On the Margin Effects of Commercial Bank Expansion into Securities and Insurance Activities under the Same Roof:
A Mathematical Swap Approach

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Abstract: The Gramm-Leach-Bliley Act (GLBA) of 1999 allows commercial bank expansion into investment banking of securities and insurance activities without limit in subsidiaries separate from commercial banks. This paper demonstrates how securities hedging, insurance underwriting, and capital regulation through the total return swap approach jointly determine the optimal bank interest margin under the same roof. We find that the bank’s interest margin and noninterest income are positively related to the bank’s securities hedging demand, and to the insurance underwriting provision, but negatively related to the bank capital requirement. We also find that the results of the bank’s noninterest income follow a similar argument as in the case of a change in interest margin. The results show that the combined production of commercial banking expansion into investment banking enhances the bank’s synergistic gains. In addition, if regulators reduce capital charges, a commercial bank will have a strong incentive to expand its investment banking activities. This suggests obvious diversification benefits from the return to investment banking under the same roof.
1 Introduction

U.S. Congress passed the Gramm-Leach-Bliley Act (GLBA) of 1999, also known as the Financial Services Modernization Act. This Act allowed bank holding companies to convert to financial holding companies and conduct investment banking of securities and insurance activities without limit in subsidiaries separate from their commercial banks (Geyfman and Yeager, 2009). After the passage of this Act of 1999, a major strategic shift was the move to create more diversified commercial banks that could reap cross-selling and diversification gains in a relatively deregulated environment. Mamun, Hassan, and Lai (2004), and Mamun, Hassan, and Maroney (2005) examine the impact of the GLBA on the banking industry and find that the industry has a welfare gain from this Act. Paradoxically, Hirtle and Stiroh (2007) argue that “The focus on the diversified model, however, was short-lived”. One observer (Business Week, 2005) also concludes that “the initial hope of many financial companies [commercial banks] that welding brokerage, insurance, and retail banking businesses would create sales synergies just didn’t pan out”. In response, U.S. banks, particularly the largest, have dramatically expanded their retail banking operations over the last few years (Hirtle and Stiroh, 2007).

A great deal of analysis has been devoted to understanding the circumstances under which each of commercial and investment banking activities might require the services of Section 20 subsidiaries within a bank holding company. Under this view, one would naturally tend to be sympathetic to “narrow banking” proposals, which call for the breaking up of a bank holding company into separate commercial and investment banking operations that would resemble commercial banks and finance companies, respectively. While much has been learned from this work, it has not addressed a fundamental question: why is it important that one commercial bank carry out investment banking functions under the same roof? For example, Allen and Jagtiani (2000) provide a convincing argument that commercial bank expansion into securities and insurance activities should be made by intermediaries, but it is hard to see in their study why the intermediary cannot be a commercial bank or a financial holding company (we use the two terms interchangeably in what follows), rather than a bank holding company.

In this paper, we argue that there may indeed be significant synergies between commercial banking and investment banking within a commercial bank. We focus on credit derivatives that, in our view, are important in distinguishing financial holding companies from banking holding companies: total return swaps. We take the central feature of total return swaps to be that the commercial bank has the option to directly trade the contract of its some risky loan repayments with counterparties without involving an intermediary. This security hedging option can help the bank manage the credit risk of its loan investment by insuring against adverse movements in the credit quality of its borrowers. The bank also has the
option to directly sell the contract of its some risk-free liquid-asset repayments to counterparties. This insurance provision option is a logical extension of the bank’s business. The bank earns a fee for selling such the insurance option. Simply put, once the decisions to extend total return swap transactions have been made, the combined production of commercial bank expansion into securities hedging and insurance providing activities is operated under the same roof.

Another distinguishing feature is capital requirements for credit derivative transactions. Suppose a bank uses a swap option to conduct a long-term hedge for credit risk of a large borrower in the narrowing banking operation. The swap reduces the risk of the bank, but under the risk-based capital standards such as the Basel Accord II, there is no recognition of the lower risk. Not only is there no reduction in the bank’s capital requirement for the risky loans, but the bank must set aside additional capital to insure against counterparty default (Neal, 1996). To the extent that there is a real synergy, the conduct of capital regulation could be noticeably altered.

The purpose of this paper is to develop a model of synthetic commercial bank behavior to examine the impact of securities hedging, insurance providing activities, and capital regulation on bank interest margins under the same roof. Our primary emphasis is the selection of the bank’s optimal interest margin, which is the difference between the rate of interest rate the bank charges borrowers and the rate the bank pays to depositors. Wong (1997) argues that earnings from the margin significantly account for bank profits. As the margin is so important to bank profitability, the issues of how it is optimally determined and how it adjusts to changes in the banking environment deserve closer scrutiny.

To do this, our model will have to incorporate two distinct banking functions in particular: commercial banking of deposit-taking and lending and investment banking of securities hedging and insurance providing. In theory, the combined production of commercial banking and investment banking could enhance or hurt a commercial bank’s returns. Commercial banks potentially achieve revenue and cost economies by providing both services. In practice, interest margin management of commercial banking is done through a “cost of goods sold” approach in which deposits are the “material” and loans are the “work in process” (see Finn and Frederick, 1992). Noninterest income management of investment banking in our model is possibly operated through a “swap transaction” approach in which credit derivatives are financial contracts that “selling” or “buying” assets with credit risk (see Neal, 1996).

In this paper, we find that an increase in the amount of the buying security hedging option or the selling insurance option to replace the swap transaction (i) decreases the loan amount at an increased margin, and (ii) increases the bank’s noninterest income from the swap transaction. In addition, an increase in bank capital requirement (i) decreases the bank’s margin, and (ii) decreases the benefit from the swap transactions. This paper suggests that the combined production of commercial banking and investment banking enhances a commercial bank’s returns through bank interest margin management. Further, if the regulatory authority allows prudent structured swap transactions to reduce bank capital requirements, this will provide a strong incentive for the bank to
adopt such options. Our bottom line conclusion is that decreasing a risk-based system of capital standards in the GLBA environment leads to the “return” to investment banking from commercial banking.

This paper is organized as follows. In section 2, we present the studies that form the background to our paper. In section 3, the framework and assumptions for the model are presented. The model is developed in section 4. Section 5 and 6 derive the solutions of the model and comparative static analysis, respectively. The final section discusses the results and implications of the model.

2 Background

The following sketch is somewhat selectively called from existing literature and is intended to provide motivation for our paper. Our theory of commercial banking firm is related to three strands of the literature.

The first is the literature on the optimal bank interest margin determination. McShane and Sharpe (1985), and Allen (1988) have provided models of bank interest margins based on the bid-ask spread model of Stoll (1978). Zarruk and Madura (1992) develop a model with random loan defaults in which changes in capital regulation and deposit insurance premiums have direct effects on the bank’s interest margin. Wong (1997) develops a model to demonstrate how cost, regulation, credit risk, and interest rate risk conditions jointly determine the optimal bank interest margin decision. While we also examine bank interest margin determination, our focus on the synthetic management of commercial bank expansion into securities hedging and insurance providing activities under the GLBA takes our analysis in a different direction.

The second strand is the impact of the GLBA on synthetic commercial banks. Some (e.g., Hogan, 2001) argue that the impact will be phenomenal; others (e.g., Barth, Brumbaugh, and Wilcox, 2000) argue it will only be marginal. These arguments are further extended to account for the most likely effects of the GLBA on bank stock price (Mamun, Hassan, and Maroney, 2005), on bank risk (Allen and Jagtiani, 2000), and on bank wealth (Mamun, Hassan, and Lai, 2004). The primary difference between our model and these studies is that we consider the impact of swap trade (between securities hedging and insurance providing contracts and earning-asset portfolio) and capital regulation on bank interest margin decision.

The third strand is the literature on synthetic profitability concerns. Wall and Eisenbeis (1984) find that during 1970s, the correlation between bank earnings and security broker/dealer earnings was negative, indicating potential gains from diversification. Allen and Jagtiani (2000) find that diversification benefits are not sufficiently large to justify expanding bank powers into securities activities. Rime and Stiroh (2003) conclude that banks do not appear to benefit from broader product mixes. This paper is in sharp contrast to the previous literature that commercial banks may shift their focus toward more banking activities like securities hedging and insurance providing ones and away from traditional commercial banking activities like deposit-taking and lending ones. This paper examines the links between both functions for commercial bank expansion under the same roof to better understand the drivers and the impact of the renewed focus on investment banking.

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3 The Framework and Assumptions

3.1 Framework

In order to get closed-form, tractable solutions, a framework and a few simplifying assumptions are made. We assume that all financial decisions are made and values are determined with a one-period horizon only, \( t \in [0,1] \). The model is designed to capture in a minimalist fashion in the following characteristics of a commercial bank.

The bank with the GLBA’s permission potentially achieves synergistic gains and risk diversification by providing the combined production of commercial banking, securities, and insurance activities under the same roof. With this permission, the bank can conduct securities and insurance activities without limit in subsidiaries separate from itself. In commercial banking activities, our primary emphasis is the selection of optimal interest margin related to deposit-taking and lending that generate interest income. In securities and insurance activities, our focus is the assessment of net diversification benefits, in particular, related to hedging cost and trading revenue that generate net noninterest income.

For a hedging purpose related to securities activities, the bank directly uses a total return swap to mitigate credit risk. In this type of transaction, the bank sends some of its loan portfolio to counterparty bank \( X \). But the bank finds it costly to conduct this swap transaction. Because the return is guaranteed, the bank has eliminated the credit risk on the swapped loan portfolio. For a revenue purpose related to insurance activities, the bank also uses a total return swap to manage credit risk that provides insurance to counterparty bank \( Y \). In this type of transaction, the bank sends some of default-free assets (for example, Treasury bills) to counterparty \( Y \). The bank anticipates some revenues from this swap transaction since the bank has taken the credit-related losses from the counterparty’s swapped risky assets.\(^1\)

Both hedging and revenue purposes imply that our model will have to incorporate two distinct cash flows. There needs to be a hedging cost of reducing the bank’s credit risk through diversification, as well as a potential benefit from insuring counterparty against credit-related losses. Total return swaps are appealing to commercial banks whose loan portfolios are concentrated in particular industries or geographic areas (see Neal, 1996). If there is little common movement in default rates in trading activities, counterparties are better off. Furthermore, this framework is a model of commercial bank behavior that integrates the risk considerations and total return swap transactions of the portfolio-theoretic approach with the market conditions and interest margin determination of the firm-theoretic approach.

3.2 Assets

The bank makes term loans \( L \) at \( t = 0 \) which mature and are paid off at \( t = 1 \). The one-period interest rate on these loans is \( R_L \). We assume that the

\(^1\) Note that securities hedging and insurance underwriting in our model are one type of the combined production of commercial bank expansion into securities and insurance activities. Other possible expansions include securities and insurance underwriting related to synergistic gains (e.g., Allen and Jagtiani, 2000), securities and insurance hedging related to risk diversification (e.g., Neal, 1996), and securities underwriting and insurance hedging related to synergistic gains and risk diversification. Without losing generality, our framework can be applicable to the three relevant cases.
bank has some market power in lending which implies \( \frac{\partial L(R_s)}{\partial R_s} < 0 \). This would be characteristic of imperfect loan markets in which financial intermediaries have a reason to exist. Empirical evidence by Slovin and Sushka (1984) and Hancock (1986) supports the presence of rate-setting behavior in loan markets. Further, the assumption of market power is only to limit the scale of lending activities, and an assumption about increasing costs of making loans would reach the same end. The details of what drive loan demand are unimportant for our purposes, so this abstraction is sufficient. In addition to term loans, the bank can also hold an amount \( B \) of risk-free liquid assets, for example, Treasury bills, on its balance sheet at \( t = 0 \). These assets earn the security-market interest rate of \( R \).

