**Negative Consequence of Motor Voltage Asymmetry and Its Influence to the Unefficient Energy Usage**

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**Abstract:** - Analysis of the effect of unbalanced voltages on the three-phase induction motor is presented in the paper. Since the unbalanced voltage of 2%, 3.5% and 5% increase in losses could reach, in the same order, the amount of 8%, 25% and 50% of nominal power losses in the motor, it is reasonable requirement to permit voltage asymmetry $\leq 2\%$, so this is the upper limit in most national and international standards. The truth is that with a smaller load, the motor could safely work also at higher values of unbalanced voltage. The literature states that the information given previously is determined from measurements and that they are higher than calculated values. However, it is explained here by the fact that the rotor inverse resistance is higher by 1.4 times compared to the rotor resistance in short circuit mode, since current frequency of the negative sequence in the rotor winding is two times higher ($f_{NS} \approx 2f_{SC}$), i.e. it is higher for 1.41 times than corresponding values given in the literature. Thus it is increasingly convinced that the requirements which are given in most appropriate standards are justified. Performed analysis show that there are some considerations that could be included in current standards. Motor operation is not generally allowed when the values of the coefficient of unbalanced voltage are $U_{NS}/U_n \geq 5\%$, because in the (rare) case when the direct and inverse component of the stator currents in (a) phase matching, increase of the current in that phase would be $\geq 1.38$ times, and the increase of the losses in the windings of that phase would be $\geq 90\%$.

**Key-Words:** - negative sequence voltage, asymetry, power losses, induction motor, energy efficiency, standard

1 Introduction

Electrical energy savings in the drive could be realized by improvements of power quality in the consumer network. Power losses and reactive loads are increased additionally due to the voltage asymmetry and (or) existence of high-order harmonics in supply voltage. Voltage asymmetry could appear due to the existence of larger monophase loads or asymmetrical capacitor bank with damaged or switched of capacitor because of blown fuse in one phase. Today, existence of high-order harmonics in supply voltage is frequent due to the increased number of consumers supplied over rectifiers and inverters like electrical drives, electro-thermal loads, etc.

Term power quality [1-3] mostly means quality of supply voltage that should meet following requirements:
- voltage value (permissive variations are in the range of $\pm 5\%$ of nominal voltage),
- permissive total harmonics distortion of voltage is $THD_u \leq 3 - 8\%$ (higher value is for low voltage networks),
- permissive voltage asymmetry is $3\%$ and has higher influence on accurate and economical motor operation.

Effect of voltage asymmetry to the three phase induction motor is equivalent to the appearance of negative sequence voltage system that create rotating field which rotates contrary to the rotation of the positive sequence field and motor rotating direction. Consequence is that small values of negative sequence voltage produce relatively high values of negative sequence currents.

Per definition, coefficient of asymmetry ($K_{NS\%}$) is ratio of negative sequence voltage ($U_{NS}$) and positive sequence voltage ($U_{DS}$). For the simplification, Standard NEMA MG1 determines coefficient of asymmetry in such a way that at first average arithmetical value of line voltages is determined. For instance, for measured voltages of 396V, 399V and 405V average value is 400V. Then, the highest variation from average voltage is determined (405V – 400V = 5V). At the end, coefficient (percent) of voltage asymmetry is calculated as quotient of highest variation and average value: $K_{NS\%} = 100 - (5/400) = 1.25\%$. 
Since percentages of negative sequence currents are higher 6-10 times, than under such a low voltage asymmetry negative sequence currents could reach up to ten percent. This causes additional motor heating and appearance of inverse torque that relates to the starting and maximum motor torque decrease and smaller increase of motor slip during operation. Because power losses and motor heating are increased, than allowable motor loading is decreased. As percent of asymmetry rises, derating factor of nominal power decreases (shown in Fig. 1 per NEMA MG1 [1]).

![Fig. 1. Relation between derating factor and voltage asymmetry](image)

With increase of voltage asymmetry coefficient, motor efficiency decreases under all load levels. Dependence of motor efficiency is given in Fig. 2 for voltage asymmetry coefficient of 0.00%, 2.50%, 5.00% and 7.50% [1].

![Fig. 2. Motor efficiency in dependence of motor load for different voltage asymmetries](image)

Electrical energy consumption is unnecessarily increased due to the lower motor efficiency (Fig. 2), so maintenance of low voltage asymmetry (≤ 2%) is a measure of efficient energy usage. In that case, the influence of voltage asymmetry (negative sequence voltages) will be in detail presented in the paper as follows. At first, a procedures for calculation and analysis of negative sequence currents and corresponding power losses and inverse motor torques will be shown. Than, evaluation of negative sequence voltage values that could be arisen in considered consumer networks is presented.

2 Equivalent circuit and motor parameters for negative sequence voltage system

2.1 Equivalent circuit of induction motor for negative sequence

When induction motor is supplied from network with asymmetrical voltages, than three phase voltage system should be decomposed to positive, negative and zero sequence. Further, using equivalent motor circuits [4-8] separately for positive (Fig. 3a) and for negative sequence (Fig. 3b), calculations and analyses of motor energy and operation characteristics (currents, power losses, torques) are performed.

![Fig. 3. Equivalent circuits of induction motor](image)
At the end, with corresponding superposition of relevance values their overall values are obtained. Only in that way a real (overall) values of motor power losses could be determined.

### 2.2 Parameters of equivalent circuit for negative sequence voltage

Stator winding resistance \( R_s \) and reactance \( X_s \) are almost the same for positive and negative voltage sequence. Parameters of equivalent circuit that are correlated to the rotor side and negative sequence voltage system (resistance \( R_{r,NS} \) and reactance \( X_{r,NS} \)) are substantially different than those for positive voltage sequence, because the frequency of the rotor currents in negative sequence are multiple higher:

\[
f_{NS} = f_1(2-s) \approx 2f_1 >> s f_1 \quad (1)
\]

Values of resistance \( R_{r,NS} \) and reactance \( X_{r,NS} \) are determined from the corresponding parameters in short-circuit regime, \( R_{r,SC} \) and \( X_{r,SC} \). It is shown in [4-6] that values of corresponding resistances and reactances are approximately equal in those regimes: \( R_{r,NS} \approx R_{r,SC} \) and \( X_{r,NS} \approx X_{r,SC} \). If we look carefully, we could find that this statement is not correct. Those parameters changes are dependent on the ratio of the rotor conductor height (\( H \)) and field penetration (\( \bar{\zeta}_{r,SC} = 2\rho/(\mu \cdot 2\pi f_1) \)), i.e. the quotient \( \zeta = H/\bar{\zeta}_{r,SC} \); where:

\( \rho \) - conductor specific resistance,
\( \mu = \mu_0 \) – magnetic conductance of conductor.

Fig. 4 is given for \( \bar{\zeta}_{r,SC} = H/\bar{\zeta}_{r,SC} = 1.2\pm3: \)

\( R_{r,SC}(f) = 1\pm3R_r \) and \( L_{r,SC}(f) = 1\pm0.50L_r \), where \( R_r \) and \( L_r \) are resistance and inductance in low slip regime, respectively, for instance in nominal regime.

Since current frequency of the negative sequence in the rotor winding is two times higher \( (f_{NS} \approx 2f_1 = 2f_{r,SC}) \), than field penetration of those currents are lower for \( \sqrt{2} \) times, so corresponding values of reactance are lower and resistance higher for \( \sqrt{2} \) times:

\[
R_{r,NS} = R_{r,SC} \cdot \sqrt{2} \quad (2)
\]

\[
X_{r,NS} = X_{r,SC} / \sqrt{2} \quad (3)
\]

The explanation is based on the fact that for the motors larger than 5 kW (or for relative field penetration \( \zeta_{r,SC} = H/\bar{\zeta}_{r,SC} \geq 1 \)) in the short circuit regime \( (f_{r,SC} = f) \), it is start not to use the whole section (or height \( H \)) of the rotor bar [8].

From that it could be concluded that for the negative sequence \( (f_{NS} = 2f) \) it is used for \( \sqrt{2} \) times lower part of the rotor conductor height and also section of the rotor conductor.

Since usually values of rotor resistance \( (R_r) \) and reactance \( (X_r) \) are known in the nominal regime, then it is necessary to know values of coefficient of rotor resistance increase \( (k_{R} > 1) \) and coefficient of rotor reactance decrease \( (k_X < 1) \), both in the short circuit regime. From (2) and (3), the values for \( R_{r,NS} \) and \( X_{r,NS} \) could be calculated as:

\[
R_{r,NS} = k_{R,NS} \cdot R_r \quad (4)
\]

\[
X_{r,NS} = k_{X,NS} \cdot X_r \quad (5)
\]

where values of coefficients \( k_{R,NS} \) and \( k_{X,NS} \) are determined from Fig. 4 for previously established value of relative field penetration \( \zeta_{r,NS} = H/\bar{\zeta}_{r,SC} \).

In such a manner are determined approximate values of coefficients \( k_{R,NS} \) and \( k_{X,NS} \), for different rotor slot deep \( H (mm) = 15; 20; 30; 40; 50 \) and given in Table 1.

From the quantitative review of corresponding values for \( k_{R,SC} \) and \( k_{R,NS} \), \( k_{X,SC} \) and \( k_{X,NS} \), it could be seen that valid relations are: \( k_{R,NS} \approx 1.41 \cdot k_{R,SC} \) and \( k_{X,NS} \approx k_{X,SC} / 1.41 \), and it could be concluded that relations (2) and (3) are correct. In that way it is confirmed author’s state that “it is not correct to believe that values of corresponding resistances and reactances for negative sequence currents are approximately equal to those for motor short
circuit regime as referred in the literature [4-6]”. Contrary, it is only correct that those values are in relation by (2) and (3). It is useful to specify common values for rotor slot deep H (mm) and frame sizes for standard induction motors, as given in Table 2. From these facts more precisely calculations and analyses of negative sequence voltage (and negative sequence currents) influence to the motor operation could be performed.

<table>
<thead>
<tr>
<th>Rotor slot deep H (mm)</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_{\text{ALSC}} = 10 \text{ mm}; \delta_{\text{ALNS}} = 7 \text{ mm} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_{R,\text{SC}} )</td>
<td>1.3</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
</tr>
<tr>
<td>( K_{R,\text{NS}} )</td>
<td>2.0</td>
<td>2.8</td>
<td>4.2</td>
<td>5.6</td>
<td>7.0</td>
</tr>
<tr>
<td>( K_{X,\text{SC}} )</td>
<td>0.9</td>
<td>0.75</td>
<td>0.50</td>
<td>0.38</td>
<td>0.30</td>
</tr>
<tr>
<td>( K_{X,\text{NS}} )</td>
<td>0.75</td>
<td>0.54</td>
<td>0.36</td>
<td>0.27</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table 2: Usual values for rotor slot deep H (mm) and frame sizes for standard induction motors

<table>
<thead>
<tr>
<th>P_n (kW)</th>
<th>1.1-2.2</th>
<th>3-7.5</th>
<th>11-18.5</th>
<th>22-45</th>
<th>55-160</th>
<th>200-355</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial heigh</td>
<td>80-90</td>
<td>100-112</td>
<td>132-160</td>
<td>180-200</td>
<td>225-280</td>
<td>315-400</td>
</tr>
<tr>
<td>Rotor slot deep, H (mm)</td>
<td>13-17</td>
<td>18-22</td>
<td>24-34</td>
<td>35-44</td>
<td>40-50</td>
<td>40-50</td>
</tr>
</tbody>
</table>

3 Negative sequence currents and power losses

Negative effect to the motor operation due to the presence of negative sequence voltage is obvious for two reasons:
- it induces occurrence of negative sequence currents that produces losses in the stator and rotor windings, i.e. on the stator (R_s) and rotor (R_r,NS) resistance, and
- appearance of inverse torque opposite to the motor operative torque that originates from direct voltages and currents system.

It is useful to express the value of negative sequence current \( I_{1,\text{NS}} \) in the units of nominal positive sequence current \( I_{1n,\text{PS}} \):

\[
\frac{I_{1,\text{NS}}}{I_{1n}} = \frac{U_{1,\text{NS}} / Z_{M,\text{NS}}}{U_{1n} / Z_{M,n}} \\
\approx \frac{U_{1,\text{NS}} / X_{\text{NS}}}{U_{1n} / X_{\text{SC}}} = \frac{U_{1,\text{NS}}}{U_{1n}} \cdot \frac{1}{X_{\text{NS}}} = \frac{U_{1,\text{NS}}}{U_{1n}} \cdot \frac{1}{X_{\text{SC}}} \\
\approx (6 + 8) \cdot \frac{U_{1,\text{NS}}}{U_{1n}}
\]

since negative sequence impedance

\[ Z_{M,\text{NS}} \approx X_{M,\text{NS}} \approx (0.8-0.9) \cdot X_{M,\text{SC}}, \] where \( X_{M,\text{SC}} \) is motor short circuit reactance and \( X_{M,\text{SC}} \approx (0.15 \div 0.20) \cdot Z_{M,n} \) (\( Z_{M,n} \) - motor impedance in nominal regime). It could be seen from (6) that negative sequence currents in stator and rotor windings are 5 to 8 times higher from the values of negative sequence voltage coefficients \( (U_{1,\text{NS}}/U_{1}) \). Since negative sequence currents are not dependent on motor load and slip, than we suggest to calculate the coefficient of asymmetry in the units of nominal motor voltage. In the following calculations and analyses the value \( X_{\text{NS}} = 0.13 \) (or \( X_{\text{SC}} = 0.16 \), i.e. \( I_{1,\text{NS}}/I_{1n} = 6.25 \) is used, so:

\[
\frac{I_{1,\text{NS}}}{I_{1n}} = 7.7 \cdot \frac{U_{1,\text{NS}}}{U_{1n}}
\]

Value of increased losses in the phase windings of stator is proportional to the square of negative sequence currents, than from (7) it could be calculated as:

\[
P_{\text{Cu,NS}}/P_{\text{Cu,n}} = (I_{1,\text{NS}}/I_{1n})^2 = 60 \cdot (U_{1,\text{NS}}/U_{1n})^2
\]

Percent of losses in rotor conductors could be higher up to 3-6 times (2-5 times due to the higher rotor resistance for negative sequence currents and further up to 1.2 times due to the additional increase of rotor winding temperature under such a high power losses), i.e. \( R_{\text{r,NS}} \approx R_{\text{r,SC}} = (2\div 6) \cdot R_r \). Equation for losses calculation in the rotor windings for \( R_{\text{r,NS}} = 5 \cdot R_r \) is:

\[
P_{\text{Cu,r,NS}}/P_{\text{Cu,r,n}} = 5 \cdot (I_{1,\text{NS}}/I_{1n})^2 = 300 \cdot (U_{1,\text{NS}}/U_{1n})^2
\]
Assuming that negative sequence impedance is \( Z_{SC,NS} \approx X_{SC,NS} = 0.13 \) (or \( I_{SC}/I_n = 7.7 \)) and negative sequence rotor resistance is \( R_{r,NS} = 5 \cdot R_r \), the power losses values are calculated for voltage asymmetry of 2.5%, 3.5% and 5%. Such a calculated values from (6.7) and (6.8), in percent of nominal losses, for particular motor parts (stator, rotor) and whole motor, are given in Table 3.

Similar data are given in [2] noting that there were based on measurements that are larger than calculated values is explained by the increase resistance rotor for inverse power by 1.5 times because of the additional increase in temperature of the rotor conductor, which is not possible at this scale. This can only be explained by significant increase of rotor resistance for another 40% (2), compared to what is stated in the literature. On this basis, the obtained values for the permitted engine load \( P/P_n(\%) \) also were similar to those provided to the appropriate standards (Fig. 1).

Given the pessimistic assumption, especially for smaller engine power (up to 10 kW) when the rotor resistance increased only by 2-3 times (up to 1.5-2.5 times higher due to the inverse resistance rotor currents and even up to 1.2 times for an additional increase in temperature of the conductor rotor in such a large loss of power), ie \( R_{r,NS} \approx 1.4R_{r,SC} = (2 \div 3) R_r \). Calculation of losses in the rotor conductors, for \( R_{r,NS} = 3R_r \), was conducted by the expression:

\[
P_{Cu, R, NS}/P_{Cu, r} = 60 \cdot (I_{1,NS}/I_{1n})^2 = 180 \cdot (U_{1,NS}/U_n)^2 \tag{10}
\]

since the inverse of impedance \( Z_{1,NS} \approx X_{NS} \approx 0.9X_{SC} = 0.13 \) (i.e., when the motor short circuit reactance \( X_{SC} = 0.143 \), or \( I_{SC}/I_n = 7 \)). Thus obtained data are given in Table 4. These data are reliable engine power below 10kW and these parameters, compared to those in Table 4 and the relevant literature [2].

### Table 3: Increased power losses in % of nominal losses for particular motor parts and motors over 100kW (or \( R_{r,NS} = 5 \cdot R_r \))

<table>
<thead>
<tr>
<th>Voltage asymmetry (%)</th>
<th>0</th>
<th>2</th>
<th>3.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative sequence current (%)</td>
<td>0</td>
<td>15</td>
<td>27</td>
<td>38</td>
</tr>
<tr>
<td>Stator current (RMS) (%)</td>
<td>100</td>
<td>104</td>
<td>107.5</td>
<td></td>
</tr>
<tr>
<td>Average value for stator winding</td>
<td>0</td>
<td>2.4</td>
<td>7.4</td>
<td>15</td>
</tr>
<tr>
<td>Rotor</td>
<td>0</td>
<td>12</td>
<td>37</td>
<td>75</td>
</tr>
<tr>
<td>Motor (overall)</td>
<td>0</td>
<td>8</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>Increased temperature (°C)</td>
<td>60</td>
<td>65</td>
<td>75</td>
<td>90</td>
</tr>
<tr>
<td>Isolation class A</td>
<td>80</td>
<td>86</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>Permissible motor load ( P/P_n(%) )</td>
<td>100</td>
<td>96.5</td>
<td>90</td>
<td>81</td>
</tr>
</tbody>
</table>

For motors of isolation class A or B

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Table 4: Increasing the power losses in % of nominal loss of some parts of the motor, over
For motor power below 10 kW (or \( R_{\text{NS}} = 3 \cdot R_e \))

<table>
<thead>
<tr>
<th>Voltage asymmetry (%)</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative sequence current (%)</td>
<td>0</td>
<td>15</td>
<td>22</td>
<td>38</td>
</tr>
<tr>
<td>Stator current (RMS) (%)</td>
<td>100</td>
<td>101</td>
<td>102</td>
<td>107.5</td>
</tr>
</tbody>
</table>

Increased losses (%)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>2.4</th>
<th>5.4</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average value for stator winding</td>
<td>0</td>
<td>7</td>
<td>16</td>
<td>45</td>
</tr>
</tbody>
</table>

Motor (overall)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>5</th>
<th>11</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increased losses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Increased temperature (°C)

<table>
<thead>
<tr>
<th>Isolation class A</th>
<th>60</th>
<th>63</th>
<th>66</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolation class B</td>
<td>80</td>
<td>84</td>
<td>89</td>
<td>106</td>
</tr>
</tbody>
</table>

Permissible motor load \( P/P_n \) (%)

| For motors of isolation class A or B | 100 | 98 | 95 | 86 |

Based on data given in Table 4, it is concluded that the effects of unbalanced increase in electricity and power loss is less when it comes to motor power \( \leq 10 \) kW, compared to the data specified in the relevant standards. Thus, the above increase, the asymmetry of 3%, approximately equal to those which are listed in Table 3 for the asymmetry of 2%, so the networks with motors below 10 kW may be eligible asymmetry coefficient of negative sequence voltage \( \left( \frac{U_{1,\text{NS}}}{U_n} \right) \leq 3\% \). In practice, allowed the nominal load of the motor, up to \( \frac{U_{1,\text{NS}}}{U_n} \leq 3\% \) (Table 4).

Although the unbalanced voltage losses in the rotor conductors is much higher than the corresponding losses in the stator winding, but increase the losses of one phase of the stator can be the greatest. Specifically, unfavorable is in case when the direct and inverse current component could be the same in one phase. Current in this phase, at the voltages unbalance of 5% would be:
- At nominal load \( I = I_{\text{PS}} + I_{\text{NS}} = I_{N} + 0.38N = 1.38N \), while
- At 80% of the motor load: \( I = I_{\text{PS}} + I_{\text{NS}} = 0.8I_{N} + 0.38N = 1.18N \), while the corresponding power losses, respectively, were higher than the nominal 100% \( (1.38^2 - 1) = 90\% \) and 100% \( (1.18^2 - 1) = 39\% \).

Then the current increase in that phase would be 1.38 times at the negative sequence voltage \( \frac{U_{1,\text{NS}}}{U_n} = 5\% \) and increase of the losses in the windings of that phase could reach 90% = 100% \( (1.38^2 - 1) \). Otherwise, in the practice it could be rarely the case when direct and inverse component of the current matching phase angle. For example, it is necessary to stress that the asymmetry is a consequence of only one phase voltage deviation of the values (not phased by the angle) and the phase angles of the direct and inverse impedance a little different, which is rarely filled because tan(\( \phi_{\text{NS}} \)) = 0.3 ÷ 0.4. But it is possible that the phase angle between these components is about 30°, and a corresponding increase in this phase will be lower:
- current increase would be in the analyzed cases 1.235-\( I_N \) and 1.079-\( I_N \), respectively and
- corresponding increase in losses in this phase would be 52.5% and 16.4%.

For these reasons, the motor operation is not allowed when the values of the coefficient of negative sequence voltage is \( \frac{U_{1,\text{NS}}}{U_n} \geq 5\% \).

Interestingly, modern motors of higher efficiency (IE3 or “premium efficiency motors”) have less inverse power) [7], at the same values of negative sequence voltage, and therefore less additional losses due to the unbalance, compared to a standard motors (Fig. 5).
Fig. 5. Dependence of inverse currents of unbalanced voltage motors for:
a) standard motor (class IE1) and b) premium efficiency motors class (IE3)

Experimental measurements [3] showed that the effect of unbalanced voltages on the iron losses increase more if the motor is powered with high voltage. Thus, Fig.6 shows:
- losses increase in iron is 25W, and the unbalanced voltage of 5% and nominal voltage,
- losses increase in iron is 35W when the unbalanced voltage of 5% and the voltage of 110%.

This additionaly influence on reducing the coefficient of nominal power (derating factor), as well as reducing motor efficiency and increasing power consumption.

4 Motor inverse torque

The system of negative sequence voltages leads to the appearance of the inverse torque (\(T_{e,NS}\)), which is opposed to the torque that drives the motor (the torque that comes from the direct system voltages and currents, \(T_{e,PS}\)). Resultant driving torque is reduced, \(T_e = T_{e,PS} - T_{e,NS}\) and the direct torque must be increased to compensate that decrease. Therefore, the slip and direct current systems are increased, and also the corresponding power losses. Direct (\(T_{e,PS}\)) and inverse (\(T_{e,NS}\)) electromagnetic torques are calculated using the equation:

\[
T_{e,PS} = \frac{3U_{PS}^2 R_s}{s\Omega (R_s + R_y/(2-s))^2 + X_{SC}^2}
\]

\[
T_{e,NS} = \frac{3U_{NS}^2 R_y/(2-s)}{s\Omega (R_s + R_y/(2-s))^2 + X_{SC}^2}
\]

assuming that \(R_{NS}=R_{PS}\) and \(R_y/(2-s) = R_y/2\) and with less neglect, leads to a relationship:

\[
\frac{T_{e,NS}}{T_{e,PS}} \approx \left(\frac{U_{1NS}}{U_{1PS}}\right)^2 \frac{Z_{PS}}{2X_{SC}}
\]

Sometimes it is convenient to express the inverse torque in units of the nominal torque and the

\[
\frac{T_{e,NS}}{T_{e,n}} \approx \left(\frac{U_{1NS}}{U_{1n}}\right)^2 \frac{Z_n}{2X_{SC}} \approx 3\left(\frac{U_{1NS}}{U_{1n}}\right)^2
\]
This means that, for the coefficient of unbalanced voltage \( U_{1,NS} / U_n = 0.04 \), would be \( T_{e,NS} / T_{em} = 3 \times 0.04^2 = 0.0048 \). Slip and power losses will also be increased for 0.0048 times, or 0.5%. But if we assume that the inverse rotor resistance for 2-5 times higher (up to 2-4 times higher because of the inverse rotor resistance currents and even up to 1.2 times for an additional increase in temperature of the rotor conductor in such a large loss of power), then the expression for the relative values of the inverse torque is:

\[
\frac{T_{e,NS}}{T_{em}} \approx (6 + 15) \left( \frac{U_{1NS}}{U_{1n}} \right)^2
\]

(14)

where the lower value of the coefficient related to the motor power up to 10 kW, and the upper value for motor power above 100 kW. When coefficient of unbalanced voltage \( U_{1,NS} / U_n = 0.05 \), inverse torque in units of the nominal torque could be at \( T_{e,NS} / T_{em} = 10 \times 12.05^2 = 0.025 \).

Slip and power losses will be increased also for 0.025 times, or 2.5% \( P_n \). As this is a medium power motor, with \( \eta_n \approx 90\% \) or with losses of \( P_m \approx 10\% P_n \) power losses are increased by \( \Delta P_\gamma = 25\% P_m \), which is less than half of the determined value of increasing losses \( \Delta P_\gamma, NS = 50\% \).

The explanation lies in the fact that in this way (i.e. through the power that is allocated to resistance \( R_{r,NS} / 2 \), Fig. 3c) covers only half the power losses in resistance \( R_{r,NS} \) while the remaining half of the compensated part of mechanical power (\( P_m \)), which is obtained through the axis of direct voltage system, Fig. 7. Thus, another procedure confirmed adequate accuracy of quantitative estimates given in Table 4.

### 5 Unbalanced voltage influences to the power factor

Lee have shown in [12] a comparison between different kinds of unbalance voltage (magnitude and angle) and their effects on the losses, efficiency and power factor, for a three phase induction motor of 3 HP. It is shown that, if \( U_{DS} \) is constant and \( U_{NS} \) increases, the power factor will be reduced less than the efficiency. In a recently work [13], authors give a mathematical equation with efficiency and power factor as a function of \( U_{DS} \) and \( U_{NS} \). With that equation it is confirmed a qualitative description in [12] about lower influence of unbalance voltage on the power factor value, and this is reason why it is not investigated in the paper.
6 Causes and evaluation of inverse voltage values

By definition, the coefficient of asymmetry is relationship between the inverse system voltage (\(U_{NS}\)) and direct voltage systems (\(U_{PS}\)). Thus, the percentage of unbalanced voltage is calculated using the formula:

\[
K_{NS} \% = 100 \frac{U_{NS}}{U_{DS}}
\]

where the direct and inverse system voltage component are calculated using the formula:

\[
U_{PS} = \frac{U_{ab} + aU_{bc} + a^2U_{ca}}{3}
\]

\[
U_{NS} = \frac{U_{ab} + a^2U_{bc} + aU_{ca}}{3}
\]

Thus, in the case of balanced (simmetrical) voltages at the motor \(U_{ab} = U_{bc} = U_{ca}\), - we get that \(U_{PS} = U_{ab} = U_{bc} = U_{ca}\) (12), and
- \(U_{NS} = 0\) (13).

Asymmetry can arise for several reasons. One is the joining of large consumers to one or two phases. Thus, if a consumer who connected to one phase, e.g. phase "a", creates a voltage drop \(\Delta U = 3\%\), then it causes the asymmetry of \(1\%\) \((U_{NS} = 1\%)\), since the unsymmetrical voltage system can be presented as the sum of the symmetric system voltage \((U_{ab} = U_{bc} = U_{ca})\) and unbalanced system voltages \((U_{ab} = \Delta U = 3\%, U_{bc} = 0, U_{ca} = 0)\). To this second voltage system, according to (17), corresponds the unbalanced system voltage of \(U_{NS} = 1\%\). If purely inductive consumer is connected between two phases, so that the voltage levels on each phase of the impedance network is \(3\%\), then a similar analysis leads to the conclusion that this causes the asymmetry of \(2\%\) \((U_{NS} = 2\%)\), at the motor connections. These cases are possible in practice, and rarely exceed the specified quantitative values.

Asymmetry may be a consequence of fuse capacitor burning. The fall out of condenser part and reduction of capacitive power is equivalent to appear of inductive loads between the two corresponding phases. This results in the appearance of the inverse voltage value which is equal to \(2/3\) reduction of the phase voltage between phases in which the reduced capacitive load.

The general assessment is that it is almost always the asymmetry coefficient \(K_{NS} < 2\%\), except for the interruption of one phase in the network, or interruption in any of the motor phase windings.

7 Conclusion

Results of analysis presented in the paper could be summarized in following:

1. The effect of unbalanced voltages on the three-phase asynchronous motor is equivalent to the appearance of an inverse system voltage creates a rotating field that rotates in the opposite direction with respect to the rotation of the direct field and the direction of rotation machines. The consequence is that the low value of the inverse voltage is inverse current relatively high values. By definition, the asymmetry coefficient \((K_{NS} \% = 100U_{1, NS}/U_{a})\) is an inverse relationship between the system voltage \((U_{NS})\) and direct voltage systems \((U_{DS})\).

2. Since the unbalanced voltage of \(2\%, 3.5\%\) and \(5\%\) increase in losses could reach, in the same order, the amount of \(8\%, 25\%\) and \(50\%\) of nominal power losses in the motor, it is reasonable requirement to permit voltage asymmetry \(\leq 2\%\), so this is the upper limit in most national and international standards. The truth is that with a smaller load, the motor could safely work also at higher values of unbalanced voltage.

3. The literature states that the information given under 2. is determined from measurements and that they are higher than calculated values. It is, however, here explained by the fact that the rotor resistance higher by \(1.4\) times compared to the rotor resistance in short circuit mode, and not to equal to the same value as described in the literature. Thus it is increasingly convinced that the requirements which are given in most appropriate standards are justified.

4. Voltage unbalance causes further increase of the motor heating, as well as the occurrence of inverse torques which leads to a reduction of the starting and maximum torque, and a smaller increase in motor slip in operation. Since by the increase of unbalanced voltage power losses are increased and motor heating, than allowed motor load decreases more with higher percentage of unbalance. Thus, for the unbalanced voltage of \(2\%, 3\%, 4\%\) and \(5\%\), values of derating factor are \(0.95, 0.93, 0.87\), and \(0.81\), respectively, according to NEMA standards.

5. Based on the actual calculation and analysis, it was found that the effects of unbalanced increase in electricity and power loss is less when it comes to motor power \(\leq 10\ kW\), compared to the data referred to the appropriate standards. Thus, the above increase, the asymmetry of \(3\%\), approximately equal to those referred to in the standards for asymmetry.
of 2%, so the networks with motors below 10 kW may be acceptable voltage asymmetry with asymmetry coefficient \((U_{1,NS}/U_n) \leq 3\%\).

6. Motor operation is not generally allowed when the values of the coefficient of unbalanced voltage are \(U_{1,NS}/U_n \geq 5\%\), because in the (rare) case when the direct and inverse component of the stator currents in (a) phase matching, increase of the current in that phase would be \(\geq 1:38\) times, and the increase of the losses in the windings of that phase would be \(\geq 90\%\).

References:


