Adaptive nonuniform sampling delta modulation - practical design studies

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Abstract: The method of design the values of the adapted parameters for 1-bit delta modulators is the subject of the paper. The method involves systems with adapting both parameters: the quantization step sizes or/and sampling intervals. The necessary values of the sampling intervals and the step sizes should be calculated before the modulation procedure starts. All of the data of the step sizes or/and time intervals are written into the Lookup Tables. Block diagram of the ANS-DM delta modulator based on the Lookup Tables is explained. Main parts of the article are devoted to the analytical relationships on the basis of which step sizes and sampling intervals can be calculated. These formulas have been used in the program SYMMOD to computation particular values of the step sizes and sampling intervals. Two parameters adaptation in the delta modulation makes the modulator and it's algorithm more complicated but decreases considerably the required number of sampling intervals and step sizes thus improving the quality of conversion (*SNR*) in relation to the solutions with only one parameter adaptation.

Key-Words: Adaptive delta modulation, non-uniform sampling, Lookup Table, Jayant's algorithm, dynamic range Lambert's function.

1 Introduction

In the ADM modulators, the variation of the input signal slope causes a change of the quantization step size (k) or sampling interval (τ) according to the adaptation algorithm. Contemporary the modulators with syllabic adaptation of the quantization step size (CVSD) are the most popular and widely applied [1, 2, 3].

The solutions called **N**on-uniform **S**ampling **D**elta **M**odulation (step size = *const*, sampling interval = *var*) and **A**daptive NS-DM (step size = *var* and sampling interval = *var* – adaptation of two parameters) are the examples of the delta modulators with the sampling interval adaptation [4, 5].

In 1-bit adaptive delta modulators (ADM) a wide dynamic range (*DR*) and high quality conversion (*SNR*) are obtained due to the adaptation of the quantization step size or/and the sampling interval [4, 5, 6]. This, in turn, generates a great number of its values to calculation during the process of coding and decoding. The necessity of performing a great number of arithmetic-logic operations in the real time during the shortest sampling intervals requires very fast signal processors. Therefore attempts at earlier design of all required adaptive parameters values are natural. Next the values of the step sizes and sampling intervals are written into the special memories called Lookup Tables (LUT). Lookup Tables are used to replace runtime computation with a simpler array indexing operation. The saving in terms of processing time can be significant. For ANS-DM it is especially important when the sampling intervals are very short.

Decreasing of the number of the adaptive parameters values, causes the simplification of the ADM modulators hardware but always leads to reduction of the conversion quality (*SNR*) or an increasing of the average bit rate (BR_{avg}).

The most effective method of 1-bit ADM modulators realization when preserving a high quality of conversion follows from analytical considerations presented in [4, 5]. It is ANS-DM method. The total dynamic range of the ANS-DM modulator is a product of the dynamic ranges from the each adapted parameter [7].

ANS-DM modulator works according to the typical backward 3-bit algorithm for the two parameters: quantization step size and sampling interval. There are two feedback loops on the functional diagram of the ANS-DM modulator (Fig.1). One is negative (covers quantizer and predictor) and its task is proper approximation. The second is positive (covers only predictor) and produces absolute values of the approximated signal. Output bits of modulation carry information not only

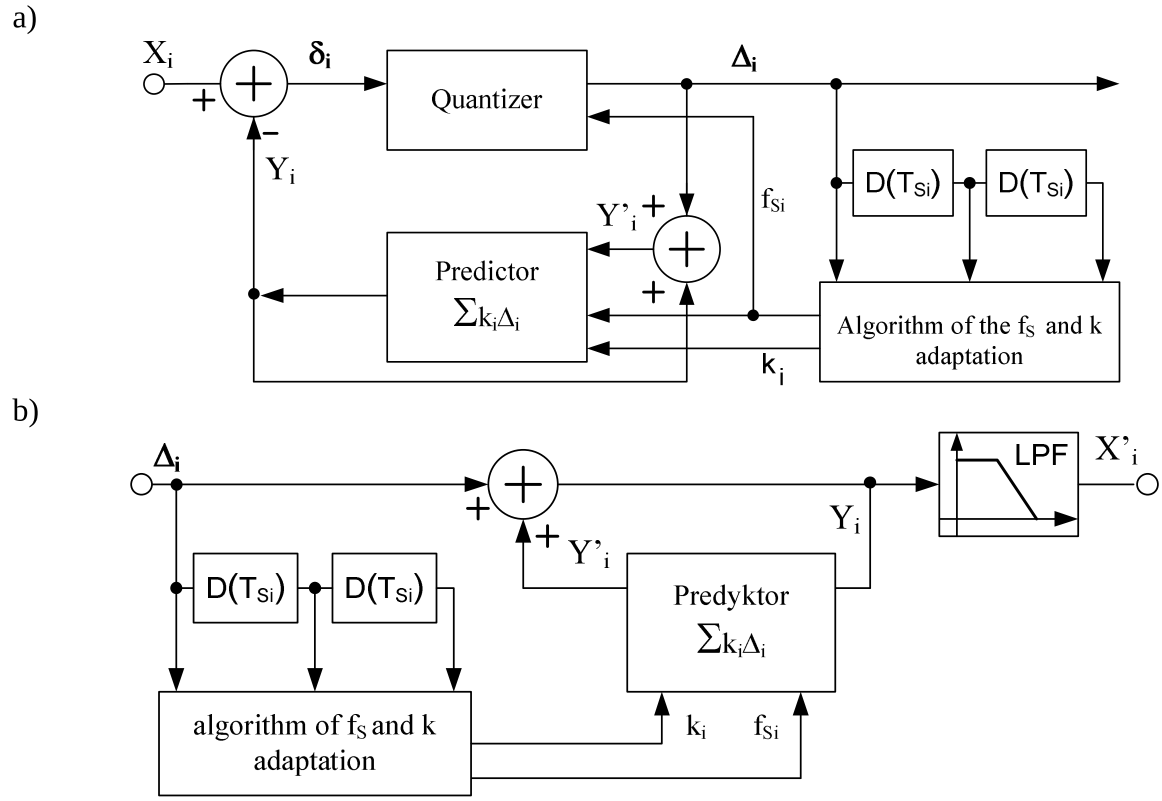

Fig.1. Functional diagram for ANS-DM converter, a) coder, b) decoder.

about the sign of comparison but also about values the sampling intervals and step sizes.

ANS-DM demodulation could have the identical algorithm as modulation. The architecture of the demodulator consist of the same blocks as these which are in the feedback loop of the modulator.

The method of design the number and values of the NS-DM and ANS-DM (Fig.1) sampling intervals is the main aim of this paper

2 3-bits Jayant's algorithm for the ANS-DM modulation

The value of the approximation staircase is a sum of the successive steps (k_i) multiplied by the sign of comparison (1, 3, 3'). Time duration of each stair depends on adaptation algorithm (5, 5').

If the approximation signal Y_i doesn't catch up the input signal X_i during 3 successive intervals (Fig. 2, 3) than the next interval has to be shorter and/or step size has to be higher (3, 3'), (5.5') and (Table 1).

For other possibilities next sampling interval has to be wider and/or step size smaller (Table1).

The staircase in the NS-DM modulator can be expressed as in (1)

$$Y_i = \sum_{k=1}^{i-1} \Delta_i k_i \qquad (1)$$

$$\Delta_i = \mathrm{sgn}[X_i - Y_{i-1}] \qquad (2)$$

where

k_i - i-th value of the step size,

X_i – i-th value of the input signal,

Y_i - i-th value of the approximation signal.

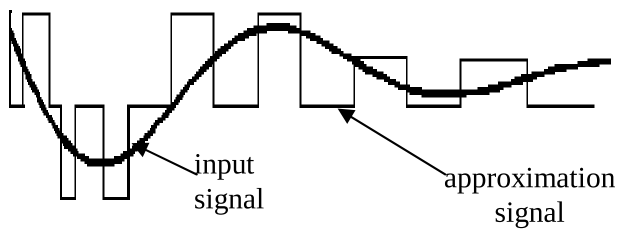

Fig. 2. Simplified diagram of the step size an sampling frequency adaptation (only granular noise).

The Jayant's algorithm of the step size changing [8]

$$k_n = Pk_{n-1} \quad \textit{when} \ b_n = b_{n-1} = b_{n-2} \qquad (3)$$

$$k_n = Qk_{n-1} \quad \textit{for remaining cases} \qquad (3')$$

Generally in ANS-DM modulator algorithm the limitation of the maximum and minimum step size values are assumed

$$k_n = k_{max} \qquad \textit{when} \quad Pk_{n-1} > k_{max}$$

and

$$k_n = k_{min} \qquad \textit{when} \quad Qk_n < k_{min}$$

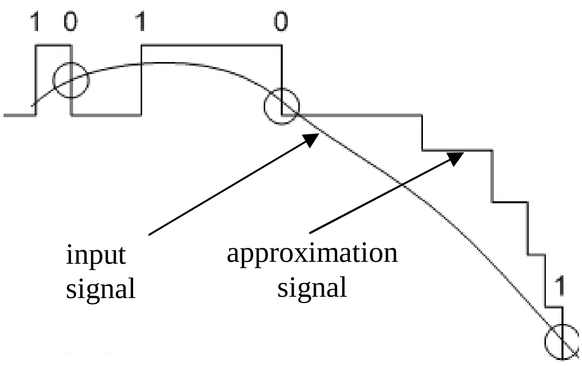

Fig.3. Both the step size and sampling interval adaptation for slope overload minimization in ANS-DM modulation.

The sampling intervals τ_i vary according to the characteristics of the input signal and the next sampling time t_{i+1} and can be expressed as

$$t_{i+1}=\tau_i+t_i \tag{4}$$

The Jayant's algorithm of the sampling interval change is:

$$\tau_i= G\,\tau_{i-1} \quad when \;\; b_n = b_{n-1} = b_{n-2}, \tag{5}$$

$$\tau_i= H\,\tau_{i-1} \quad for\ remaining\ cases \tag{5'}$$

Generally in ANS-DM modulator algorithm the limitation of the maximum and minimum sampling interval values are assumed

$$\tau_n = \tau_{max} \quad when \quad H\tau_{n-1} > \tau_{max}$$

and

$$\tau_n = \tau_{min} \quad when \quad G\tau_n < \tau_{min}$$

where

b_i –digital value of the output bit,
P, Q –constant factors of the steps size modification, $PQ \le 1$ and $Q<1<P$;
G, H – constant factors of the sampling interval modification, $G<1<H$
τ - sampling interval.

Details of the step sizes and sampling intervals changes are shown in the Table 1.

Table 1. Jayant's algorithm for ANS-DM modulator

b_i	b_{i-1}	b_{i-2}	S_i flag	T_i flag	S_{i-1} flag	T_{i-1} flag	Step size	Sampling interval
0	0	0	0	1	0	0	=	↘
			1	0	0	1	↗	=
			1	1	1	0	↗	↘
			1	1	1	1	↗	↘
0	0	1	0	0	H	H	↘	=
0	1	0	0	1	H	H	=	↗
0	1	1	0	0	H	H	↘	=
1	0	0	0	0	H	H	=	↗
1	0	1	0	1	H	H	↘	=
1	1	0	0	0	H	H	=	↗
1	1	1	0	1	0	0	1	↘
			1	0	0	1	↗	1
			1	1	1	0	↗	↘
			1	1	1	1	↗	↘

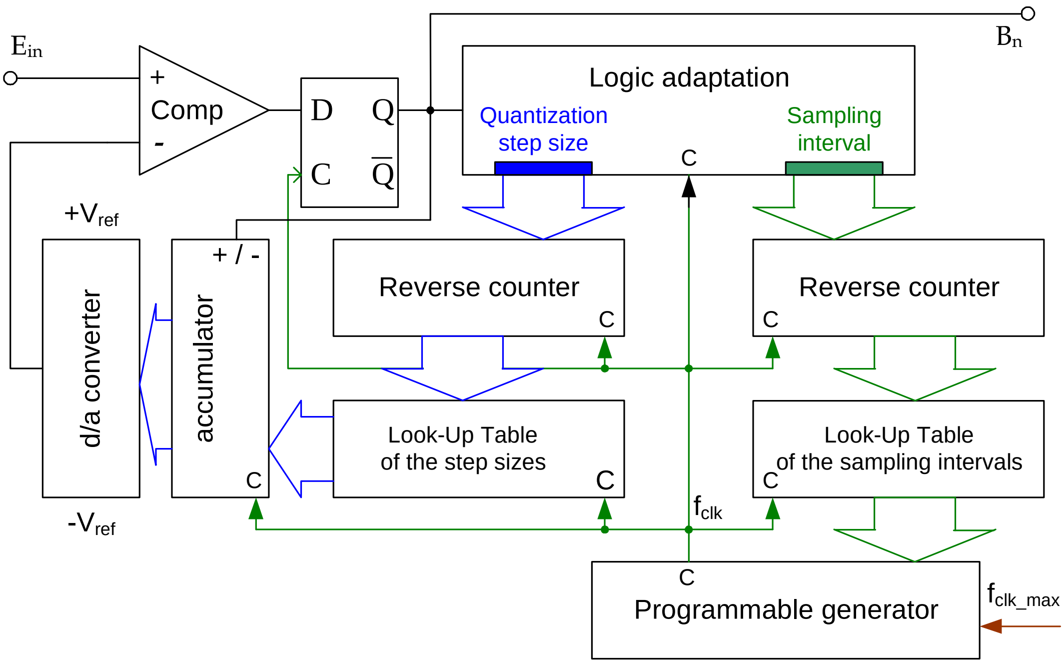

Fig.4. Block diagram of the ANS-DM delta modulator based on the Lookup Tables.

2.1 ANS-DM modulator hardware and software implementation

The block of logic adaptation (Fig. 4) monitors output bits and on the basis of the last three bits finds address of the step size and sampling interval in the LUTs.

The value of the step size is add to the accumulator.

The sampling intervals are used to the switching and synchronization of the digital circuits.

The step sizes and the sampling intervals are a'priori written to the LUTs with the help of the special control program (SYMMOD) which computes all values of the sampling intervals and quantization step sizes (Fig. 5).

3 Number of step sizes and sampling intervals - design problems

Direct influence on the total number of necessary step sizes or/and sampling intervals is determined by [9, 10]

- the required dynamic range *DR*,
- the algorithm of the step size *k* or sampling intervals τ modifying,
- values of the adaptation factors *G*, *H*, *P*, *Q*,
- the number of the factors which are adapted.

The ADM systems usually are used to conversion the analogue input signals (with a wide dynamic range) into the digital form. Therefore ADM converters to reach the assumed coding quality entail a wide changing of the step sizes or sampling intervals. "On-line" calculating their values come across the relative computational difficulty because the time complexity of determining the functions (3, 3' and 5, 5') occurs. The number of data that processor takes to solve is in some moments too high for a real time computing, even the DSP technique is used.

Fortunately, the necessary number of the step size and sampling interval values is limited and can be calculated before the coding procedure starts. All of the data of the step sizes or/and time intervals are written to the Lookup Tables. This solution is more effective and meets the assumption concerning the quality (*SNR*), the input signal dynamic range (*DR*) and average bit rate (BR_{avg}).

The results of the analytical investigations and the practical experiments show that different functions of the step size and sampling interval modification give a similar coding quality [1, 6, 8]. However applying an exponential algorithm (3, 3' and 5, 5') brings the least numbers of the sampling intervals, so it is most popular [8].

Among all mentioned above, the input signal dynamic range *DR* and the quality of the reconstruction signal have a dominating influence on the step sizes or/and sampling intervals number.

Next parts of the paper are devoted to the analytical relationships on the basis of which step sizes and sampling intervals can be calculated. These formulas have been used in the program SYMMOD to computation particular values of the step sizes and sampling intervals.

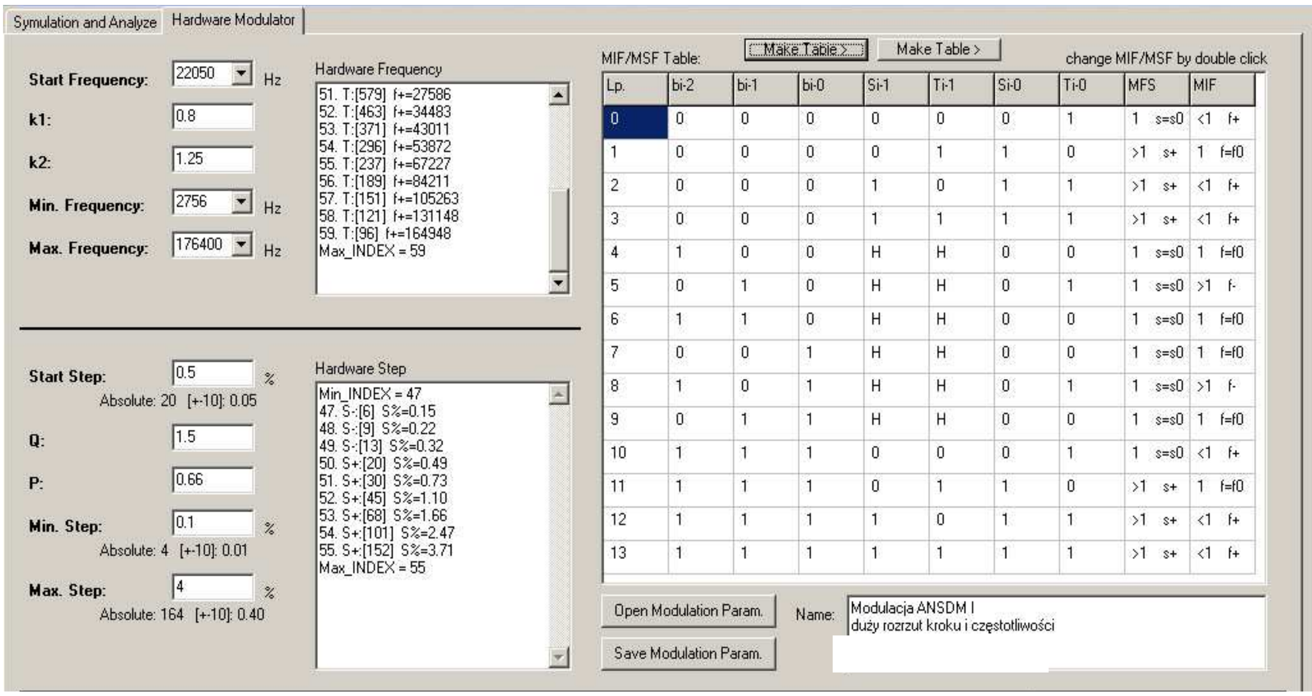

Fig. 5. User interface of the control program SYMMOD for delta modulators

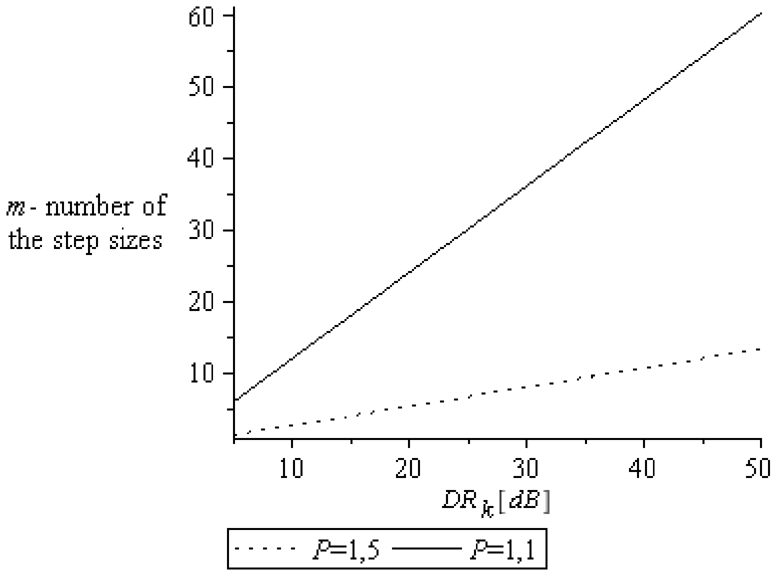

Fig.6. The number of the step sizes as a function of CFDM modulator dynamic range DR_k for two step size modification factors.

In ADM modulators the quantization step size (CFDM, CVSD modulators [3, 6]), the sampling rate (NS-DM modulators) or both these parameters simultaneously (ANS-DM modulators) can be adapted [11, 12, 13]. Knowing the range of the adapted parameters and optimal values of the adaptation factors (*G*, *H*, *P*, *Q*) it is possible to calculate the number and values of the quantization step sizes and the sampling intervals. It permits to achieve the desired dynamic range of the constant *SNR* value in ADM modulator.

3.1 Design the values of the step sizes for CFDM modulators

The dynamic range of the input signal (*DR*) decides about the number of the quantization step size values.

Abate's [6] conditions (6, 6') have to be accomplished in order to achieve the desired dynamic range DR_k of the constant *SNR* value in CFDM delta modulator

$$k_{\min} B \;/ \chi\sqrt{S_1} = \ln(2B) \qquad (6)$$

$$k_{\max} B / \chi\sqrt{S_2} = \ln(2B) \qquad (6')$$

where

$B=f_s/2f_c$ – oversampling ratio

χ – constant factor characterizing kind of the input signal

S – mean power of the input level

f_c cut-off frequency of the input signal.

k_{min}, k_{max} – minimal, maximal quantization step size;

To maintain a constant maximum *SNR* value of the CFDM modulator each change of the input level must be accompanied by the adequate change of step size.

At first the set of equations (6, 6') should be solved as a function of the $k_{\max}/k_{\min}$.

$$\left(\frac{k_{\max}}{k_{\min}}\right) \stackrel{\text{def}}{=} K_k = \frac{\sqrt{S_2}}{\sqrt{S_1}} = \sqrt{DR_k} \qquad (7)$$

where

K_k - extension ratio of the quantization step sizes to achieve the desired dynamic range DR_k of the constant *SNR* value in CFDM modulation and

$$K_k\,[dB] \stackrel{def}{=} 20\log(K_k) \qquad (7')$$

The number of the quantization step sizes can be calculated on the basis of

- K_k definition (7, 7')
- step size changes algorithm (3, 3')

At last the number of the quantization step size *m* of CFDM modulator can be determined from equation (8)

$$K_k = \frac{k_{\max}}{k_{\min}} = (P)^m \qquad (8)$$

Taking the factor *P* from the range (1,1-1,5) and assuming that $1/P = Q$, the required step sizes number decrease twice and $SNR \cong SNR_{max}$. Finally on the basis of formulas (7', 8) the number of the step sizes *m* can caculated

$$m = \frac{K_k\,[\text{dB}]}{20 \cdot \log(P)} \qquad (9)$$

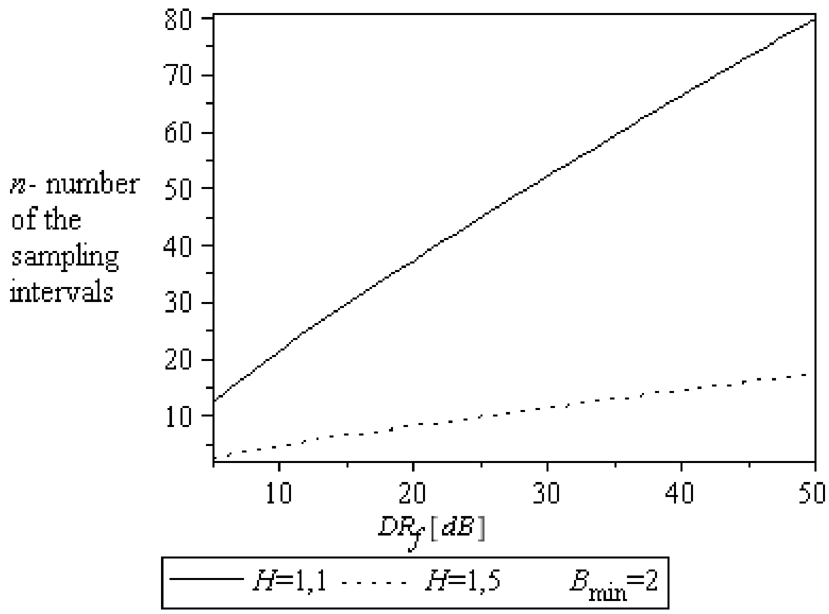

Fig. 7. The number of the sampling intervals as a function of NS-DM modulator dynamic range DR_f for two sampling intervals modification factors *H*.

The number of the step sizes as a function of CFDM modulator dynamic range DR_k for two step size modification factors is shown in Fig 6.

The particular values of the quantization step sizes can be determined from the formula (8) knowing k_{max} or k_{min}.

As can be seen from the formulas (7') every doubling of the extension ratio of the quantization step sizes K_k increases CFDM dynamic range DR_k by 6 dB [14].

3.2 Design the values of the sampling intervals for NS-DM modulators

For NS-DM and ANS-DM modulators calculation the number of the sampling intervals basis on the analytical works [4, 5, 7, 9] and on the results of simulation investigations [10, 12, 14, 15].

The dynamic range of the input signal (*DR*) decides about the number of the sampling intervals values.

Abate's conditions (10, 10') have to be accomplished in order to achieve the desired dynamic range DR_f of the constant *SNR* value in NS-DM delta modulator.

$$kB_{\max} / \chi \sqrt{S_2} = \ln(2B_{\max}) \quad (10)$$

$$kB_{\min} / \chi \sqrt{S_1} = \ln(2B_{\min}) \quad (10')$$

where

$B_{max}=f_{s\ max}/2f_c$, ($B_{min}=f_{s\ min}/2f_c$) – oversampling ratio of the maximum (minimum) sampling frequency.

To maintain a constant maximum *SNR* value of the NS-DM modulator each change of the input level must be accompanied by the adequate change of sampling interval.

The solution of equations (10, 10') is

$$DR_f = \left(\frac{K_f \ln(2B_{\min})}{\ln(2B_{\min}) + \ln(K_f)} \right)^2 \quad (11)$$

where

$DR_f = \dfrac{\sqrt{S_2}}{\sqrt{S_1}}$ - dynamic range of the NS-DM modulator

K_f - the extension ratio of the sampling intervals to achieve the desired dynamic range DR_f of the constant *SNR* value in NS-DM modulation and

$$\frac{B_{\max}}{B_{\min}} = \frac{f_{s\_\max}}{f_{s\_\min}} = \frac{\tau_{\min}}{\tau_{\max}} \overset{def}{=} K_f \,. \quad (12)$$

In many aplications we should find solution of the equation (11) as a function of the K_f.

But the thing is the equation (11) is transcendental for K_f. However in many computer mathematical programs special function called Lambert*W* is implemented. The -1'st branch of this function

Lambert$W$(-1,-x)[1] [16], enable us to receive the closed form solution of the equation (11) for K_f.

$$K_f = -\frac{\sqrt{DR_f}}{\ln(2B_{\min})}\text{Lambert}\,W\left(-1, -\frac{\ln(2B_{\min})}{2B_{\min}\sqrt{DR_f}}\right)$$

(13)

Equation (13) has real solutions [5] when

$$0 < \frac{\ln(2B_{\min})}{2B_{\min}\sqrt{DR_f}} < \frac{1}{e}. \qquad (13')$$

The number of the sampling intervals can be calculated on the basis of

- K_f definition (12)
- sampling interval changes algorithm (5, 5')

Finally the number of the sampling intervals n of NS-DM modulator can be found from equation (14)

$$K_f = (H)^n \qquad (14)$$

Taking the factor H from the range (1,1-1,5) and assuming that $1/H = G$, the required number of the sampling intervals decrease twice and $SNR \cong SNR_{max}$. Next on the basis of formulas (11, 14) we have the number of the sampling intervals n

$$n = \frac{\log(K_f)}{\log(H)} \qquad (15)$$

For known f_{s_max} or f_{s_min} (τ_{min} or τ_{max}) the particular values of f_{si} (τ_i) can be calculated from equations (5, 11, 14).

Finally on the basis of the (13) and (15):

$$n = \frac{\log\left(-\frac{\sqrt{DR_f}}{\ln(2B_{\min})}\,LambertW\left(-1, -\frac{\ln(2B_{\min})}{2B_{\min}\sqrt{DR_f}}\right)\right)}{\log(H)} \qquad (16)$$

The number of the sampling intervals as a function of NS-DM modulator dynamic range DR_f for two sampling intervals modification factors H is shown on Fig. 7.

As can be seen from the formulas (7, 7', 8, 10, 11) every doubling of the maximal sampling frequency range K_f increases value of the NS-DM dynamic range (DR_f) less than 6 dB. The value of the maximal sampling frequency range (8) is dependent from minimal oversampling ratio B_{min}. In practice the gain of the NS-DM dynamic range changes from about 3 to 5 dB per octave of K_f ratio. As results from the equation (8), at least for $B_{min} \to \infty$ the dynamic $DR_f \to 6$ *dB/oct* of K_f ratio.

3.3 Design the values of the step sizes and sampling intervals for ANS-DM modulators

The number of the sampling intervals and step sizes we can found in similar manner as for CFDM and NS-DM.

After solution set of equation 17, 17' we get dependency (18) which shows that the total dynamic range of the ANS-DM modulation.

For ANS-DM modulation (k=var, f_s=var), in order to maximize SNR in some range of the input signal level the Abate's conditions [1] have to be obeyed

$$k_{\max} B_{\max} / \chi\sqrt{S_2} = \ln(2B_{\max}) \qquad (17)$$

$$k_{\min} B_{\min} / \chi\sqrt{S_1} = \ln(2B_{\min}) \qquad (17')$$

In similar manner as for CFDM and NS-DM modulations basing on the equation (12, 12') the DR range may be written as

$$DR = \frac{S_2}{S_1} = \left(\frac{k_{\max}}{k_{\min}}\right)^2 \left(\frac{B_{max}}{\ln(2B_{max})}\frac{\ln(2B_{min})}{B_{min}}\right)^2 = (K_k)^2\left(\frac{[\ln(2B_{min})]K_f}{\ln(2B_{min}) + \ln(K_f)}\right)^2 = DR_k DR_f. \qquad (18)$$

The total dynamic range of the ANS-DM modulation can be calculated as a product of the dynamic range derived both from sampling frequency DR_f and step size adaptation DR_k dynamic range [3]. It is very important performance. It shows decreasing of the indispensible number of the ANS-DM adaptive parameters.

As seen from (18) an infinite number of pairs of the coefficients DR_k, DR_f that help to provide the equation (18) occurs. Only in practice, definite chosen applications help to determine the useful pairs.

From the equation (18) the following relation is obtained in the logarithmic measure

$$DR\,[\text{dB}] = DR_k\,[\text{dB}] + DR_f\,[\text{dB}] \qquad (19)$$

The total dynamic range of the ANS-DM (DR) in dB is the sum of the dynamic range from step size changes (DR_k) and from sampling frequency changes (DR_f).

As the designing aspect is considered, in many cases, the dynamic range DR is given. Then the equations $K_k = f_1(DR, K_f, B_{min})$ and $K_f = f_2(DR, K_k, B_{min})$

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[1] The *LambertW* - The Lambert's W function, named after Johann Heinrich Lambert, also called the Omega function or Product Log. This special function denotes also as $W(x)$ satisfies the condition: $W(x)exp(W(x))=x$. Similarly to the equation $yexp(y)=x$ that has infinite number of solutions y for every non-zero value x, also the W has an infinite number of branches. The principal branch (m=0) is denoted as $W(x)$ and all the remaining as $W(m, x)$. The –1'st branch, $W(-1,x)$ is real valued on the interval –1/e…0.

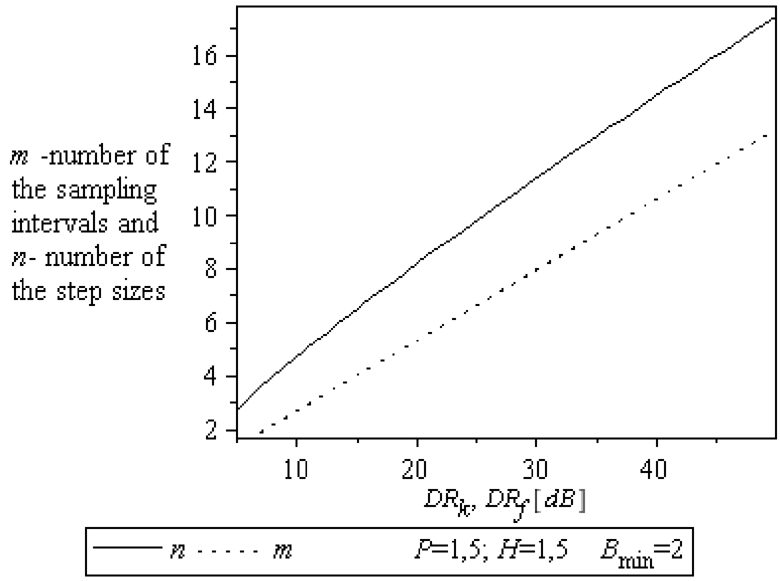

Fig. 8. The number of the sampling intervals (n) and the step sizes (m) as a function of dynamic range from step size changes (DR_k) and from the sampling interval changes (DR_f) in ANS-DM modulation.

have to be determined from (18). The solutions have a form

$$K_k = \sqrt{DR}\left(\frac{\ln(2B_{min}) + \ln(K_f)}{K_f \ln(2B_{min})}\right) \tag{20}$$

$$K_f = -\frac{\sqrt{DR}}{K_k \ln(2B_{min})} LambertW\left(-1, -\frac{K_k \ln(2B_{min})}{2B_{min}\sqrt{DR}}\right). \tag{21}$$

The number of the sampling intervals (n) and the step sizes (m) as a function of dynamic range from step size changes (DR_k) and sampling interval changes (DR_f) in ANS-DM is shown on Fig.8.

4 Example of the number of discrete parameters values for 1-bit adaptive delta modulators

Optimal ranges of *P*, *Q* and *G*, *H* factors have been found in simulation and analytical research [7, 10, 14, 17].

The number of the sampling intervals (n) and the step sizes (m) have been calculated on the basis of (9), (16). The results are shown in Tab.1.

Simulation investigations have been helped to establish the optimum values of adaptive factors *G*, *H*, *P*, *Q* [7, 10, 14]. These values are usually approximate to a unity (Table 1). It is the direct cause of a great number of required quantization step sizes and sampling intervals. The overall number of the step sizes and sampling intervals as a function of the share (%) of the dynamic range from the sampling interval changes (DR_f) in the total *DR* of the ANS-DM modulation is shown on Fig. 9.

Higher values of the share (%) of the dynamic range from the sampling interval changes (DR_f) increase overall number of sampling intervals and step sizes esspecially when the *H* factor is close to unity. The best results (*SNR*, BR_{avg}, *DR*) are achieved for about 25% to 50 % of the share of the dynamic range from the sampling interval changes in the total *DR* [10, 14].

Table 1. The number of the step sizes and sampling intervals. Dynamic range *DR* =36 [dB].

Kind of ADM modulator	Optimal values range of *P*, *Q* and *G*, *H* factors	Number of step sizes values *m*	Number of sampling intervals values *n*
CFDM DR_k=36 dB	*P*=1,1-1,5 *Q*=0,6-0,9	11-44	1
NS-DM DR_f=36 dB	*G*=0,6-0,9 *H*=1,1-1,5	1	15-61
ANS-DM DR_f=DR_k =0,5*DR*	*P*=1,1-1,5 *Q*=0,6-0,9 *G*=0,6- 0,9 *H*=1,1-1,5	6-22	9-35

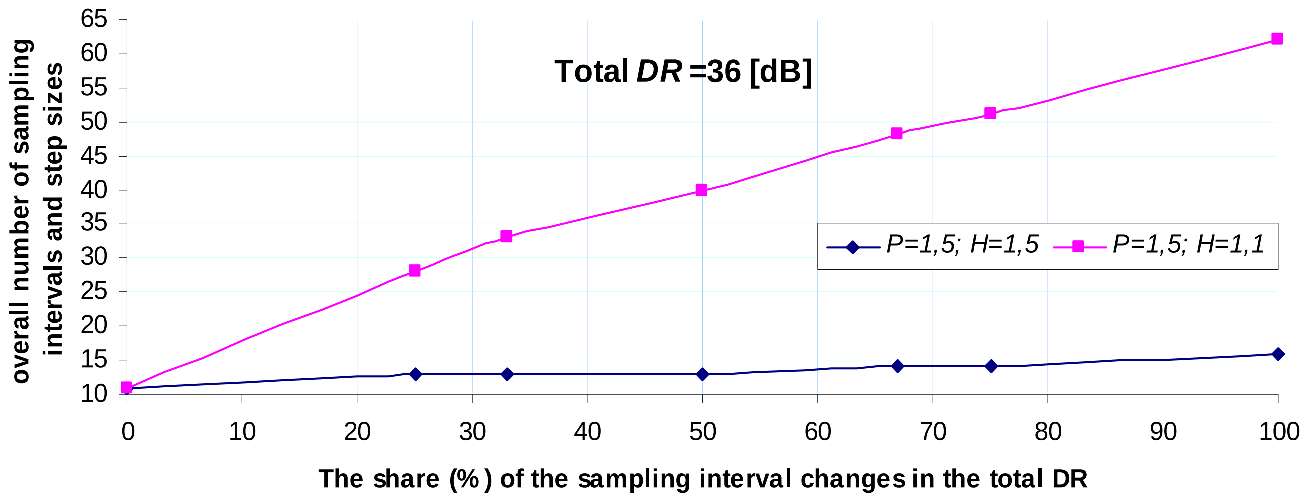

Fig.9. The overall number of the step sizes and sampling intervals as a function of the share (%) of the dynamic range from the sampling interval changes (DR_f) in the total *DR* of ANS-DM modulation

5 Conclusions

To verify the analytical results a computer simulation has been performed and a hardware prototype built.

The presented principles of the NS-DM and ANS-DM parameters design, help to determine appropriate performances indispensable for correct functioning of the 1-bit delta modulators with LUTs.

The form of the Abate's conditions of the Signal to Noise maximum (SNR_{max}), makes the special function Lambert*W* extremely suitable for the determination of the sampling intervals in non-uniform delta modulations with LookupTables.

Adaptation of two parameters (ANS-DM modulation) makes the hardware solution of the delta modulator and it's algorithm more complicated but decreases considerably the required number of values of every parameter thus improving the quality of conversion (*SNR*) in relation to the solutions with one parameter adaptation.

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