Dynamic Stability Evaluation for an Electromagnetic Contactor

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Abstract: - This paper aims at researching the relationship between the dynamic stability of an electromagnetic contactor (abbreviated as contactor below) and the magnetic energy stored in the contactor. Explored contactors are served as an equivalent nonlinear and time-varying inductor. Some basic and important characteristics of the contactor are theoretically researched and discussed. Several critical results are found during closing phase and voltage sag events occurred simply by using the basic definitions of passivity conditions theorem. By means of magnetic circuit analyzing method, a mathematic model and its numerical computer program of a contactor are first established in our laboratory. Therefore, some typical and important experiments such as the dynamic stability of contactor during the closing process and voltage-sag events are carried out by using this model. Later, these simulation results are compared with the experimental results for validating the correctness of this contactor model too. The final comparing results are in agreement with well. For purpose of further to know the voltage sensitivity of the experimental contactor, the CBEMA curves of contactor are used and obtained as well. Finally, the dynamic stability of the contactor during closing process of the contactor. Lots of simulation and experimental results make sure that the contactor model is correct and proposed dynamic stability evaluation method is effective.

Key-Words: - CBEMA curve, passivity energy, contactor, voltage sag, dynamic stability

1 Introduction

Contactors are devices that are very sensitive to power disturbances like voltage sags [1][2]. Until now, these devices have been widely used in many industrial applications. Contactors often act as switches in a variety of electrical systems for both power and control purposes. During power disturbances, for example, voltage-sags events, if the accidental events occurred; the contactors have possibly been disengaged because the power line voltage is lower than the normal voltage value. As a result of the accidental events produced by the power disturbances like voltage sags is outside the system's control and are random, therefore, the disengagements may lead resulting to an uncontrolled and possibly expensive process shutdown. In this paper, for brevity, the voltage sags will be characterized by the sag amplitude, duration, and its point-in-wave where a simulated voltage-sag event occurs, and the effect of the voltage sag on the performance of contactor will be studied and demonstrated the results by using the CBEMA curves [3].

In addition, there is another unstable state for a contactor often occurs when it is initiated by an ac

voltage source and situated in the closing process. In the normal closing process of contactor, there are two kinds of forces, such as the magnetic force and the spring force, act on the movable part of contactor at the same time. So that, the movement and the moving direction of the movable part of contactor that follows the net force or the difference of these two forces.

Indeed, if the electrical character of contactors is considered, contactors are nonlinear and timevarying elements. There is a general fact describes that any nonlinear passive time-varying RLC network made of flux-controlled inductors is a stable network. In other words, the energy stored in inductors will decrease as long as the resistive network absorbs energy. In many applications this process will force the stored energy to go to zero as the time tends to infinity. Hence, for the purpose of investigating the dynamic stability property of contactor during closing process and voltage sag event occurred, the characteristics of contactor situated in these states will respectively be considered in terms of the energy stored in the contactor by using the position versus time varying curves and the flux linkage versus instantaneous coil current varying curves (here, the coil current and armature position are assumed as two independent variables).

2 Mathematical Models

Contactors are typical electromechanical device. Fig. 1 indicates that the basic structure of a popular electromagnetic contactor is partitioned into three portions, such as the electrical portion, the magnetic energy-conversion portion, and the mechanical portion. For simplifying the theoretical analysis, the loss included in the magnetic energy-conversion portion is assumed to be considered in the electric portion and mechanical portion, respectively. Therefore, the magnetic energy-conversion portion can be therefore viewed directly as a lossless and conservative sub-system. It is also referred to as a lossless magnetic coupling system. Based on the law of conservation [4] in physics, the energy balance relationship inside an electromagnetic contactor system is given by

$$W_e = W_f + W_m + W_h \tag{1}$$

where

- W_e : The input electrical energy is supplied by the external voltage source u(t);
- W_f : The magnetic coupling energy is stored in the magnetic energy-conversion sub-system;
- W_m : The total work done is dissipated by the mechanical sub-system;
- W_h : An amount of heating loss is dissipated in all the system.



Fig. 1. Sketch the equivalent configuration of an electromagnetic contactor.

2.1 Electrical model

The external voltage source u(t) supplies to the electric energy over a time interval dt. By Kirchihoff's voltage law and work-energy principle, we derive equations (2) and (3) as follows,

$$u(t) = ir + \frac{d\lambda}{dt}$$
(2)

$$dW_e = i u \, dt \tag{3}$$

where, the symbol, λ , represents the flux linkage of contactor coil. As we known, the external voltage source u(t) equals the number of windings of coil N times the derivative of flux with respect to time, that is $N(d\phi/dt)$. Substitute u(t) into (3), we obtain

$$dW_e = i N \, d\phi \tag{4}$$

When the electromagnetic force F_e produced by the magnetic circuit imposed upon the armature and larger than the spring anti-force, the armature certainly begins moving forward the fixed iron core and the displacement is assumed to be dx. The differential work done on the armature by the mechanical system can be given as follows

$$dW_m = F_e \, dx \tag{5}$$

During the closing process, the amount of the flux in the magnetic circuit and the coil-current value are almost not affected [5]. This phenomenon means that the instantaneous change of total flux and coil current approximates zero. Equation (5) yields the following form.

$$F_e \, dx = -dW_f \tag{6}$$

The simplified expression shown in (6) indicates the differential work done by the electromagnetic force that is determined by the decrease in the energy stored in magnetic coupling field. The field energy stored in the lossless magnetic energy conversion system commonly equals the stored energy in the inductor. It can be described by

$$W_f = \frac{1}{2}L(x)i^2$$
(7)

Substituting (7) into (6), the electromagnetic force dynamically imposed upon on the armature can be determined by the coil current and the derivative of the self-inductance with respect to armature displacement and shown as follows:

$$F_e = -\frac{dW_f}{dx} = -\frac{1}{2}i^2 \frac{dL(x)}{dx}$$
(8)

In theory, the basic characteristic of self-inductance L(x) varies inversely proportional with the armature displacement х so that the electromagnetic force shown in (8) illustrates that the value of electromagnetic force is a function of the square of coil current and the change of selfinductance with respect the to armature displacement. Obviously, the change of electromagnetic force forced on the armature is always a nonlinear value during the closing process.

2.2 Mechanical model

The mechanism appearance of the electromagnetic contactor shown in Fig. 2 consists of an armature and a fixed iron core. One complete closing course of the electromagnetic contactor can basically be divided into two stages: the first stage is the state transition from the initial opening position to the contacts closing state. The movable iron core is normally linked with one or triple sets of contacts moves toward the fixed iron core by simple mechanism. If the electromagnetic force overcomes the spring anti-force, the armature or the movable contacts moves forward to the direction of fixed iron core. After the movable contacts have engaged with the fixed contacts, the second stage begins. The armature continues moving toward the fixed iron core until they are actually closed with the fixed iron core. During this stage, the anti-force produced by the return spring will be incorporated into the tension force of contacts' spring. Therefore, the mechanical model should be considered by two stages, respectively [5,6].

2.2.1 First stage: $(d_A - d_C) \le x \le d_A$

The symbol d_C is the aperture between the movable contacts and the fixed contacts. In contrast, the symbol d_A represents the aperture between the movable iron core of armature and the fixed iron core. By using the Newton's law of motion, the mathematical model of mechanical system of contactor can be expressed as follows:

$$\sum F = ma \tag{9}$$

The resultant force $\sum F$ is equivalent to the armature mass multiplies its acceleration and it is the addition of three different forces, such as electromagnetic force, spring anti-force and viscous friction force. The moving acceleration item in (9) can be described as the second order time differentiation of armature displacement. Equation (9) can be rearranged and shown as follows:

$$\sum F = F_e - F_f - F_b = m \frac{d^2 x}{dt^2}$$
(10)

where

- F_e : electromagnetic force;
- F_f : counter force produced by return spring;
- F_h : viscous friction force;
- *a* : moving acceleration of armature;
- *m* : armature mass;
- *x* : armature displacement.



The representation of an electromagnetic force shown in (8) is a function of the coil current and the armature displacement, that is $F_e = -f(i,x)$. The spring tension force varies proportionally with the change of armature displacement. This anti force is come from the two return springs is assumed to be expresses as $F_f = 2K_1x + a_1$, where a_1 is a constant initial pressure force of the spring system imposed upon the armature. Furthermore, the viscous friction force F_b , in principle, is a function of the moving velocity of armature during the considered time interval. The representation of this item is represented as $F_b = -b\frac{dx}{dt}$, where the symbol b is constant coefficient. The armature mass is the contribution of the mass of movable iron core, $m_{4,2}$ and the mass of one or three movable contacts' mass, m_C .

2.2.2 Second stage: $0 \le x \le (d_A - d_C)$

During the second stage of mechanical motion, movable contacts would be engaged with fixed contacts first, and then the armature continues to move toward the fixed iron core. As a result of the disappearance of the contact aperture, this moving process is also called as the super path of the armature. As stated in the previous section, the total counter force imposed upon the armature is the addition of the former working stage and the tension force of contacts' spring. It can be written as below:

$$F_f = (2K_1 + 3K_2)x + a_1 + 3d \tag{11}$$

where K_2 is the elastic coefficient of three contacts' spring. *d* is the initial pressure of contacts' spring forced on the armature. During the super path of armature, the movable iron core uniquely continues to move toward the fixed iron core, but not all the armature mechanism. Therefore, the total armature mass is only equivalent to the mass of movable iron core m_A .

3 Dynamic Behavior

In fact, the dynamic behavior of contactor is energy transition from the electrical power to the mechanical movement. Almost the basic closing process of contactor, the energy transition process is a nonlinear and time-varying as mentioned earlier [7]. In the following, we will discuss the corresponding property variations in the respective electro-magnet part and mechanical part during transient events occurred.

3.1 Electro-magnet part

Fig. 3 shows the equivalent electrical circuit of the contactor from the view point of its coil. By employing the Kirchhoff's voltage laws (KVL), the applied coil voltage of contactor should be dropped on each load in the same electrical loop. Respectively, the voltage drop terms occur due to the coil resistance, coil inductance, $d\lambda/dt$ (= $f_L(x,i)$), and the motion voltage, $e (= f_v(x,i))$, and so on. It is noted that the latter two terms are always varied with the position or displacement of movable-part and the value of coil current.



Fig. 3. Equivalent electrical circuit as the contactor is energized by a voltage source.

The equations of motion are obtained for the contactor operating in the closing phase can be derived by a straightforward application of Kichhoff's voltage law on a generalized coordinates. The contactor can be theoretically described by a set of four differential equations and given in the following:

$$u(t) = ir + \frac{d\lambda}{dt}$$

$$= \sqrt{2}U_{RMS} \sin(wt + \varphi)$$

$$\frac{d^{2}x}{dt^{2}} = \frac{\sum F}{m}$$

$$\frac{dx}{dt} = v$$

$$F_{e} = -\frac{1}{2}i^{2}\frac{dL}{dx}$$
(12)

where the symbols used in (12) are defined as follows, respectively:

- U_{RMS} : is the root-mean-square value of the applied coil voltage source,
- *r* : is the resistance of the coil,
- m: is the mass of the movable part,
- w: is the angular frequency of ac source,
- *t* : is the operating time,
- ϕ : is the initial phase angle of applied AC voltage,
- F_f : is the occurred anti-force due to the return springs,
- F_e : is the electromagnetic force, which acts on the movable part or armature,
- λ : is the flux linkage of coil, its value is equal to the coil inductance multiplied by the instantaneous coil current or the number of coil windings multiplied by the magnetic flux, one can write as follows:

$$\lambda = L(x) \times i = N \times \phi \tag{13}$$

As we known, the moving velocity of armature or movable part equals the time rate of change of position, v = dx/dt. By using the chain rule, lots of valuable information could be found. From (2) or (12), dL/dt = (dL/dx)(dx/dt) = v(dL/dx), therefore, the loop voltage equation shown in (7) can be rewritten as:

$$u(t) = ir + L\frac{di}{dt} + vi\frac{dL}{dx}$$
(14)

where the *ir* term represents the ohmic voltage drops across the coil, the L di/dt term represents the inductive voltage drop in a coil, and the *vi dL/dx* term represents the motional voltage drop produced by the moving velocity of armature and it can also be rewritten as follows:

$$e = vi \frac{dL(x)}{dx}$$

$$= f_V(x,i)$$
(15)

Note that the motional voltage of the movable part can be expressed in terms of its velocity, instantaneous coil current, and the position rate of the change of the coil inductance. As seen in (15), the motional voltage e is a function of the value of the moving velocity or armature position and the instantaneous current value which flows in the coil. In the following, the transient behaviors of the contactor worked in the closing phase and the close phase are going to be studied, respectively, but the opening phase will not be discussed here because the latter phase does not concern with our investigating topic.

3.1.1 Closing phase

Differentiates (13) with time and the chain rule is introduced in basic differential theory, the results can be written by

$$\frac{d\lambda}{dt} = \frac{d}{dt} [L(t)i(t)]$$

$$= L\frac{di}{dt} + i\frac{dL}{dt}$$
(16)

We may interpret the right-hand side of (16) as follows. The term L(t)(di/dt) gives the voltage at time t across a time-invariant inductor whose inductance is equal to the number L(t). In the second term of (16) the voltage at time t is proportional to the current at time t; hence it can be interpreted as the voltage across a linear timevarying resistor whose resistance is equal to $\dot{L}(t)$. The series connection of these elements describes the voltage-current relation of a linear time-varying inductor. Clearly, if $\dot{L}(t)$ is negative, we have an active resistor; thus, it is necessary that $\dot{L}(t)$ be nonnegative for the time-varying inductor to be passive (namely it means that never delivers power to the outside world).

According to (14), we realize that the relation between the applied coil voltage and instantaneous coil current is time-varying and nonlinear during the closing process of contactor. Consequently, it is a very difficult for people to accurately predict the exact dynamic behaviours of contactor during this phase stage.

3.1.2 Close phase

Once the working status of contactor has entered into the close phase, the value of the coil inductance becomes a constant as well as the motional voltage is zero. Because that the behavior of the movable part worked in this phase is stationary, so that, the equation given in (14) and (15) can be rearranged and written using as follows:

$$\frac{d\lambda}{dt} = L\frac{di}{dt}, \qquad e_l = 0 \tag{17}$$

A general expression for the contactor coil, as seen in (12), can be further expressed by the following simplified form:

$$u(t) = ir + L\frac{di}{dt} \tag{18}$$

Clearly, (18) is a one order differential equation and the solution of the instantaneous coil current certainly is an exponential form:

$$i(t) = \frac{u}{r}(1 - e^{-\frac{t}{\tau}})$$
(19)

Physically, (19) means that the value of instantaneous coil current should exponentially increase and become a constant value after approximately five times time constant, $\tau = L/r$.

3.2 Mechanical part

In general, the mechanical part of the contactor is consists of movable part, stationary electromagnet, two return springs, and one or three contacts' springs. In fact, the mechanism of contactor is really equivalent to a mass-spring-damp system. Therefore the dynamic behaviours of the mechanical part can be naturally represented by a second order ordinary differential equation and like as follows.

$$F_e = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx$$
(20)

Most of the working time of the contactor, the viscous coefficient *B* is small, so that it is reasonable if it is ignored in our research work. For brevity, the spring tension term, which acts on the movable part, is defined as F_f here. Finally, (20) can be simplified and given by:

$$F_e - F_f = m \frac{d^2 x}{dt^2} = \Delta F \tag{21}$$

In case of the contactor coil is energized by an external voltage source, the general expression of the electromagnetic force can be written as follows:

$$F_e = \frac{1}{2}i^2 \frac{dL(x)}{dx}$$
(22)

Note that the electromagnetic force is a function of the square instantaneous value of the coil current and the armature position rate of change of the coil inductance. Strictly speaking, the value of the electromagnetic force is dynamically varied with the position of movable part. If there is a movement is produced by the movable part, the value electromagnetic force should change as well.

4 Passivity and Stability

Consider a one-port which include nonlinear and time-varying elements. Let us drive the one-port by a generator, as shown in Fig. 4 [8]. From basic physics then we know that instantaneous power entering the one-port at time t is

$$p(t) = e_l(t)i(t) \tag{23}$$

and the energy delivered to the one-port from time t_0 to t is

$$W(t_0, t) = \int_{t_0}^t p(t')dt' = \int_{t_0}^t e_l(t')i(t')dt'$$
(24)

Since the exploring target is going to be a contactor in this paper and its characteristic in nature is nonlinear time-varying inductor, and the characteristic is given for each t by a curve similar to that shown in Fig. 5, the curves changes as t varies. For convenience, let us assume that at all times t the characteristic curve of one port goes through the origin; thus, the inductor is in the zero state when the flux is zero. In addition, we also assume that at all times t the inductor is flux-controlled. Furthermore, we may represent the nonlinear time-varying inductor by

$$i = \hat{i}(\lambda, t) \tag{25}$$

Note that *i* is an explicit function of both λ and *i*. The voltage across the inductor is given by Faraday's law as follows:

$$e_l = \frac{d\lambda}{dt} \tag{26}$$

Substitutes (25) and (26) into (24), the energy delivered by the generator to the inductor from time t_0 to t can be written as

$$W(t_0,t) = \int_{t_0}^t e_l(t')\dot{i}(t')dt' = \int_{\lambda(t_0)}^{\lambda(t)} \hat{i}(\lambda',t)d\lambda' \quad (27)$$

Equation (27) shows that $W(t_0,t)$ is a function of the flux at the starting time t_0 and at the observing time t. If we assume that the state of flux is zero, that is, $\lambda(t_0) = 0$, and if we choose the state of zero flux to correspond to zero stored energy, then, recalling that an inductor stores energy but does not dissipate energy, according to the energy conservation the energy stored ε must be equal to the energy delivered by the generator from t_0 to t, that is, $W(t_0,t)$, must be equal to the energy stored

$$\varepsilon[\lambda(t), t] = W(t_0, t) = \int_0^{\lambda(t)} \hat{i}(\lambda', t) d\lambda'$$
(28)

Note that ϕ' is the dummy variable of integration and that *t* is considered as a fixed parameter during the integration process.

Equation (16) can also be written in the form

$$W(t_0, t) = \varepsilon[\lambda(t), t] - \varepsilon[\lambda(t_0), t_0] - \int_{t_0}^t \frac{\partial}{\partial t'} \varepsilon[\lambda(t'), t')] dt'$$
(29)

The first two terms give the difference between the energy stored at time t and the energy stored at t_0 . The third term is the energy delivered by the circuit to the agent. This term changes the characteristic of the inductor; thus, it is the work done by the electrodynamic for forces during the changes of the configuration of the inductor. If the input of contactor is assumed as a time varying inductor, we use a time-varying inductance L(t) to describe its time-varying characteristic. Thus,

$$i = \frac{\lambda}{L(t)} \tag{30}$$

According to (12), the energy delivered to the timevarying inductor by a generator from t_0 to t is

$$W(t_0,t) = \frac{1}{2}L(t)i^2(t) - \frac{1}{2}L(t_0)i^2(t_0) + \int_{t_0}^{t} \frac{1}{2}\dot{L}(t')i^2(t')dt'$$
(31)

where the last term in the right-hand side is the energy delivered by the circuit to the agent. This term changes the characteristic of the time-varying inductor. It is important to note that this last term depends both on the waveform $\dot{L}(\cdot)$ and the waveform $i(\cdot)$.

A one-port is said to be passive if the energy is always nonnegative, this is also called the passivity condition.

$$W(t_0, t) + \varepsilon(t_0) \ge 0 \tag{32}$$

for all initial time t_0 , for all time $t \ge t_0$, and for all possible input waveforms. From (32) passivity requires that the sum of the stored energy at time t_0 and the energy delivered to the one-port from t_0 to t be nonnegative under all circumstances. So that, the passivity condition for time-varying inductors require that for all time t, and for all possible inductor current and inductor voltage waveforms. From (31) and (32), the condition for passivity is

$$\frac{1}{2}L(t)\dot{i}^{2}(t) + \int_{t_{0}}^{t} \frac{1}{2}\dot{L}(t')\dot{i}^{2}(t')dt' \ge 0$$
(33)

for all time t, for all starting times t_0 , and for all possible currents $i(\cdot)$. Therefore we can assert that a nonlinear time-varying inductor is passive if and only if

 $L(t) \ge 0 \tag{34}$

and

$$\dot{L}(t) \ge 0 \tag{35}$$



Fig. 4. A one-port is driven by a generator.



Fig. 5. Characteristics curve of a typical nonlinear time-varying inductor. (Providing that is timeinvariant for simplicity)

An inductive one-port is said to be locally passive at an operating point if the slope of its characteristic in the $i\lambda$ plane is nonnegative at the point. An inductive one-port is said to be locally active at an operating point if the slope of its characteristic in the $i\lambda$ plane is negative at the point. If the inductive one-port is passive, the instantaneous power entering it is nonnegative. Thus, the energy stored in the network is a nonincreasing function of time. In other words,

$$\varepsilon(t) \le \varepsilon(t_0)$$
 for all $t \ge t_0$ (36)

We can say that a network is passive if all the elements of the network are passive. And, however, any nonlinear passive time-varying network made of flux-controlled inductors is a stable network.

5 Simulation and Discussion

In this paper, the experimental contactor is manufactured by a domestic and professional company, Shilin. This device is a tripolar contactor and its type is S-C21L. The coil will be applied an ac voltage source, their nominal power frequency is 60 Hz, and its rated root-mean-square coil voltage is 220 V_{RMS} . Rated contacts' capacity is 5.5 KW, 24 A, the number of coil windings are 3750, the coil resistance is 285 Ω , and the mass of the movable part is 0.115 Kg.

5.1 Establishing simulation model

In general, the dynamic behavior of the contactor can be predicted by using the simulation technique. From the preceding statement in this paper, we know that the governing equations of the contactor are basically composed by the electrical circuit equations and mechanical motion equations. Therefore, five individual simulation sub-modules are firstly established based on the respectively governing equations. At last, a complete contactor simulation model could be built by means of combining the preceding obtained five sub-modules; into a resulting model. The completed simulation model of contactor is shown in Fig. 6.

In order to verify the correctness of the simulation model of the contactor, further to compare of the concerning parameters between the experimental results and the simulation results are essential and necessary work. Hence, the instantaneous coil current, electromagnetic force, counter force and movable-part position, are all used as the compared parameter items. In Fig. 6, it is shown that the simulation results using contactor model are basically in agreement with well the experimental results. To a word, the validation of the correctness and precision of the simulation model of the contactor are successfully done. This simulation model can then be used to simulate and predict the performance of contactor under different operating assumed working status.



Fig. 6. Completed simulation model of the electromagnetic contactor.





Fig. 7. Simulated parameter waveforms by model compared with those measured by experimental contactor.

5.2 CBEMA curves analysis

The dynamic performance of contactor is to be studied and expressed by using CBEMA curves when it is operated under different voltage-sags conditions like amplitude, duration, and initial voltage sags phase. This kind of curve, CBEMA, was originally introduced to represent the computers voltage-tolerance performance for over-voltage and under-voltage. They show minimum voltage against maximum duration, which ensure the non-stop operation of contactor. Provided that a special pointin-wave where sag occurs, the boundary between succeed and fail to close the electrical contacts are measured and plotted, as shown in Fig. 8. The abscissa of Fig. 8 is voltage-sags duration in cycle, while the ordinate is voltage-sags amplitude in percentage. CBEMA curves of contactor in terms of initial voltage sags phase 0° , 45° , 90° and 135° are respectively obtained, notice that the border of the 45° initial voltage phase is similar to one of the 135°, but there are not completely identical.





Fig. 8. CBEMA curves of the contactor tolerance under phase angle 0° , 45° , 90° and 135° where sag occurs.

(a)

5.3 Flux linkage versus coil current varying curves

From the characteristic of the contactor in terms of input driving impedance, it is indeed an inductor element. As mentioned above, if the inductive oneport is passive, the instantaneous power entering it is nonnegative. Thus, the energy stored in the network is going to be a nonincreasing function of time, as seen in (21). However, any nonlinear passive time-varying network made of fluxcontrolled inductors is a stable network. We can say that a network is passive if all the elements of the network are passive. Therefore, we say that the dynamic behavior of contactors is either stable or unstable can be completely decided by the net energy stored in the contactor during the dynamical period. If the net energy stored is nonincreasing, the dynamic stability of contactor will be situated at stable state. As a matter of fact, in many practical applications, most the characteristic of the contactor is passive, but locally active at some operating points occur because the slope of the characteristic in the $i\lambda$ plane is negative. So that the electrical contacts is whether remain closed or dropped out, completely relies on the energy delivered by the circuit to the agent $\int_{t_0}^{t} \frac{1}{2} \dot{L}(t') \dot{i}^2(t') dt'$ during the dynamical period. If the stored energy within this period is negative, the electrical contacts are anticipated to be dropped out, as shown in Fig. 9; contrarily, if the stored energy within this period is nonnegative, the electrical contacts will therefore remain closed or disengagement and re-engagement, as shown in Fig. 10 and 11. In one word, the electrical contacts are either remain closed or dropped out after the considered dynamical period past depends on the value of the net stored energy is negative or nonnegative within the dynamical





Fig. 9. Coil inductance $\dot{L}(t)$ and position varying curves when the voltage sag whose amplitude is 10%, duration is one cycles, and point-in-wave 0° where sag occur.



Fig. 10. Coil inductance $\dot{L}(t)$ and position varying curves when the voltage sag whose amplitude is



20%, duration is three cycles, and point-in-wave 0° where sag occur.

Fig. 11. Coil inductance $\dot{L}(t)$ and position varying curves when the voltage sag whose amplitude is 20%, duration is two cycles, and point-in-wave 90° where sag occur.

As commented earlier, the dynamic characteristic of the contactor is like a nonlinear time-varying inductor. Nevertheless, for a nonlinear time-varying inductor to be passive, its characteristic (in the $i\lambda$ plane) must pass through the origin and lie in the first and third quadrants in the neighbourhoods of the origin. Also, if the characteristic of a nonlinear time-varying inductor is monotonically increasing and lies in the first and third quadrants, it is passive.

Fig. 9 shows that the energy stored in the contactor is zero during the voltage sag occurs because the position of the movable part never leaves away the electromagnet. However, the characteristic of a nonlinear time-varying inductor is

monotonically increasing and lies in the first and third quadrants, it is passive. Therefore, the electrical contacts is remain closed during the voltage sag occurs.

Fig. 10 shows that the total energy stored within the position increasing part is -0.04368 Joules, while the total energy stored within the position decreasing part is 0.06493 Joules during the voltage sag occurs. The characteristic of a nonlinear timevarying inductor is monotonically increasing and lies in the third quadrants, it is passive. Note that the there is partial characteristic occurs the time rate of the change of coil inductance $\dot{L}(t)$ is negative, as shown in Fig. 10(a), where denoted by a dash circle; namely, the operating feature of these points is locally active. Consequently, the energy stored in the contactor within this dynamic period becomes a negative value and the magnetic circuit produces an interesting result that the movable part disengage and then re-engage without any disengagement of the electrical contacts. In addition, considering the net energy stored in the contactor during voltage sag occurs is still a nonnegative value; hence, the contactor is to be passive and stable.

Fig. 11 shows that the total energy stored within the position increasing part is -0.004604 Joules, while the total energy stored within the position decreasing part is 0.05413 Joules during the voltage sag occurs. Since the characteristic of a nonlinear time-varying inductor is monotonically increasing and lies in the first and the third quadrants, it should be passive. Note that the there are some operating points in the characteristic occurs the time rate of the change of coil inductance $\dot{L}(t)$ is negative, as shown in Fig. 11(a), where is denoted by a dash circle; namely, the operating feature of these points is locally active. The sustaining time of the negative coil inductance $\dot{L}(t)$ is longer than preceding case. As seen in (37), because of the net energy stored in the contactor during voltage sags has become a nonnegative value, and therefore the unintended disengagement of the electrical contacts occurs. For this situation, the characteristic of the contactor can be said to be an active and the action is considered as an unstable operation.

6 Conclusion

Up to now, there is nothing works concerning the dynamic stability of the contactor by researching of the characteristic of contactor can be found. In this paper, according to the basic definitions of the stability and passivity condition, we provide a new approach, which can be used to predict the dynamic stability of contactor in terms of its characteristic curve in the $i - \lambda$ plane. We are successful to obtain some valuable results by using the established simulation model of the contactor. The contactor model followed by the electromagnetic circuit analysis approaches is established. In the following the contactor voltage sag sensitivity is also simulated on this model and the simulation results are shown in CBEMA curves. Additionally, as seen in the characteristic of contactor in the $i - \lambda$ plane, provided that there exists locally active point during closing process or where voltage sag occurs, the final state of contacts may be remain closed, disengagement or disengagement and reengagement, which depends only on the net energystoring within the dynamic period. From those simulation results, we can ensure the dynamic stability of contactor such as during closing process or where voltage sags occur, can be predicted by using the characteristic of a contactor in the $i - \lambda$ plane.

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