

# Design, implementation and evaluation of an optimal iterative learning control algorithm

V. VITA

A. VITAS

G.E. CHATZARAKIS

Department of Electrical Engineering Educators  
 ASPETE-School of Pedagogical and Technological Education  
 N. Heraklion, 141 21 Athens  
 GREECE  
 vasvita@yahoo.co.uk

*Abstract:* Iterative learning control (ILC) is used to control systems that operate in a repetitive mode, improving tracking accuracy of the control by transferring data from one repetition of a task, to the next. In this paper an optimal iterative learning algorithm for discrete linear systems is designed and implemented. The design and implementation that have been done using Matlab® 7 and Simulink are described in detail. The algorithm is applied on several representative discrete systems cases in order to be evaluated and to reveal its capabilities and limitations.

*Key-Words:* Discrete linear systems; iterative learning control; Matlab® 7; simulation.

## 1 Introduction

Iterative Learning Control (ILC) is a relatively new concept in the control theory. It evolved from the need to control dynamical systems that are supposed to carry out a given task repetitively and more specifically for systems where the desired output values are a function of time and the task is carried out repeatedly. A classical example is the robot in the car industry that follows a given trajectory and welds at specific points along the trajectory. Normally the robot would be tuned once through feedback or feedforward control or even a combination of both. After the tuning, it would carry out the repetitions performing in the same way. The obvious drawback of this approach is the fact that if there is an error between the measured trajectory and the reference trajectory due to a wrong selection of the control input trajectory, the robot will repeat this error at each trial, i.e., if there is an error in the performance it will repeat the same error at each iteration.

To overcome this problem, Arimoto, one of the inventors of ILC [1, 2], suggested that both the information from the previous tasks or “trials” and the current task should be used to improve the control action during the current trial. In other words, the controller should learn iteratively the correct control actions in order to minimize the difference between the output of the system and the given reference signal. He called this method “betterment process” [3]. This approach is more or less an imitation of the learning process of every

intelligent being. Intelligent beings tend to learn by performing a trial (i.e. selecting a control input) and observing what was the end result of this control input selection. After that they try to change their behaviour (i.e. to pick up a new control input) in order to get an improved performance during the next trial. Based on the overall idea of ILC, the procedure which results in a controller that learns the correct control input and that learning is done through iteration, the term Iterative Learning Control is nowadays used to describe control algorithms that result in the “betterment process” as suggested by Arimoto [3]. In this work an optimal iterative learning algorithm for discrete linear systems is designed and implemented. The algorithm is applied on several representative discrete systems cases in order to be evaluated and tested, producing very good and useful results.

## 2 Norm-optimal iterative learning control algorithm

The aim of the ILC algorithm [1, 5], is to find iteratively an optimal input  $u^*$  for the plant under investigation. This input, when applied to the plant, should generate an output  $y^*$  that tracks the desired output  $y_d$  “as accurately as possible”. The use of the phrase “as accurately as possible” states the significance of obtaining the smallest possible difference between the actual  $y^*$  and desired  $y_d$  output. This difference is actually the error  $e$  and the

above lead to the conclusion that the error  $e$  is desired to be minimal. The ultimate aim of the ILC is to push the error  $e$  to zero. Thus it is clear that applying an ILC algorithm will result in a two-dimensional system because the information propagates both along the time axis  $t$  and the iteration axis  $k$ .

The two-dimensionality of the iterative learning control introduces problems in the analysis as well as the design of the system and hence the two-dimensions are an expression of the systems dependent dynamics: a) the trial index  $k$  and b) the elapsed time  $t$  during each trial. In order to overcome such difficulties an algorithm for Iterative Learning Control is introduced, which has the property of solving the two-dimensionality problem. For the solution of this problem the cost function for the system under investigation is introduced. The minimization of the cost function will provide an effective algorithm for iterative learning control (ILC).

The following cost function is proposed [4]:

$$J(u_{k+1}) = \|u_{k+1} - u_k\|_R^2 + \|e_{k+1}\|_Q^2 \quad (1)$$

where:

$J(u_{k+1})$  is the cost function with respect to the current trial input,

$\|u_{k+1} - u_k\|^2$  is the norm of the difference between the current and previous trial inputs,

$\|e_{k+1}\|^2$  is the norm of the current error and

$R, Q$  are symmetric and positive definite weighing matrices. It is assumed that  $R$  and  $Q$  are diagonal matrices, and for simplicity  $R = rI$  and  $Q = qI$ , where  $q$  and  $r$  are positive real numbers ( $q, r \in \mathfrak{R}$ ).

In ILC literature, researchers have suggested different cost functions to solve the ILC problem. The reasons that lead to the realization that the selected cost function (1) is effective and appropriate are the following: a) The  $\|u_{k+1} - u_k\|^2$  factor represents the importance of keeping the deviation of the input between the trials small. Intuitively this should result in smooth convergence behaviour. This requirement could also be stated as the need for producing smooth control signals, in order to obtain smooth manipulation of actuators. b) The  $\|e_{k+1}\|^2$  factor represents the main objective of reducing the tracking error at each iteration. c) In order to state which of the above two factors plays a more significant role in the cost function the weighting matrices  $R$  and  $Q$  are used. If the interest is focused in retaining the deviation of the input between the trials small, then the ratio  $\beta = r/q$  has to be “large”. On the other hand, if keeping the error

small is more significant, then the ratio  $\beta = r/q$  has to be “small”. The actual meaning of “small” and “large” depends on the system being considered and the units measured. d) The optimal value of the cost function is bounded. If the cost function is evaluated with  $u_{k+1} = u_k$  then (1) becomes:

$$J(u_k) = \|u_k - u_k\|_R^2 + \|e_k\|_Q^2 = \|e_k\|_Q^2 \quad \text{and hence the optimal value: } J(u_{k+1}) \leq \|e_k\|_Q^2.$$

It is also clear that the optimal cost function has a lower bound:

$$J(u_{k+1}) = \|u_{k+1} - u_k\|_R^2 + \|e_{k+1}\|_Q^2 \geq \|e_{k+1}\|_Q^2.$$

Hence combining the above, the upper and lower bounds are expressed by (2).

$$\|e_{k+1}\|^2 \leq J_{k+1}(u_{k+1}) \leq \|e_k\|^2 \quad (2)$$

The differentiation of the cost function (1) with respect to  $u_{k+1}$  produces the solution used to update the input, the norm-optimal ILC algorithm, which is investigated in this work. The input up-date law is the following [4]:

$$\frac{\partial J}{\partial u_{k+1}} = 0 \Rightarrow u_{k+1} = u_k + R^{-1}G^T Q e_{k+1} = u_k + G^* e_{k+1} \quad (3)$$

or

$$u_{k+1} - u_k = R^{-1}G^T Q e_{k+1} \quad (4)$$

where:

$u_{k+1}$  is the current trial input,

$u_k$  is the previous trial input,

$e_{k+1}$  is the current trial error and

$G^* = R^{-1}G^T Q$  is the gain matrix-adjoint of the plant - that represents the relative weighting of the cost function requirements (error-input deviation).

The causal solution to the above problem is given by introducing the following proposed algorithm for Norm-Optimal ILC, which consists of the following terms:

Term I: The gain matrix  $K(t)$ . Given in the form of the discrete Ricatti equation [6, 7]:

$$K(t) = \Phi^T K(t+1)\Phi - \Phi^T K(t+1)\Gamma[\Gamma^T K(t+1)\Gamma + R]^{-1} \cdot \Gamma^T K(t+1)\Phi + C^T Q C \quad (5)$$

for  $t \in [0, N-1]$  and with the terminal condition  $K(N) = 0$ .

Term II: The feedforward (predictive) term.

$$\xi_{k+1}(t) = [I + K(t)\Gamma R^{-1}\Gamma^T]^{-1} \{ \Phi^T \xi_{k+1}(t+1) \} + C^T Q e_k(t+1) \quad (6)$$

for  $t \in [0, N-1]$  with the terminal condition  $\xi_{k+1}(N) = 0$ .

Term III: The input update law.

$$u_{k+1}(t) = u_k(t) - (\Gamma^T K(t)\Gamma + R)^{-1} \Gamma^T K(t) \cdot \Phi [x_{k+1}(t) - x_k(t)] + R^{-1}\Gamma^T \xi_{k+1}(t) \quad (7)$$

Easily can be observed that Term I is independent of the inputs, outputs and states of the system, Term II, the predictive term  $\xi_{k+1}(t)$  is dependent on the previous trial error  $e_k(t+1)$  and Term III, the input update law depends on the previous trial input  $u_k(t)$ , the current state  $x_{k+1}(t)$ , the previous trial state  $x_k(t)$ , and the predictive term  $\xi_{k+1}(t)$ . This is hence a causal iterative learning control algorithm consisting of current trial-full state-feedback along with feedforward of the previous trial error data.

### 3 Design and implementation of the ILC algorithm

The development of the software for the implementation of the "Norm-Optimal ILC" algorithm includes the following stages: a) Simulation requirements: A breakdown of the specific tasks required to develop the proposed system and b) Construction and verification: Coding and testing the various systems' components and eventually testing it as an integrated unit.

As mentioned in section 2, the three terms which constitute the causal form of the ILC are: a) the gain matrix (5). b) the feedforward (predictive) term (6) and c) the input update law (7). These terms constitute the software's required outputs as well as its inputs and thus have to be written in the appropriate programming language for implementation. The following phases represent a possible approach for designing the modules of the proposed software:

**Phase I:** The modeling of the Riccati equation solution for the gain matrix  $K(t)$ . Since  $K(t)$  is independent of the inputs and states of the system, it may be independently simulated for all trials of  $K$  and trial steps  $0 \leq t \leq N-1$ . It is to be noted that this phase undergoes only one iteration and this iteration occurs at the beginning of the implementation. All the values of  $K(t)$  are stored in some type of array structure (within the Workspace) so that they may be referenced when necessary.

**Phase II:** The modeling of the predictive (feedforward) term  $\xi_{k+1}(t)$ , which regulates the plant's operation along with the corresponding sampling times. It should be observed that the predictive term  $\xi_{k+1}(t)$  is dependent on the error quantum generated by the previous trial  $e_k(t+1)$ , thus the error data for each trial is fed into the following trial iteration / simulation  $\xi_{k+1}(t) = f(e_k(t+1))$ . Since the final condition of the

feedforward term is known  $\xi_{k+1}(N)=0$ , the simulation will solve the term in a recursive fashion and subsequently store the data. This second phase will be iterated for every trial.

**Phase III:** Modeling of the input update law (7) to produce new input data for each sample time of the corresponding trial. In order to simulate (7), a deviation of the term in three parts is required. Part I -  $u_k(t)$ : this represents the feedforward data of the previous trial input. Part II -  $(\Gamma^T K(t) \Gamma + R)^{-1} \Gamma^T K(t) \Phi [x_{k+1}(t) - x_k(t)]$ : the feedforward of the previous trial state as well as the feedback of the current trial input. Part III -  $R^{-1} \Gamma^T \xi_{k+1}(t)$ : the feedforward of the predictive term.

This phase uses data generated from the first two phases and generates the necessary input for the current trial (i.e., for every instance of  $t$ ). The data accumulation from phases one and two is fed forward to the current phase and permits *on-line* simulation of the controller and the plant's operation. The coding of the phases is as follows: *Phase I* - It is coded 'from the ground up' in the Matlab® 7 [8, 9] programming environment, *Phase II* - It follows a similar process to the first but also incorporates error data produced by the first and third phase during the previous trial, *Phase III* - It is implemented in the Matlab's Simulink environment incorporating Matlab function code for certain Blocks and data produced by the first and second phase.

The inputs for the implementation were: a) the Discrete System Matrices, namely  $\Phi$ ,  $\Gamma$  and  $C$ , with  $D$  being set to 0. If the matrices' parameters are known, these parameters can be incorporated as actual values in the source code. In the event that only the discrete plant's transfer function is known, then - with the assistance of an inbuilt Matlab function - the matrices can be extracted and directly expressed in the code. The code also provides the ability to change the system's order and parameters, b) the number of iterations  $T$ , c) the sampling length  $N$  for each trial, d) the sampling interval  $T_s$ , e) the desired output, which is the reference signal  $r$ , f) the ratio  $\beta$  of the influence degrees  $q$  and  $r$  of the weighting matrices  $Q$  and  $R$  respectively, which can only take positive values, g) the initial conditions (IC) of the discrete plant and h) the initial input  $u_0$  estimate.

The outputs of the implementation were: a) the current trial output  $y_{new}$  which is used in error calculation, b) the current trial error  $e_{new}$ , which is saved in an area of system main memory that Matlab® 7 [8, 9] has configured, commonly referred to Workspace. This Workspace data is also used as

an input for Phase II in the form of the previous error  $e_{old}$ , c) the current state, which is fed back to the system as well as being further used as  $x_{old}$ , thus it is also stored in the Workspace. The flow-chart in Fig. 1 illustrates the basic steps of the algorithm's implementation.

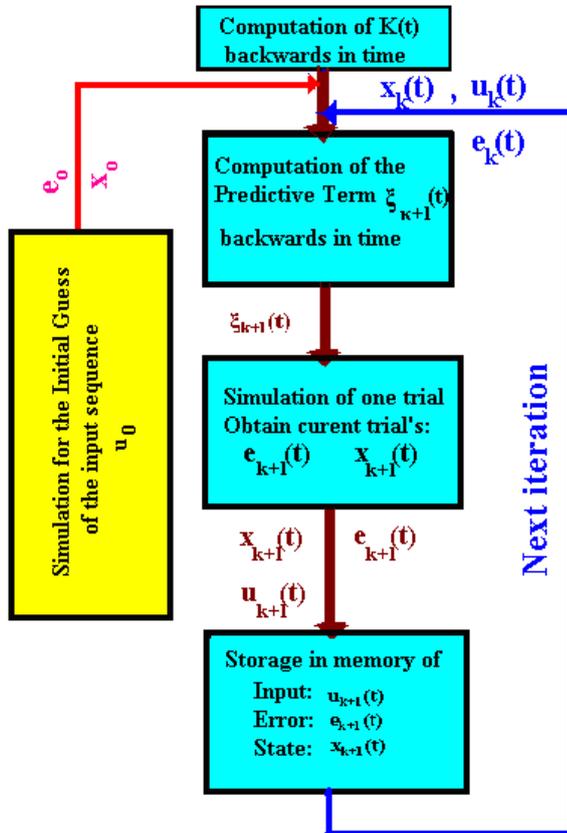


Fig. 1: Flow chart of the algorithm's implementation

As can be observed, the initial estimated error values are fed into the Workspace, simultaneously the data from the first phase - the  $K(t)$  gain matrix computation - is also being passed to this Workspace. The second phase (represented by the second turquoise box) takes all the data that has been stored so far in the Workspace from these two input sources and initializes. During the second phase's operation, it generates the values for the predictive term sequence that, in turn, are fed to the third phase (the third box). Finally, the third phase references all the Workspace values and performs an on-line operation to regulate the plant.

A representation of a typical Phase III Simulink model sequence of operations during an on-line plant regulation is shown in Fig. 2, where the components of this system are presented as a block diagram. The controller is fully implemented at this point to regulate the plant and part of its algorithm is responsible for training the plant in order to follow the desired trajectory-reference  $r$ . For clarity and

ease of reference, the three parts of the up-date law modeled in Fig. 2 have been specifically encircled and designated via blue lettering. The red thick line indicates the feedback flow for the current states. The Simulation model even if it is typical for an on-line regulation for a discrete plant, it is not common in a real world environment where it is more usual to have continuous system control [10].

## 4 Evaluation of the ILC algorithm

### 4.1 Effect of the relative degree

The Norm-Optimal algorithm is applied to a 1<sup>st</sup>, a 2<sup>nd</sup> and a 3<sup>rd</sup> order discrete linear system, with a relative degree of one, two and three respectively. At the end of the implementation an assessment in terms of reference tracking error and control effort is done. The selected systems, expressed in the Laplace domain, are the following: 1<sup>st</sup> order system:

$$H_1(s) = \frac{1}{(s+2)} \text{ with a relative degree one, 2<sup>nd</sup> order}$$

$$\text{system: } H_2(s) = \frac{6}{(s+2)(s+3)} \text{ with a relative degree}$$

$$\text{two and 3<sup>rd</sup> order system: } H_3(s) = \frac{24}{(s+2)(s+3)(s+4)}$$

with a relative degree three.

When the Norm-Optimal ILC algorithm is applied to all the above systems the response in terms of reference tracking emerges as shown in Fig. 3. Fig. 3 shows that the systems under investigation are controlled successfully and their output shows the ability of tracking the reference signal in an acceptable way. It can be observed that the 1<sup>st</sup> order system with the relative degree of one achieves the most satisfactory final trial trajectory in terms of reference tracking. The worst tracking performance amongst the three systems belongs to the 3<sup>rd</sup> order system, with the relative degree of three. In order to accomplish a more objective assessment for the three systems, the characteristic performance is plotted (Fig. 4).

It is clear from the simulation results of Fig. 4 that the convergence rate increases in inversely proportional with the relative degree of the system. The 1<sup>st</sup> order system demonstrates the fastest convergence of the error sequence  $e_k(t)$  to zero. On the other hand, the 3<sup>rd</sup> order system demonstrates the slowest convergence to zero. It can also be observed that the smallest the relative degree of the system the less trials it takes for the algorithm to train the system, i.e., the error to reach small bounds with respect to zero.

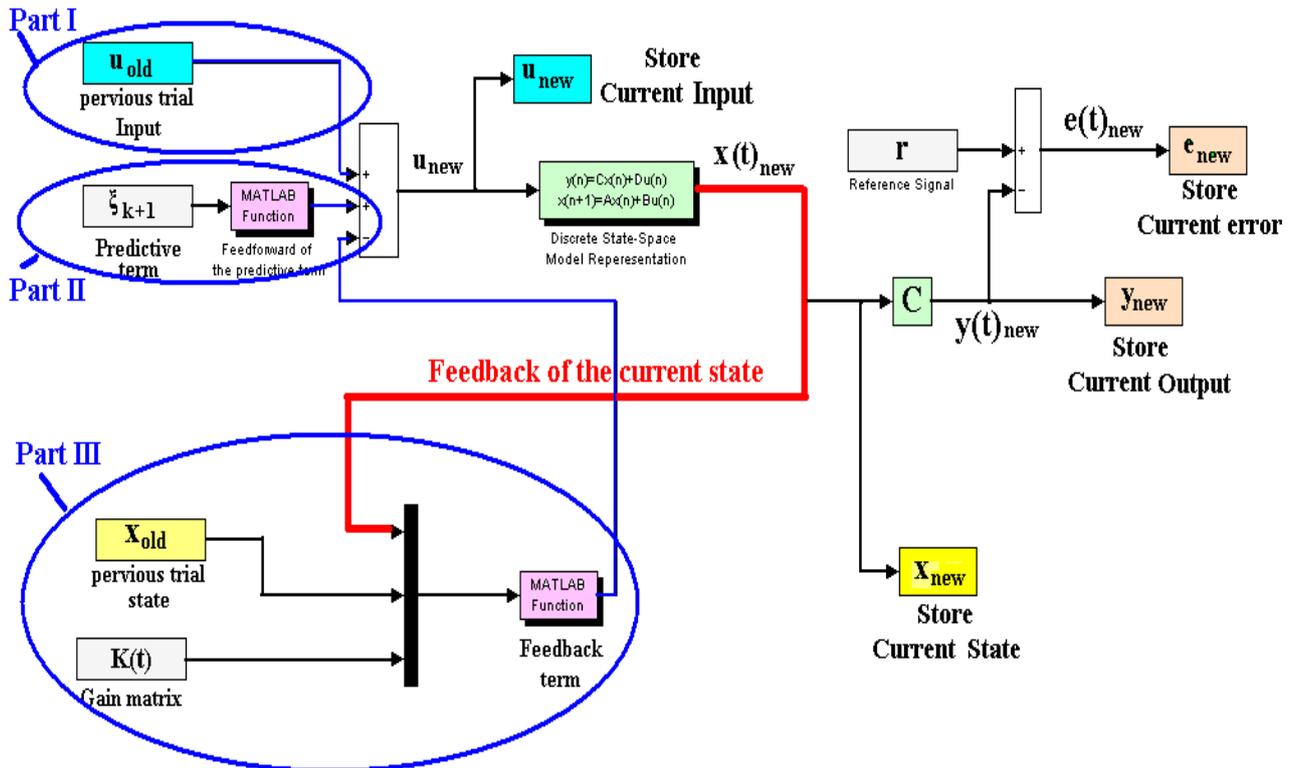


Fig. 2: On-line simulation for control of a discrete linear plant

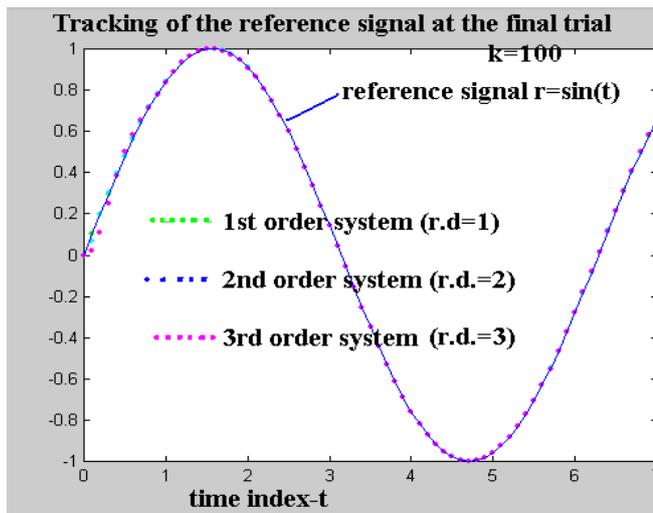


Fig. 3: Tracking of reference signal.

### 4.2 Varying reference signal $r$

In this case an attempt to commend on the dependence of algorithm on the reference signal is made. As a desired output is selected the signal  $r_k(t)$  that has the property of changing from trial to trial. In practice this could be translated as an attempt of a robot arm to work on a moving target. In this simulation, the algorithm is applied to the following 2<sup>nd</sup> order system  $H(s)$ , expressed in the Laplace- domain:  $H(s) = \frac{6}{(s+2)(s+3)}$ , with zero

initial  $r_k(t) = \sin(t) + (1-b_k)\cos(t)$ , where  $b_k, k \in \mathbb{R}$  and where  $k = Trials$  has been selected. The conditions. In order to validate the algorithm's performance the reference signal implementation of the Norm-Optimal ILC algorithm to the above system produces Fig. 5, which illustrate the performance of the algorithm with respect to the varying reference  $r_k$ .

These surface plots demonstrate the change of the reference  $r_k(t)$  and the plant output  $y_k(t)$  (Fig. 5) for each sample time and trial. It can easily be observed that the plant shows a good behaviour in

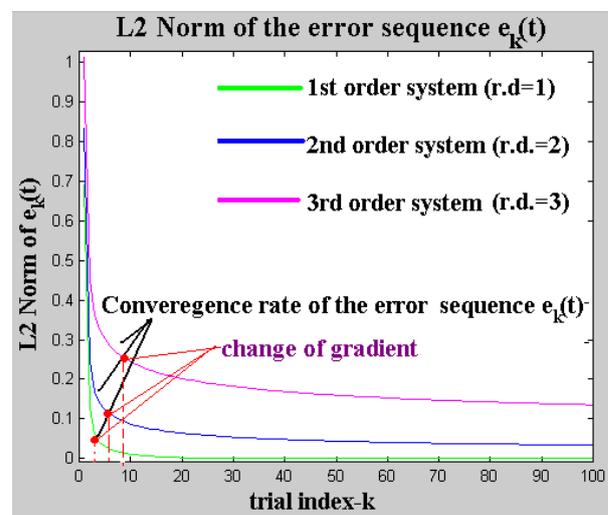


Fig. 4: L2 Norm of the error sequence.

terms of reference tracking, despite the reference variation over the trials. Especially at the last trial the plant output demonstrates an improved reference tracking behaviour. In order to assess the overall algorithms performance the performance criterion (i.e. L2 Norm of the error) is plotted (Fig. 6).

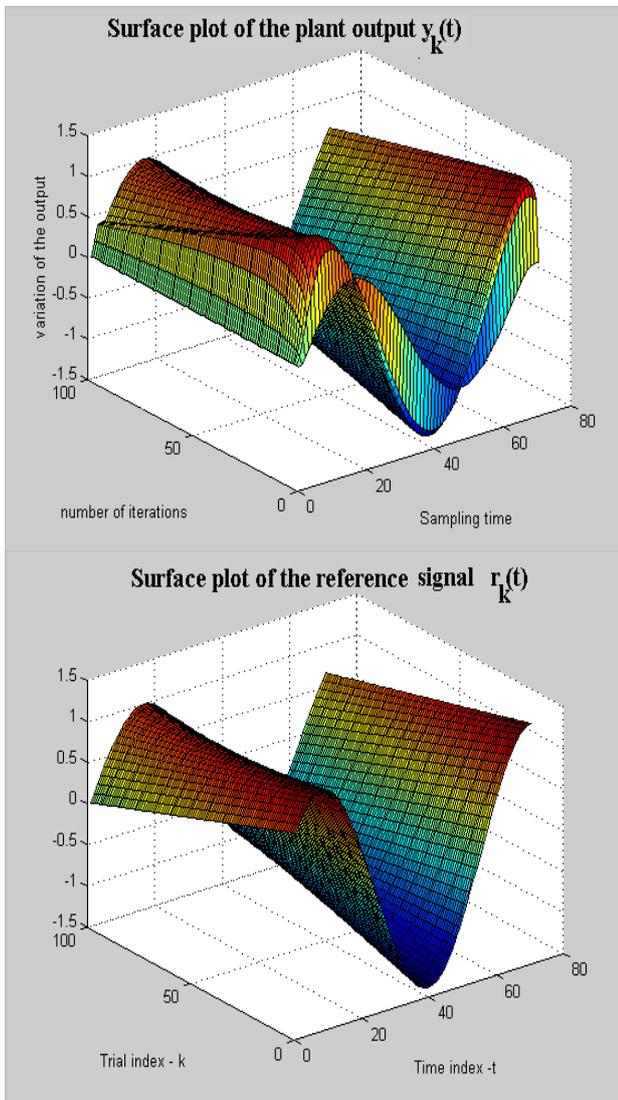


Fig. 5: Surface plot of the reference signal and plant output.

As it can be seen in Fig. 6 the error convergence is very fast for the first seven trials. After that, it still continues to converge with a reduced speed until the 82<sup>th</sup> trial, where the error bound reaches desirable limits. The above deduction is reinforced through observation of the error sequence surface plot. It is obvious that as the number of trials grows, the performance of the plant improves and the error decreases. However, the number of trials required for the error to reach desirable bounds, is much greater than the previous examples. This is not odd, since the reference is changing over trials. But the

algorithm adapts to the changes and as the number of trials increases, the algorithm begins to predict the change of the reference. It uses then the prediction to modify the control actions suitably in order to track the reference in an optimal way.

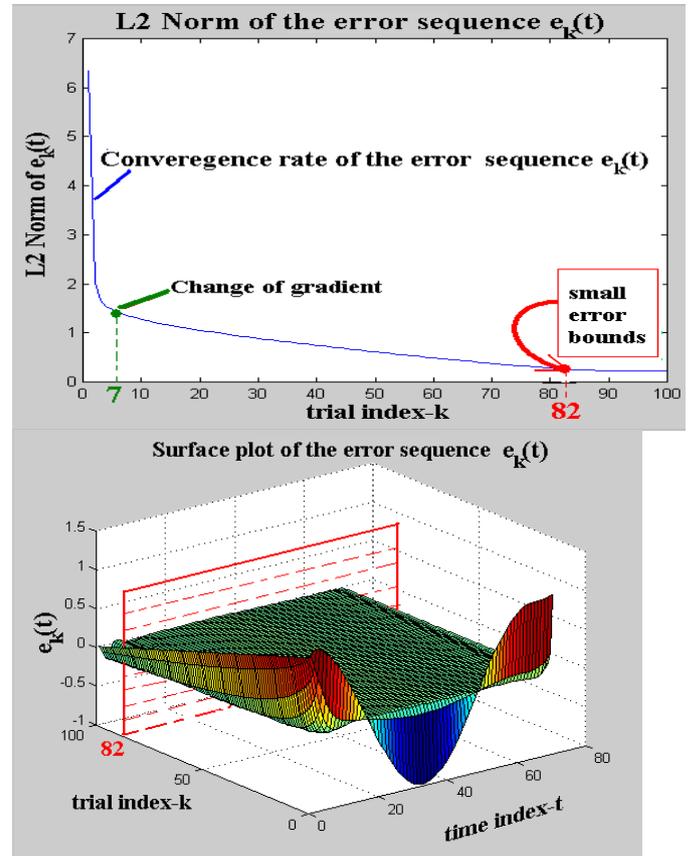


Fig. 6: L2 Norm of the error and surface plot.

### 4.3 Disturbance acting on the plant

In this case the effect of noise presence in the plant is examined. It is assumed that the model of the plant is known and it is the following 2<sup>nd</sup> order system  $H(s)$  given in the Laplace-domain as:

$$H(s) = \frac{6}{(s+2)(s+3)}$$

with zero initial conditions. The

selected form of disturbance is bounded white noise with sampling time equal to the discrete systems sampling time.

#### 4.3.1 Study case I: Disturbance acting on the input of the plant

The disturbance act is expressed with the presence of white noise in the input. The point of the noise entrance in the plant is presented in the relevant Simulink model used for the current simulation. The effect of the disturbance on the error evolution is given in Fig. 7.

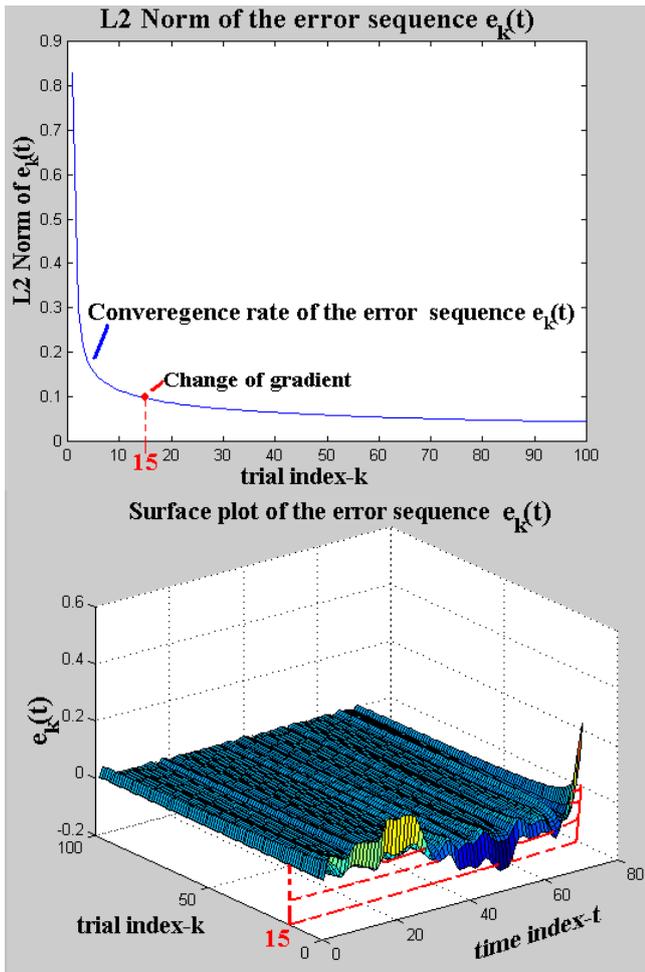


Fig. 7: L2 Norm of the error sequence and surface plot

It shows that the error decreases exponentially even with the influence of the white noise in the input. The convergence of the error is very fast for the first trials and for the next trials the convergence get slower. But even the first 15 trials are adequate for the tracking error bound to reach desirable limits. This assumption is reinforced through the observation of the error surface plot, where it is apparent that the error bounds, after the 15<sup>th</sup> trial, are of small size. Hence, the algorithm succeeds to control the plant, despite the noise presence in the input.

**4.3.2 Study case II: Disturbance acting on the output**

The disturbance act is expressed with the presence of white noise in the output. The point of the noise entrance in the plant is presented in the relevant Simulink model used for the current simulation. The effect of the disturbance on the error evolution is presented in the following Fig. 8.

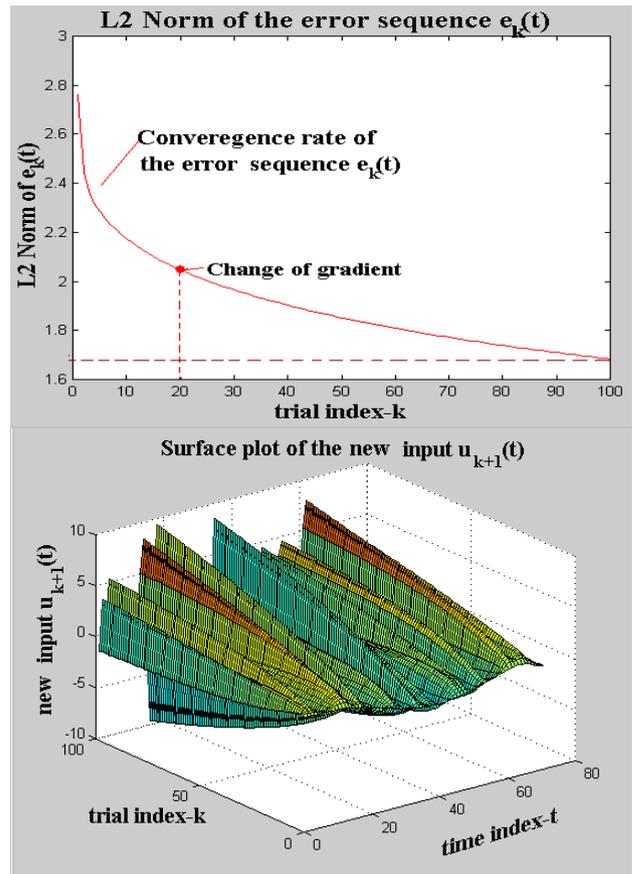


Fig. 8: L2 Norm of the error sequence and surface plot of the new input.

The convergence rate is fast for the first 20 trials, but becomes slow for the following ones. Furthermore the error does not reach satisfactory bounds within a reasonable number of trials. However, eventually it will converge to zero. In order to commend on the overall performance of the algorithm (and despite the errors zero convergence), the controlled input sequence is plotted, where excessive manipulations are required. It can be assumed that the noise that affects on the output is a critical phenomenon and seriously affects the plants zero tracking performance. Hence, the algorithm does not adequate cope with this effect.

**4.3.3 Comparison of the two study cases**

Another aspect of testing the algorithms performance is to compare its sensitivity towards the two studied cases of disturbance. In order to compare the ability of the algorithm to overcome the disturbances effect the performance criterion (L2Norm) is used. Fig. 9 demonstrates the convergence rate of the error sequence in both the cases of noise disturbance (input-output acting disturbance).

Based on Fig. 9 it is evident that the rate of the error convergence is faster for the case where the noise enters in the plant from the input. On the other hand, the algorithm accommodates the presence of noise in the output in a harder way and requires more trials in order to train the plant. One approach to validate this behaviour is the fact that the entrance of the noise from the input allows the plant to filter it out through the current state feedback. This filtering-action is not possible when the state enters the plant in the output. The algorithm has then to wait until the next sampling time and trial in order to cope with the disturbance [11].

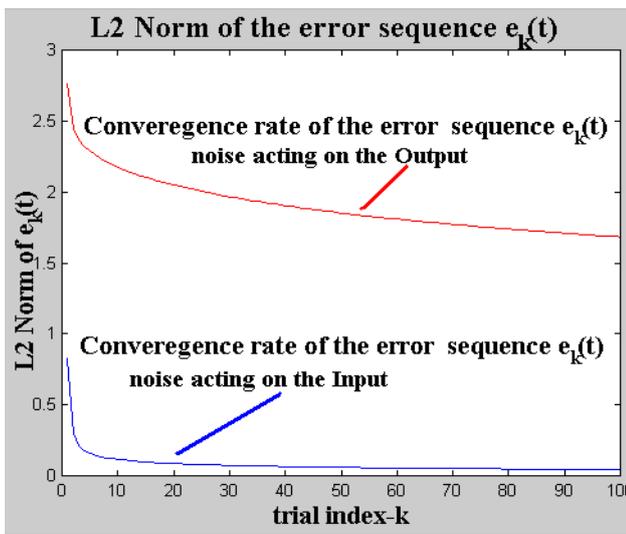


Fig. 9: L2 Norm of the error.

#### 4.4 Plant uncertainty

The plant uncertainty lies in the fact that the nominal plant model differs from the true one. Representative cases for plant uncertainty are selected and through their examination, the suitability and behaviour of the Norm-Optimal algorithm implementation is commended. The actual plant to be controlled is given in the Laplace domain as:  $H(s) = \frac{21}{(s+1)(s+3)(s+7)}$ , with zero initial conditions. This 3<sup>rd</sup> order plant possess three stable poles  $s_1, s_2, s_3$ . The poles have certain properties in terms of classic control theory: The  $s_1 = -1$  pole: The dominant pole, which plays the most significant role in terms of plant stability and also affects the speed of the plants response in a decelerating way [12, 13], The  $s_2 = -3$  pole. Finally, the  $s_3 = -7$  pole, which represents the fast response mode of the system [12].

Study Case I: The nominal plant  $\overline{H_1(s)}$  neglects the fast mode (with pole  $s_3 = -7$ ) of the true plant

$H(s)$ . The nominal plant selected is the following 3<sup>rd</sup> order system (The nominal plant is chosen with the same relative degree as the true one, for implementation simplicity):

$$\overline{H_1(s)} = \frac{12}{(s+1)(s+3)(s+4)}, \quad \text{with zero initial conditions.}$$

It can easily be observed that the nominal plant retains the first two poles of the true plant, with the difference that the fast response pole,  $s_3 = -7$  is replaced with  $\overline{s_3} = -4$ . In practice, this can happen when a fast subsystem is wrongly modeled [4]. The main interest is to test if the algorithm converges despite the true plant and nominal model mismatch.

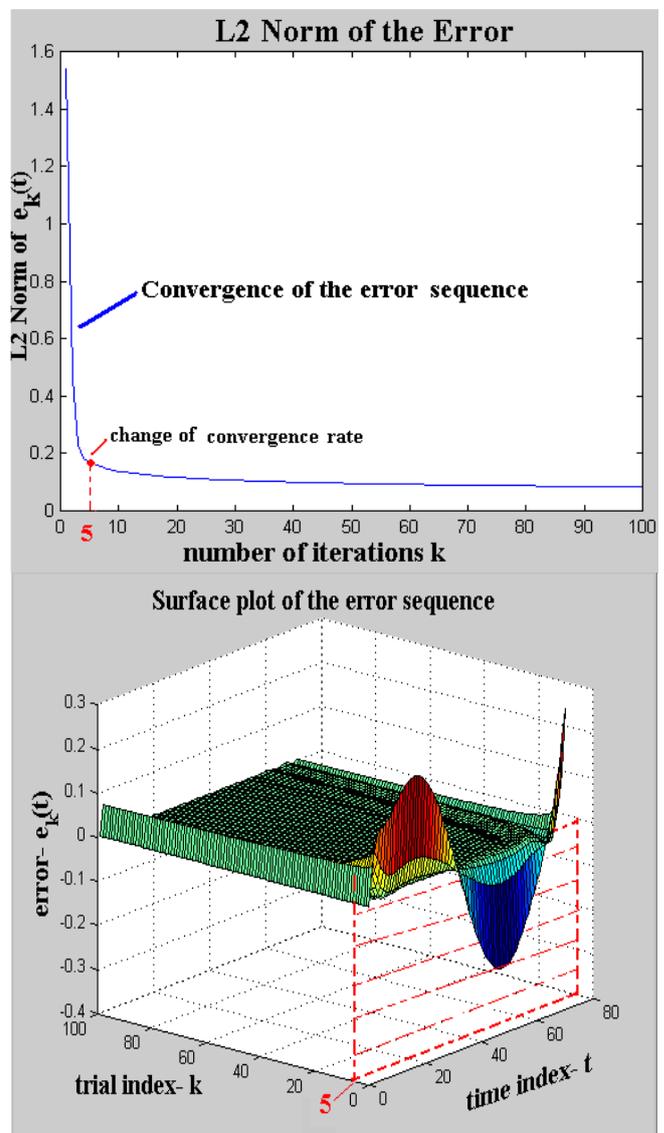


Fig. 10: L2 Norm of the error sequence and surface plot.

Fig. 10 shows that the error sequence converges practically to zero. The rate of the convergence is

fast for the first five trials and after that point becomes slower. It is clear that the band of error has reached satisfying small limits within the first five trials.

Study Case II: The nominal plant  $\overline{H}_2(s)$  neglects the dominant pole ( $s_1 = -1$ ) of the true plant  $H(s)$ . As the nominal plant  $\overline{H}_2(s)$  the following 3<sup>rd</sup> order system is selected (The nominal plant is once again chosen with the same relative degree as the true one, for implementation simplicity): 
$$\overline{H}_2(s) = \frac{84}{(s+4)(s+3)(s+7)}$$
 with zero initial conditions. It can easily be observed that the nominal plant retains two poles of the true plant:  $\overline{s}_2 = s_2 = -3$  and  $\overline{s}_3 = s_3 = -7$  with the design difference that the dominant pole  $s_1 = -1$  is replaced with  $\overline{s}_1 = -4$ . The main interest is to test if the algorithm converges despite the mismatch between the true plant and nominal model. The simulation produces Fig. 11, that shows that the convergence rate of the error is exponential and the convergence gets much slower after the 10<sup>th</sup> trial. Hence, the learning successfully takes place within the first 10 trials and after those; the error converges practically to zero.

## 5 Results and discussion

In section 4, the Norm-Optimal Iterative Control algorithm has implemented to various systems. The algorithms control performance was tested and its potentials and limitations were studied. At the beginning, the dependence of the error convergence rate on the systems relative degree was tested. It was shown that the higher the relative degree of the system the slower the error sequence convergence. It has been shown also the ability of the algorithm to cope with a considerable true plant and nominal model mismatch. Furthermore the degree of robustness towards exogenous disturbances was examined. Two cases of disturbance were considered: the case of white noise entering the plant from the input and the case of white noise affecting the output. The algorithm demonstrated high convergence performance in the case of the input disturbance. For the case of the disturbance acting in the output, the implementation resulted in geometrical error convergence; however, the excessive control input manipulations reduced the algorithms performance. Another case revealed the algorithms ability to successfully train a plant in order to track a varying reference signal. On the whole, the Norm-Optimal ILC algorithm exhibited a

high degree of robustness in the simulations. The most important property of the algorithm, its geometrical convergence of the error sequence, was proved also in practise. In other words, the error sequence is guaranteed to converge to zero in the limit when the algorithm is implemented. Finally it has been observed that the performance of the algorithm can be tuned to a high degree, through variation of design parameters and improvements.

On the other hand, limitations of the algorithm can be considered the following. The algorithm cannot effectively cope a noisy output, as it is downstream of the control action. The robustness of the algorithm is still theoretically unproven. The implementation of the algorithm is possible only for cases where the plant model is known in advance and that the algorithm is effective only for invertible systems.

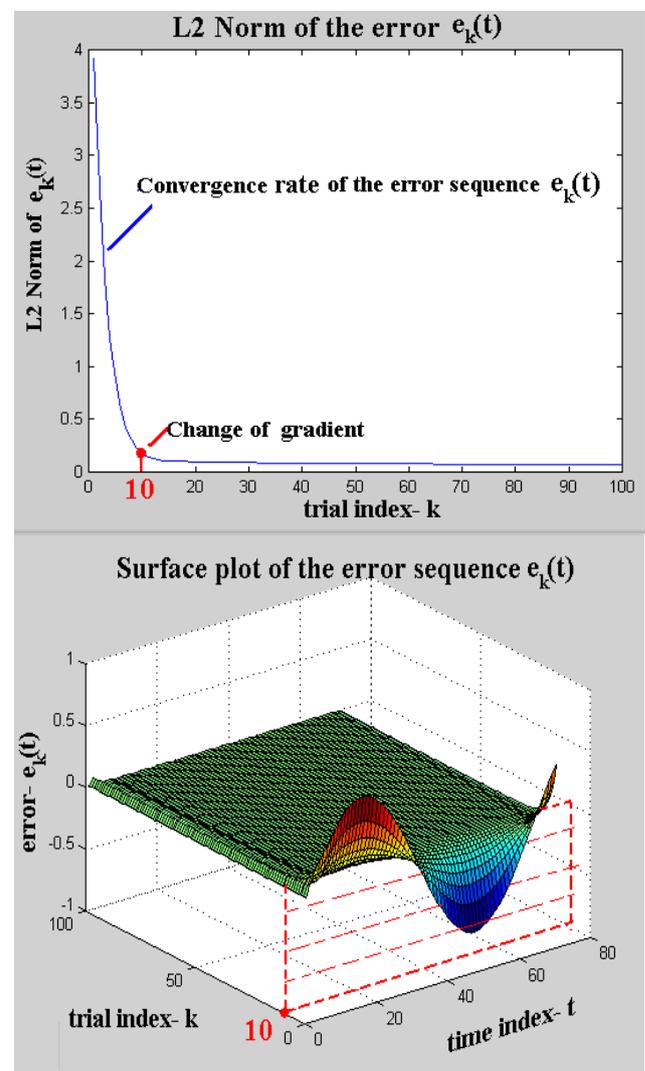


Fig. 11: L2 Norm of the error sequence and surface plot.

## 6 Conclusions

In this paper an optimal iterative learning algorithm for discrete systems has been designed and implemented. Its design and implementation in Matlab® 7 and Simulink have been described in detail. The algorithm has been applied on several representative discrete systems cases in order to evaluate its performance and to reveal its capabilities and limitations.

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### Vitae:

**Vassiliki Vita** was born in Athens, Greece. She received a BEng degree in Electrical Engineering from the faculty of Higher School of Pedagogical & Technical Education (ASETEM/SELETE), Athens, Greece in 2000. In 2002 she received a Master of Science in Control Systems from the Department of Automatic Control & Systems Engineering, at the University of Sheffield in U.K. Since 2000 she has worked as a free-lancer engineer and since 2002 as an Electrical Engineering Lecturer.

**Athanasios Vitas** was born on January 9, 1975 in Athens, Greece. He received the Dipl. Eng. degree in Electrical Engineering from the National Technical University of Athens (NTUA) in 2000. Since 2000 he has worked as a free-lancer engineer and since 2004 as a Traffic Analysis Engineer at Wind's Network Development Department / Core Network.

**George E. Chatzarakis** was born in Serres, Greece, on May 20, 1961. He received the Dipl. Eng. and the PhD degrees in Electrical Engineering from the National Technical University of Athens (NTUA) in 1986 and 1990, respectively. Currently, he is a Professor in the School of Pedagogical and Technological Education (ASPETE) in Athens, Greece.