$ (namely d and s is the same station or d is in the neighbor vertices union of s). If yes, we stop searching. And else we make the matrix multiplication to get a new matrix $A_{(T_{s,d}-1)}=A\times A\times A\cdots=A^{(T_{s,d}-1)}$, judging and recording all the vertices $k_{(T_{s,d}-1)}$ which satisfy the conditions that $A_{(T_{s,d}-1)_{s,k}}\neq 0$ and $A_{k,d}\neq 0$. Then we repeat this process using all the vertices $k_{(T_{s,d}-1)}$ instead of the target vertex d until $T_{s,d}=1$. So we can obtain all the transfer routes that $s\to k_1\to k_2\cdots\to k_{(T_{s,d}-1)}\to d$. Because we can know the multiplication times of any two vertices and the max multiplication times from the matrix T, we can get all the transfer routes between every two vertices in the max multiplication times, namely it is a parallel algorithm. The flow chart is shown in Fig. 7.

A according to the concept of matrix multiplication searching algorithm, we suppose to find the least transfer routes from station 1 to station 8, namely searching vertex 8 from vertex 1. From the matrix D represented upon, we can see that $D_{1,8}=3$. It means that there must be two transfer stations. And by the matrix multiplication searching algorithm represented upon, we can get

$$A_2=A\times A=A^2=\begin{bmatrix}2&1&1&1&0&1&0&0\\1&4&1&1&1&1&1&1\\1&1&2&1&0&1&0&0\\1&1&1&2&1&1&1&1\\0&1&0&1&3&2&2&2\\1&1&1&1&2&5&2&2\\0&1&0&1&2&2&3&2\\0&1&0&1&2&2&2&3\end{bmatrix}$$

.

From matrix A_2 and A, we can get that only vertex $k_2=6$ can satisfy the conditions that $A_{2_{1,k}}\neq 0$ and $A_{k,8}\neq 0$. So vertex 6 is the second transfer vertex in the route from vertex 1 to 8. To use vertex 6 instead of 8 as the target, we can obtain that vertex 2 is the first transfer vertex through the same method using upon.

So we have searched vertex 8 from vertex 1, and the transfer vertices: vertex 2 and 6 have been recorded. The transfer route from vertex 1 to vertex 8 is 1→2→6→8. Namely the transfer route from station 1 to station 8 is obtained, and this route is a least transfer route with transfer station 2 and 6. If there are several transfer routes which have the same number of transfers, namely there are several choices of transfer stations, we record all the transfer routes and transfer stations. Then we obtain all the transfer routes which have the same number of transfers (the least number of transfers).

3.2 Straight-line distance

By using the matrix multiplication searching algorithm, we obtain all the least transfer routes. Now we introduce the vertex weight, namely every station's longitudes and latitudes. The edge weight—the distance between every two stations—will be got by computing the vertex weight. So we can model the PTN to a weighted network, and the value of weight is the distance between every two stations. Here the distance between every two stations represents the straight-line distance between them, is not the real traveling distance. Because the paths which buses travel on are curves, the real traveling distance is the sum of the distance of these curves. But the straight-line distance is an important reference of the real traveling distance between two stations. If the straight-line distance between two stations is shorter than other's, mostly the real traveling distance between these stations is also shorter.

The straight-line distance between two stations is computed by every station's longitudes and latitudes. The computing formulation is presented as follow.

$$x=(jd2-jd1)\times PI\times R\times \cos(((wd1+wd2)/2)\times PI/180)/180$$

$$y=(wd2-wd1)\times PI\times R/180$$

$$\text{Distant}=\sqrt{x^2+y^2}$$

Here, jd1, wd1 and jd2, wd2 are the longitudes and latitudes of two stations respective, the unit is degree. PI=3.14159265, R is the semi diameter of the earth.

There are three rules must be obeyed when we compute the total straight-line distance $Dis_{i,j}$ between every two stations:

A. $Dis_{i,j}$ is computed by the formulation presented above, when station i and j are adjacent stations.

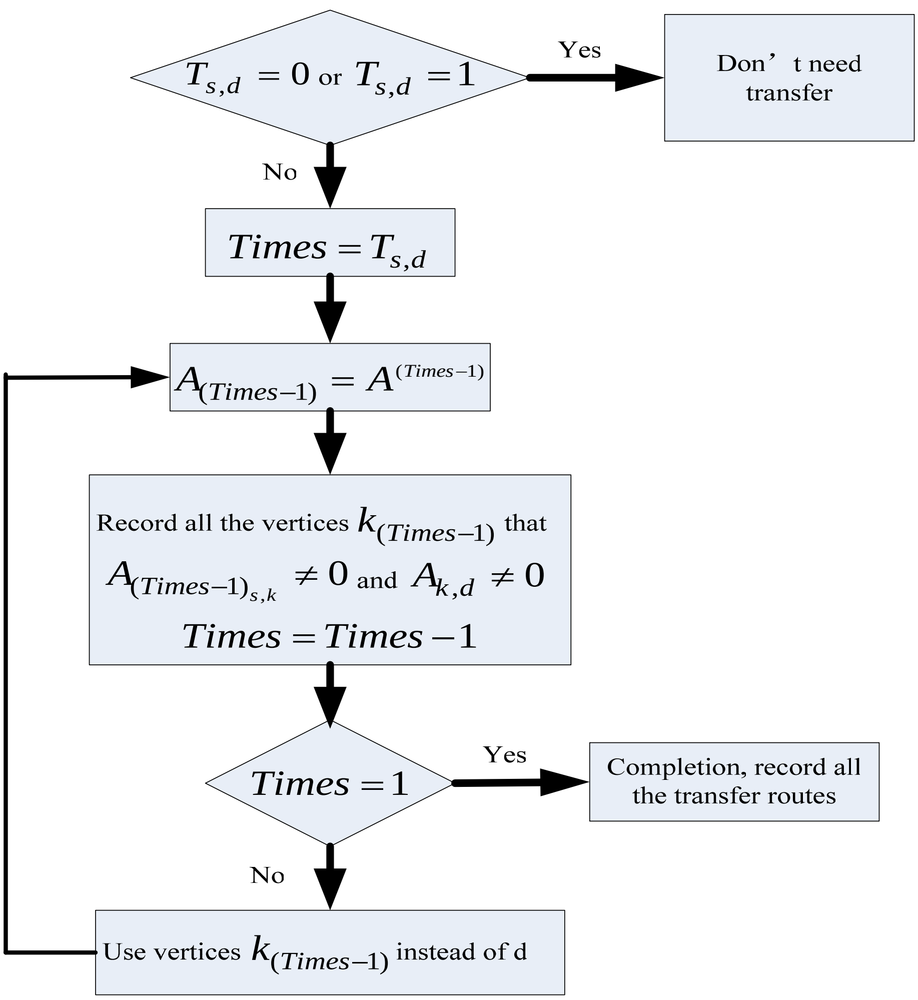

Fig. 7 The flow chart of searching transfer routes using matrix multiplication searching algorithm

B. If station i and j are not adjacent, the straight-line distance between i and j is the sum of the straight-line distance between the adjacent stations of these two stations. For example, to compute the total straight-line distance ($Dis_{1,6}$) between station 1 and 6 which shown in Fig. 5, is $Dis_{1,6} = Dis_{1,2} + Dis_{2,4} + Dis_{4,6}$.

C. $Dis_{i,j}$ can be computed if there is at least one bus line that will halt at station i and j , namely station i and j needn't transfer. Otherwise, $Dis_{i,j} = \infty$.

By using the formulation and rules presented above, we can obtain a weighted matrix of PTN. The value of weight is the straight-line distance between every two stations. Comparing the straight-line distance of every least transfer routes which are got in section 3.1, we can get a transfer route which not only have the least transfer times but also the shortest straight-line distances. And we have discussed that if the straight-line distance between two bus stations is shorter than other's, mostly the real traveling distance between these stations is also shorter. So this transfer route got by us is essentially the same to the route which has the shortest traveling distance.

4 Application

We obtain the real data of PTN of Hangzhou from Internet [17]. Besides, we get the longitude and latitude of the bus stations of Hangzhou by GPS and electronic map [20].

We use the real data of Hangzhou to testify our method which described in section 3:

(a). From bus station 20 (Jiangcun business house section) to bus station 25(Dianzi university). Do not need transfer, namely these two bus stations are at least on one same bus line. Actually, we can travel from bus station 20 to 25 by line 3.

(b). From bus station 1 (Xiache crossing) to bus station 50 (Jiulisong). The transfer route is 1→1367→50 (Xiache crossing→Midu bridge→Jiulisong). The number of transfer times is one. The shortest straight-line distance is 11.577 km.

(c). From bus station 100 (Zhejiang university of technology) to bus station 110 (Sudi). The transfer route is 100→218→110 (Zhejiang university of technology→Yanan road→ Sudi). The number of transfer times is one. The shortest straight-line distance is 13.258 km.

(d). From station 5 (Song city) to station 999 (Number 5 crossing of number 2 road). The transfer route is 5→1386→33→999 (Song city→East bus station→ Number 3 crossing of number 2 road→ Number 5 crossing of number 2 road). The number of transfer is two. The shortest straight-line distance is 21.738 km.

(e). From station 692 (Yujia) to station 1361 (Fang mountain). The transfer route is 692→311→325→326→1361 (Yujia→Zhuantang→Guanxiang crossing→Zhangjia bridge→ Fang mountain). The number of transfer is three. The shortest straight-line distance is 36.562 km.

For these experimental stations, we make the real test of transfer and measure the real traveling distance. The results show that the transfer routes we got have the least number of transfer times and the shortest straight-line distance. Besides, they are the routes which also have the shortest traveling distance. So the practical data of Hangzhou has testified our method's efficiency.

5 Conclusion

In this paper, we have proposed a public transport transfer method based on matrix multiplication searching algorithm and weighted networks. PTN is modeled to an unweighted network and adjacency matrix is used to denote the connections of the network. The statistical characteristics of real PTN of Hangzhou are analyzed. All the least transfer routers between any two bus stations are obtained by the matrix multiplication searching algorithm. Then we model the PTN to a weighted network by the straight-line distances between the bus stations which are computed by every station's longitudes and latitudes. Comparing the straight-line distance of every least transfer routes which are got by the unweighted network model, we get a transfer route which not only have the least transfer times but also the shortest straight-line distances at last. The practical data of Hangzhou has shown that these routes also have the shortest real traveling distance and testified the method's efficiency.

We focus on the bus transport networks in this paper, and we find the transfer routes from one bus station to another. Actually there are more public transportation tools such as subway in the cities. So we will study on the transfer methods from more than one public transportation tools in the future. And we will expand the public transport network from one city to one country, from one country to the world.

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