Eddy Current Testing in Height Measurement of Copper Cylinder And Its 2D Electromagnetic Field Simulation

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Abstract: - In this article, this measurement result was found that the fluctuation amplitude of the coil’s inherent inductance L gradually reduces with increasing of frequancecy while the fluctuation amplitude of the coil’s inherent resistance R gradually increases with the increasing of frequancecy. According to the basic principle of eddy current testing, some experiments in this paper proved that the equivalent resistance R and the equivalent impedance Z of the eddy current testing equivalent circuit is unstable in repeated measurements, but the equivalent inductance L has a good stability and can be used to measure the height of copper cylinder at an appropriate frequancecy after doing a calibration between the equivalent inductance L and the height of copper cylinder. Besides, the abnormal measurement result of the equivalent inductance L was illustrated by the finite element electromagnetic field simulation when the copper cylinder is too high.

Key-Words: - Eddy current testing; Height measurement of copper cylinder; The finite element

1 Introduction
Since it appeared in 1970s, eddy current testing has become one of the most frequently used non-destructive technology, for example, it not only can be used to detect corrosion, cracks and other defects of metal, but also can be used to measure surface hardness, coating thickness and other parameters of metal. In this paper, it is used to measure the height of copper cylinder.

2 The principle of eddy current testing
The principle of eddy current testing is shown in Fig. 1. An AC current i_1 flowing through the circular coil produces a time-varying magnetic field H_1. If a tested conductor is put close to the coil, the time-varying magnetic field H_1 will introduce a current i_2 within the conductor. Due to Lenz's law, the eddy current i_2 flows in the opposite direction to i_1 and the i_2 also introduces an associated magnetic field H_2 which is directly opposite to H_1 and interacts with the coil to change the coil equivalent impedance Z. From the change of Z, we can get some the properties of the conducting material[1].

The equivalent circuit diagram of eddy current testing is shown in Fig. 2. Where, R_1 and L_1 is the coil impedance, R_2 and L_2 is the tested conductor impedance. U is the external excitation signal. M is the coefficient of mutual inductance which is related to the lift-off x.
Based on above three equations, the equivalent coil impedance $Z$ can be expressed as the following function.

$$Z = f(\rho, \mu_r, f, x)$$

(4)

Where, $\rho$ is the resistivity of the tested conductor, $\mu_r$ is the relative permeability of the tested conductor, $f$ is the excitation frequency and $x$ is the lift-off.

The depth that eddy currents penetrate into a material is affected by the frequency of the excitation current and the electrical conductivity and magnetic permeability of the conductor. As shown in Fig.3 and Fig.4, the depth of penetration become small in high frequency or high conductivity or high permeability. The depth at which eddy current density has decreased to 1/e, or about 37% of the surface density, is called the standard depth of penetration $\delta$.

$$\delta = \sqrt{\frac{\rho}{\mu_0 \mu_r \pi f}} = \sqrt{\frac{1}{\mu_0 \mu_r \pi f \sigma}}$$

(5)

Where, $\mu_0$ is the vacuum permeability, $\mu_r$ is the relative permeability of the tested conductor, $f$ is the excitation frequency and $\sigma$ is the electrical conductivity of the tested conductor.

Although eddy currents penetrate deeper than one standard depth of penetration, they decrease rapidly with depth. At two standard depths of penetration $2\delta$, eddy current density has decreased to 1/e squared or 13.5% of the surface density[3]. At three depths $3\delta$, the eddy current density is down to only 5% of the surface density.
the function about the height of copper cylinder. According to equation 1, 2 and 3, R and L are also the function about the height of copper cylinder [2].

In this paper, I use the experimental method as shown in Fig. 5 to research the functions between the coil equivalent impedance $Z$, equivalent resistance $R$, equivalent inductance $L$ and the height of copper cylinder. Then, I can know the height of copper cylinder according to the coil equivalent impedance $Z$, equivalent resistance $R$ or equivalent inductance $L$ due to these functions.

Now, I do some experiments as the way shown in Fig. 5. The internal diameter, the external diameter, the height and the number of turns of the coil which I used are respectively $4.08 \text{ mm}$, $6.36 \text{ mm}$, $0.75 \text{ mm}$ and 235 turns.

First, I use Agilent instrument 4294A to get the inherent $L$ and the inherent $R$, and the inherent $Z$ of the excitation coil at different frequencies.

Form the Fig. 6, we can see the inherent $L$ of the excitation is $223.4 \text{ uH}$, and, in the frequency range of $1\sim30 \text{ KHZ}$, the fluctuation amplitude of inductance $L$ gradually reduces with the increasing of frequency.

Form the Fig. 7, we can see the original $R$ of the excitation is $31.88 \Omega$, and, in the frequency range of $1\sim30 \text{ KHZ}$, the fluctuation amplitude of resistance $R$ gradually increases with the increasing of frequency.

Form the Fig. 8, we can see impedance $Z$ is not fluctuate and gradually increases with the increasing of frequency.

Now, I pick out seven copper cylinders and they are all the same except their height. The height of the seven copper cylinders are respectively $2 \text{ mm}$, $2.5 \text{ mm}$, $3 \text{ mm}$, $3.5 \text{ mm}$, $4 \text{ mm}$, $4.5 \text{ mm}$, and $5 \text{ mm}$. Then, I measure the equivalent inductance $L$ of the excitation coil when the different copper cylinders are put into its central hole.

According to equation 5, $\delta$ is a function about $f$, so we need to pick out a low frequency to get a large $\delta$ by which we can measure the higher copper cylinder. But, form the Fig. 6, we know that the fluctuation amplitude of inductance $L$ is very large in low frequency which is harmful to our measurement. So we need do some experiments at different frequency in order to select an appropriate frequency.
As shown in Fig. 9 - Fig. 13, we can not see the change of equivalent inductance $L$ at the frequency of 1KHZ after different copper cylinder were put in the central hole of excitation coil, because the fluctuation amplitude of inherent inductance $L$ is too large. At the frequency of 5KHZ, we begin to be able to distinguish the height of copper cylinder, although the fluctuation amplitude of inherent inductance $L$ is also very large. At the frequency of 10KHZ, if the height of copper cylinder is not higher than 4.5mm, we have already be able to measure the height of copper cylinder very well by measuring the equivalent inductance $L$ after doing a
calibration between the equivalent inductance $L$ and the height of copper cylinder.

At the frequency of 20KHZ and 30KHZ, the fluctuation amplitude of inherent inductance $L$ become very small. We can get a better height measurement when the height of copper cylinder is not higher than 4.5mm, but the copper cylinder whose height is 5mm is abnormal, this is because $\delta$ become smaller with the increasing of frequency.

As we have known that equivalent resistance $R$ is also the function about $h$, some experiments about the function $R$ on the height of copper cylinder are need to see whether $R$ can be better used to measurement.

As shown in Fig.14, the inherent resistance of the excitation coil is different in repeated measurements at a fixed frequency, so it can not be used to measure the height of copper cylinder as the way shown in Fig.5.

As $R$ is a part of $Z$ and the inherent impedance of the excitation coil is unstable, $Z$ is also unstable as shown in Fig.15 and can not be used to measure the height of copper cylinder as the way shown in Fig.5.

4 2D finite element method simulation and analysis

Electromagnetic analysis is actually solving maxwell equations in the given boundary conditions. Maxwell's equations are a group of equations which can illustrate all the macroscopic electromagnetic phenomena. There can be written in the the form of differential equations and can be also written in the form of integral equations, but in this paper, we only give their differential form, as they can derive some differential equations which are suitable for analyzing electromagnetic problems on the finite element method[4].

For the general time-varying fields, differential forms of maxwell's equations can be written as follows:

\[ \nabla \times E + \frac{\partial B}{\partial t} = 0 \quad (6) \]
\[ \nabla \times H - \frac{\partial D}{\partial t} = J \quad (7) \]
\[ \nabla \cdot D = \rho \quad (8) \]
\[ \nabla \cdot B = 0 \quad (9) \]

Where: $E$ is electric field strength(V/m), $D$ is electric flux density(C/m$^2$), $H$ is magnetic field intensity(A/m), $B$ is magnetic flux density, $J$ is current density(A/m$^2$), $\rho$ is charge density(C/m$^2$).

Another basic equation is a continuity equation and can be written as:

\[ \nabla \cdot J = -\frac{\partial \rho}{\partial t} \quad (10) \]

Charge conservation is described in this equation.

Among the equations(6)–(10), there are only three one that are independent and they are called independent equations. The first three equations(6)–(8), or the first two equations(6) and (7) and equation (10), can be selected as such independent equations. The other two equations (9) and (10), or (9) and (8) can be derived from the independent equations, so they are called auxiliary equations[5].

In static field(not changing with time), equations(6),(7) and (10) can be written as:

\[ \nabla \times E = 0 \quad (11) \]
\[ \nabla \times H = J \quad (12) \]
\[ \nabla \cdot J = 0 \quad (13) \]
Equation (8) and (9) remain the same. In this case, there is obviously no interaction between the electric and magnetic fields. Therefore, we can separately consider the electrostatic field and magnetostatic field. Equation (8) and (11) describe the situation of electrostatic field while Equation (9) and (12) describe the situation of magnetostatic field, Equation (13) is the certain result of equation (12).

From the above, we have know that there are only three independent equations among five maxwell's equations and the number of independent equations is less than the number of the unknown parameters, so we can not get the definite solution of maxwell's equations, but, after the constitutive relations of the electromagnetic parameters were known, we can get the definite solution. The macroscopic nature of the medium is described by the constitutive relations.

For simple medium, they can be written as:

\[ D = \varepsilon E \]  
\[ B = \mu H \]  
\[ J = \sigma E \]

Where: \( \varepsilon \), \( \mu \) and \( \sigma \) are respectively the dielectric constant (F/m), Permeability (H/m) and conductivity (S/m). For the anisotropic medium, these parameters are vectors. For the isotropic medium, these parameters are scalar. For the non-homogeneous medium, they are a function of position. For the homogeneous medium, they do not change with the position[6].

In order to solve the maxwell equations in the finite element method, the scalar potential and vector potential are introduced to maxwell equations. The magnetic vector potential is defined as:

\[ B = \nabla \times A \]  
\[ J = \sigma E \]

But, this equation can not uniquely determine \( A \), if \( A \) is a solution of this equation, Arbitrary function \( A' = A + \nabla f \) is also a solution of this equation, no matter what the \( f \) is. Therefore, in order to get \( A \), we must add a condition on the divergence of \( A \), and this condition is called standard conditions[7]. A natural choice for this condition is

\[ \nabla \cdot A = 0 \]

At present, the best software for the finite element method simulation is ANSYS. ANSYS is a general purpose finite element modeling package for numerically solving a wide variety of mechanical problems. These problems include: static/dynamic structural analysis (both linear and non-linear), heat transfer and fluid problems, as well as acoustic and electromagnetic problems[8].

In ANSYS, the magnetic vector potential (MVP) formulation is a nodal-based method and it has three degrees of freedom per node than the scalar method: \( A_x \), \( A_y \), and \( A_z \), the magnetic vector degrees of freedoms in the X, Y, and Z directions. Voltage-fed or circuit coupled analysis may add up to three additional degrees of freedom to the magnetic vector degree of freedoms: current, electromotive force drop, and electric potential. You must use the MVP formulation for a 2-D static magnetic analysis; doing so results in a single magnetic vector potential degree of freedom, \( A_z \).

With the MVP formulation, you model current sources (current conducting regions) as an integral part of the finite element model. However, the MVP formulation runs more slowly than the scalar formulation because of the added degree of freedoms.

The procedure for doing a static magnetic analysis using the software ANSYS consists of five main steps: create the physics environment, build and mesh the model and assign physics attributes to each region within the model, apply boundary conditions and loads (excitation), obtain the solution, review the results.

4.1 Creation of 2D models
As shown in Fig.16, due to the symmetry of the geometry and the boundary conditions, the geometry model (coil, tested conductor and air) for computation can be simplified as 1/4 portion to analyze the magnetic field in the tested copper cylinder. In the model, copper cylinder was simulated by the box(A1), excitation coil was simulated by the box(A2), and the air field was simulated by the box(A4). The geometric items of simulation models are the same as the physical models except that the height of copper cylinder simulation model is 7.5mm.
4.2 Selection of element type
The ANSYS element library contains more than 150 different element types. Each element type has a unique number and a prefix that identifies the element category. There are several element types can be used for 2D electromagnetic analysis, such as PLANE13, PLANE53. In this paper, PLANE53 was adopted and its element behavior is axisymmetric in this simulation. PLANE53 is based on the magnetic vector potential formulation and is applicable to the following magnetic field analyses: eddy currents, magnetostatics, electromagnetic-circuit coupled fields, and voltage forced magnetic fields.

4.3 Material properties
Most element types require material properties. In this simulation, there are only three materials: the tested conductor, the coil and the air field, the relative permeability of which are 1H/m, 0.9999H/m and 1H/m, respectively.

4.4 Meshing
There are three meshing methods in ANSYS: free meshing, mapped meshing and sweep meshing. In this paper, free meshing was adopted. Smart element sizing (SmartSizing) gives the mesher a better chance of creating reasonably shaped elements during automatic mesh generation. In this paper, smartsizing was set at 3. The results of meshing of the model is shown in Fig.17.

4.5 Boundary conditions and load (excitation)
We can get many solutions in solving differential equations given in the above context, but only one of them is the true solution. In order to obtain the true solution, the boundary conditions of the corresponding region should be known. In other words, a complete description of an electromagnetic problem should include all the information about differential equations and boundary conditions. In this paper, the boundary conditions are shown as Fig.18. Then, use the operation “Calc Geometric Items of Areas” to get the area (A2) and apply 1A current on the area.

4.6 Solution and simulation results analysis
In this paper, static analysis was adopted. Fig.19 is the 2D magnetic flux lines distribution and Fig.20 is magnetic flux density vector sum. In Fig.19 (a) and Fig.20 (a) the height of the height of copper cylinder simulation model is 7.5mm. In Fig.19 (b), the height of the height of copper cylinder simulation model is 5mm. In Fig.20 (b), the height of the height of copper cylinder simulation model is 3.5mm. Fig.19 (c) is the value of the magnetic flux lines. Fig.20 (c) illustrates the magnetic flux density vector sum represented by different colors. From Fig.19, we can see that the magnetic flux lines can not reach 5mm of the height of copper cylinder in 1A excitation current and there are no magnetic flux through the center of the copper cylinder. In the above measurement, the excitation current that agilent instrument 4294A exported is not less than 1A, so the magnetic flux lines can also reach 5mm of the height of copper cylinder. From Fig.20, we can see that the magnetic flux density vector sum have already no change when the height of copper cylinder is higher than 3.5mm.

Fig.19. 2D magnetic flux lines

Fig.20. Magnetic flux density vector sum
Now, we can know why the measurement is abnormal when the height of the copper cylinder is higher than 4.5mm, there are not magnetic flux lines that can reach 5mm of the height and the magnetic flux density vector sum have already become very weak when the height of copper cylinder is higher than 3.5mm.

5 Conclusion
In this article, this measurement result was found that the fluctuation amplitude of the coil inherent inductance $L$ gradually reduces with increasing of frequency while the fluctuation amplitude of the coil inherent resistance $R$ gradually increases with the increasing of frequency. Then, according to the basic principle of eddy current testing, some experiments in this paper prove that the equivalent resistance $R$ and the equivalent impedance $Z$ of the eddy current testing equivalent circuit is unstable in repeated measurements, but the equivalent inductance $L$ has a good stability and can be used to measure the height of copper cylinder at an appropriate frequency after doing a calibration between the equivalent inductance $L$ and the height of copper cylinder. Besides, the abnormal measurement result of the equivalent inductance $L$ was illustrated by the finite element electromagnetic field simulation when the copper cylinder is too high.

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