A survey on modeling and simulation of a signal source with controlled waveforms for industrial electronic applications

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Abstract: - In industrial electronic systems, power converters with power components are used. Each controlled component has its own control circuit. In this paper, the authors propose an original control circuit in order to replace the different existing circuits. The proposed circuit is the representation of an elliptical trigonometry function. Thus, by varying the value of one of its parameter, the output waveform will change. Finally, Labview and Matlab simulation results of the studied circuit are presented and discussed.

Key-Words: - Power converters, power electronics, power components, mathematics and trigonometry.

1 Introduction

In motor drives, robotics, or other industrial electronic applications, the use of power converters is essential to improve the control and, therefore, the efficiency of the studied system [1]. Power converters are generally composed of power components with different characteristics [2]. These components are divided in two categories: the controlled components and the uncontrolled components [3]. The controlled components, like thyristors and transistors, need controlled signals with specified waveforms in order to be applied on their controlled terminals (base or gate) [4]. Thus, each component has its own control source [5].
This paper underlines the importance of the elliptical trigonometry functions in generating different waveforms by varying one parameter value.
In fact, the elliptical trigonometry is an original study regarding the trigonometry literature [6],[7]. The existed trigonometry is a particular case of the elliptical trigonometry [8],[9]. The last one describes an elliptic curve [6], but the existed one describes a circular form [10],[11]. The mathematical topics of Fourier series and Fourier transforms rely heavily on knowledge of trigonometric functions [12],[13] and find application in a number of areas, including statistics [14],[15]. The trigonometric functions are also very important in technical subjects like science, engineering, and even medicine.
Regarding the elliptical trigonometry topic, its important functions are the elliptical cosine and the elliptical sine. In this paper, generalities on the proposed function are presented in section two. In section three, a revue on the elliptical trigonometry is given. In section four, the block diagram of the elliptical sine function is presented. Labview simulation results of the studied function are illustrated in section five. Programming the used function in Matlab is treated in section six. Finally, a conclusion about the elliptical trigonometry functions is presented in section 7.

2 Generality

The used controlled circuits for power transistors (MOSFET, IGBT, etc) differ from those used for Thyristors (GTO, Triac, etc). Designing and modeling circuits for all these controlled components taking into account their different characteristics, take time, and realizing them practically, take time and money. In this paper, one electronic circuit representing an elliptical trigonometry function is proposed to be used in simulation (Labview, Matlab, etc), or practically, in order to control the different existed power components (figure 1). Tow parameters, a and/or b, are used as input variable parameters for the proposed function. ‘α’ is equal to ‘ω.t’.

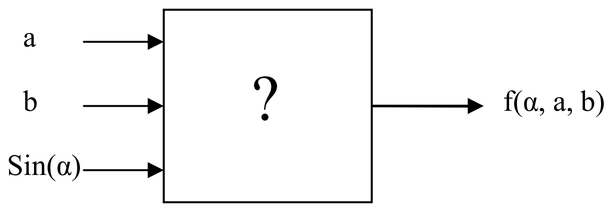


Fig. 1: The studied function.

3 Revue on the elliptical trigonometry

In order to make a review on the elliptical trigonometry, it is necessary to introduce the definition of the angular functions. In fact, the angular functions are new mathematical functions that produce a rectangular signal, in which period is function of angles [6]. Similar to trigonometric functions, the angular functions have the same properties as the precedent, but the difference is that a rectangular signal is obtained instead of a sinusoidal one [16],[17],[18]. Moreover, one can change the frequency, the amplitude and the width of any period of the signal by using the general form of the angular function.

- The expression of the angular function related to the (ox) axis is defined, for $K \in Z$, as [6]:

$$\mathrm{ang}_x[\beta(\alpha+\gamma)] = \begin{cases} +1 \quad \text{for} \\ (4K-1)\dfrac{\pi}{2\beta} - \gamma \le \alpha \le (4K+1)\dfrac{\pi}{2\beta} - \gamma \\ \\ -1 \quad \text{for} \\ (4K+1)\dfrac{\pi}{2\beta} - \gamma \le \alpha \le (4K+3)\dfrac{\pi}{2\beta} - \gamma \end{cases} \tag{1}$$

For $\beta = 1$ and $\gamma = 0$, the expression of the angular function becomes:

$$\mathrm{ang}_x(\alpha) = \begin{cases} +1 & \text{for} \quad \cos(\alpha) \ge 0 \\ -1 & \text{for} \quad \cos(\alpha) \prec 0 \end{cases} \tag{2}$$

Its waveform is illustrated in figure 2.

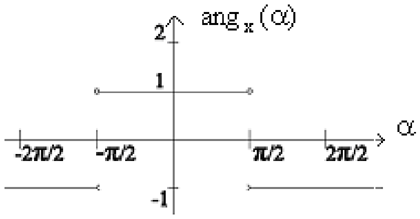


Fig. 2: The $\mathrm{ang}_x(\alpha)$ waveform.

- The expression of the angular function related to the (oy) axis is written as [6]:

$$\mathrm{ang}_y[\beta(\alpha+\gamma)] = \begin{cases} +1 \quad \text{for} \\ 2k.\dfrac{\pi}{\beta} - \gamma \le \alpha \le (2k+1).\dfrac{\pi}{\beta} - \gamma \\ \\ -1 \quad \text{for} \\ (2K+1)\dfrac{\pi}{\beta} - \gamma \le \alpha \le (2K+2).\dfrac{\pi}{\beta} - \gamma \end{cases} \tag{3}$$

For $\beta = 1$ and $\gamma = 0$, the simplified expression of the angular function, which is presented in figure 3, is:

$$\mathrm{ang}_y(\alpha) = \begin{cases} +1 & \text{for} \quad \sin(\alpha) \ge 0 \\ -1 & \text{for} \quad \sin(\alpha) \prec 0 \end{cases} \tag{4}$$

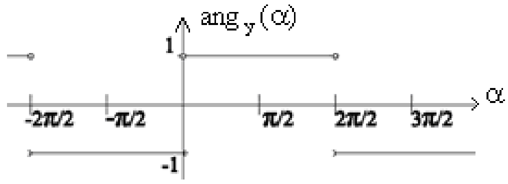


Fig. 3: The $\mathrm{ang}_y(\alpha)$ waveform.

3.1 The elliptical trigonometry unit

The Elliptical Trigonometry Unit is an ellipse with a center O (x = 0, y = 0) and has the equation form:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \tag{5}$$

with:
'a' is the radius of the ellipse on the (x'ox) axis,
'b' is the radius of the ellipse on the (y'oy) axis.

The elliptical trigonometry unit is used to define functions that describe the ellipse (figure 4) as the elliptical cosine and the elliptical sine [6].

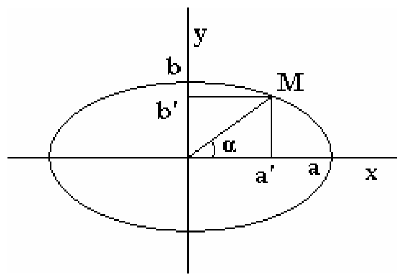


Fig. 4: The elliptical trigonometry unit.

It is essential to note that 'a' and 'b' must be positive. In this paper, 'a' is fixed to 1. 'b' is a variable parameter.

3.2 The elliptical trigonometry functions

- The elliptical cosine is the function that gives the ratio of $\frac{a'}{a}$ with a' is the projection of the point M (onto the ellipse) on the (x'ox) axis (figure 4):

$$\cos\mathrm{el}_b(\alpha) = \frac{a'}{a} \tag{6}$$

with α is the angle defined by the position of the point M:

$\alpha = (\widehat{OX;OM})$.

The elliptical cosine function is written as [6]:

$$\cos el_b(\alpha) = \frac{ang_X(\alpha)}{\sqrt{1+\left[\frac{a}{b}.tg(\alpha)\right]^2}} \quad (7)$$

For specific parameter values, this equation gives the classic cosine expression [15].

- The elliptical sine is the function that gives the ratio of $\frac{b'}{b}$. Therefore:

$$\sin el_b(\alpha) = \frac{b'}{b} \quad (8)$$

The elliptical sine expression is [6]:

$$\sin el_b(\alpha) = \frac{a}{b} tg(\alpha) \frac{ang_X(\alpha)}{\sqrt{1+\left[\frac{a}{b} tg(\alpha)\right]^2}} \quad (9)$$

- The elliptical tangent function is written [6]:

$$tg\, el_b(\alpha) = \frac{\sin el(\alpha)}{\cos el(\alpha)} = \frac{a}{b}.\frac{\sin(\alpha)}{\cos(\alpha)} = \frac{a}{b}.tg(\alpha) \quad (10)$$

3.3 Derivative of the elliptical functions

- The derivative of the elliptical cosine function is:

$$(Cosel_b(u))' = -\frac{b}{a} u' Sinel_b(u). \left(\left(\frac{a}{b}\right)^2 Cosel_b{}^2(u) + Sinel_b{}^2(u) \right) \quad (11)$$

- The derivative of the elliptical sine is:

$$(Sinel_b(u))' = \left(\frac{a}{b} \cdot \frac{ang_X(u) \cdot tg(u)}{\sqrt{1+(\frac{a}{b} tg(u))^2}}\right)'$$

$$= \frac{b}{a} u' Cosel_b(u) \left(\left(\frac{a}{b}\right)^2 Cosel_b{}^2(u) + Sinel_b{}^2(u) \right) \quad (12)$$

- The elliptical tangent derivative is:

$$(tgel_b(u))' = \frac{a}{b}(tg(u))' = \frac{a}{b} u'(1+tg^2(u))$$

$$= \frac{a}{b} u'. \left[1 + \left(\frac{b}{a}\right)^2 tgel_b{}^2(u) \right] \quad (13)$$

3.4 Original formula

To complete the set of formulas that describe the elliptical trigonometric functions, it should be noted that [6]:

$$Cosel_b{}^2(u) + Sinel_b{}^2(u) = 1 \quad (14)$$

4 Block diagram of the elliptical sine function

In this paper, the elliptic sine function, which is defined in equation (9), is chosen to be treated in the following sections. Its block diagram is illustrated in figure 5. There are three inputs connected to this diagram, two variable parameters, ‘a’ and ‘b’, and one sine wave.

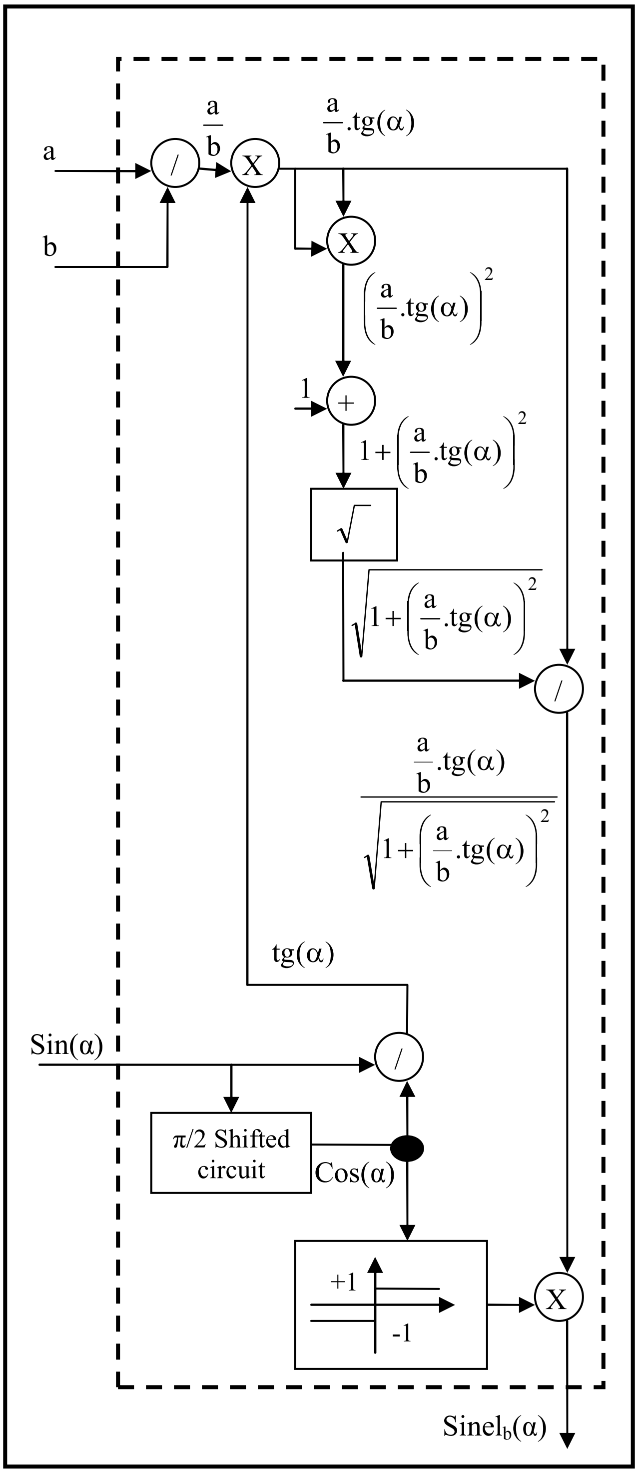


Fig. 5: The elliptical sine block diagram.

5 Simulation results in Labview

Consider ‘a = 1’ and ‘b’ the variable parameter. This choice is proposed to simplify the following study of the elliptical sine function. Its representation in Labview is illustrated in figure 6.

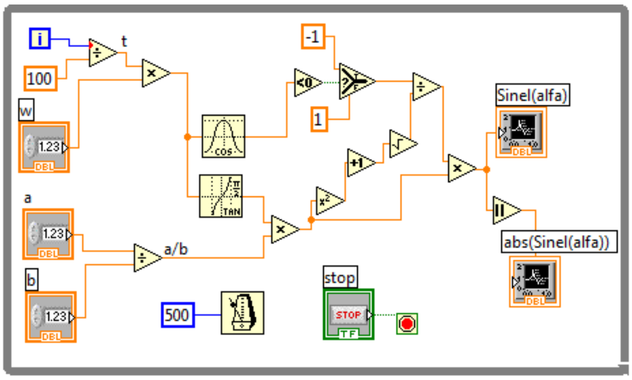


Fig. 6: Representation of Sinel$_b$ (α) in Labview.

5.1 First case (b = 1)

When a = b = 1, the ellipse equation, defined in (5), becomes:

$$x^2 + y^2 = 1 \qquad (15)$$

which is the equation of a circle (figure 7).

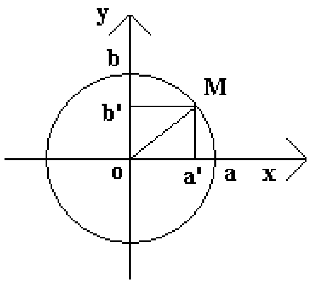


Fig. 7: The circle trigonometry unit.

As: $\sin el_b(\alpha) = \frac{b'}{b} = \frac{b'}{1}$

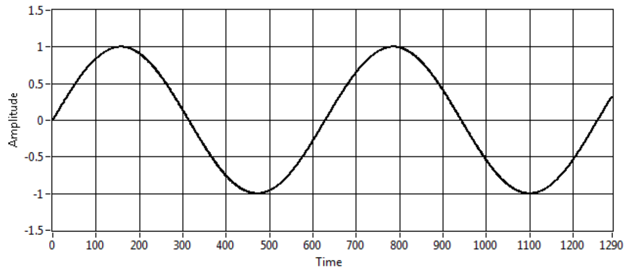
and

$$\cos(\alpha) = \frac{b'}{OM} = \frac{b'}{1},$$

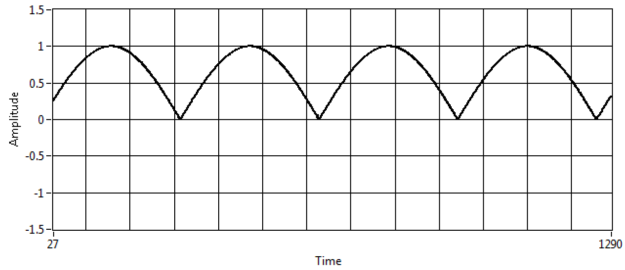
because OM is the radius of the circle and is equal to unity, therefore, the elliptical sine expression (9) becomes:

$\sin el_b(\alpha) = \sin(\alpha)$.

Figures 8.a and 8.b represent the waveforms of the elliptical sine function and its absolute value for $a = b = 1$.



a) sin el$_b$ (α)



b) | sin el$_b$ (α) |

Fig. 8: Elliptical sine waveforms for b = 1.

5.2 Second case (b >> 1)

From equation (9), two configurations are studied:

1. Consider $\alpha = \frac{(2.k+1)\pi}{2}$ and $b \prec\prec a.tg(\alpha)$.

In this case, $tg(\alpha) = \pm\infty$. As ‘b’ is a constant, therefore: $\left[\frac{a}{b}.tg(\alpha)\right]^2 \succ\succ 1$. Thus, the elliptical sine becomes:

$$\sin el_b(\alpha) = \frac{a}{b}.tg(\alpha).\frac{ang_X(\alpha)}{\sqrt{1+\left[\frac{a}{b}.tg(\alpha)\right]^2}} \approx ang_X(\alpha).$$

Thus, the obtained signal is a square signal.

2. Consider $\alpha \neq \frac{(2.k+1)\pi}{2}$ and $b \succ\succ a.tg(\alpha)$.

In this case, $tg(\alpha) \neq \infty$. As ‘b’ is a constant, therefore: $\left[\frac{a}{b}.tg(\alpha)\right]^2 \approx \varepsilon$ is very much smaller than unity. The obtained elliptical cosine expression is:

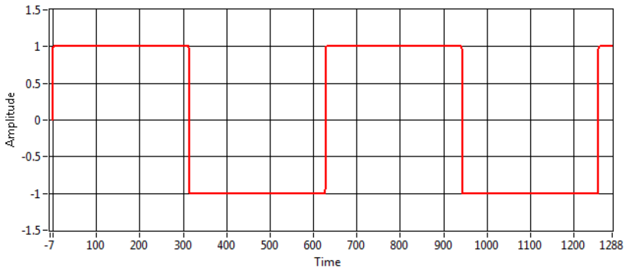
$$\sin el_b(\alpha) = \sqrt{\varepsilon}.\frac{ang_X(\alpha)}{\sqrt{1+\varepsilon}} \approx 0..$$

Thus, the output signal is a signal of zero amplitude.

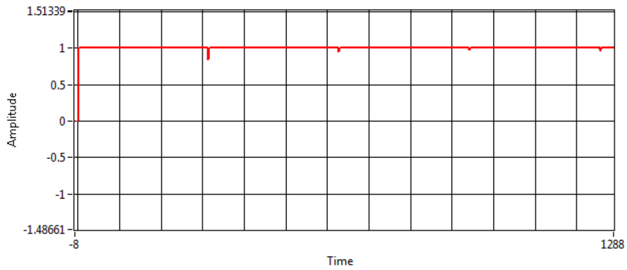
- Consequently:

$$Cosel_b(\alpha) = \begin{cases} \pm 1 & \text{for} \quad \alpha = \frac{(2k+1)\pi}{2} \\ \pm 0 & \text{for} \quad \alpha \neq \frac{(2k+1)\pi}{2} \end{cases} \qquad (16)$$

Thus, a purely square signal can be obtained (figure 9.a). By using its absolute value (figure 9.b), the obtained signal is a continuous or dc signal.



a) sin el_b (α)



b) | sin el_b (α) |

Fig. 9: Elliptical sine waveforms for b = 80 >> 1.

5.3 Third case (b << 1)

From equation (9), two configurations are also studied:

1. Consider $\alpha \neq k.\pi$ and $b \prec\prec a.tg(\alpha)$:

In this case $tg(\alpha) \neq 0$, Therefore $\left[\frac{a}{b}.tg(\alpha)\right]^2 \succ\succ 1$.

Thus:

$$\sin el_b(\alpha) = \frac{a}{b}tg(\alpha).\frac{ang_X(\alpha)}{\sqrt{1+\left[\frac{a}{b}tg(\alpha)\right]^2}} \approx ang_X(\alpha).$$

2. Consider $\alpha = k.\pi$ and $b \succ\succ a.tg(\alpha)$.

In this case $tg(\alpha) = 0$, Therefore $\left[\frac{a}{b}.tg(\alpha)\right]^2 = 0$,

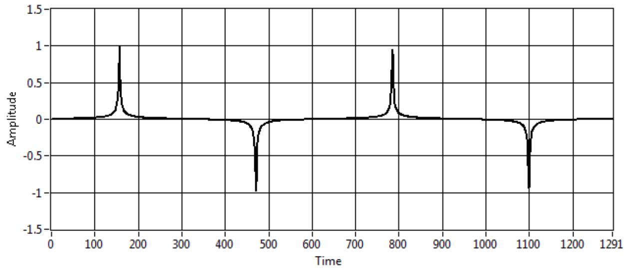
thus:

$$\sin el_b(\alpha) = 0.\frac{ang_X(\alpha)}{\sqrt{1+0}} \approx 0.$$

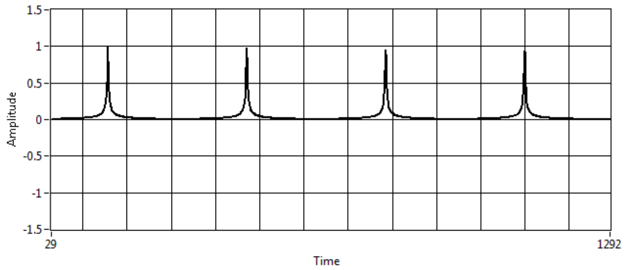
- As conclusion:

$$\sin el_b(\alpha) = \begin{cases} \pm 1 & \text{for} \quad \alpha \neq k.\pi \\ +0 & \text{for} \quad \alpha = 2k\pi \\ -0 & \text{for} \quad \alpha = (2k+1).\pi \end{cases} \qquad (17)$$

For this configuration, a Dirac signal with positive and negative pulses or with only positive pulses can be obtained (figures 10.a and 10.b).



a) sin el_b (α)

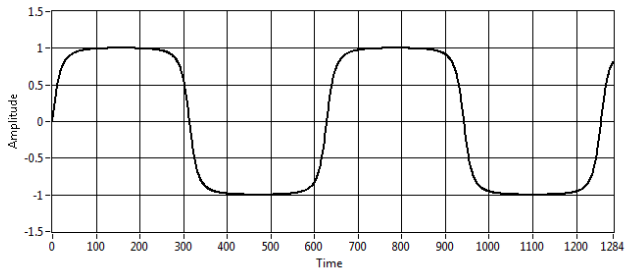


b) | sin el_b (α) |

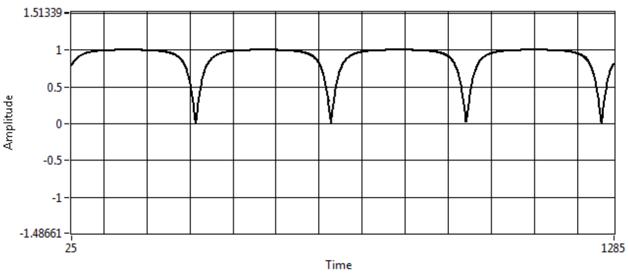
Fig. 10: Elliptical sine waveforms for b =0.001 << 1.

5.4 Fourth case (b < 1)

In this case, an elliptical deflated form is obtained (figure 11.a). The absolute value of this signal (figure 11.b) has an average value greater than that of an absolute value of the sinusoidal signal. Hence, the advantage is that the average of the signal can be increased by only varying the value of the parameter 'b' without any use of any other functions.



a) sin el_b (α)

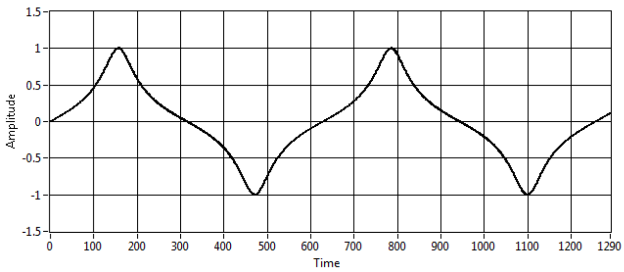


b) | sin el_b (α) |

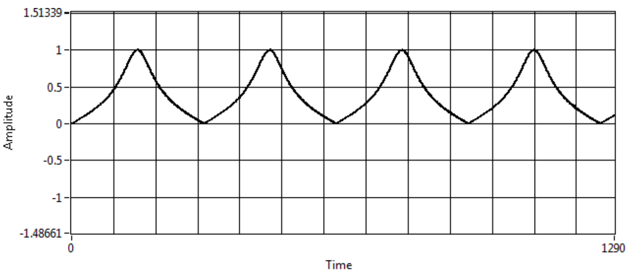
Fig. 11: Elliptical sine waveforms for b = 0.2 < 1.

5.5 Fifth case (b > 1)

For this configuration, the signal takes the elliptical swollen form (Figure 12.a). The average value of its absolute signal (figure 12.b) is less than that obtained by the absolute value of the sinusoidal wave. Therefore, decreasing the average of a signal by varying only one parameter 'b', is also another advantage of the elliptical trigonometric functions.



a) $\sin el_b(\alpha)$



b) $|\sin el_b(\alpha)|$

Fig. 12: Elliptical sine waveforms for b = 3 > 1.

5.6 Determining the value of 'b' for a quasi-triangular signal

The main objective of this part is to obtain a signal closed to a triangular one. Therefore, the following method is proposed in order to calculate the value of 'b' for which the error between the desired signal and the obtained one is minimized. This study is limited to the angular interval $\left[0, \frac{\pi}{2}\right]$.

1. Consider the equation of a straight line:

$$y = c.\alpha + d \tag{18}$$

This line is supposed to contain the following two points: $(\alpha = 0, y = 0)$ and $\left(\alpha = \frac{\pi}{2}, y = 1\right)$.

For $c = \frac{2}{\pi}$ and d = 0, the expression (18), which is drawn in figure 13, becomes:

$$y = \frac{2}{\pi}.\alpha \tag{19}$$

2. For the same interval $\left[0, \frac{\pi}{2}\right]$, the angular function, $ang_x(\alpha)$, is equal to one, therefore, the elliptical sine becomes:

$$Sinel_b(\alpha) = \frac{a}{b}.tg(\alpha).\frac{+1}{\sqrt{1+\left(\frac{a}{b}.tg(\alpha)\right)^2}}$$

The wave form of this function is also illustrated in figure 13.

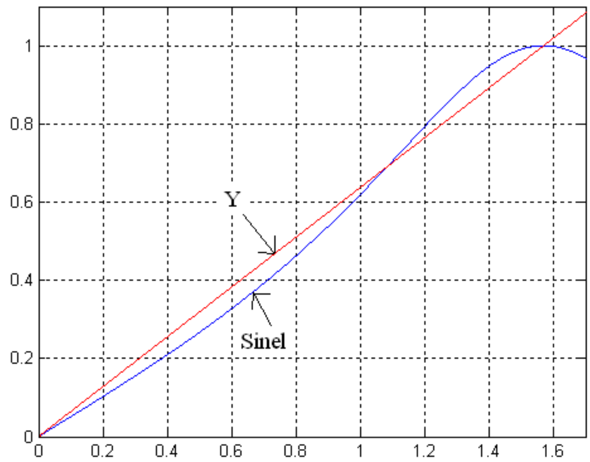


Fig. 13: Obtained signal $sinel_b$ and the desired signal (y) waveforms.

The difference between these two functions, $Sinel_b$ and y, is:

$\varepsilon = \left\|Sinel_b(\alpha) - y\right\|$.

It is considered that, for $\alpha = \frac{\pi}{4}$ (the center of the studied interval $\left[0, \frac{\pi}{2}\right]$), the error ε is equal to zero. Thus, b takes the value of $\sqrt{3}$. To calculate the maximum and the minimum error values, ε_{max} and ε_{min}, the derivative of ε must be equal to zero.

$$\text{Therefore: } \begin{cases} \varepsilon_{max} = 0.0737 \approx 7.4\% \\ \varepsilon_{min} = 0.0178 \approx 1.8\% \\ \varepsilon_{avg} = 0.0215 \approx 2.2\% \end{cases}$$

Figure 14 represents the variation of this error in the interval $\left[0, \frac{\pi}{2}\right]$.

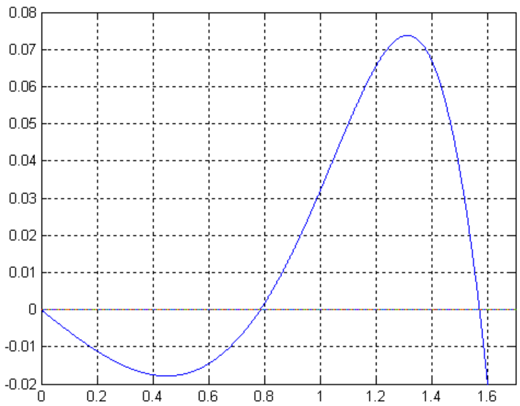
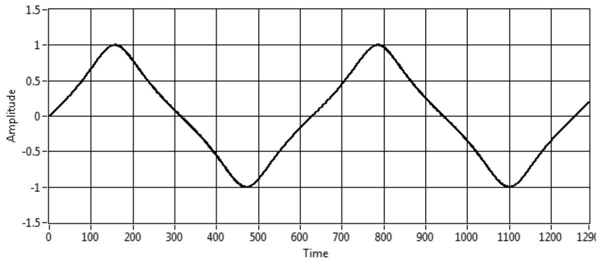
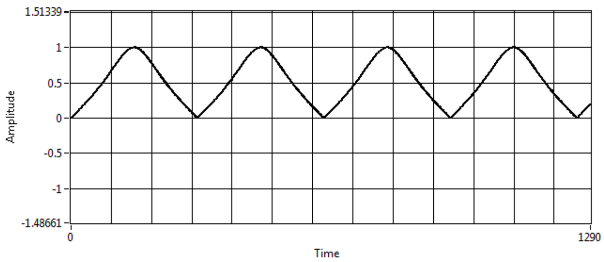


Fig. 14: Error between the desired signal and the obtained one.

For $b = \sqrt{3}$, it is difficult to reduce the error to zero. Therefore, the elliptical sine function takes a quasi-triangular signal waveform as illustrated in figure 15.



a) sin el_b (α)



b) | sin el_b (α) |

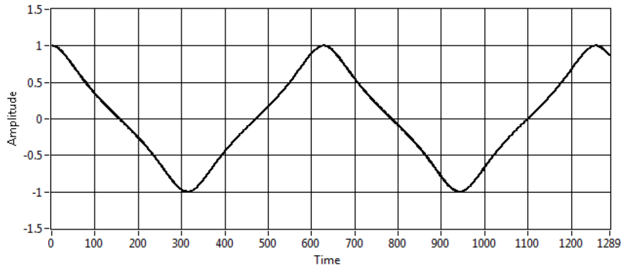
Fig. 15: Elliptical sine waveforms for $b = \sqrt{3}$.

In spite of the presence of the error, its amplitude is small and can be considered in some applications as negligible.

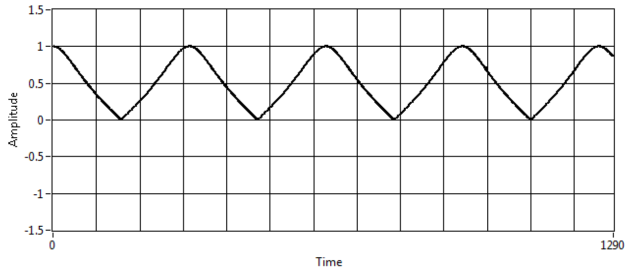
Consequently, by varying b between 0 and 1 in the elliptical sine function, different signal forms are obtained. Thus, the elliptical trigonometry functions will have important influences in engineering, especially in power electronics [19],[20].

5.7 First conclusions

As presented previously, the elliptical sine function takes different forms by varying the input parameter 'b'. The same analysis can be treated using the parameter 'a'. Therefore, same waveforms will be obtained but with different value of 'a'. Then, 'a' and 'b' can be tow different potentiometers, changed manually in an analog circuit, or by programming in a digital circuit. This study can be generalized to the second elliptical function which is the $cosel_b$ (α). Thus, for a = 1 and $b = \sqrt{3}/3$, the obtained elliptical cosine waveforms are given in figure 16.



a) cos el_b (α)



b) | cos el_b (α) |

Fig. 16: Elliptical cosine waveforms for $b = \sqrt{3}/3$.

Consequently, it should be noted that:

1. By varying 'b' between 0 and 1, all signals with different waveforms can be obtained from the $sinel_b$ (α) or the $cosel_b$ (α) functions.

2. By varying 'b' between 0 and +∞, the obtained signal changed from the pulse of Dirac ($v \approx 0$) to a dc signal ($v = V_{max}$).

3. By using three inputs (a, b and sin (α)), and for different values for the two variable parameters (a and b), four different signals can be obtained in the same time. These signals are those of the following functions: $sinel_b$ (α), $cosel_b$ (α), | $sinel_b$ (α) | and | $cosel_b$ (α) |.

Table 1 resumes the output signal waveforms of the elliptical trigonometry functions for 'a' equal to 1 and for different values for b.

Table 1: Waveform of the elliptical trigonometry functions.

b	$cosel_b$ (α)	$sinel_b$ (α)
b >> 1	Square signal	Pulse of Dirac (positive and negative)
b >> 1	Continuous signal (with absolute value)	Pulse of Dirac (with absolute value)
b > 1	Elliptical swollen	Elliptic deflated
b = 1	Sinusoidal signal (cosine)	Sinusoidal signal (sine)
b < 1	Elliptic deflated	Elliptical swollen
b << 1	Pulse of Dirac (positive and negative)	Square Signal
b << 1	Pulse of Dirac (with absolute value)	Continuous signal (with absolute value)
b=$\sqrt{3}/3$	Quasi-triangular signal	
b = $\sqrt{3}$		Quasi-triangular signal

6 Programming the elliptical sine function in Matlab

As presented and analyzed in the previous section, the elliptical sine function can be also programmed and written in the Matlab software. Thus, the elliptical trigonometry functions can be used in any industrial applications.

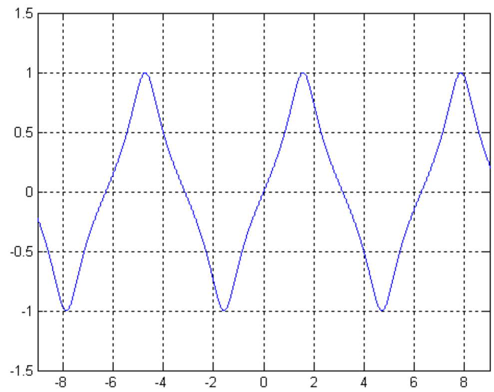
The following program, illustrated in figure 17, represents the detailed steps in writing the elliptical sine function in Matlab. This program is proposed to be divided in tow parts. In the first one, the program contains the definition and the expression of the angular function, defined previously in equation (2). The elliptical sine expression, defined in equation (9), is written the second part.

For a = 1, and for the same values taken for b in Labview examples, the Matlab simulation results give the same waveforms for the elliptical sine function. As example, figure 18 represents the waveform of the elliptical sine function and its absolute value for b = $\sqrt{3}$.

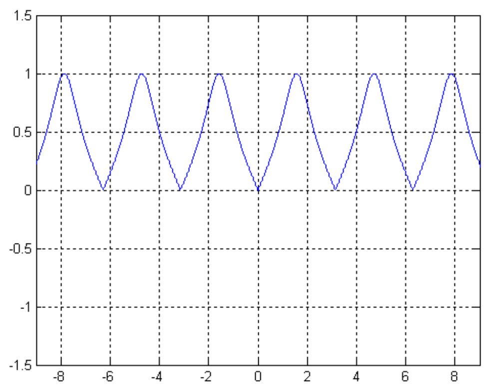
```
% Programming the angular function related
% to the x'ox axis: angx = angx (α)
angx = sign(cos(α));

% Programming the elliptical sine function:
% sinel = sinel (α, a, b)
a = 1;
b = sqrt(3);
α = -15:0.0001:15;
sinel = angx(α). * (a/b). * tan(α) *
        (1. / (sqrt (1. + ((a/b). * tan(α)). ^2) ) ).
y = sinel ;
plot (α, sinel)
axis ([-9 9 -1.5 1.5]);
% Domain of the window or zoom for good vision
grid MINOR;
```

Fig. 17: Elliptical sine program in Matlab



a) sin el_b (α)



b) |sin el_b (α)|

Fig. 18: Elliptical sine waveforms for b = $\sqrt{3}$.

7 Conclusion

In this paper, the elliptical unit and its trigonometry functions are presented. The proposed elliptical trigonometry is a new form of trigonometry that permits producing multiple forms of signals by varying some parameters; it can be used in numerous scientific domains and particularly in mathematics and in engineering.

The elliptical sine function is treated in this paper. The model of the studied function is represented in Labview and in Matlab. In general, a connection cable, with specific transmission data protocol, connects any industrial system to the computer. One can use the studied function in order to generate control signal in need for the power components of the industrial system.

For the studied function, more than eight different signals are produced by varying one parameter. These signals are: sinusoidal signal, rectangular, square, elliptical swollen, elliptical deflated, quasi-triangular, impulse of Dirac with positive and negative part, impulse of Dirac positive part only, continuous signal, etc.

The elliptical trigonometry functions will be widely used in electronic domain especially in power electronics. Thus, several studied will be improved and developed after introducing the new functions of the elliptic trigonometry. Some mathematical expressions and electronic circuits will be replaced by simplified expressions and reduced circuits.

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