Design and Implementation of a Negative Feedback Oscillator Circuit Based on a Cellular Neural Network with an Opposite Sign Template

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Abstract: - In this paper, explicit amplitude and frequency expressions for a Cellular Neural Network with an Opposite-Sign Template (CNN-OST) under oscillation condition are derived and a novel inductorless oscillator circuit with negative feedbacks, based on this simple structure is designed and implemented. The system is capable of generating quasi-sine signals with tuneable amplitude and frequency which can’t be provided at the same time in the classical oscillator circuits.

Key-Words: - Oscillator Circuits, Cellular Neural Networks, Quasi-Sine Signals, Curve and Surface Fitting, Operational Amplifiers, Circuit Design.

1 Introduction
Oscillators are electronic circuits that produce periodical signals (sinusoids, square waves etc.), which play very important roles, especially in communication systems. Colpitts/Hartley, Phase-shift and Wien are widely used sinusoidal oscillator types [1]. The block diagram of a conventional sinusoidal oscillator is shown in Fig. 1.

Here, the \( v_o(t) \) is the periodical output voltage, \( v_i(t) \) is the feedback voltage. \( \alpha = \frac{v_o(t)}{v_i(t)} \) is the voltage gain of the feedforward, \( \beta = \frac{v_i(t)}{v_o(t)} \) is the voltage gain of the feedback networks. The operation principle is as follows:

The noise in the medium which can be considered as the initial condition \( v_o(0) \) for the output voltage, is captured and amplified by the feedforward block employing an active component such as a transistor or an Operational Amplifier (OpAmp), while a 180° phase shift is realized by the passive feedback network, until a periodical sine with constant amplitude and frequency is generated as shown in Fig. 2. The feedback network also sets the frequency of the output signal.
The oscillation constraint is strict ($\alpha \cdot \beta = 1$) and is determined by voltage gains of both blocks. It can be set to a greater value, however in this case the active component at the feedforward network will saturate. Therefore, the amplitude of oscillation depends only on the source voltage of the active component. Despite the system proposed in this paper, in classical oscillators, the tuning of the amplitude by varying the values of circuit components is not possible because of the positive feedback.

Papers about negative feedback oscillators also exist in literature [2], however our system depends on a Cellular Neural Network (CNN) architecture, therefore having a simpler design, since it only contains fundamental OpAmp application blocks.

The paper is organized as follows; first, a brief information about the CNNs are given and a special CNN system, CNN-OST and its oscillation constraint are introduced. The amplitude and frequency surfaces with respect to template elements are sketched and two explicit analytical approaches are found for each. Secondly, a circuit based on this simple topology is designed, simulated and implemented. The output voltages are compared with the numerical solutions of the nonlinear differential equation system defining the structure. Finally, the advantages and drawbacks of the proposed circuit and future works that can be done are discussed.

2 CNNs

2.1 CNNs and Their Applications

CNNs are a class of dynamical neural networks and were first proposed by Chua and Yang [3] which find very effective applications in image processing [4], as well as in chaotic signal generation because of their nonlinear manner [5]. The block diagram of a CNN neuron (cell) is sketched in Fig. 3.

As a dynamic neural network, CNN cells include an integration unit, however unlike the Hopfield nets, here, cells only interact with each other by the $r$ neighbourd hood definition given below:

$$N_{ij} = \{C_{ik}| \max\{k-i|, l-j| \leq r; 1 \leq k \leq M, 1 \leq l \leq N\}$$

(1)

According to (1), at a neighbohood dimension of $r = 1$, a cell will only interact with its nearest neighbours. This constraint will dramatically decrease the number of weight coefficients between cells $C_{ij}$ and $C_{il}$ at an $M \times N$-cell system, enabling a simple implementation. The nonlinear differential equation for a CNN cell $C(i,j)$ shown in Fig. 3 is given by,

$$\dot{x}_{i,j} = -x_{i,j} + A(i,j; k,l) y_{k,l} + B(i,j; k,l) u_{k,l} + I$$

(2)

Where $x_{i,j}$ is the “state” of the cell $C(i, j)$ which we will mostly deal throughout the paper; $y_{k,l}$ is the “output”, $u_{k,l}$ is the “input” of its neighbour $C(k,l)$; $A(i,j; k,l)$ is the “weight coefficient” between $C(i,j)$ and the output of $C(k,l)$; $B(i,j; k,l)$ is another “weight coefficient” between $C(i,j)$ and the input of $C(k,l)$; and finally, $I$ is a “threshold” value common for all cells. The activation function namely, “Piecewise Linear Function” (PWL) shown in Fig. 4 can be written as follows

$$y_{i,j} = \frac{1}{2} \left[ |x_{i,j} + I| - |x_{i,j} - I| \right]$$

(3)
The PWL activation function of a CNN cell is given in Fig. 4. Here, the \( v_{xi,j} \) is the “state” \( x_{i,j} \); \( v_{yi,j} \) is the “output” \( y_{i,j} \); \( A(i,j;k,l) \) and \( B(i,j;k,l) \) are the “weight coefficients” between the cells \( C(i,j) \) and \( C(k,l) \). The voltage controlled current sources that can easily be realized by OpAmp blocks, and the cascaded non-inverting amplifier and voltage divider at the output providing the PWL activation function, will be discussed at Section 3.1 in detail.

The circuit model of the cell \( C(i,j) \) is given in Fig. 5. Here, the \( v_{xi,j} \) is the “state” \( x_{i,j} \); \( v_{yi,j} \) is the “output” \( y_{i,j} \); \( A(i,j;k,l) \) and \( B(i,j;k,l) \) are the “weight coefficients” between the cells \( C(i,j) \) and \( C(k,l) \). The voltage controlled current sources that can easily be realized by OpAmp blocks, and the cascaded non-inverting amplifier and voltage divider at the output providing the PWL activation function, will be discussed at Section 3.1 in detail.

The nonlinear differential equations written in matrix form that define the system is given below:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix} x_1 \\
 x_2 
\end{bmatrix} + \begin{bmatrix} \alpha & -\beta \\
\beta & \alpha 
\end{bmatrix} \begin{bmatrix} y(x_1) \\
 y(x_2) 
\end{bmatrix} ; \quad \alpha > 0, \beta > 0 \quad (4)
\]

Here \( A \) is the template matrix that contains the weight coefficients. The \( \alpha \) and \( \beta \) are, \( A(i,j;i,j) \) the self weight coefficient of a cell and \( A(i,j;k,l) \) the weight coefficient with its neighbor, respectively. As seen from (4), the system contains neither \( B(i,j;k,l) \) nor \( I \). The \( x_1 \) and \( x_2 \) solutions can be considered as the outputs. Under oscillation conditions [8], they will have exactly the same amplitude and frequency, however with phase shifts, therefore analyzing only one of them will be sufficient [9].

### 3 Amplitude and Frequency Surfaces

The variations in \( A \) change the stability of the system. Savaci and Vanderwelle proved that \( x_1 \) and \( x_2 \) solutions are oscillations under the following constraint [8]:

\[
\beta > \alpha - 1 \quad (5)
\]

The oscillations are in quasi-sine waveforms because of the nonlinear characteristic of the PWL activation function. The first goal is to assign explicit formulas for the amplitudes and frequencies of these oscillations with respect to the template components. This process is carried out by sketching the 3D surfaces as functions of \( \alpha s \) and \( \beta s \) under the constraint in (5). Mentioned graphics are shown in Fig. 7a and b.
As seen from Fig. 7a, the amplitude depends on \( \alpha \) and is independent from \( \beta \), specifically, linearly increasing with \( \alpha \), therefore can be expressed with the form below:

\[
G(\alpha) = A\alpha + B \quad (6)
\]

It is clear that, this enables the tuning of the \( G(\alpha) \) amplitude by changing the \( \alpha \) values. The following \( A \) and \( B \) coefficients are found by utilizing curve fitting methods within the limitations chosen for the mesh plot:

\[
A = 2.0013 \quad (7a)
\]

\[
B = -0.8332 \quad (7b)
\]

One can see that by analyzing the graphic in Fig. 7b, the frequencies of the oscillations depend on both \( \alpha \) and \( \beta \) values. Therefore the following analytical approach will be convenient to describe the dealt parameter:

\[
F(\alpha, \beta) = (C/\alpha)\beta + D(\alpha) \quad (8)
\]

Here, the \( D(\alpha) \) function is a polynomial with a chosen order of seven which is found by trial and error as the optimum. After a surface fitting process for Fig. 7b, the numerical values for \( C \) and the \( D(\alpha) \) polynomial are determined as:

\[
C = 0.14 \quad (9a)
\]

\[
D(\alpha) = 1.2651 + 2.8768\alpha - 2.5932\alpha^2 + 1.2839\alpha^3 - 0.3961\alpha^4 + 0.0812\alpha^5 - 0.0115\alpha^6 + 0.0011\alpha^7 \quad (9b)
\]

These analytical approaches prove that, both amplitudes and frequencies of the signals generated by the system are controllable unlike the conventional oscillators. In order to find the template component values for a chosen amplitude and frequency pair, one must solve the two nonlinear equations (6) and (8) for \( \alpha \) and \( \beta \).

3. Oscillator Circuit Based on CNN-OST

3.1 Circuit Design

The topology proposed in Fig. 8 is suitable for the realization of the equation system that defines the CNN-OST, where the \( \alpha \) and \( \beta \) components are represented by the conductances of the \( R_\alpha \) and \( R_\beta \) resistors between the cells [10,11]. The circuit will generate quasi-sine signals under specific component values, however without employing positive feedbacks seen in the conventional oscillator circuits, since all the feedbacks are applied to inverting-inputs of the OpAmps as seen from the schematic. The resistances simulating the components can be computed as follows:

\[
R_\alpha = \frac{R_x}{|\alpha|} \quad (10a)
\]

\[
R_\beta = \frac{R_x}{|\beta|} \quad (10b)
\]

The node-voltage-equation for the inverting input of the OpAmp Op11 at cell C(1,1), can be written as in (11), if the OpAmp is assumed to be ideal where \( v_{x1} \) is the state of this cell, \( v_{y1} \) is its output and \( v_{y2} \) is the output of the neighbouring cell.

\[
\dot{v}_{x1} = \frac{1}{R_xC_x} \left( -v_{x1} + \alpha v_{y1} - \beta v_{y2} \right) \quad (11)
\]

It is clear that by choosing \( 1/R_xC_x = 1 \), the above equation becomes the first row of (4). Here, the Op12-Op13 and Op22-Op23 inverting and non-inverting amplifier pairs provide the PWL characteristics of the activation functions for both cells. In order to obtain the negative \( \beta \) value in (4), an extra inverting amplifier circuitry (Op24) with a gain of “-1” is attached at the output of the cell C(1,2). The quasi-sine oscillations are the \( v_{x1} \) and \( v_{x2} \) “states” taken from the outputs of the OpAmps Op11 and Op21 providing the load independency as well, since the input resistances of the OpAmps can be assumed as open-circuits.
3.2. Frequency Denormalization

The $1/R_C$ coefficient at (11) determines the range of the oscillations’ frequency. Specifically, as $1/R_C$ increases the frequency band will increase and decrease vice-versa [12]. By varying either $R_i$ or $C_i$ values, a denormalization process can be carried out by the multiplication of the right hand side of the equation, with the frequency therefore, it is possible to generate oscillations on a wide frequency range without ever changing the resistors for $\alpha$ and $\beta$. It is better to vary $C_i$ instead of $R_i$ to achieve this purpose otherwise, as seen from (10), $R_\alpha$ and $R_\beta$ therefore the mentioned components will be changed for varied $R_i$ values.

One can use stereopotentiometers in order to adjust the values of $\alpha$s and $\beta$s simultaneously, to set the desired amplitude and frequency.

4 Simulation and Implementation

In this section, first, an analysis is performed for a chosen $R_\alpha$ and $R_\beta$ pair, by comparing the theoretical values of the amplitude and frequency computed by the numerical solutions of (6) and (8) by using a 2-cell CNN model constructed in MatLab Simulink software given in Fig. 9 [11], with the ones obtained from the simulated and implemented circuits. Here, the A and B coefficients in the state equation blocks correspond to $1/R_C$, and the K gains denote the $\alpha$s and $\beta$s between the cells, furthermore Cs and Ds must be chosen as 1 in this model. The PSpice software is used for the circuit simulations. The measurement set up for the implementation is given in Fig. 10.

Then as a synthesis procedure, for a specific amplitude and frequency pair, the design is verified by comparing the measured outputs of the oscillator circuit with the simulation as well as with the numerical solutions of (6) and (8). As depicted...
before, the two components of the state set, \( v_1 \) and \( v_{12} \), have exactly the same characteristic and waveform however with phase shifts, therefore analyzing a single state (\( v_{11} \)) is enough.

4.1. Analysis

If standard resistors \( R_a=2.2k\Omega \), \( R_b=470\Omega \) and \( R_c=10k\Omega \) are chosen, \( \alpha \) and \( \beta \) become 4.545 and 21.276 respectively from (10). Then, by using the analytical approaches given in (6) and (8), the amplitudes and frequencies of the oscillations are computed as, \( G=7.918V \) and \( f=0.625Hz \) with the model given in Fig. 9. Here, the \( C_x \) value is chosen as 1\( \mu \)F, which will denormalize the frequency by \( 1/R_cC_x=100 \) and change it to 62.500Hz as depicted in Section 3.2, for the ease of monitoring the oscillations with a standard oscilloscope. The piecewise activation relationship between the state and the output of the cell C(1,2) is given in Fig. 12a. The numerical, simulation and implementation results are given in Fig. 11, 12 and 13, respectively.
Fig. 12: Circuit simulations for analysis process:
(a) The simulation of the piecewise linear activation function at cell C(1,2)
(b) Time-domain simulation,
(c) Frequency domain simulation
for the analysis process when $R_a=2.2\,k\Omega$, $R_b=470\,\Omega$.

Fig. 13: (a) Implementation result in time domain,
(b) In frequency domain when $R_a=2.2\,k\Omega$, $R_b=470\,\Omega$ by using the set up in Fig. 10.

4.2. Synthesis

Here, the amplitude is chosen as $G=12\,V$ and the frequency as $f=1\,Hz$. By solving (6) and (8) simultaneously, the $\alpha$ and $\beta$ values are found as 6.410 and 47.030, which make the corresponding interconnected resistances $R_a$ and $R_b$ 1560.062$\,\Omega$ and 212.630$\,\Omega$, respectively according to (10). Standard resistors 1500$\,\Omega$ and 220$\,\Omega$ are employed in the simulation and implementation. Moreover, the $1/R_C$ coefficient is made 100 again by choosing $R_C=10k\Omega$ and $C_C=1\mu\text{F}$ which will denormalize the oscillation frequency to 100Hz. Numerical, simulation and implementation results in time and frequency domains results are sketched in Fig. 14, 15 and 16.
Fig. 14: The numerical results at the synthesis process by using the model given in Fig. 9.  
(a) Time domain simulation,  
(b) Frequency domain simulation.

Fig. 15: Circuit simulations for synthesis process:  
(a) Time-domain simulation,  
(b) Frequency domain simulation for chosen amplitude of 12V and frequency of 100Hz.

Fig. 16: (a) Implementation result in time domain,  
(b) In frequency domain for chosen amplitude of 12V and frequency of 100Hz.

Since the oscillations are quasi-sines, one can see that by analyzing all of the frequency domain results, the peak value of the amplitude is not exactly equal to the desired value because of the effect of side harmonics. However, as seen from the operations in time domain, the amplitudes of the oscillations take satisfactory values pretty close to the proposed ones.

Results of the numerical computations, simulations and implementations are summarized in Table-I.

5 Conclusions
In this paper, a CNN based oscillator circuit is designed and implemented. Unlike the classical oscillators, the proposed circuit has negative
Table - I: Summary of numerical, simulation and implementation results for analysis and synthesis processes.

<table>
<thead>
<tr>
<th>Analysis ( (R_a=2.2k\Omega, R_f=470\Omega; ; \alpha=4.545, \beta=21.276) )</th>
<th>Simulation (PSpice)</th>
<th>Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical (Simulink)</td>
<td>G=7.918V</td>
<td>G=8.281V</td>
</tr>
<tr>
<td>Amplitude: G=7.918V</td>
<td>G=7.972V</td>
<td></td>
</tr>
<tr>
<td>Frequency: f=62.500Hz</td>
<td>f=65.013Hz</td>
<td>f=70Hz</td>
</tr>
<tr>
<td>Synthesis: Amplitude=12V, Frequency=100Hz ( (\alpha=6.410, \beta=47.030; ; R_a=1.5k\Omega, R_f=220\Omega) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Numerical (Simulink)</td>
<td>G=11.678V</td>
<td>G=11.090V</td>
</tr>
<tr>
<td>G=11.678V</td>
<td>G=12.668V</td>
<td></td>
</tr>
<tr>
<td>f=97.656Hz</td>
<td>f=90.018Hz</td>
<td>f=110Hz</td>
</tr>
</tbody>
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| feedbacks and tuneable amplitude. Since it is based on a 1x2-cell structure CNN-OST, it is capable of generating two quasi-sine signals at the same time with phase shifts. Furthermore, the design is inductorless. The classical 741 BJT OpAmp ICs are employed in the simulation and implementation. If high performance FET OpAmps are used, it is obvious that even more accurate results can be obtained. Having too many components when compared to conventional oscillators is a drawback, it also seen that the closer the \( \alpha \) and \( \beta \) values the worse the output wave shape which must be taken into account in the design. Moreover, because of the nonlinear activation function at the CNN cells, the outputs are not exact sinewaves, therefore contains side harmonics, however, the design is believed to be employed in many applications and suitable for integration.

References:
