Modeling Bank Interest Margin and Loan Quality under the Troubled Asset Relief Program: An Option-Pricing Approach

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Abstract: The troubled assets on U.S. banks’ books could grow to as much as $5 trillion, one Goldman Sachs analyst estimates [10]. Will setting up the Troubled Asset Relief Program (TARP) be good move for bank loan quality? The answer is yes. In an option-pricing model where the bank’s book value of loans is above its market price, an increase in loan amount sold, exactly what the TARP is meant to target, increases the bank’s interest margin. The gap where carrying value is above market price is shrinking by decreasing the risky loans held by the bank and thus the bank’s loan portfolio quality is improved.

Key-words: Troubled Asset Relief Program, Bank Interest Margin, Loan Quality

1 Introduction

There is a recent report in the Economist (March 28th, 2009a, p.70): “In aggregate the carrying value of the top-ten banks’ loan books [in America] was 3% above the market price in
December [2008]. That gap may not seem much, but it amounts to over $110 billion; if it were crystallized, it would wipe out a quarter of these banks’ tangible common equity, their purest form of capital.”

Asset quality problems have plagued banks in particular when their carrying value of loan books is above the market price. Concerns about bank asset quality and bank failures have prompted regulatory authorities to bailout some categories of risky loans. For example, the financial authority in the United States announced on March 23rd 2009 marks a revival of sorts for the asset-buying component of the Troubled Asset Relief Program (TARP), a $700 billion rescue fund created last October (Economist, 2009b). For example, Henry, Goldstein, and Farzad (2009) report that Bank of America gets $15 billion in October 2008 and $20 billion in January 2009 from the TARP. Will setting up the TARP be a good move for loan quality reflected by shrinking the gap where carrying value is above market value? As loan quality is so important to bank profits, the issues of how it is optimally determined and how it adjusts to changes with government help deserve closer scrutiny.

We argue that the answer to the question above lies in the relation of bank interest margin and loan quality under government help. In practice, loan quality management can be related to bank margin management, which is done through a “cost of goods sold” approach. It is the approach that deposits are the “material” and loans are the “work in process” (see Finn and Frederick, 1992). Two divergent bases in this approach are employed to model bank behavior. The book basis means that revenues and expenses are recognized when they are received and paid. Only certainty and/or linear risk preferences have been taken into account in this book basis. Alternatively, the market basis means that revenues and expenses are recognized when they are earned or incurred, regardless of when cash is received or paid. This market basis explicitly treats uncertainty and/or nonlinear risk preferences in discussions of bank behavior.

A growing body of recent literature documents that future equity returns can be predicted by earnings quality (see Sloan, 1996, and Chan, Chan, Jegadeesh, and Lakonishok, 2006). Chan, Chan, Jegadeesh, and Lakonishok (2006) focus on measure, accounting accruals, which is a potentially important indicator related to earnings quality. Accruals represent the difference between a firm’s accountings and its underlying cash flow. In this paper, we apply the concept of Chan, Chan, Jegadeesh, and Lakonishok (2006) and demonstrate that the positive difference between equity valued in the book basis and equity valued in the market basis partially wipes out the market value of bank equity, the form of capital. Bank capital positions reflect asset portfolio risks. Zarruk and Madura (1992) point out that the principal advantage of a risk-based system of capital standards, for instance, is the explicit treatment of uncertainty which has played a role in discussions of asset quality. Accordingly, we can argue that shrinking the positive difference allows the inclusion of loan quality management along with the behavioral mode of loan rate-setting.

Our theory of loan quality management is related to three strands of the literature. The first is the literature on the determination of bank interest margin, in which Stoll (1978), McShane and Sharpe (1985), Allen (1988), Zarruk and Madura (1992), and Wong (1997) are major contributors. In particular, McShane and Sharpe (1985), and Allen
(1988) provide models of bank interest margins based on the bid-ask spread model (Stoll, 1978). Unlike previous formulations, In Zarruk and Madura (1992) where loan losses are the source of uncertainty, changes in capital regulation or deposit insurance premiums have direct effects on the bank’s interest margin. Wong (1997) explores the determinants of optimal bank interest margins based on a simple firm-theoretic model under multiple sources of uncertainty and risk aversion. While we also examine bank interest margin, our focus on the loan quality management aspects of loan rate-setting takes our analysis in a different direction.

The second strand is the modern loan quality management literature. Lending taking into account quality generates proprietary information about the borrower is common in the banking literature (Rajan, 1992). Zarruk (1989) and Zarruk and Madura (1992) provide firm-theoretical models with only a single source of uncertainty to explain bank spread behavior: funding risk as in Zarruk (1989) and credit risk as in Zarruk and Madura (1992). Wong (1997) uses credit risk and interest risk to determine the optimal bank interest margin decision. The primary difference of our model is that we consider the effect of government help on loan quality management by decreasing the difference between the carrying value of the bank’s equity book and its market price.

The third strand is the literature on political concerns. Kroszner and Strahan (1996) show how regulators deferred the reckoning of costs in failing Savings and Loan associations in the United States. Bongini, Claessens, and Ferri (2001) examine the role of political connections in government intervention to failing banks in four Asian crisis countries. Brown and Dinç (2005) study the role of the electoral cycle in the timing of government interventions. This paper differs from the ones above in both scope and focus. It studies the government insure a bank’s rotten loans without nationalizing the bank. It also focuses on the role of the bank’s optimal interest margin determination in loan quality management under government help.

Our primary emphasis is the selection of the bank’s optimal interest margin, which is the difference between the rate of the bank charges borrowers and the market rate the bank pays to depositors. Banks are in the business of lending and borrowing money. Earnings from the margin typically account for significant bank profits. This is in particular true when the U.S. banking industry is experiencing a renewed focus on retail banking, a trend often attributed to the stability and profitability of retail activities (Hirtle and Stiroh, 2007). Lin, Chang, and Lin (2009) develop on option-pricing model to determine the market value of bank equity and its default probability in equity return explicitly incorporating the TARP. However, their model ignores the asset quality management incurred in the bank’s operations management.

As an alternative, unlike previous formulations, the model developed here assumes a setting in which the bank is subject to government help by the TARP. The bank’s objective is to set the rate of the bank charges borrowers in order to minimize the positive gap between its carrying value and its market value. A comparative static result shows that an increase in the government’s help increases the bank’s interest margin and thus increases the bank’s equity in the asset quality management. This paper concludes that setting up the TARP for a “bad bank”
solution may be a good move for loan quality.\(^1\)

This paper is organized as follows. Section 2 develops the basic structure of the model. Section 3 derives the solution of the model and the comparative static analysis. The final section concludes.

2 The Model

In order to get closed-form, tractable solutions, a few simplifying assumptions are made. We shall indicate when these assumptions affect the qualitative results derived in this paper.

Our model is myopic in the sense that all economic decisions are made and values are determined with a one-period horizon only, \(0 \leq t \leq 1\). Consider a single-period of a banking firm. At \(t = 0\), the bank accepts \(D\) dollars of deposits. The bank provides depositors with a market rate of return, \(R_D\). Equity capital \(K\) held by the bank at the \(t = 0\) is tied by regulation to be a fixed proportion \(q\) of the bank’s deposits, \(K \geq qD\). The required capital-to-deposits ratio \(q\) is assumed to be an increasing function of the amount of the loans \(L\), held by the bank at \(t = 0\). We have \(\partial q / \partial L = q' > 0\) (see Zarruk and Madura, 1992 and Lin, Lin, and Jou, 2009). This assumption implies a risk-based system of capital standards.

The bank makes term loans \(L\) at \(t = 0\) which mature and are paid off at \(t = 1\). Loan demand faced by the bank is specified as \(L(R_L)\) where \(R_L\) is loan interest rate. We assume that the bank has some market power in lending (see Hannan, 1991) and Wong (1997)) which implies that \(\partial L / \partial R_L < 0\). The assumption of market power is to limit the scale of lending activities, and an assumption about increasing administrative costs of making loans would achieve the same end. The details of what drive loan demand are unimportant for our purposes, so this abstraction is sufficient (see Kashyap, Rajan, and Stein, 2002). In addition to term loans, the bank can also hold an amount \(B\) of liquid assets, for example, Treasury bills, on its balance sheet during the period. These assets earn the security-market interest rate of \(R\).

When the capital constraint is binding, the bank’s balance-sheet constraint is given by:

\[
L + B = D + K = K \left( \frac{1}{q} + 1 \right)
\]  \(1\)

The balance-sheet constraint demonstrates the bank’s operations management in lending activities since the total assets in the left-hand side are financed by demandable deposits and equity capital in the right-hand side. Further, equation (1) explains deposits are renewed each period, based on the status of the bank at that time. The bank can also change its capital structure at the start of each period based on the past performance of its assets and its future prospects. Our model is, however, dynamic in nature, although the focus of this paper is on one period valuation.

\(^1\) Here is one way of creating a “bad bank” to take on toxic assets suggested by Francis (2009). The first step is that the Treasury Department establishes the bad bank, capitalizing it with some remaining funds from TARP. The second step is that the bad bank raises additional funds, either by borrowing from the Fed or selling shares to private investors. The third step is that the bad bank uses funds to buy toxic assets from target banks. It holds the assets to maturity, or sells them as markets revive. Losses are split among the bad bank’s investors and taxpayers. The last step is that the Treasury and/or investors commit additional capital to target banks, compensating for losses realized from selling toxic assets.
The government (the bad bank) uses funds to buy toxic assets from the bank that the TARP is meant to target. If the bad bank overpays, the selling bank gets a windfall at taxpayers’ expense. If the bad bank underpays, the bank will not have an incentive to sell. We assume that the bank is willing to sell the amount of toxic loan repayments \( \theta(1 + R_L)L \), \( 0 < \theta < 1 \), to the government. With government help when the carrying value of the bank’s toxic loan books is far above its market price, it is reasonable to assume that the binding value of toxic loans sold to the government is set to equal its book value.3

Our approach in calculating equity value uses Merton’s (1974) model. The equity of the bank is viewed as a call option on its risky-loan repayments. The reason is that equity holders are residual claimants on the bank’s risky-loan repayments after all other obligations have been met. The strike price of the call option is the book value of the bank’s net liabilities. When the value of the bank’s risky-loan repayments is less than the strike price, the value of equity is zero.4 Equation (1) demonstrates that the capital structure of the bank includes both equity and debt. We use the theoretical distribution implied by Merton’s (1974) model. With the financial help from the TARP, the market value of the bank’s risky-loan repayments specified as \( V = (1 - \theta) \times (1 + R_L)L \) follows a geometric Brownian motion of the form:5

\[
\frac{dV}{V} = \mu dt + \sigma dW
\]

where \( V \) is with an instantaneous drift \( \mu \) and an instantaneous volatility \( \sigma \). \( W \) is a standard Wiener process.

To gain the essence of earnings (loan repayments in our model) quality, let us consider a model due to Kihlstrom and Levhari (1977). They make a fundamental assumption that there is a “linear technology”.6 We use this assumption to define earnings quality as the positive gap between its carrying value of equity return and its market value. The principal advantage of this setting is the explicit treatment of bank earnings quality which has long played an important role in discussions of bank behavior. Our setting also includes two aspects of more realistic bank behavior. First, the setting is applicable to loan markets since such markets are virtually always highly concentrated where banks set loan rates and face random loan levels (see Zarruk and Madura, 1992, and Wong, 1997). Second, the setting allows the inclusion of the resource costs incurred in bank operations (see Sealey, 1980).7

The bank’s objective is to set \( R_L \) to minimize this gap, subject to equation (1). Equivalently, the bank

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2 “There should be no shortage of buyers for American banks’ rotten assets . . . . Sellers will be harder to entice.” (See Economist, 2009b, p.69) However, we argue that “add in the toxic-asset plan, and the total climbs to as much as $608 billion.” (See Economist, 2009b, p.70).

3 Note that the government (the bad bank) might revive trading of mortgage-related securities, for example, by establishing valuations for these assets. As the market recovers, or if housing prices begin to rise, the bad bank could break even or even turn a profit. The role of how the bad bank plays is unimportant for our purposes, so this simple reduced-form approach is sufficient.

4 Crouhy and Galai (1991) and Mullins and Pyle (1994) also use this approach to discuss capital regulation.

5 Fat tails are reflected by the customer acceptance (see Asosheha, Bagherpour, and Yahyapour, 2008). For simplicity, this particular case is ignored.

6 Chan, Chan, Jegadeesh, and Lakonishok (2006) use this assumption to examine the relationships between earnings quality and stock returns.

7 Sealey (1980) argues that the portfolio-theoretic approach has been employed in the literature to discuss related issues of earnings quality. However, this approach omits these two key aspects in our setting.
can maximize its loan quality when the objective of this positive gap is minimized. The bank’s objective can be stated as:

$$\min_{R_L} Q = (V - Z) - (VN(d_1) - Ze^{-\delta}N(d_2)) \quad (3)$$

where

$$Z = (1 + R_D)K/q - (1 + R)[K(1/q + 1) - L] - \theta(1 + R_d)L,$$

$$d_1 = \sigma^{-1}(\ln(V/Z) + \delta + \sigma^2/2), \quad d_2 = d_1 - \sigma,$$

and

$$\delta = R - R_D.$$

$Z$ is the book value of the bank’s net liabilities defined as the difference between the total promised interest payments to depositors and the amounts of repayments from the risk-free liquid assets and toxic loans bailed out by the government. $N(\cdot)$ is the cumulative density function of the standard normal distribution. $\delta$ is the spread rate between $R$ and $R_D$.

The first term on the right-hand side of equation (3) can be interpreted as the book-basis equity, while the second term can be interpreted as the market-basis equity. Specifically, the second term demonstrates that the equity holders are the residual claimants on $V$ after $Z$ has been met where $Z$ is the strike price of the call option. Equation (3) demonstrates a model of loan quality management that integrates the risk considerations and market conditions of the market-basis valuation with that of the book-basis valuation.

### 3 Solution and Comparative Static Result

Partially differentiating equation (3) with respect to $R_L$, the first-order condition is given by:\(^8\)

\[
\frac{\partial Q}{\partial R_L} = \left( \frac{\partial V}{\partial R_L} - \frac{\partial Z}{\partial R_L} - \frac{\partial V}{\partial R_L} N(d_1) + V \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial R_L} \right) \\
- \frac{\partial Z}{\partial R_L} e^{-\delta}N(d_2) - Ze^{-\delta} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial R_L} = 0 \quad (4)
\]

where

\[
\frac{\partial V}{\partial R_L} = (1 - \theta)[L + (1 + R_L) \frac{\partial L}{\partial R_L}] < 0, \\
\frac{\partial Z}{\partial R_L} = \frac{(R - R_D)Kq'}{q^2} \frac{\partial L}{\partial R_L} + (1 + R) \frac{\partial L}{\partial R_L} - \theta[L + (1 + R_L) \frac{\partial L}{\partial R_L}] < 0,
\]

\[
V \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial R_L} = Ze^{-\delta} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial R_L}, \quad \text{and}
\]

\[
\frac{\partial d_1}{\partial R_L} = \frac{\partial d_2}{\partial R_L}.
\]

We require that the second-order condition $\partial^2 Q/\partial R_L^2 > 0$ be satisfied. Equation (4) can be rewritten as:

\[
\frac{\partial V}{\partial R_L} - \frac{\partial Z}{\partial R_L} = \frac{\partial V}{\partial R_L} N(d_1) - \frac{\partial Z}{\partial R_L} e^{-\delta}N(d_2) \quad (5)
\]

where

\[
\frac{\partial V}{\partial R_L} - \frac{\partial Z}{\partial R_L} = \frac{(R - R_D)Kq'}{q^2} \frac{\partial L}{\partial R_L} + (1 + R) \frac{\partial L}{\partial R_L} - L(1 + \frac{1}{\eta}), \quad \text{and}
\]

\[
\eta = \frac{L}{(1 + R_L)} \frac{\partial (1 + R_L)}{\partial L}.
\]

$\eta$ is the interest rate elasticity of loan demand.

The equilibrium condition demonstrates that the book-basis difference between the marginal loan repayments and the marginal net-obligation payments equals the market-basis difference.
between the risk-adjusted value of the marginal loan repayments and the risk-adjusted value of the net-obligation payments. We note that both the differences are positive when loan demand faced by the bank is very elastic. This note implies that the bank sets its optimal loan rate, \( R^L \), at the point where the risk-adjusted value of marginal loan repayments exceeds the risk-adjusted value of marginal net-obligation payments.

It is of interest to compare the optimal loan rate (the optimal bank interest margin) under loan quality maximization with that under equity return maximization when the bank’s carrying value of equity return is above its market value. In the equity return maximization case, the first-order condition (5) becomes

\[
\frac{\partial V}{\partial R_L} N(d_1) - \frac{\partial Z}{\partial R_L} e^{-\delta} N(d_2) = 0 \quad (6)
\]

Condition (6) implies that the bank sets its optimal loan rate, \( R^L \), at the point where the risk-adjusted value of the marginal loan repayments equals the risk-adjusted value of the marginal net-obligation payments. Comparing conditions (5) and (6) and using the second-order condition of the equity return maximization case, we have \( R^S > R^Q \). The optimal bank interest margin is larger when the bank’s objective is equity return maximization than when the bank’s objective is loan quality maximization. This result is intuitive because an opportunity cost of decreasing bank interest margin arises when the choice of a loan quality goal in modeling the bank’s optimization problem is made. Our paper supports Santomero’s (1984) argument that the choice of an appropriate goal in modeling the bank’s optimization problem remains a controversial issue.

Having examined the solution to the bank’s optimization problem, we consider the effect on the optimal loan rate (and thus the optimal bank interest margin) from changes in \( \theta \) of the mode. Implicit differentiation of equation (4) with respect to \( \theta \) yields:

\[
\frac{\partial R^O}{\partial \theta} = -\frac{\partial^2 Q}{\partial R_L \partial \theta} \left| \frac{\partial^2 Q}{\partial R^L} \right| (7)
\]

where

\[
\frac{\partial^2 Q}{\partial R_L \partial \theta} = (L + (1 + R_L) \frac{\partial L}{\partial R_L})(N(d_1) + e^{-\delta} N(d_2))
\]

\[-(\frac{\partial V}{\partial R_L} \frac{\partial N(d_1)}{\partial d_1} - \frac{\partial Z}{\partial R_L} e^{-\delta} \frac{\partial N(d_2)}{\partial d_2}) \frac{\partial d_1}{\partial \theta} \text{ and } \frac{\partial d_1}{\partial \theta} = -\frac{(1+R_L)L}{\sigma} \left( \frac{1}{V} - \frac{1}{Z} \right) < 0.
\]

The first term of \( \partial^2 Q / \partial R_L \partial \theta \) represents the mean profit effect on \( \partial Q / \partial R_L \) from a change in \( \theta \). This term is negative in sign. If loan demand is relatively rate-elastic, a larger negative mean profit effect is possible at an increased government bailout. The second term represents the variance effect on \( \partial Q / \partial R_L \) from a change in \( \theta \). The sign of the second term is governed by the difference between \((\partial V / \partial R_L)(\partial N(d_1) / \partial d_1)\) and \((\partial Z / \partial R_L)e^{-\delta}((\partial N(d_2) / \partial d_2))\), which reflects the bank’s underlying preference in the call option-pricing valuation. A conventional explanation of the positive difference indicates that the bank has an
increasing risk magnitude for \( Q \) according to the first-order condition in equation (5). In this case, we say that the bank is operating liquidity management under less risk. Under the circumstances, the variance effect is positive. This in turn indicates \( \partial^2 Q / \partial R_L \partial \theta < 0 \) and then \( \partial R_L / \partial \theta > 0 \).

Intuitively, as the bank’s toxic loans are increasingly bailed out by the government, the bank must now provide a return to a higher level of loan quality. One way the bank may attempt to augment its total returns is by shifting its investments to the liquid assets and away from its loan portfolio. If loan demand is relatively rate-elastic, a less loan portfolio is possible at an increased margin. We note that an increase in the government’s help decreases the bank’s unsold risky-loan amount and has an indeterminate effect on the bank’s toxic-loan amount sold to the government. “To date [March 28th, 2009], just under half of the TARP’s $700 billion has been disbursed.” (Economist, 2009b) The results of equation (7) provide an alternative explanation for this observation and indicate that setting up the TARP is a good move for loan quality since the risky loans held by the bank decreases.

4 Conclusion

We develop an option-pricing model to determine the bank’s optimal interest margin when its carrying value of loan books is above its market price. Within the setting, we argue that an increase in the toxic loans bailed out by the government, exactly what the TARP is meant to target, increases the bank’s optimal interest margin (and thus the bank’s loan quality). The effectiveness of the TARP is largely explained by our results. Of course, it may be recognized that banks may be reluctant to sell their loans and their most toxic securities. The possible biggest challenges would include how to price the assets and whether the bad bank would buy from any institution or just a select few. There is also an open question: who would run the bank? The possible options range from the Federal Deposit Insurance Corporation to privates contractors. Accordingly, the effectiveness of the rescue plan may be re-evaluated. Such concerns are beyond the scope of this paper and so not addressed here. What this paper does demonstrate, however, is that banks may take advantage of the rescue plan to increase their margins and asset quality.

Appendix

First, the sign of the term \( \partial V / \partial R_L \) is governed by the following term:

\[
L + (1 + R_L) \frac{\partial L}{\partial R_L} = L \left[ 1 + \frac{(1 + R_L)}{L} \frac{\partial L}{\partial R_L} \right] = L \left[ 1 + \frac{(1 + R_L)}{L} \frac{\partial L}{\partial (1 + R_L)} \right] = L \left( 1 + \frac{1}{\eta} \right) \quad \text{(A1)}
\]

We note that \( \eta \) is the interest rate elasticity of loan demand. There is \( \eta < -1 \) since loan demand faced by the bank is \( L(R_L) \) where \( \partial L / \partial R_L < 0 \). Thus, we have \( (1 + 1/\eta) < 0 \) and hence \( \partial V / \partial R_L < 0 \). As pointed out by Wong (1997), the one-plus interest rate elasticity of loan demand can be treated as the interest rate elasticity of loan demand. Based on the first-order condition,
the term $\partial Z / \partial R_L$ is negative in sign.

Second, a problem in applying equation (3) is in calculating the cumulative normal distribution function $N(\cdot)$. In equation (3), there are

$$d_1 = \frac{1}{\sigma} (\ln \frac{V}{Z} + \delta + \frac{1}{2} \sigma^2) \quad (A2)$$

$$d_2 = d_1 - \sigma \quad (A3)$$

$$d_2^2 = d_1^2 + \sigma^2 - 2d_1\sigma = d_1^2 + \sigma^2 - 2(\ln V/Z + \frac{1}{2} \sigma^2)$$

$$= d_1^2 - 2(\ln V/Z + \delta) \quad (A4)$$

Following Hull (1993), $N(d_2)$ can be evaluated directly using numerical procedures. One such approximation is

$$N(d_2) = 1 - (a_1 + a_2 k^2 + ak^3) \frac{\partial N(d_2)}{\partial d_2} \quad (A5)$$

where

$$k = (1 + ad_2)^{-1}, \quad \alpha = 0.33267, \quad a_1 = 0.436183,$$

$$a_2 = -0.1201676, \quad a_3 = 0.9372980, \text{ and}$$

$$\frac{\partial N(d_2)}{\partial d_2} = (\sqrt{2\pi})^{-1} e^{(-d_2^2/2)} > 0.$$  

In our model, we rewrite the term $\partial N(d_2) / \partial d_2$ as follows.

$$\frac{\partial N(d_2)}{\partial d_2} = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}(d_1^2 - 2(\ln V/Z + \delta))} = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}(d_1^2 - 2(\ln V/Z + \delta))}$$

$$= \frac{\partial N(d_1)}{\partial d_1} V e^\delta \quad (A6)$$

Further,

$$V \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial R_L} - Z e^{-\delta} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial R_L}$$

$$= (V \frac{\partial N(d_1)}{\partial d_1} - Z e^{-\delta} \frac{\partial N(d_2)}{\partial d_2}) \frac{\partial d_1}{\partial R_L}$$

$$= (V \frac{\partial N(d_1)}{\partial d_1} - Z e^{-\delta} \frac{\partial N(d_2)}{\partial d_2} V e^\delta) \frac{\partial d_1}{\partial R_L} = 0 \quad (A7)$$

Accordingly, we have the result of equation (A7), expressing $V(\partial N(d_1) / \partial d_1)(\partial d_1 / \partial R_L) = Z e^{-\delta}(\partial N(d_2) / \partial d_2)(\partial d_2 / \partial R_L)$ in equation (4).

References:


