Symbolic equation for linear analog electrical circuits using Matlab

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Abstract: - In this paper is presented a program which generates the modified nodal equation for electric analog circuits in a symbolic, partial symbolic and numerical mode. The program is an application, made in the environment of the program MATLAB version 7.1, which has a powerful symbolic math toolbox. MATLAB is a high-performance software package dedicated for numerical analysis and graphic representations in engineering applications. In this paper we try to explore the capabilities of this program in symbolic domain.

Key-Words: - analog circuits, circuit functions, symbolic equations, Matlab, netlist, modified nodal analysis.

1 Introduction
The modified nodal methods have been implemented in a program on a Pentium Dual Core computer compatible to obtain symbolic forms, partly symbolic or numerical equations for electronic linear analog circuits. Between the input gate and the output one specified by the user, any of the four types of circuit functions (transfer impedance - $Z_{ei}$, admittance transfer function - $Y_{ei}$, voltage transfer factor (amplifier) - $A_{ei}$ and current transfer factor (amplifier) – $B_{ei}$) for linear analog electric circuits around a functionary point. Starting from the description of the circuit through to a file-type input netlist, according to the window shown in figure 1, the following capabilities:
- generates symbolic, partly symbolic or numerical equations using modified nodal analysis for linear electrical circuits;
- generates symbolic form of the extensive matrix of the system equations and determines the symbolic solution, partly symbolic or numerical.

![Fig.1 Application window](image-url)
to the circuit system;

Data entry is a set of lines describing the circuit branch. Each branch of the circuit is described by a proper letter which indicates the type of circuit elements, the first node, final node, the nominal value and the value of the tolerance. In the case of controlled sources we should also introduce nodes of command branch. The netlist is similar to those described in the program SPICE.

2 Modified Nodal Analysis (MNA)

To solve the circuit and obtain the desired circuit function we use the modified nodal analysis.

In the modified nodal analysis the circuit matrix is obtained by the augmentation of the nodal conductance (admittance) matrix corresponding to the NA-compatible circuit elements with additional rows and columns for non-NA-compatible circuit elements.

The circuit equations in operational corresponding to the modified nodal analysis method, have the following form:

\[
\begin{bmatrix}
Y_{n-1,n-1}(s) & B_{n-1,m}(s) \\
A_{m,n-1}(s) & Z_{m,m}(s)
\end{bmatrix}
\begin{bmatrix}
V_{n-1}(s) \\
I_{m}(s)
\end{bmatrix}
= \begin{bmatrix}
I_{sc,n-1}(s) \\
E_{m}(s)
\end{bmatrix}.
\]

Where: \( Y_{n-1,n-1}(s) \) - is the admittance matrix corresponding to the n-1 independent nodes; \( B_{n-1,m}(s) \) is an (n-1)xm matrix and it contains the elements -1, 0, +1 and the current gains of the CCCS’s; \( A_{m,n-1}(s) \) is an mx(n-1) matrix containing the elements -1, 0, +1 and the voltage gains of the VCVS’s; \( Z_{m,m}(s) \) is a mxm matrix having the transfer impedances of the CCVS’s; \( V_{n-1}(s) \) is node voltage vector corresponding to the n-1 independent nodes. The vector \( I_{n}(s) \) represents the current vector corresponding to the non-NA compatible circuit branches and it has the following structure:

\[
I_{m}(s) = \begin{bmatrix}
I_{e}(s) & I_{c}(s) & I_{sc}(s) & I_{C}(s) & I_{L}(s)
\end{bmatrix},
\]

where: \( I_{e}(s) \) is the ideal voltage source current vector; \( I_{c}(s) \) the controlled (output) branch current vector of all controlled voltage sources; \( I_{sc}(s) \) the controlling (input) branch current vector of the CCVS’s; \( I_{C}(s) \) the controlling branch current vector of the CCCS’s. The vectors \( I_{sc,n-1}(s) \) and \( E_{m}(s) \) represent the contributions of the excitation sources (independent current and voltage sources).

3 Description of the application

To explain and understand this application we consider the following circuit (figure 2).
application describes how to use a lowpass Sallen-Key filter with a dual supply voltage. The circuit provides a Butterworth response with a 30MHz bandwidth, and is ideal for video-reconstruction filtering in HDTV applications. In HDTV applications, lowpass filters are used for reconstruction of RGB and component video (Y, Pb, and Pr) signals. They are placed following the video DAC to remove the higher frequency replicas of the signals, as well as before the ADC for anti-aliasing. Figure 2 shows a one-channel, dual-supply configuration incorporating the MAX4382. It is a three-pole, Sallen-Key Butterworth lowpass filter, in which the current DAC generates the video signal, and the resistor (RL) sets the amplitude. With the MAX4382, the RL, R1, R2, C1, and C2 form a two-pole, Sallen-Key lowpass filter having a gain of 2. The driving load (75Ω) at the output, plus RT and Cp, sets the real pole.

In the Figure 2 circuit, the -3dB bandwidth is about 30MHz. The attenuation is approximately 14dB at 44.25MHz, and 28dB at 74.25MHz. The group delay is roughly 6.5ns. If the current DAC load is different than 75Ω, just use the following relationship to set the value of R1: R1 + RL = 150Ω. For RL greater than 150Ω, C1, C2, R1, and R2 will need to be adjusted.

The input file for this example has the following structure:

The first row from this net list file represents the number of nodes of the circuit. After that each line describes a circuit element which is defined by the first letter (ex. R for resistor, C for capacitor etc). This application, in order to solve the circuit with symbolic method, needs to contain the following instruction lines, particularized for each circuit element – in the example we have the code for resistors.

```
for i = 1:n
%daca avem resistor
if (findstr(latura(i).tip, 'R' ) == 1)
    nr_r_k = nr_r_k + 1;
    k1 = int8(str2double(num2str(latura(i).ni)));
    k2 = int8(str2double(num2str(latura(i).nf)));
    if  ((k1 > 0) & (k2 > 0))
        exp1 = [A(k1, k1) '1/'  latura(i).tip];
        exp2 = [A(k1, k2) '-1/'  latura(i).tip];
        exp3 = [A(k2, k1) '-1/'  latura(i).tip];
        exp4 = [A(k2, k2) '1/'  latura(i).tip];
        str1 = sym(exp1);
        str2 = sym(exp2);
        str3 = sym(exp3);
        str4 = sym(exp4);
        C(k1, k1) = C(k1, k1) + str1;
        C(k1, k2) = C(k1, k2) + str2;
        C(k2, k1) = C(k2, k1) + str3;
        C(k2, k2) = C(k2, k2) + str4;
```
elseif (k1 > 0)
    exp1 = [A(k1, k1) '1/' latura(i).tip];
    str1 = sym(exp1);
    C(k1,k1) = C(k1, k1) + str1;

elseif (k2 > 0)
    exp4 = [A(k2, k2) '1/' latura(i).tip];
    str4 = sym(exp4);
    C(k2, k2) = C(k2, k2) + str4;
end

The A coefficient matrix obtained with the program for this example will be as above.
The admittance transfer function for this example in a symbolic form:

\[
Y_{ei} = \frac{GA*(R5+R4)/(1+s*C3*R6)/(R4*R5+R5*Rf+R4* Rf+R5*R3+R5*s*C2+R2*R3- R2*R5*GA*Rf*s*C2+R2*R5*Rf*s*C2+R5*Rf*s*C1 *R3+ R2*R5*Rf*s*C1+R2*R5*Rf*s^2*C1*R3*C2+R2*R5 +R2*Rf*R4*s*C2+Rf*R4*s*C1*R3+R2*Rf*R4*s*C1+R2*Rf*R4*s^2*C1*R3+C1+R2*Rf*R4*s*C2+R2*R4*s*C2+R2*R4+GA*Rf*s *C1*R3*R4+R5*s*C1*R3*R4+R2*GA*Rf*s*C1*R4 +R2*R5*s*C1*R4+R2*GA*Rf*s^2*C1*R3*C2+R4+ R2*R5*s^2*C1*R3*C2*R4)*Rf*E1}{\ldots}
\]

In fig. 3 is represented the output waveform for Lowpass Sallen-Key filter obtained with this Matlab application after replacing symbolic values with nominal ones.

4 Examples of applications

Bandpass Sallen-Key filter

The parameter of this filter is Q of 10 and
amplification is 10 at $f_o=500$ Hz. The nominal values of the filter’s components are: $R1=15800$; $R2=5110$; $R3=2610$; $R4=3320$; $R5=13300$; $C1=1e^{-7}$; $C2=C1$;
The input file for this example has the following structure:

```
5
R1 1 2 15800 0.02
R2 2 4 5110
C1 2 0 0.00000001 0.1
C2 2 3 0.00000001 0.1
R3 3 0 2610 2
Rf 3 5 10000000000 0.02
R4 5 0 3200 0.02
R5 4 5 13300 0.02
E1 1 0 1
G1 5 3 4 0 10000
```

The nominal waveform obtained after replacing the symbolic letter with values is illustrated below:

The transfer function for the filter has the following form:

$$Y_{ei}=K(s)\frac{C2G1}{(R5s+1)(R3s+1)}\left(\frac{Rf}{R4s+1}\right)\left(\frac{1}{R1s+1}\right)$$

The extend $A_{ei}$ coefficient matrix for this filter is like below:

$$A_{ei} = \begin{bmatrix}
1/R1 & -1/R1 & 0 & 0 & 0 & 1 & 0 \\
-1/R1 & 1/R1+1/R2+s*C1+s*C2 & -s*C2 & -1/R2 & 0 & 0 & 0 \\
0 & -s*C2 & s*C2+1/R3+1/Rf & 0 & -1/Rf & 0 & 0 \\
0 & -1/R2 & 0 & 1/R2+1/R5 & -1/R5 & 0 & 1 \\
0 & 0 & -1/Rf & -1/R5 & 1/Rf+1/R4+1/R5 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -A & 1 & A & 0 & 0
\end{bmatrix}$$
In order to be sure that our application generates correct symbolic equation, and does not introduce some deviation in the comportment of the circuits, we have simulated the same circuit with the same nominal values in ORCAD program, and we obtained the following waveform which is similar, if not identical, with the one obtained with our algorithm.

**Bandpass Deliyannis filter**

The input data of this bandpass filter are: $f_0 = 500$ Hz; $Q = 20$; $R_1 = 4.93K$; $R_2 = 205K$; $R_a = 10K$; $R_b = 252K$; $C_1 = 0.01uF$; $C_2 = 0.01uF$

The input file for this filter has the following structure:

```
5
R1 1 2 4930 0.02
R2 3 4 205000 0.02
C1 2 3 0.0000001 0.1
C2 2 4 0.00000001 0.1
R5 3 5 10000 0.2
R3 4 5 252000 0.2
R4 5 0 10000 0.2
E1 1 0 1 0.1
G1 3 5 4 0 100000
```

The transfer function for the filter has the following form:
The nominal waveform obtained after replacing the symbolic letter with values is illustrated below:

![Fig. 8 Output waveform for bandpass Delyiannis filter](image)

The extend $A_{el}$ coefficient matrix for this filter is as above.

$$
A_{el} = \begin{bmatrix}
\frac{1}{R1} & -\frac{1}{R1} & 0 & 0 & 0 \\
-\frac{1}{R1} & \frac{1}{R1} + s^*C1 + s^*C2 & -s^*C1 & -s^*C2 & 0 & 0 & 0 \\
0 & -s^*C1 & \frac{1}{R2} + s^*C1 + \frac{1}{R5} & -\frac{1}{R2} & -\frac{1}{R5} & 0 & 0 \\
0 & 0 & -\frac{1}{R2} & 1/R2 + s^*C2 + 1/R3 & -1/R3 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & A & 1 & -A & 0 & 0
\end{bmatrix}
$$

In order to be sure that our application generates correct symbolic equation, and does not introduce some deviation in the comportment of the circuits, we have simulated the same circuit with the same nominal values in ORCAD program, and we obtained the following waveform which is similar, if not identical, with the one obtained with our algorithm.

![Fig. 9 Output waveform from ORCAD simulation](image)
5 Conclusion

The main advantage of the MNA method is that the contributions of all individual circuit elements can be inserted directly in appropriate places in (1), independently of whether the primary variable of a particular element is a voltage or a current.

MNA method associated with an efficient nod tearing leads to an important reduction in computing time and to an increase of the accuracy.

This is a new software environment that allows the schematic representation, analysis and design for linear analog circuits.

This application is very useful in circuits design because it is dedicated only for circuit analysis and combined with the power of Matlab platform we can test very complex circuits.

Next steps are to develop this application with new capabilities like tolerance analysis.

Programs like ORCAD have limited number of circuits nodes that they can solve.

The speed of this application is not the main issue only the size of the circuit analysis it’s important

6 Instruction lines

function tolerereffectfin
% clear;
if nargin<1,
    action='start';
end;
%coordonatele ferestrei grafice si a axelor pe ecran
ps=0.01;   % distanta ferestrei fata de stanga
pj=0.3;    % distanta ferestrei fata de jos
lg=0.98;   % lungimea ferestrei
hg=0.65;   % inaltimea ferestrei
global DISP
global maximuri
global count
count=1;
maximuri(count)=0;

DISP=1;
S=brighten([zeros(8,2) (3:10)/10];
prism(56),1/3);
if strcmp (action, 'start'),
    clf reset;
set(gcf,'Units','normalized','backingstore','off',
    'Color','blue','NumberTitle','off','colormap','S',
    'Position',[0.125 0.3 0.98 0.65],
    'Name','ANALIZATOR DE TOLERANTA a CIRCUITELOR ELECTRICE ANALOGICE');
end;
n_semn=1;   % numarul de tipuri de semnale
n_buts=n_semn+1; % numarul de butoane
n_inchidire=4; % numarul de butoane pentru reprezentari
n_buttoane=n_buts+n_inchidire+3; % numarul de butoane
% echivalente (+spatiile)
% se plaseaza butoanele menului in partea stinga
h_g=1-(n_buttoane+1)*.01;  % inaltimea echivalenta pentru butoane
h_buttoane=.9/n_buttoane
l_buttoane=0.37; % lungimea butoane
l_d_s=0.01;     % distanta fata de marginea din stanga
l_d_j1=0.01;    % distanta fata de marginea de jos
% se pune butonul de inchidere "stop"
pos = [d_s1 d_j1*5 l_buttoane h_buttoane]; % pozitia butoanei
%f=figure('Visible','off','Position',[360,500,450,350]);
hc1 = uicontrol (gcf,
    'Units','normal','Position',[d_s1 d_j1 l_buttoane h_buttoane],
    'String','STOP','Callback',{@stop_binary_Callback});
%pos = [d_s1 (h_buttoane+6+d_j1) l_buttoane*.95+h_buttoane];
% hincarca=uicontrol(gcf,'units','normal','Style','pushbutton','String','Incarca circuit','Position',[d_s1 (h_buttoane+6+d_j1) l_buttoane*.95+h_buttoane],
% 'Callback',[@incarca_Callback]);
hfuncie=uicontrol(gcf,'Style','pushbutton','units','normal','String','Incarca functie','Position',[d_s1 (h_buttoane+6+d_j1) l_buttoane*.95+h_buttoane],
% 'Callback',[@funciebutton_Callback]);
haruncab=uicontrol(gcf,'Style','pushbutton','units','normal','String','grafic','Position',[d_s1 (h_buttoane+6+d_j1) l_buttoane*.95+h_buttoane],
% 'Callback',[@aruncab_Callback]);
latura = struct('tip',{''},'ni',{},'nf',{},'valoare',{},'tola r',{});

val(1)=0;
nod1(1)=0;
nod2(1)=0;
frec=[];
functietran=[''];
old=-1;
nrlaturi=0;
y=[];
hold on;
set(f,'Visible','on')

mx=[''];

function
incarcabutton_Callback(source,eventdata)

[filename,pathname]=uigetfile('*.cir');

if isequal(filename,0)
    disp('Utilizatorul a ales Cancel')
else
    disp(['Utilizatorul a ales',fullfile(pathname,filename)])
    %open(file)

fid=fopen(fullfile(pathname,filename),'r');

sr=fgetl(fid);

[mx, sir] = strtok(sir);

nm = str2num(mx)

k=1;

while 1
    sir=fgetl(fid);
    if ~ischar(sir), break, end

    [mx,sir]=strtok(sir);
    latura(k).tip=mx;
    [mx,sir]=strtok(sir);
    latura(k).ni=str2num(mx);
    [mx,sir]=strtok(sir);
    latura(k).nf=str2num(mx);
References: