Odd/Even Scroll Generation with Inductorless Chua's and Wien Bridge Oscillator Circuits

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Abstract: - An inductorless Chua's circuit, and a scroll generator are presented in this paper. The scheme employs only RCs and op-amps, and provides a very reliable generation of multiple odd and even chaotic scrolls. The design formulas and examples including simulation and practical results are illustrated. The principle has been applied successfully to generate some odd/even scrolls with the Wien bridge oscillator as illustrated by the simulation and the practical results.

Key-Words: - Chaos, Chua's circuit, inductorless, Wien bride oscillator, scroll generation.

1 Introduction

CHAOTIC phenomenon has received much interest for a few decades. Such behaviour has been successfully applied to signal transmission and cryptography [1]-[3]. Several types of oscillators have been studied and applied for generating chaos, e.g. Collpits, Wien bridge, Chua, Lorenz, etc.. Among those, Chua's circuit stands out, and always provides a reliable result. The original Chua's circuit has been modified to have only RCs and op-amps [4]-[6], whose structures are compact and open for simple adjustments. More complicated dynamics of the Chua's circuit have been investigated. These include antimonotonicity, and bubble formation [7]-[8]. It is also possible to replace the piecewise linear characteristic of the Chua's diode with a smooth cubic function [9].

After successful applications of the double scroll oscillators, n-scroll chaotic circuits producing a more complicated behaviour have been developed. The key concept of the scroll generator is to add multiple breakpoints into the characteristics of the Chua's diode [10]-[12]. The generator has not been applied with the inductorless Chua's circuit, and the Wien bridge oscillator before. In this paper, the backgrounds of the Chua's, Wien bridge, and scroll generating circuits are reviewed in the next section. Section 3 presents the design examples, simulation and practical results based on the inductorless Chua's circuit as well as the Wien bridge oscillator. Conclusion follows in section 4.

2 Backgrounds

2.1 The Chua's circuit

The Chua's circuit as shown in Fig.1 is a rich 3rd-oder dynamical system. The NR stands for the nonlinear resistor which is commonly implemented via 2 op-amps and 6 resistors known as the Chua's diode.

$$C_1 \frac{dv_1}{dt} = \frac{1}{R}(v_1 - v_2) - f(v_1) \tag{1}$$

$$C_2 \frac{dv_2}{dt} = \frac{1}{R}(v_1 - v_2) + i_3 \tag{2}$$

$$L \frac{di_3}{dt} = -v_2 \tag{3}$$

$$f(v_1) = G_1 v_1 + \frac{1}{2}(G_1 - G_0)\{|v_1 + E_1| - |v_1 - E_1|\} \tag{4}$$

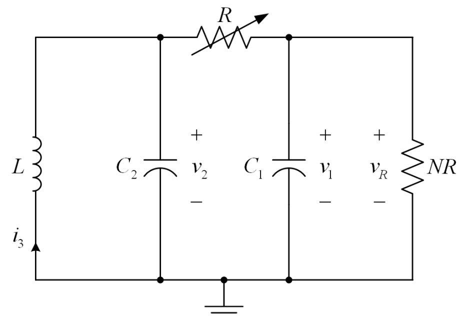


Fig.1 Chua's circuit.

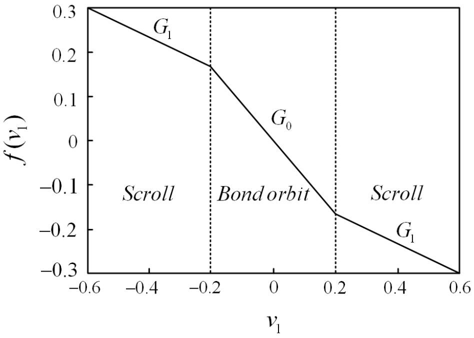


Fig. 2 V-I characteristic of Chua's diode.

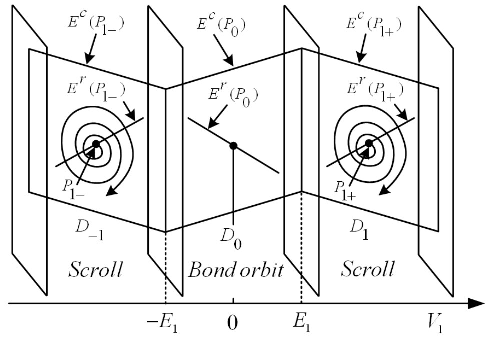
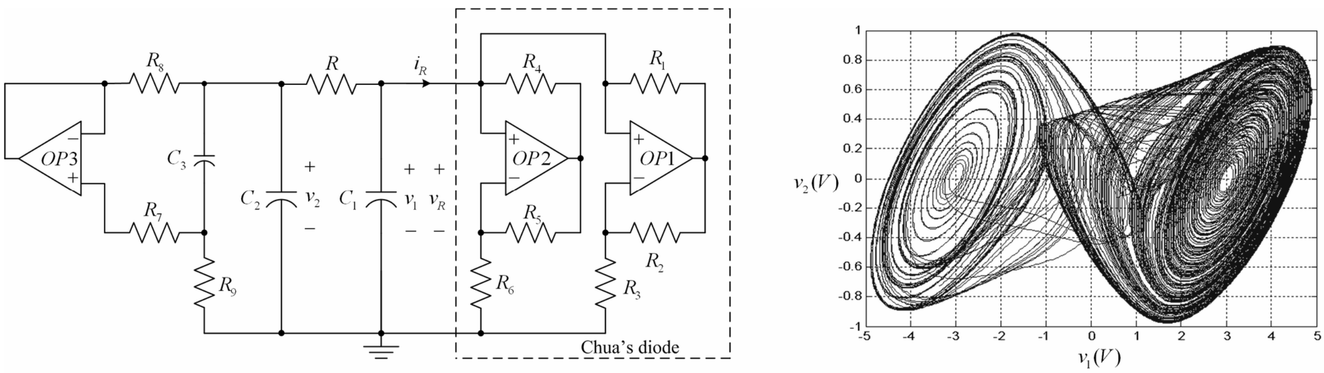


Fig. 3 Vector field regions of double scroll.

The dynamics of the Chua's circuit are described by Eqs. (1)-(3). Eq. (4) expresses the nonlinear function $f(v_1)$ whose characteristic curve is depicted in Fig. 2. Fig. 3 represents the vector regions of a double scroll case. The attractors D_1 and D_{-1} slide from one plane to another alternately through the bond orbit. The inductor in the original Chua's circuit can be replaced by a simulated inductor. Fig. 4 illustrates the inductorless Chua's circuit with its simulated behaviour obtained from PSIM.

2.2 The chaotic Wien bridge oscillator

Wien bridge is a very well-known oscillator. When it is connected to the Chua's diode, the circuit is represented by the diagram in Fig. 5, and produces a double scroll chaotic behaviour. The circuit is very sensitive to initial conditions while the interconnected resistor, R, plays a key role for the adjustment of the initial conditions. The dynamic of the circuit can be represented by Eqs. (5)-(7). The previously described piecewise linear characteristic of the Chua's diode is still applied. The simulated responses of the circuit are shown in Fig. 6.



(a) circuit diagram (b) simulated phase portrait

Fig. 4 An inductorless Chua's circuit.

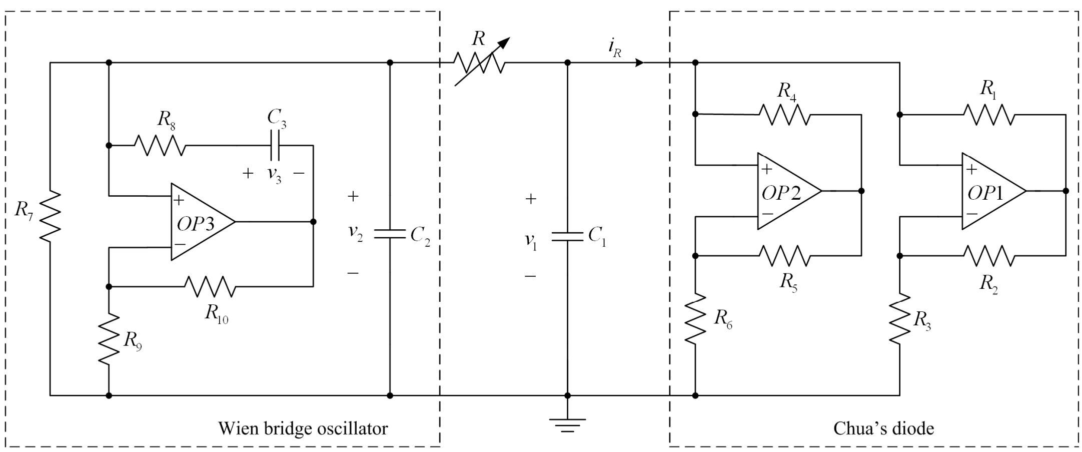
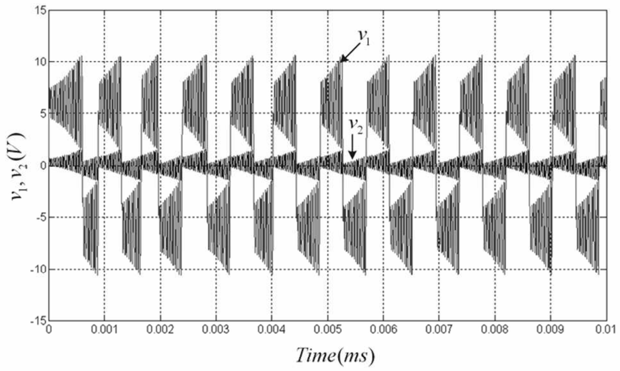


Fig. 5 A chaotic Wien bridge oscillator.

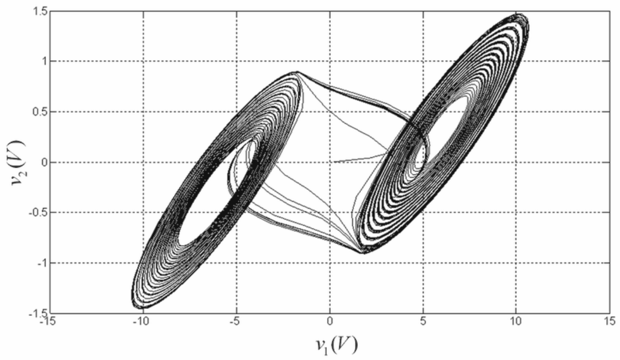
$$C_1 \frac{dv_1}{dt} = \frac{1}{R}(v_1 - v_2) - f(v_1) \tag{5}$$

$$C_2 \frac{dv_2}{dt} = \frac{1}{R}(v_1 - v_2) + \frac{1}{R_8} v_3 \tag{6}$$

$$C_3 \frac{dv_3}{dt} = -\frac{R_3}{R_1 R_4} v_2 - \frac{1}{R_1} v_3 \tag{7}$$



(a) time-domain



(b) phase portrait

Fig. 6 Dynamic responses of the Wien bridge oscillator.

2.3 The scroll generators

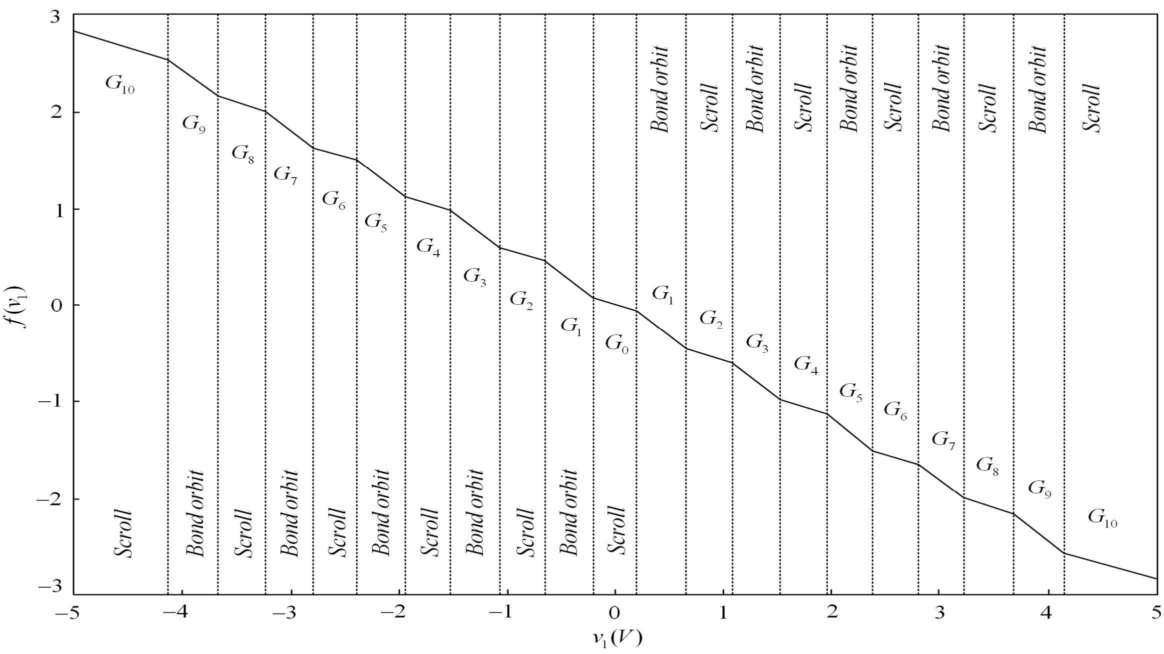
In the paper, it is shown that the inductorless Chua's circuit can be used as the core to generate the chaotic behaviour for multi-scroll generation. Referring to Fig. 3, the left and right boundaries of the bond orbit are separated by the breakpoint voltages, $-E_1$ and E_1 respectively. More break-points can be inserted to generate multiple attractors and bond orbits based on the piecewise linear function concept [11]-[12]. According to this principle, the nonlinear function, $f(v_1)$, of the n-scroll chaotic attractor can be expressed by

$$f(v_1) = G_{n-1} v_1 + 0.5 \sum_{i=1}^{n-1} (G_{i-1} - G_i) \left\{ |v_1 + E_i| - |v_1 - E_i| \right\}$$

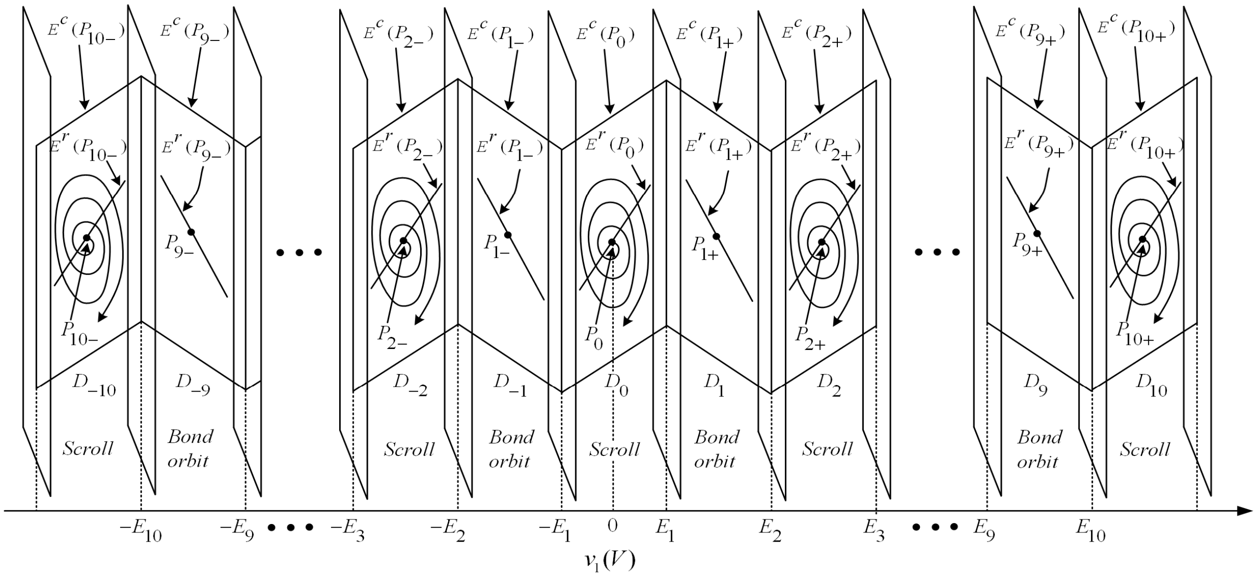
$$= \begin{cases} G_0 v_1 \\ G_i v_1 + \sum_{j=1}^{i} (G_{j-1} - G_j) E_j \\ G_{n-1} v_1 + \sum_{j=1}^{i} (G_{j-1} - G_j) E_j \end{cases} \tag{8}$$

As an example, an 11-scroll attractor requires 21 segments of the piecewise-linear function as shown in Figs. 7(a)-(b).

Referring to Fig. 7(b), the center manifold contains the attractor D_0 with the equilibrium point $P_0 = 0$. For a given breakpoint E_j, the equilibrium points, P_{i+} and P_{i-}, in different regions can be calculated according to Eq. (9),



(a) piecewise linear function



(b) vector field

Fig. 7 The vector manifold of 11-scroll attractor.

$$\begin{cases} P_{i+} = [v_{1i+} \quad v_{2i+} \quad i_{3i+}] \\ \quad = \begin{bmatrix} \dfrac{\sum_{j=1}^{i}(G_{j-1}-G_j)E_j}{(G+G_i)} & 0 & \dfrac{-G\sum_{j=1}^{i}(G_{j-1}-G_j)E_j}{(G+G_i)} \end{bmatrix} \\ P_{i-} = [v_{1i-} \quad v_{2i-} \quad i_{3i-}] \\ \quad = \begin{bmatrix} \dfrac{-\sum_{j=1}^{i}(G_{j-1}-G_j)E_j}{(G+G_i)} & 0 & \dfrac{G\sum_{j=1}^{i}(G_{j-1}-G_j)E_j}{(G+G_i)} \end{bmatrix} \end{cases} \quad (9)$$

where $i = 1, 2, 3, ...$; and v_{1i+}, v_{2i+}, i_{3i+}, v_{1i-}, v_{2i-}, i_{3i-} are the projections of the points P_{i+} and P_{i-} to the corresponding v_1, v_2 and i_3 axes, respectively. The condition $E_i < v_{1i+} < E_{i+1}$ must hold for symmetrical scrolls, and the breakpoint voltages can be determined from (10) where G = 1/R = 1/1.65k = 0.6061 S.

$$E_{i+1} = \frac{2\sum_{j=1}^{i}\left(G_j - G_{j-1}\right)E_j}{\left(G+G_i\right)} - E_i \quad (i = 1, 2, 3, \ldots) \quad (10)$$

3 Realization, Simulations, and Experiments

Realization of the scroll generating circuit follows the procedures described in [10]. After specifying the conductance Gs, and the breakpoint E_1, the other breakpoints can be calculated according to Eq. (10). The circuit uses op-amps, and Rs as the building block shown in Fig. 8.

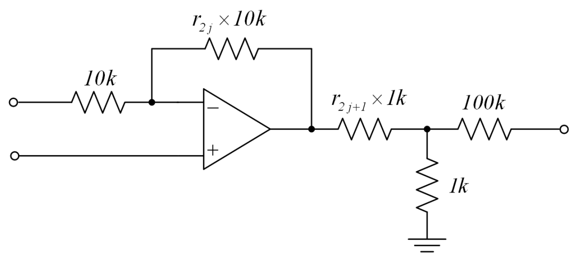


Fig. 8 Building block of the scroll generator.

As an example, the components of the Chua's circuit are $C_1 = 5.06\mu F$, $C_2 = 47.9\mu F$, $R = 1.65k\Omega$, and $L = 9.3mH$. The inductor is realized by an op-amp with $C_3 = 0.01\mu F$, $R_4 = 1\Omega$, $R_5 = 980k\Omega$, and $R_6 = 10k\Omega$. To achieve the odd numbers of scroll of 3, 5, 7 and 9, based on the double scroll Chua's circuit, the following breakpoint (E_1), and the conductances are assigned: $E_1 = 0.2V$, $G_0 = G_2 = G_4 = ... = G_8 = -0.32mS$, and $G_1 = G_3 = ...G_7 = -0.852$ mS . Hence, the obtained breakpoints are $E_i = [E_1 \ E_2 \ E_3 ... E_8] = [0.2000 \ 0.6654 \ 1.0654 \ 1.5308 \ 1.9308 \ 2.3962 \ 2.7962 \ 3.2616]$.

$$r_{2j} = \frac{R_{2j,2}}{R_{2j,1}} = \frac{E_{sat}}{E_{11-j}} - \frac{1+(-1)^j}{2} \quad (j = 1,2,3,...,10) \tag{11}$$

$$\begin{cases} G_8 = -\dfrac{1}{R_3}\left(\dfrac{R_{02}}{R_{01}}\right) & (|v_1| \geq E_8) \\ G_{8-j} = G_{9-j} + \dfrac{(-1)^j}{R_3}\left[\dfrac{1+(-1)^j}{2} + r_{2j}\right]\left(\dfrac{1}{1+r_{2j+1}}\right) & (E_{8-j} \leq |v_1| < E_{9-j},\ j = 1,2,3,...,10) \end{cases} \tag{12}$$

$$\begin{cases} r_0 = \dfrac{R_{02}}{R_{01}} = -R_3 G_8 \\ r_{2j+1} = \dfrac{R_{2j+1,2}}{R_{2j+1,1}} = \dfrac{(-1)^j\left[\dfrac{1+(-1)^j}{2} + r_{2j}\right]}{R_3\left(G_{8-j} - G_{9-j}\right)} - 1 \quad (j = 1,2,3,...,10) \end{cases} \tag{13}$$

The + and – input pins of the op-amp used as the building block are alternately tied to ground. The normalized resistances r_{2j} and r_{2j+1} can be calculated using the formulas in Eqs. (11)-(13), respectively. Consequently, the following normalized resistances of the scroll generator can be obtained.

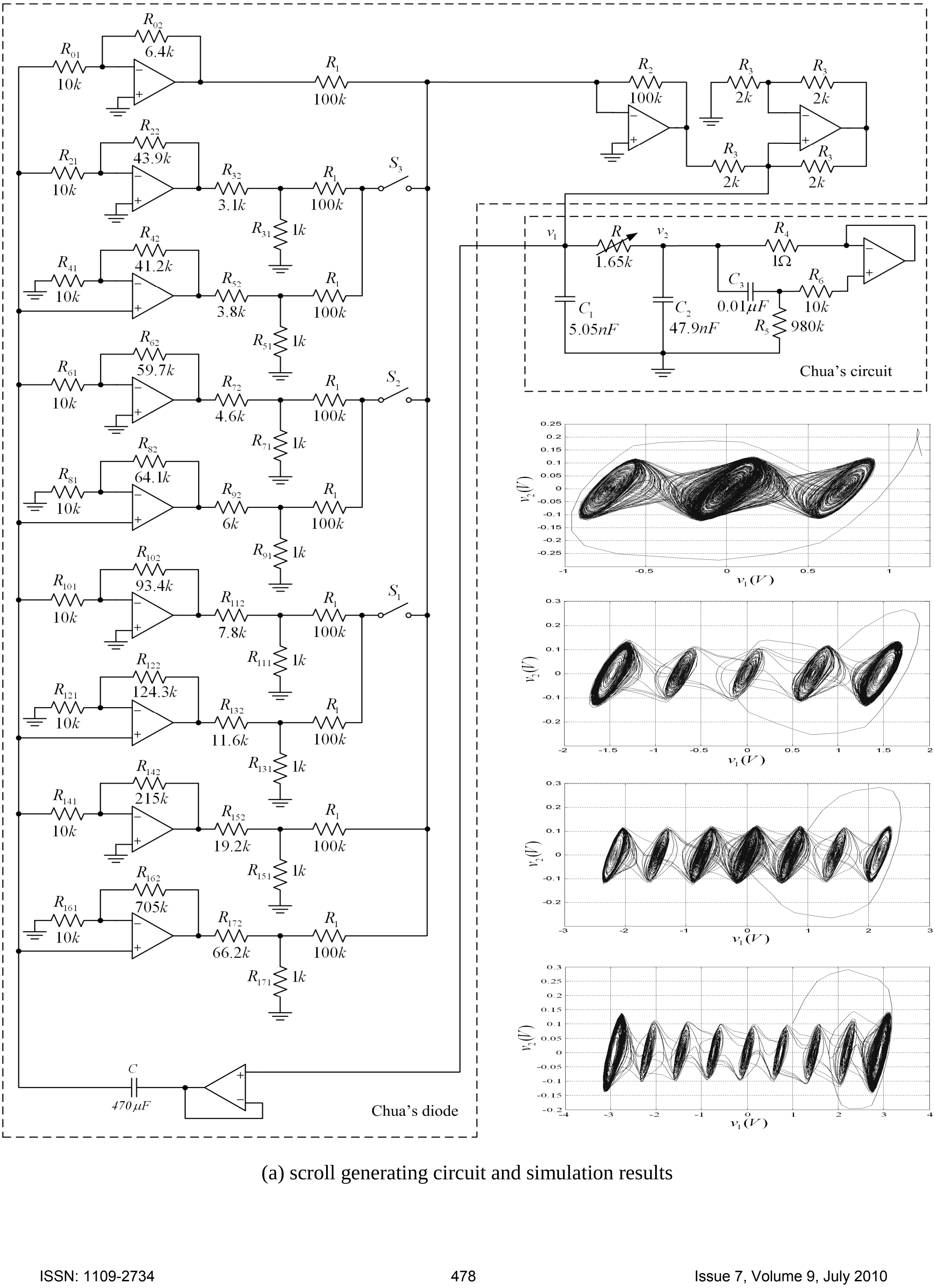
$$\begin{cases} r_{2j} = [r_0 \quad r_2 \quad r_4 \quad r_6 \quad r_8 \quad r_{10} \quad r_{12} \quad r_{14} \quad r_{16}] \\ \quad = [0.6400 \quad 4.3844 \quad 4.1141 \quad 5.9678 \quad 6.4063 \\ \qquad 9.3416 \quad 12.4223 \quad 21.4911 \quad 70.5000] \\ r_{2j+1} = [r_3 \quad r_5 \quad r_7 \quad r_9 \quad r_{11} \quad r_{13} \quad r_{15} \quad r_{17}] \\ \quad = [3.1207 \quad 3.8065 \quad 4.6089 \quad 5.9608 \quad 7.7797 \\ \qquad 11.6149 \quad 19.1984 \quad 66.1992] \end{cases}$$

Fig. 9 illustrates the scroll generating circuit to achieve 3, 5,7 and 9 scrolls based on the inductorless Chua's circuit. The statuses of the switches (S_1, S_2, S_3) are as follows: (off, off, off) for 3, (on, off, off) for 5, (on, on, off) for 7, and (on, on, on) for 9 scrolls, respectively. Simulation and experimental results are also depicted in the same figure. The same principle and the circuit are applied to the case of Wien bridge oscillator. As shown in Fig. 10, both simulation and experimental results confirm the successful applications.

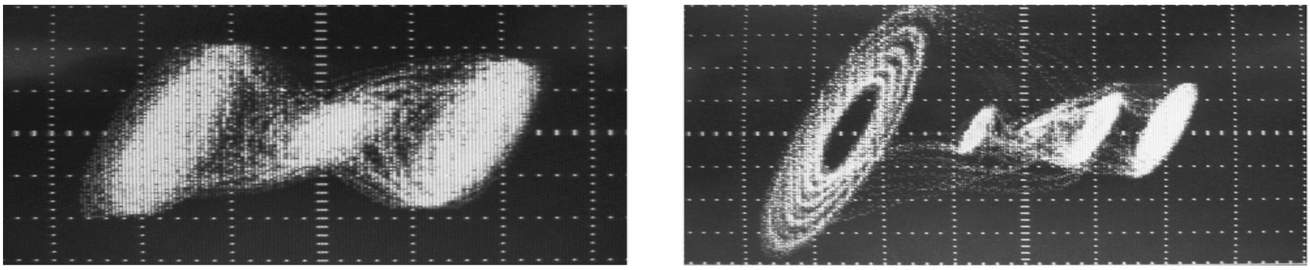
The circuit structure as well as the design formulas can be readily applied to generate even numbers of scroll. Consider the cases of 4 and 6 scrolls, the following conductances are assigned: $G_0 = G_2 = G_4 = -0.852mS$, and $G_1 = G_3 = G_5 = -0.32mS$. Eqs. (10)-(13) are pplied to calculate the breakpoints, and the normalized resistances obtained as: $E_i = [E_1\ E_2\ E_3 ... E_5] = [0.2000\ 0.5441\ 0.9441\ 1.2881\ 1.6881]$, and

$$\begin{cases} r_{2j} = [r_0 \quad r_2 \quad r_4 \quad r_6 \quad r_8 \quad r_{10}] \\ \quad = [0.6400 \quad 8.4736 \quad 10.1060 \quad 15.1516 \quad 25.2966 \\ \qquad 71.5000] \\ r_{2j+1} = [r_3 \quad r_5 \quad r_7 \quad r_9 \quad r_{11} \quad r_{13} \quad r_{15} \quad r_{17}] \\ \quad = [6.9639 \quad 9.4380 \quad 13.2402 \quad 23.7149 \quad 66.1992]. \end{cases}$$

Using the same building block shown in Fig. 8, one can realize the even scroll generating circuit as illustrated in Fig. 11. The circuit diagrams in the insets of Fig. 11(a) represent the Chua's diode, the inductorless Chua's circuit, and the Wien bridge oscillator. The first chaotic core is the inductorless Chua's circuit of which simulated responses are depicted in Fig. 11(b). When the Wien bridge oscillator is in use, the inductorless Chua's circuit is disconnected at the point "a" at which the Wien bridge oscillator is connected instead. There are 2 main switches for scroll generation. To generate 4 and 6 scrolls, the switches (S1,S2) must be (on,off), and (on,on), respectively. Fig. 12 illustrates the experimental results of even scroll generation with the inductorless Chua's circuit, and the Wien bridge oscillator used as the chaotic core circuits. Both simulated and experimental results show very nice agreements, and confirm the successful applications of the scroll generating technique. During the experiments, setting up the initial conditions through adjusting the corresponding components was not difficult providing that good quality components were used.

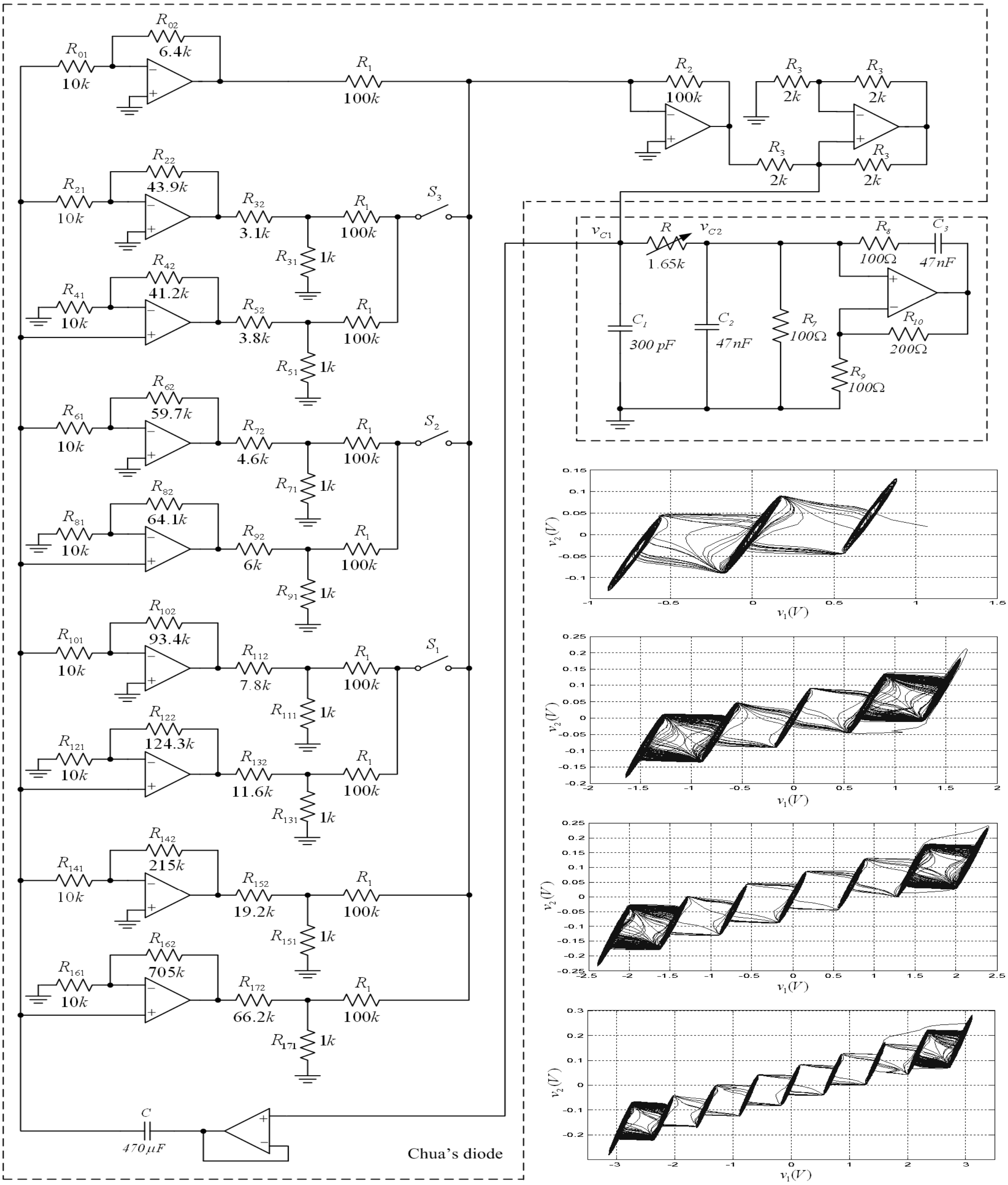


(a) scroll generating circuit and simulation results

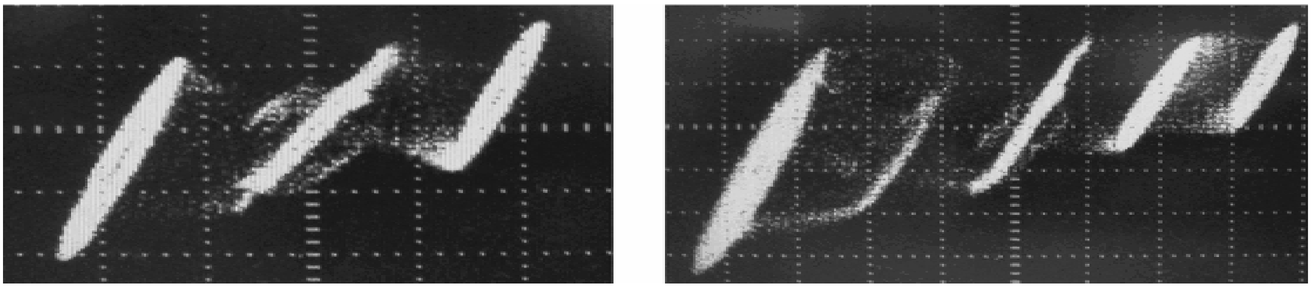


(b) experimental results

Fig. 9 Odd-scroll generating circuit based on the inductorless Chua’s circuit.

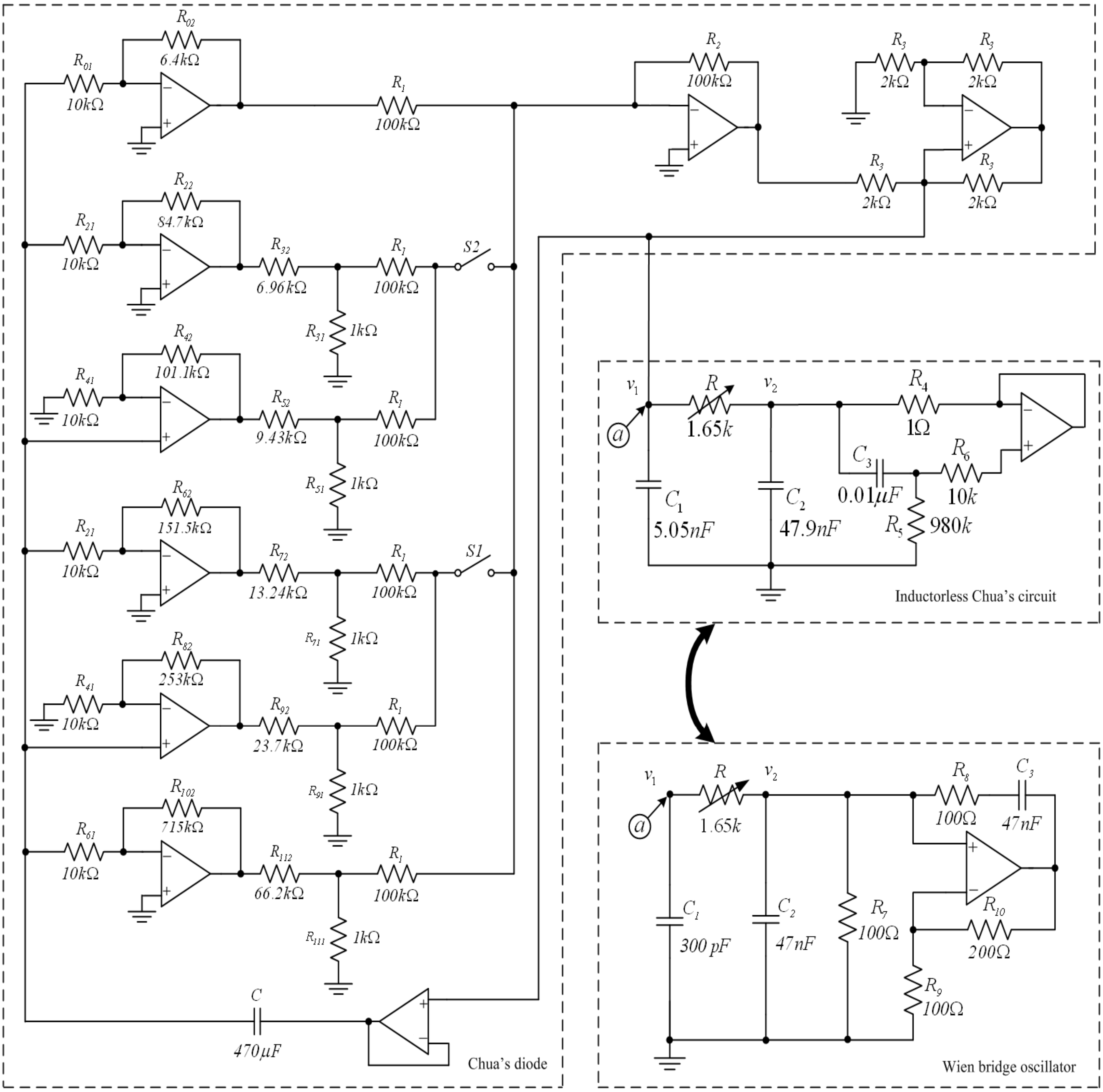


(a) scroll generating circuit and simulation results

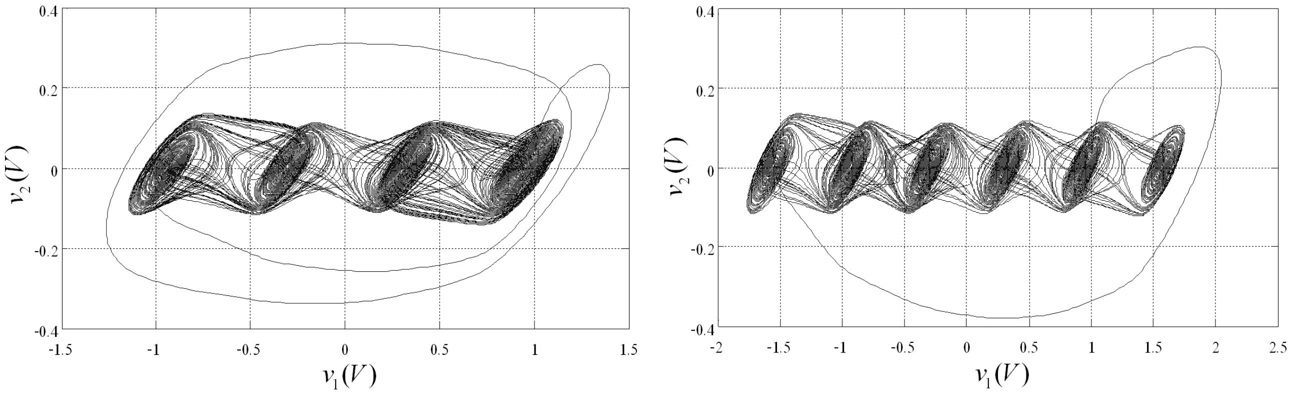


(b) experimental results

Fig. 10 Odd-scroll generating circuit based on the Wien bridge oscillator.

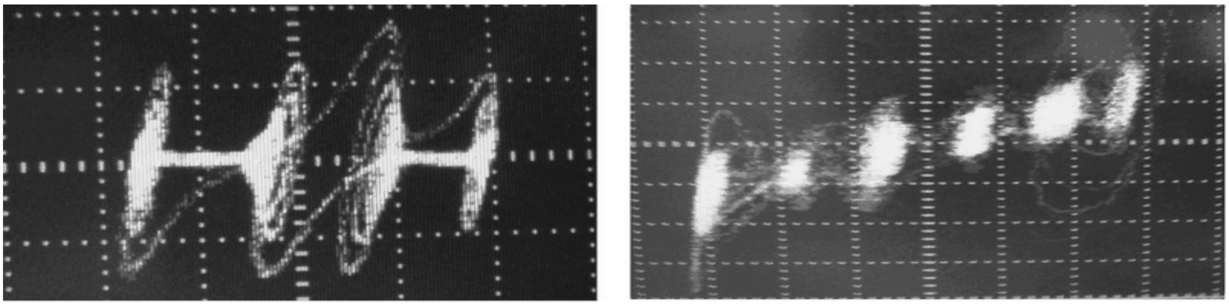


(a) scroll generating circuit

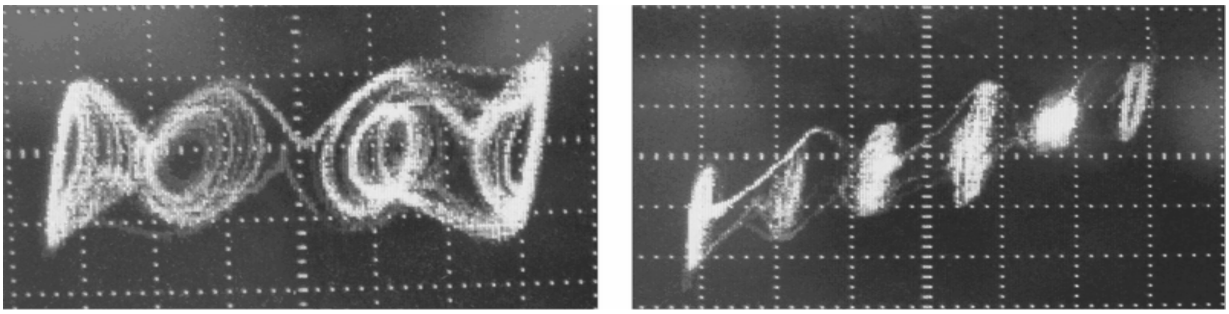


(b) simulation results

Fig. 11 Even-scroll generating circuit based on the inductorless Chua's circuit and the Wien bridge oscillator.



(a) 4 and 6 scrolls generated from the inductorless Chua's circuit.



(b) 4 and 6 scrolls generated from the Wien bridge oscillator.

Fig.12 Experimental results.

4 Conclusion

The principles of the scroll generating circuit, the Chua's circuits, and the Wien bridge oscillator have been explained in the paper. The paper presents the development of scroll generating circuits capable of generating both odd and even numbers of scroll. Both simulation and practical results are illustrated for successful scroll generations with the inductorless Chua's circuit, and the Wien bridge oscillator. Setting up the initial conditions for the circuits can be simply done through adjusting some interconnected resistances.

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