### 3.3 Liabilities

The total assets to be financed at \( t = 0 \) are \( L + B \). They are financed partly by demandable deposits \( D \). The bank is fully insured by the Federal Deposit Insurance Corporation (FDIC) and it is assumed that the bank pays a zero deposit insurance premium, for the sake of simplicity. The bank provides depositors with a rate of return equal to the risk-free rate \( R_D \). In addition to deposits, the bank can also issue claims in the public market at \( t = 0 \), denoted by \( K \). These claims mature at \( t = 1 \), and can be thought of equity capital. Equity capital held by the bank at \( t = 0 \) is tied by regulation to be a fixed proportion \( q \) of the bank’s deposits, \( K \geq qD \). The required capital-to-deposits ratio \( q \) is assumed to be an increasing function of \( L \) held by the bank at \( t = 0 \), \( \frac{\partial q}{\partial L} = q' > 0 \) (see Zarruk and Madura, 1992). This system is designed to force bank’s capital positions to reflect their asset portfolio risks.

### 3.4 Total return swap transactions

Potential counterparties can attempt to come to some agreement about the degree of default risk and use that knowledge to price their positions. The most common credit swap is called a total return swap (Neal, 1996). To do this, we take three parties: the bank, counterparty \( X \), and counterparty \( Y \) (see Fig. 1). For the sake of simplicity, our model ignores the transaction costs incurred in the hedging and underwriting operations since they are operated under the same roof of the bank. For a security hedging purpose, the amount of the option for the bank to replace the swap is \( \alpha(1 + R_s)L \), where \( 0 < \alpha < 1 \). The option to replace the swap from counterparty \( X \)’s point of view is \( \alpha(1 + R_s - R_b)L \), where \( \alpha R_sL \) is the bank’s hedging cost and is the counterparty \( X \)’s expected revenue based on the agreement about the degree of default risk in the contract of \( \alpha(1 + R_s)L \). For an insurance revenue purpose, the amount of the option for the bank to replace the swap is \( \beta(1 + R)B \), where \( 0 < \beta < 1 \). The option to replace the swap from counterparty \( Y \)’s point of view is \( \beta(1 + R + R_b)B \), where \( \beta R_bB \) is the bank’s expected revenue and is the counterparty \( Y \)’s hedging cost of the contract of \( \beta(1 + R)B \). Note that the yield curve is flat since the option relaxed (the original) fixed coupon is assumed to be equal to the option pay (the original) fixed coupon. The direction of the adjustment depends on \( R_s \) or \( R_b \) in our model.

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2 Results to be derived from our model do not extend to the case where the bank is a price taker in the loan market (see Baltensperger, 1980 and Wong, 1997).
4 Model Setup

4.1 The bank’s objective

With all the assumptions in place, we are now ready to set up the bank’s objective. The bank seeks to maximize its expected equity value, denominated in \( t = 1 \) dollars:

\[
S = \text{Max} \left[ 0, (1 - \alpha)(1 + R_c)L + \alpha(1 + R_c - R_s)L + (1 - \beta)(1 + R)B + \beta(1 + R + R_d)B - (1 + R_d)D \right] \quad (1)
\]

In equation (1), we can demonstrate that the bank’s interest income is the revenue from loan and liquid assets net of the cost of deposits denoted by \((1 + R_c)L + (1 + R)B - (1 + R_d)D\), and its noninterest income is the revenue from insurance underwriting net of the cost of security hedging specified by \(\beta R_d B - \alpha R_s L\). In so doing, it faces the following balance sheet constraint:

\[
L + B = D + K = K(1/q + 1) \quad (2)
\]

Constraint (2) is simply a balance sheet identity illustrating the bank’s liquidity management since the total assets in the left-hand side are financed by deposits and equity capital in the right-hand side. Note that the total liabilities in the identity also imply a risk-based system of capital standards.

As noted by Santomero (1984), the choice of an appropriated goal in modeling the bank’s optimization problem remains a controversial issue. Much of the literature follows Black and Scholes (1973) and Merton (1974) by viewing the market value of firm equity as the standard call option on the underlying assets with exercise price equal to the promised payment of liabilities. In other words, the value of equity has the features of a contingent claim written on the value of the firm’s assets. Setting the objective of our model, we apply Black and Scholes’ (1973) option-based framework and further set the bank’s optimal loan rate (and thus the optimal margin). Based on equations (1) and (2) with this approach, we specify the underlying risky assets \(V\) and the promised payment of net obligations \(Z\) as follows:

\[
V = (1 - \alpha)(1 + R_c)L + \beta(1 + R + R_d)[K(1/q + 1) - L] \quad (3)
\]

---

3 The administrative costs and the fixed costs in the bank’s objective function are omitted for simplicity.

4 Lin, Lin, and Jon (2009a, b) and Lin Chang, and Jou (2010) also provide option-based models to explain bank margin (spread) behavior.
The first term on the right-hand side of equation (3) is the amount of loan repayment left without carrying out securities hedging operations, while the second term is the amount provided by the option for counterparty Y to replace the insurance providing operations. The first term on the right-hand side of equation (4) is the amount of deposit payment, the second term is the amount provided by the option for counterparty X to replace the securities hedging operations, and the third term is the amount of liquid-asset repayment left without carrying out insurance providing operations.

This option-based approach describes $V$ in terms of a stochastic differential equation, which follows a geometric Brownian motion. Specifically, the nonlinear dynamics of the bank’s asset value $V$ and its strike price $Z$ follow:

$$
\begin{align*}
    dV &= \mu V dt + \sigma dW \\
    dZ &= \delta Z dt
\end{align*}
$$

where $\mu$ and $\sigma$ are, respectively, the expected return and volatility of $V$, $W$ is a Wiener process, and $\delta$ is the spread rate, the difference between $R$ and $R_D$.

Given the conditions of vector (5), the market value of the bank’s equity with the call option pricing when default only occurs at maturity can be written as $\hat{S} = \hat{S}(\text{Max}[0, V - Z])$. Based on the risk-neutral valuation argument, the call option pricing is the value of this discounted rate of $\delta$, that is, $S = e^{-\delta} \hat{S}$, where $\ln V \sim \phi()$, denoting a normal distribution with mean $\ln V + \mu - \sigma^2/2$ and standard deviation $\sigma$. With this approach, the market value of equity is given by:

$$
\text{Max} S = VN(d_1) - Ze^{-\delta} N(d_2)
$$

where

$$
\begin{align*}
    d_1 &= \sigma^{-1}(\ln(V/Z) + \delta + \sigma^2/2), \\
    d_2 &= d_1 - \sigma, \\
    N() &= \text{the standard normal cumulative distribution function.}
\end{align*}
$$

The value of equity $S$ in equation (6) is expressed as the difference between the risk adjusted present value of the bank’s risky assets $VN(d_1)$ and the risk adjusted present value of the risk-default net obligation $Ze^{-\delta} N(d_2)$. The risk adjustment factors are $N(d_1)$ and $N(d_2)$, respectively. When the first term on the right-hand side of equation (6) is less than the second term, the value of $S$ is zero in the call pricing valuation.

### 4.2 The non-interest cost-benefit framework

Commercial bank expansion into securities hedging and insurance providing activities is valued by the option approach of total return swap transaction in our model. First, define $C(R_L, R, \sigma_C)$ to be the Black and Scholes’ (1973) value of the call option, written on $\beta(1 + R + R_y)B$ and with an exercise price equal to $\beta(1 + R)B$, which the bank effectively purchases from counterparty Y. $R$ and $\sigma_C$ are, respectively, the expected return and volatility of $\beta(1 + R + R_y)B$. Second, define $P(R_L, R, \sigma_p)$ to be the Black and Scholes’ (1973) value of the put option, written on $\alpha(1 + R_y)L$ and
with an exercise price equal to \( \alpha (1 + R_L - R_X) L \), which counterparty \( X \) effectively written to the bank. \( R \) and \( \sigma_p \) are, respectively, the expected return and volatility of \( \alpha (1 + R_L) L \). From the bank’s point of view, the expression for the call option is interpreted at the non-interest benefit from the insurance providing transaction, while the expression for the put option is interpreted as the non-interest cost of securities hedging transaction. The value of the cost-benefit option framework can be specified as follows:

\[
E = C(R_L, R, \sigma_c) - P(R_L, R, \sigma_p)
\]  

(7)

where

\[
C = \beta (1 + R + R_y) BN(c_i) - \beta (1 + R) Be^{-\delta} N(c_i)
= \beta (1 + R) [K (1 / q + 1) - L] [N(c_i) - e^{-\delta} N(c_i)]
+ \beta R_y [K (1 / q + 1) - L] N(c_i),
\]

\[
c_1 = \sigma_c^{-1} [\ln ((1 + R + R_y) / (1 + R)) + R + \sigma_c^2 / 2],
\]

\[
c_2 = c_1 - \sigma_c,
\]

\[
P = \alpha (1 + R_L - R_X) Le^{-\delta} N(-p_2) - \alpha (1 + R_L) LN(-p_1)
- N(-p_1) - aR_L Le^{-\delta} N(-p_2)
= \alpha (1 + R_L) L (e^{-\delta} N(-p_2),
\]

\[
p_1 = \sigma_p^{-1} [\ln ((1 + R_L) / (1 + R_L - R_X)) + R + \sigma_p^2 / 2],
\]

and \( p_2 = p_1 - \sigma_p \).

In equation (7), the first term associated with \( [N(c_i) - e^{-\delta} N(c_i)] \) of \( C(R_L, R, \sigma_c) \) is the risk-adjusted yield difference between the two parties’ option costs, and the second term associated with \( N(c_i) \) is the bank’s risk-adjusted premium paid by counterparty \( Y \) in the total return swap transaction. In addition, the first term associated with \( [e^{-\delta} N(-p_2) - N(-p_1)] \) of \( P(R_L, R, \sigma_p) \) is the risk-adjusted yield difference between the two parties’ option costs, and the second term associated with \( N(-p_1) \) is the bank’s risk-adjusted hedging cost paid to counterparty \( X \).

### 5 Solutions

With the framework and all the assumption in place, we are now ready to solve for the bank’s optimal choice of \( R_L \). Partially differentiating equation (6) with respect to \( R_L \), the first-order condition is given by:

\[
\frac{\partial S}{\partial R_L} = \frac{\partial V}{\partial R_L} N(d_1) + V \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial R_L} - \frac{\partial Z}{\partial R_L} e^{-\delta} N(d_2) = 0
\]  

(8)

Proof 1: \( V \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial R_L} = Ze^{-\delta} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial R_L} \)

A problem in applying objective (6) is in calculating the cumulative normal distribution \( N(\cdot) \). \( d_2 \) in equation (6) can be rewritten as:

\[
d_2^* = (d_1 - \sigma)^2 = d_1^2 - 2 (\ln (V / Z) + \delta).
\]

We follow Abramowitz and Stegun (1972), and Hull (1993) and use the numerical procedures to directly calculate \( N(d_2) \). One such approximation is

\[
N(d_2) = \frac{1}{\sqrt{2 \pi}} e^{-d_2^2 / 2} \sum_{i=1}^{\infty} a_i m_i \quad \forall \ d_2 \geq 0
\]

where

\[
m_1 = 1 / (1 + 0.2316419d_2), \quad a_1 = 0.319381530,
\]

\[
a_2 = -0.356563782, \quad a_3 = 1.781477937,
\]

\[
a_4 = -1.821255978, \quad a_5 = 1.330274429, \quad \text{and}
\]

\[
\frac{\partial N(d_2)}{\partial d_2} = \frac{1}{\sqrt{2 \pi}} e^{-d_2^2 / 2} = \frac{1}{\sqrt{2 \pi}} e^{-d_2^2 - 2 \ln (V / Z) - \delta}.
\]
\[ \frac{\partial N(d_z)}{\partial d_z} V = Ze^x > 0. \]

Further, \( \frac{\partial d_1}{\partial R_L} \neq \frac{\partial d_1}{\partial R_L} \neq 0. \)

Thus, \( V \frac{\partial N(d_z)}{\partial d_1} \frac{\partial d_1}{\partial R_L} = Ze^{-x} \frac{\partial N(d_z)}{\partial d_2} \frac{\partial d_2}{\partial R_L} \)

Q.E.D.

According to the proof above, we can have the simplified form of the first-order condition as follows:

\[ \frac{\partial S}{\partial R_L} = \frac{\partial V}{\partial R_L} N(d_1) - \frac{\partial Z}{\partial R_L} e^{-x} N(d_2) = 0 \quad (9) \]

where

\[ \frac{\partial V}{\partial R_L} = (1 - \alpha)(L + (1 + R_L) \frac{\partial L}{\partial R_L}) \]

\[ - \beta(1 + R + R_y)(\frac{Kq'}{q^2} + 1) \frac{\partial L}{\partial R_L}, \quad \text{and} \]

\[ \frac{\partial Z}{\partial R_L} = -(1 + R_y)Kq' \frac{\partial L}{\partial R_L} - \alpha[L + (1 + R_L - R_y) \frac{\partial L}{\partial R_L}] + (1 - \beta)(1 + R)(\frac{Kq'}{q^2} + 1) \frac{\partial L}{\partial R_L}. \]

The second-order condition for a maximum of objective (6) is specified as \( \frac{\partial^2 S}{\partial R_L^2} < 0. \) The first term on the right-hand side of \( \frac{\partial V}{\partial R_L} \) can be interpreted as the direct effect on \( V \) from changes in \( R_L \), while the second term can be interpreted as the indirect effect. The sign of the first term is governed by the interest rate elasticity of loan demand. The bank operates on the elastic portion of its loan demand curve, just as a monopolistic firm does. Thus, the direct effect is negative in sign. In addition, the indirect effect represents the reallocation effect on \( V \) from changes in \( R_L \) between \( (1 - \alpha)(1 + R_L)L \) and \( \beta(1 + R + R_y)B \). This reallocation effect is unambiguously negative in sign in our model. Since the indirect effect is in general insufficient to offset the direct effect, the sign of \( \frac{\partial V}{\partial R_L} \) is negative.

Given the condition of the first-order condition in equation (9), the term \( \frac{\partial Z}{\partial R_L} \) is negative in sign. Equation (9) then implies that the bank sets the optimal loan rate where the marginal risk-adjusted repayment from the asset portfolio equals the marginal risk-adjusted net-obligation payment. We can further substitute the optimal loan rate to obtain the equity maximization in equation (6). We substitute the optimal loan rate derived from equation (9) to obtain the noninterest income from equation (7) staying on the maximization optimization.

### 6 Comparative Static Results

Having examined the solutions to the bank’s optimization problem, in this section we consider the effect on the optimal loan rate (and thus on the interest income) and the noninterest income from changes in the parameters of the model.

#### 6.1 Impact on \( R_L \) from increasing \( \alpha \)

Implicit differentiation of equation (9) with respect to \( \alpha \) yields:

\[ \frac{\partial R_L}{\partial \alpha} = -\frac{\partial^2 S}{\partial R_L \partial \alpha} \frac{\partial^2 S}{\partial R_L^2} \quad (10) \]

where

\[ \frac{\partial^2 S}{\partial R_L \partial \alpha} = \frac{\partial^2 V}{\partial R_L \partial \alpha} N(d_1) \frac{\partial N(d_1)}{\partial d_1} e^{-x} N(d_2) \]

\[ + \frac{\partial V}{\partial R_L} \frac{\partial N(d_1)}{\partial d_1} \left(1 - \frac{VN(d_1)}{Ze^{-x} N(d_2)} \right) \frac{\partial d_1}{\partial \alpha}. \]
\[ \frac{\partial^2 V}{\partial R_i \partial \alpha} = [L + (1 + R_i) \frac{\partial L}{\partial R_i}] > 0, \]
\[ \frac{\partial^2 Z}{\partial R_i \partial \alpha} = [L + (1 + R_i - R_s) \frac{\partial L}{\partial R_i}] > 0, \]
\[ \frac{\partial^2 V}{\partial R_i \partial \alpha} > \frac{\partial^2 Z}{\partial R_i \partial \alpha} > 0, \]
\[ \frac{\partial d_i}{\partial \alpha} = \frac{\alpha \frac{\partial V}{\partial \alpha} - \alpha \frac{\partial Z}{\partial \alpha}}{\sigma \frac{\partial V}{\partial \alpha} - \sigma \frac{\partial Z}{\partial \alpha}} = \frac{\partial d_i}{\partial \alpha} > 0, \]
\[ \frac{\partial V}{\partial \alpha} = -(1 + R_i)L < 0, \]
\[ \frac{\partial Z}{\partial \alpha} = -(1 + R_i - R_s)L < 0, \text{ and} \]
\[ \frac{\partial V}{\partial \alpha} < \frac{\partial Z}{\partial \alpha} < 0. \]

The sign of equation (10) is governed by its numerator since \( \frac{\partial^2 S}{\partial R_i^2} < 0 \). The first term \([\cdot]\) on the right-hand side of \( \frac{\partial^2 S}{\partial R_i \partial \alpha} \) can be explained as the mean profit effect on \( \partial S/\partial R_i \) from a change in \( \alpha \), while the second term can be explained as the variance or “risk” effect. The mean profit effect capture the change in the optimal loan rate due to an increase in \( \alpha \), holding the risk effect constant. The sign of this mean profit effect is determined by how changes in \( \alpha \) affect the bank’s marginal risk-adjusted risky-asset repayments of \( R_i \), and its marginal risk-adjusted net-obligation payments of \( R_i \). The term \( L + (1 + R_i) \frac{\partial L}{\partial R_i} \) can be defined as the reciprocal of the interest rate elasticity of loan demand evaluated at the optimal loan rate. Changes in \( \alpha \) influence the bank’s lending activities by buying a total return swap contract that is used to manage the credit risk of its loan investments by insuring against adverse movements in the credit quality of the borrowers. In our model, these changes are directly related to risky-asset portfolio, but indirectly related to net obligation in the securities hedging transaction. Thus, the mean profit effect is positive in sign since \( \frac{\partial^2 V}{\partial R_i \partial \alpha} > \frac{\partial^2 Z}{\partial R_i \partial \alpha} > 0 \) in our model.

The sign of the risk effect depends on the term \( \partial d_i / \partial \alpha \), which is the difference between the reciprocal of \( \alpha \) elasticity of risky-asset repayments evaluated at the optimal loan rate and the reciprocal of \( \alpha \) elasticity of net-obligation payments. The former is expressed as \( (\alpha / V)(\partial V / \partial \alpha) \) while the latter is expressed as \( (\alpha / Z)(\partial Z / \partial \alpha) \). Both the elasticities are negative in sign. In terms of both the elasticities rather than both the reciprocal elasticities, \( \alpha \) elasticity of risk-asset repayments is in general insufficient to be offset by \( \alpha \) elasticity of net-obligation payments. Thus, we have \( \partial d_i / \partial \alpha > 0 \). Since the positive risk effect reinforces the positive mean profit effect to give an overall response of \( R_i \) to an increase in \( \alpha \), we establish the follow proposition.

**Proposition 1**: An increase in the commercial bank expansion into securities hedging activities increases the bank’s interest margin.

As the bank increases the securities hedging against credit-related losses by carrying out the total return swap transaction under the same roof, it must now provide a return to a larger hedging cost base. One way the bank may attempt to augment its total returns is by shifting its investments to liquid assets and away from its loan portfolio. If loan demand is relatively rate-elastic, a smaller loan portfolio is possible at an increased margin. Accordingly, greater reliance on commercial bank expansion into securities hedging services is associated with higher margin and lower risk. This suggests obvious
diversification benefits from the ongoing expansion into securities hedging; Santomero and Chung (1992) find that bank expansion into nonbanking businesses reduces risk in general. Proposition 1 is consistent with this empirical observation.

6.2 Impact on \( R_L \) from increasing \( \beta \)

Implicit differentiation of equation (9) with respect to \( \beta \) yields:

\[
\frac{\partial R_L}{\partial \beta} = -\frac{\partial^3 S}{\partial R_L \partial \beta} \left/ \frac{\partial^2 S}{\partial R_L^2} \right. \tag{11}
\]

where

\[
\frac{\partial^3 S}{\partial R_L \partial \beta} = \left[ \frac{\partial^2 V}{\partial R_L} N(d_1) - \frac{\partial^2 Z}{\partial R_L} e^{\delta d_1} \right] + \frac{\partial V}{\partial R_L} \frac{\partial N(d_1)}{\partial d_1} \left(1 - \frac{VN(d_1)}{Ze^{\delta d_1}}\right) \frac{\partial d_1}{\partial \beta},
\]

\[
\frac{\partial^2 V}{\partial R_L \partial \beta} = -(1 + R + R_g) \left(\frac{Kq}{q} + 1\right) \frac{\partial L}{\partial R_L} > 0,
\]

\[
\frac{\partial^2 Z}{\partial R_L \partial \beta} = -(1 + R) \left(\frac{Kq}{q} + 1\right) \frac{\partial L}{\partial R_L} > 0,
\]

\[
\frac{\partial^2 V}{\partial R_L \partial \beta} > \frac{\partial^2 Z}{\partial R_L \partial \beta} > 0,
\]

\[
\frac{\partial d_1}{\partial \beta} = \frac{\beta}{\sigma} \frac{\partial V}{\partial \beta} + \frac{\beta}{Z} \frac{\partial Z}{\partial \beta} = \frac{\partial d_2}{\partial \beta} > 0,
\]

\[
\frac{\partial V}{\partial \beta} = (1 + R + R_g) \left[K(1 - \frac{1}{q}) - L \right] > 0,
\]

\[
\frac{\partial Z}{\partial \beta} = (1 + R) \left[K(\frac{1}{q} - 1) - L \right] > 0, \quad \text{and}
\]

\[
\frac{\partial V}{\partial \beta} > \frac{\partial Z}{\partial \beta} > 0.
\]

The first term [·] on the right-hand side of the numerator term \( \frac{\partial^3 S}{\partial R_L \partial \beta} \) in equation (11) can be identified as the mean profit effect, while the second term can be identified as the risk effect. The mean profit effect captures the change in the optimal loan rate due to an increase in \( \beta \), holding the risk effect constant. Following a similar argument as in the case of an increase in \( \alpha \), the mean profit effect is positive because an increase in \( \beta \) makes liquid assets more profitable to invest. In response to this, the bank has an incentive to reduce the amount of loans it grants by charging a higher loan rate, ceteris paribus. The risk effect arises because a one dollar increases in \( \beta \) increases the bank’s profit by \( L(R_L) \) evaluated at the optimal loan rate in every possible state. Again, following a similar argument as in the case of an increase in \( \alpha \), the risk effect is positive.

The rationale is that as operating the total return swap transaction to bearing counterparty risk, the liquid asset is demanded increasingly. As a result, the bank raises its loan rate to cut its lending. Since the risk effect reinforces the mean profit effect to give an overall positive response of the optimal loan rate (and thus of the margin) to an increase in \( \beta \), we can establish the following proposition.

**Proposition 2**: An increase in the commercial bank expansion into insurance underwriting activities increases the bank’s interest margin.

Commercial bank expansion into insurance activities by conducting the total return swap transactions may increase the expected value and variance of the bank’s profits. The interpretation of this result follows a similar argument as in the case of a change in \( \alpha \). Basically, increases in the noninterest income from insurance business

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5 Santomero and Chung (1992) use option-pricing techniques to simulate the volatility of asset returns from combinations of 123 bank holding companies and 62 nonbank financial firms.
encourage the bank to shift investments to liquid assets such as Federal funds from its loan portfolio. In an imperfect loan market, the bank can increase the size of its margin in order to decrease the amount of loans. In the meanwhile, an increase in the noninterest income from insurance activities may increase the bank’s overall risk while enhancing its margin. Commercial bank expansion into insurance activities increases the bank’s interest and noninterest incomes at the expense of its risk concentration. We conclude that this expansion banking may be a relatively high return activity, but also a relatively unstable one. This result is consistent with the empirical findings of DeYoung and Roland (2001) that increased fee-based activities increase the volatility of bank revenue and bank earnings, and of Acharya, Hasan, and Saunders (2002) that diversification of bank assets does not typically reduce risk.\footnote{DeYoung and Roland (2001) examine the link between bank profitability, volatility, and different revenue shares for 472 large commercial banks from 1988 to 1995. Acharya, Hasan, and Saunders (2002) use bank level data for Italian banks from 1993 to 1999.}

6.3 Impact on $R_L$ from increasing $q$

Implicit differentiation of equation (9) with respect to $q$ yields:

$$\frac{\partial^3 V}{\partial R_L \partial q} = \frac{2\beta(1 + R + R_L)Kq'}{q'} \frac{\partial L}{\partial R_L} < 0,$$

$$\frac{\partial^2 Z}{\partial R_L \partial q} = 2[(1 + R) - (1 - \beta)(1 + R)]Kq' \frac{\partial L}{\partial R_L} > 0,$$

$$\frac{\partial d_i}{\partial q} = \frac{1}{\sigma^2} \left( \frac{1}{V} \frac{\partial V}{\partial q} - \frac{1}{Z} \frac{\partial Z}{\partial q} \right) < 0,$$

$$\frac{\partial V}{\partial q} = -\frac{\beta(1 + R + R_L)K}{q^2} < 0,$$

$$\frac{\partial Z}{\partial q} = -\frac{[(1 + R) - (1 - \beta)(1 + R)]K}{q^2} > 0.$$

Note that the sign of $\frac{\partial R_L}{\partial q}$ in equation (12) will be the same as the sign of $\frac{\partial^2 S}{\partial R_L \partial q}$ since $\frac{\partial^2 S}{\partial R_L^2} < 0$. The first term $[\cdot]$ on the right-hand side of $\frac{\partial^2 S}{\partial R_L \partial q} < 0$ can be interpreted as the mean profit effect, while the second term can be interpreted as the risk effect. The sign of this first term is determined by how changes in $q$ affect the bank’s marginal equity value of loan rate, holding the risk effect constant. An increase in $q$ decreases the bank’s insurance provision for counterparties on demand since the capital-to-deposits ratio is designed to force bank’s capital positions to reflect asset portfolio risks. Thus, the mean profit effect is negative in sign. The risk effect arises because a one dollar increases in $q$ decreases the bank’s profit by $L(R_L)$ evaluated at the optional loan rate in every possible state. As usual, the sign of this risk effect is indeterminate. However, equation (12) provides us with a hunch that the risk effect will be negative since $\frac{\partial d_i}{\partial q} < 0$. Since the risk effect reinforces the mean profit effect to give an overall negative response of the optional loan rate to an increase in $q$, we establish the following proposition.

**Proposition 3**: An increase in the capital-to-deposits ratio decreases the bank’s interest margin.
Intuitively, as the bank is forced to increase its capital relative to its deposit level (the capital-to-deposits ratio), it must now provide a return to a larger equity base. One way the bank may attempt to augment its total returns is by shifting its investments to its loan portfolio and away from the Federal funds market. If loan demand is relatively rate-elastic, a larger loan portfolio is possible at a reduced margin. We note that an increase in loan granted by the bank indicates its increasing swap transaction for the security hedging, while a decrease in the liquid asset indicates its decreasing swap transaction for the insurance providing. So both interest and noninterest incomes decrease with an increase in capital-to-deposits ratio. We conclude that bank capital requirement may be a relatively stable regulatory policy, but it is also a low return one under commercial bank expansion into securities and insurance activities.

6.4 Impact on $E$ from increasing $\alpha$

Total differentiating equation (7) evaluated at the optional loan rate, we can obtain the following expression for how changes in $\alpha$ affect $E$.

$$\frac{\partial E}{\partial \alpha} = \frac{\partial E}{\partial \alpha} + \frac{\partial E}{\partial R_L} \frac{\partial R_L}{\partial \alpha}$$

where

$$\frac{\partial E}{\partial \alpha} = \frac{\partial C}{\partial \alpha} - \frac{\partial P}{\partial \alpha}$$

and

$$\frac{\partial P}{\partial \alpha} = \left[(1 + R_L)LN(p_i) - (1 + R_L - R_s)L e^{-s}N(p_i)\right]$$

and

$$\frac{\partial E}{\partial R_L} = \frac{\partial C}{\partial R_L} - \frac{\partial P}{\partial R_L}.$$

The first-term on the right-hand side of equation (13) can be interpreted as the direct effect, while the second term can be interpreted as the indirect effect. The direct effect captures the change in $E$ due to an increase in $\alpha$, holding the optimal loan rate constant. The sign of the direct effect is governed by $\frac{\partial P}{\partial \alpha}$ since $\frac{\partial C}{\partial \alpha} = 0$. The marginal put-option value of $\alpha$, $\frac{\partial P}{\partial \alpha}$, can be expressed as the difference between the following terms: the former is the call option on the market value of the bank’s loan repayments with strike price equal to its counterparty effectively written to the bank; the latter is market value when loan market is perfect. Accordingly, this difference is negative ($\frac{\partial P}{\partial \alpha} < 0$) and hence the direct effect is positive in sign ($\frac{\partial E}{\partial \alpha}$). The indirect effect of equation (13) demonstrates the optimal loan rate effect on $E$ from a charge in $\alpha$. The term $\frac{\partial E}{\partial R_L}$ of this indirect effect is expressed as the difference between the marginal call-option value of $R_L$, $\frac{\partial C}{\partial R_L}$, and the marginal put-option values of $R_L$, $\frac{\partial P}{\partial R_L}$. The sign of this difference is positive since $\frac{\partial C}{\partial R_L} > 0$ and $\frac{\partial P}{\partial R_L} < 0$. As stated in Proposition 1, $\frac{\partial R_L}{\partial \alpha} > 0$. Thus, the indirect effect is positive in sign. Since the indirect effect
reinforce the direct effect to give an overall positive response of $E$ to an increase in $\alpha$, we establish the following proposition.

**Proposition 4**: Noninterest income increases with commercial bank expansion into securities activities.

A narrow banking proposal effectively calls for the breaking up of synthetic bank into commercial banking and securities operations that would resemble commercial bank and finance company, respectively. Under the view, an increase in the amount of the hedging security for the finance subsidiary of the bank to replace the total return swap increases its noninterest income. Hedging financial risk is a logical extension of the bank’s security business. We then can conclude that commercial bank expansion by hedging securities may be relatively a stable or low-risk activity, but it is also a high return one.

### 6.5 Impact on $E$ from increasing $\beta$

Differentiation of equation (7) evaluated at the optional loan rate with respect to $\beta$ yields:

$$\frac{dE}{d\beta} = \frac{\partial E}{\partial \beta} + \frac{\partial E}{\partial R_L} \frac{\partial R_L}{\partial \beta}$$

(14)

where

$$\frac{\partial E}{\partial \beta} = \frac{\partial C}{\partial \beta} - \frac{\partial P}{\partial \beta},$$

$$\frac{\partial C}{\partial \beta} = [K\left(\frac{1}{q} + 1\right) - L][1 + R + R_p]N(c_1)$$

$$- (1 + R)e^{-R} N(c_1)] > 0, \quad \text{and} \quad \frac{\partial P}{\partial \beta} = 0.$$

The first-term on the right-hand side of equation (14) can be interpreted as the direct effect, while the second term can be interpreted as the indirect effect through the adjustment of the optimal loan rate. The direct effect is positive in sigh since $\frac{\partial C}{\partial \beta} > 0$ and $\frac{\partial P}{\partial \beta} = 0$. The sign of the indirect effect is also positive since $\frac{\partial E}{\partial R_L} > 0$ and $\frac{\partial R_L}{\partial \beta} > 0$. The result of equation (14) is written in the following proposition.

**Proposition 5**: Noninterest income increases with commercial bank expansion into insurance activities.

Under the viewpoint of narrow banking, an increase in the amount of the insurance provision for the insurance subsidiary of the bank to replace the total return swap increases its noninterest income. Insuring financial risk is a logical extension of the bank’s insurance business and hence increases its noninterest income. We conclude that insurance providing may be relatively a unstable activity, but it is also a high return one.

### 6.6 Impact on $E$ from increasing $q$

Total differentiating equation (7) evaluated at the optional loan rate, we can obtain the following expression for how changes in $q$ affect $E$.

$$\frac{dE}{dq} = \frac{\partial E}{\partial q} + \frac{\partial E}{\partial R_L} \frac{\partial R_L}{\partial q}$$

(15)

where
\[ \frac{\partial E}{\partial q} = \frac{\partial C}{\partial q} - \frac{\partial P}{\partial q}, \]

\[ \frac{\partial C}{\partial q} = \frac{BK}{q^2} \left[ (1 + R + R_B)N(c_1) - (1 + R)e^{-R} N(c_2) \right] < 0, \]

and \[ \frac{\partial P}{\partial q} = 0. \]

The first term on the right-hand side of equation (15) is the direct effect which is unambiguously negative. This negative effect is only expressed by decreasingly selling the call-option total return swap to the synthetic commercial bank’s counterparty for a reduction of its business due to an increase in \( q \). The second term is the indirect effect which is also negative since \( \frac{\partial E}{\partial R_L} > 0 \) known as in equation (13) and \( \frac{\partial R_L}{\partial q} < 0 \) known as in Proposition 3. The result of equation (15) is stated in the following proposition.

**Proposition 6**: An increase in the capital-to-deposits ratio decreases the bank’s noninterest income.

Form the narrow banking viewpoint, an increase in bank capital requirement reduces the expected value of the bank’s noninterest income. The interpretation of this result follows a similar argument as in the case of the synthetic banking proposal in Proposition 3.

### 7 Conclusion

Recent research on financial operations has remained largely silent on the question of what ties together the traditional commercial banking function of lending and the investment banking of securities and insurance activities particularly after the passage of the GLBA of 1999. Our main point is that in a sense, market forces cast their ballots for financial services integration. This is especially true to the extent that commercial banks are heavily involved in their expansion into securities and insurance activities. After all, once the decisions to extend investment banking activities have been made, it is a further step to argue that the combined production of commercial banking and investment banking can enhance or hurt a bank’s return, at least a major one.

The results in this paper argue that changes in expansion into securities and insurance activities and the regulatory parameters such as capital-to-deposits ratio, have a direct effect on the bank’s optimal interest margin. In particular, greater commercial bank expansion into securities hedging and insurance providing activities are associated with higher the bank’s interest margin and noninterest income. Regulatory authority reduces bank capital requirement charges; the bank will have a strong incentive to expand its investment banking under the same roof. Both suggest obvious diversification benefits from the ongoing shifting toward high-return investment banking. Our findings provide alternative explanations for the return to investment banking concerning synthetic commercial bank behavior.

Hirtle and Stiroh (2007) argue that U.S. banks, particularly the largest, have dramatically expanded their retail commercial banking operations over the last few years. Geyfman and Yeager (2009) also argue that the financial crisis that began in 2007 has led some to question the wisdom of the GLBA. Although the question of whether GLBA should be repealed is beyond the scope of this paper, we do explore the potential benefits that accrue to synthetic...
commercial banks relative to retail commercial banks. A finding of significant margin and noninterest income increase with portfolio diversification in our paper would be at least a partial justification for the GLBA.

References:


