# On Structure of the Control Vector for Minimal-Time Networks Design Strategy

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*Abstract:* - The generalized methodology for the electronic networks optimization was elaborated by means of the optimal control theory approach. In this case the problem of the electronic system design is formulated as a classical problem of functional minimization of the optimal control theory. The minimal time system design algorithm was defined as a controllable dynamic process with an optimal control vector. By this methodology the aim of the system design process with minimal computer time is presented as a transition process of some dynamic system that has the minimal transition time. The optimal position of the control vector switch points was determined as a principal characteristic of the minimal-time system design algorithm. The special function that is a combination of Lyapunov function of the design process and its time derivative was proposed to predict the optimal control vector to construct an optimal system design algorithm.

*Key-Words:* - Minimal-time system design, circuit optimization, control theory application, Lyapunov function.

## 1 Introduction

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement. Besides the traditionally used ideas of sparse matrix techniques and decomposition techniques [1-2] some another ways were proposed t o reduce the total computer design time [3-7]. The above described system design ideas can be named as the traditional approach or the traditional strategy because the analysis method is based on the Kirchhoff laws.

The other formulation of the circuit optimization problem was developed on heuristic level some decades ago [8-9]. This idea was based on the Kirchhoff laws ignoring for all the circuit or for the circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization [10] and for the synthesis of high-performance analog circuits [11] in extremely case, when the total system model was eliminated. The authors of the last papers affirm that the design time was reduced significantly. This last idea can be named as the modified traditional design strategy. The generalized approach for the analog system

design on the basis of the control theory formulation was elaborated in some previous works, for example [12]. This approach serves for the minimal-time design algorithm definition. On the other hand this approach gives the possibility to analyze with a great clearness the design process while moving along the trajectory curve into the design space. The main conception of this theory is the introduction of the special control vector, which generalizes the design process and gives the possibility to control the design process to achieve the optimum of the design cost function for the minimal computer time. This possibility appears due to an infinite number of the different design strategies that exist within this theory. The different design strategies have the different operation number and executed computer time. By this approach the traditional design strategy is only a one representative of the enormous set of different design strategies. As shown in [12] the potential computer time gain that can be obtained by the new design problem formulation increases when the size and complexity of the system increase.

We can define the formulation of the main properties of the optimal design strategy as one of

the first problems that needs to be solved for the optimal algorithm construction.

## 2  Problem Formulation

The design process for any analog system design can be defined in discrete form [12] as the problem of the generalized cost function $F(X,U)$ minimization by means of the system (1) with the constraints (2):

$$X^{s+1} = X^s + t_s \cdot H^s \qquad (1)$$

$$\left(1 - u_j\right) g_j(X) = 0 \qquad (2)$$

$$j = 1,2,\ldots,M$$

where $X \in R^N$, $X = (X', X'')$, $X' \in R^K$ is the vector of the independent variables and the vector $X'' \in R^M$ is the vector of dependent variables ($N = K + M$), $g_j(X)$ for all $j$ presents the system model, $s$ is the iterations number, $t_s$ is the iteration parameter, $t_s \in R^1$, $H \equiv H(X,U)$ is the direction of the generalized cost function $F(X,U)$ decreasing, $U$ is the vector of the special control functions $U = (u_1, u_2, \ldots, u_m)$, where $u_j \in \Omega$; $\Omega = \{0;1\}$. The generalized cost function $F(X,U)$ can be defined for example as:

$$F(X,U) = C(X) + \psi(X,U) \qquad (3)$$

where $C(X)$ is the non negative cost function of the design process, and $\psi(X,U)$ is the additional penalty function:

$$\psi(X,U) = \frac{1}{\varepsilon} \sum_{j=1}^{M} u_j \cdot g_j^2(X) \qquad (4)$$

This formulation of the problem permits to redistribute the computer time expense between the solution of problem (2) and the optimization procedure (1) for the function $F(X,U)$. The control vector $U$ is the main tool for the redistribution process in this case. Practically an infinite number of the different design strategies are produced because the vector $U$ depends on the optimization procedure current step. The problem

of the search of an optimal design strategy is formulated now as the typical problem for the functional minimization of the control theory. The functional that needs to minimize is the total CPU time $T$ of the design process. This functional depends directly on the operations number and on the design strategy that has been realized. The main difficulty of this definition is unknown optimal dependencies of all control functions $u_j$.

The continuous form of the problem definition is more adequate for the control theory application. This continuous form replaces Eq. (1) and can be defined by the next formula:

$$\frac{dx_i}{dt} = f_i(X,U) \qquad (5)$$

$$i = 1,2,\ldots,N$$

This system together with equations (2), (3) and (4) composes the continuous form of the design process. The structural basis of different design strategies that correspond to the fixed control vector includes $2^M$ design strategies. The functions of the right hand part of the system (5) are determined for example for the gradient method as:

$$f_i(X,U) = -\frac{\delta}{\delta x_i} F(X,U) \qquad (6)$$

$$i = 1,2,\ldots,K$$

$$f_i(X,U) = -u_{i-K} \frac{\delta}{\delta x_i} F(X,U) + \frac{(1 - u_{i-K})}{t_s} \left\{ -x_i^s + \eta_i(X) \right\}$$

$$(6')$$

$$i = K+1, K+2, \ldots, N$$

where the operator $\frac{\delta}{\delta x_i}$ hear and below means

$$\frac{\delta}{\delta x_i} \varphi(X) = \frac{\partial \varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M} \frac{\partial \varphi(X)}{\partial x_p} \frac{\partial x_p}{\partial x_i}, \quad x_i^s$$

is equal to $x_i(t - dt)$; $\eta_i(X)$ is the implicit function $(x_i = \eta_i(X))$ that is determined by the system (2).

The control variables $u_j$ have the time dependency in general case. The equation number $j$ is removed from (2) and the dependent variable $x_{K+j}$ is transformed to the independent when $u_j = 1$. This independent parameter is defined by the formulas (5), (6'). In this case there is no difference

between formulas (6) and (6'). On the other hand, the Eq. (5) with the right part (6') is transformed to the identity $\dfrac{dx_i}{dt} = \dfrac{dx_i}{dt}$, when $u_j = 0$, because $\eta_i(X) - x_i^s = x_i(t) - x_i(t - dt) = dx_i$. It means that at this time moment the parameter $x_i$ is dependent one and the current value of this parameter can be obtained from the system (2) directly. This transformation of the vectors $X'$ and $X''$ can be done at any time moment. The function $f_0(X, U)$ is determined as the necessary time for one step of the system (5) integration. This function depends on the concrete design strategy. The additional variable $x_0$ is determined as the total computer time $T$ for the system design. In this case we determine the problem of the time-optimal system design as the classical problem of the functional minimization of control theory. In this context the aim of the optimal control is to result each function $f_i(X, U)$ to zero for the final time $T$, to minimize the cost function and the total computer time $x_0$.

It is necessary to find the optimal behavior of the control functions $u_j$ during the design process to minimize the total design computer time. The functions $f_i(X, U)$ are piecewise continued as the temporal functions and the structure of these functions can be found by approximate methods of the control theory [13-14].

Now the analog system design process is formulated as a dynamic controllable system. The time-optimal design process can be defined as the dynamic system with the minimal transition time in this case. So we need to find the special conditions to minimize the transition time for this dynamic system.

## 3 Lyapunov Function Definition

On the basis of the analysis in previous section we can conclude that the minimal-time algorithm has one or some switch points in control vector where the switching is realize among different design strategies. As shown in [15] it is necessary to switch the control vector from like modified traditional design strategy (MTDS), when $u_j = 1$ for all $j$ to like traditional design strategy (TDS), when $u_j = 0$ for all $j$ with some adjusting. Some principal features of the time-optimal algorithm were determined previously. These are: 1) an additional acceleration effect that appeared under special circumstances [16]; 2) the start point special

selection outside the separate hyper-surface to guarantee the acceleration effect, at least one negative component of the start value of the vector $X$ is can be recommended for this; 3) an optimal structure of the control vector with the necessary switch points. The two first problems were discussed in [16-17]. The third problem is discussed in the present paper.

The main problem of the time-optimal algorithm construction is unknown optimal sequence of the switch points during the design process. We need to define a special criterion that permits to realize the optimal or quasi-optimal algorithm by means of the optimal switch points searching. A Lyapunov function of dynamic system serves as a very informative object to any system analysis in limits of the control theory. We propose to use a Lyapunov function of the design process to detect the optimal algorithm, particularly for the optimal switch points searching. The Lyapunov function properties can help us solve this problem.

There is a freedom of the Lyapunov function choice because of a non-unique form of this function. Let us define the Lyapunov function of the design process (2)-(6) by the following expression:

$$V(X) = \sum_i (x_i - a_i)^2 \qquad (7)$$

where $a_i$ is the stationary value of the coordinate $x_i$, in other words the set of all the coefficients $a_i$ is the main objective of the design process. The function (7) satisfies all of the conditions of the standard Lyapunov function definition for the variables $y_i = x_i - a_i$. In fact the function $V(Y) = \sum_i y_i^2$ is the piecewise continue. Besides there are three characteristics of this function: i) $V(Y) > 0$, ii) $V(0) = 0$, and iii) $V(Y) \to \infty$ when $\|Y\| \to \infty$. Inconvenience of the formula (7) is an unknown point $a = (a_1, a_2, ..., a_N)$, because this point can be reached at the end of the design process only. We can use this form of the Lyapunov function if we already found the design solution someway. On the other hand, it is very important to control the stability of the design process during the optimization procedure. In this case we need to construct other form of the Lyapunov function that doesn't depend on the unknown stationary point. Let us define two new forms of the Lyapunov function by the formulas:

$$V(X,U) = [F(X,U)]^r \qquad (8)$$

$$V(X,U) = \sum_i \left( \frac{\partial F(X,U)}{\partial x_i} \right)^2 \qquad (9)$$

where $F(X,U)$ is the generalized cost function of the design process. The formula (8) can be used when the general cost function is non-negative and has zero value at the stationary point $a$. Other formula can be used always because all derivatives $\partial F / \partial x_i$ are equal to zero in the stationary point $a$. Besides, the function $V$ is the function of the vector $U$ too, because all coordinates $x_i$ are the functions of the control vector $U$.

We can define now the design process as a transition process for controllable dynamic system that can provide the stationary point (final point of the optimization procedure) during some time. The problem of the time-optimal design algorithm construction can be formulated now as the problem of the transition process searching with the minimal transition time. There is a well-known idea [18-19] to minimize the time of the transition process by means of the special choice of the right hand part of the principal system of equations; in our case these are the functions $f_i(X,U)$. It is necessary to change the functions $f_i(X,U)$ by means of the control vector $U$ selection to obtain the maximum speed of the Lyapunov function decreasing (the maximum absolute value of the Lyapunov function time derivative $\dot{V} = dV/dt$). Normally the time derivative of the Lyapunov function is non-positive for the stable processes. However, we can define now more informative function as a time derivative of Lyapunov function relatively the Lyapunov function: $W = \dot{V}/V$. In this case we can compare the different design strategies by means of the function $W(t)$ behavior and we can search the optimal position for the control vector switch points.

## 4 Optimal Strategy Prediction

The optimal structure of the control vector $U$ is the principal aim of the analysis of design process based on generalized methodology. All examples were analyzed for the continuous form of the optimization procedure (5). Functions $V(t)$ and $W(t)$ were the main objects of the analysis and its behavior has been analyzed during the design process. As shown in [20] the behavior of the

functions $V(t)$ and $W(t)$ can define the total computer time for each design strategy. It is very interesting to analyze the behavior of the function $V(t)$ for determine the optimal position of the switch points of the control vector. This function serves as a sensitive criterion to detect the optimal switching of the control vector $U$. The Lyapunov function $V(t)$ for all examples was calculated by formula (8) for $r = 0.5$.

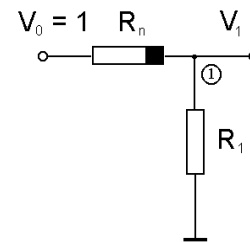The analysis of the design process for one-node passive nonlinear network in Fig.1 is presented below.



Fig. 1. One-node nonlinear passive network.

The nonlinear element has the following dependency: $R_n = a + b \cdot V_1^2$. The vector $X$ includes two components: $x_1^2 = y_1$, $x_2 = V_1$, where $y_1 = 1/R_1$. The model of this network (2) includes one equation and the optimization procedure (5) includes two equations. This network is characterized by one dependent parameters and the control vector includes one control function: $U = (u_1)$. Structural basis includes two different strategies with corresponding control vector: (0), and (1). Behavior of the functions $V(t)$ and $W(t)$ help us to determine the switch point optimal position of the control vector.

We suppose that the quasi optimal design strategy consists of two parts: the first part corresponds to MTDS with $U = (1)$ and the second part corresponds to TDS with $U = (0)$. The optimal switch point between two these strategies was a principal objective of this analysis. The consecutive change of the switch point was realized for the integration step number from 2 to 25.

The behavior of the functions $V(t)$ and $W(t)$ during the design process after the switch point is shown in Fig. 2. The corresponding total iteration number and computer time are presented in Table 1. The analysis shows that the optimal switch point corresponds to the iteration step 15 (graph 4 with dots in Fig. 2).
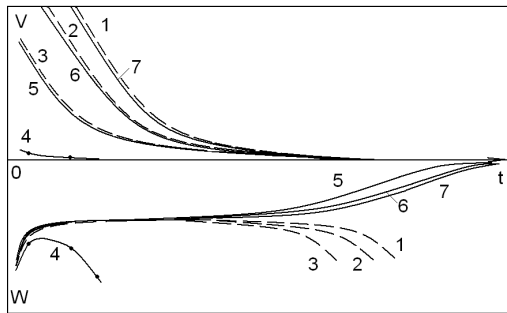
Fig. 2. Behavior of the functions *V(t)* and *W(t)* during the design process for seven different switch points (from 12 to 18) for network in Fig. 1.

Table 1. Iterations number and computer time for strategies with different switch points for network in Fig. 1.

| N | Switch point | Iterations number | Total design time (sec) |
|---|---|---|---|
| 1 | 12 | 1327 | 0,00506 |
| 2 | 13 | 1254 | 0,00478 |
| 3 | 14 | 1132 | 0,00431 |
| 4 | 15 | 283 | 0,00108 |
| 5 | 16 | 1668 | 0,00635 |
| 6 | 17 | 1790 | 0,00682 |
| 7 | 18 | 1861 | 0,00709 |

The curves 1, 2, and 3 correspond to the switch point position before the optimal switch point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one. There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4 corresponds to the optimal position of the switch point. This optimal switch point can be defined by means of the analysis of functions *V(t)* and *W(t)* behavior from 22-th integration step of the optimization procedure (5). These curves lie below all other corresponding curves. This is a very strong criterion for the optimal switch point definition.

The analysis of the design process for two-node passive nonlinear network in Fig. 3 is presented below. The nonlinear element has the following dependency: $y_{nl} = a_{nl} + b_{nl} \cdot (V_1 - V_2)^2$. The vector $X$ includes five components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$. The model of this network (2) includes two equations (*M*=2) and the optimization procedure (5) includes five equations. This network
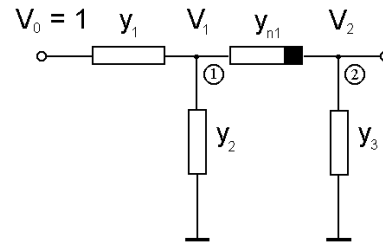


Fig. 3. Two-node nonlinear passive network.

is characterized by two dependent parameters and the control vector includes two control functions: $U = (u_1, u_2)$. Structural basis includes four different strategies with corresponding control vector: (00), (01), (10), and (11). Behavior of the functions *V(t)* and *W(t)* help us to determine the switch point optimal position of the control vector.

Taking into account the preliminary reasons about the optimal algorithm structure [15] we have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (11) that corresponds to MTDS and the second part is defined by the control vector (00) that corresponds to TDS. So, the switching is realized between two strategies, (11) and (00).

The behavior of the functions *V(t)* and *W(t)* during the design process after the switch point is shown in Fig. 4.
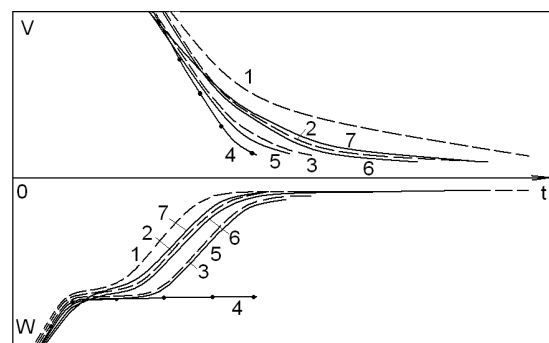


Fig. 4. Behavior of the functions *V(t)* and *W(t)* during the design process for seven different switch points (from 147 to 267) for network in Fig. 3.

The corresponding total iteration number and computer time are presented in Table 2.

The integration of the system (5) was realized by the constant integration step. The step for switch point increment is equal 20 to improve the identification of the difference between all curves.

Table 2. Iterations number and computer time for strategies with different switch points for network in Fig. 3.

| N | Switch point | Iterations number | Total design time (sec) |
|---|---|---|---|
| 1 | 147 | 8319 | 0,221 |
| 2 | 167 | 6501 | 0,172 |
| 3 | 187 | 3697 | 0,096 |
| 4 | 207 | 2860 | 0,073 |
| 5 | 227 | 3383 | 0,087 |
| 6 | 247 | 5429 | 0,142 |
| 7 | 267 | 6682 | 0,175 |

The analysis shows that the optimal switch point corresponds to the step 207 (graph 4 with dots in Fig. 4). The curves 1, 2, and 3 correspond to the switch point position before the optimal switch point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one. There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4 corresponds to the optimal position of the switch point.

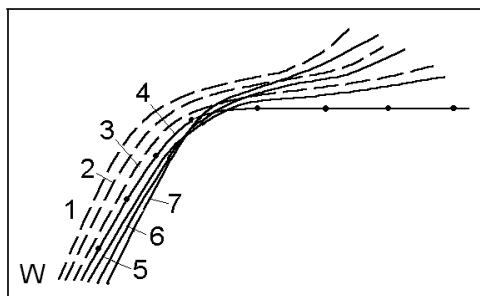The initial parts of *W(t)* dependencies of Fig. 4 are shown in Fig. 5 in large scale.



Fig. 5. Behavior of the functions *V(t)* and *W(t)* during the initial part of design process for network in Fig. 3.

We can see that the curves 1, 2, and 3, which correspond to the switch points before the optimal point (4) have not intersections. On the other hand, the curves 5, 6, and 7 that are based on the switch point after the optimal one have intersections and each this curve lies upper the curve 4 till some time point. It means that from this time moment the graph *W(t)* for the optimal switch point lies below all of other graph. So, from one hand the optimal switch point corresponds to a minimal computer time, from the other hand, this point corresponds to the graph of *W(t)* function that lies below all of other graphs. This property serves as a principal criterion for the optimal switch point selection.

The function *W(t)* that corresponds to the optimal switch point has a maximum absolute value leading off the 340th integration step. It means that from this integration step we can confidently predict the optimal switch point position that leads to the minimal computer design time.

The analysis of the design process for three-node passive nonlinear network in Fig. 6 is presented below.
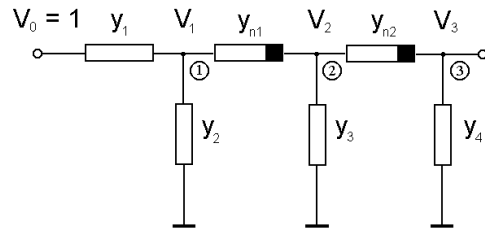


Fig. 6. Three-node nonlinear passive network.

The nonlinear elements are defined as: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The vector $X$ includes seven components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$. The model of this network (2) includes three equations ($M=3$) and the optimization procedure (5) includes seven equations. This network is characterized by three dependent parameters and the control vector includes three control functions: $U = (u_1, u_2, u_3)$. Structural basis includes eight different strategies with corresponding control vector: (000), (001), (010), (011), (100), (101), (110), and (111). Behavior of the functions *V(t)* and *W(t)* help us to determine the switch point optimal position of the control vector.

Taking into account the preliminary reasons about the optimal algorithm structure [15] we have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (111) that corresponds to MTDS and the second part is defined by the control vector (000) that corresponds to TDS. So, the switching is realized between two strategies, (111) and (000).

The optimal switch point was a principal objective of this analysis. The consecutive change of the switch point was realized for the integration step number from 2 to 20.

The behavior of the functions $V(t)$ and $W(t)$ during the design process after the switch point is shown in Fig. 7.
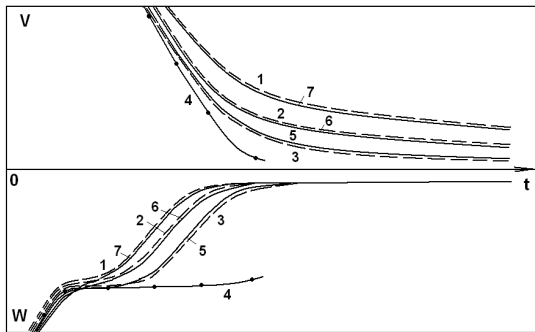


Fig. 7. Behavior of the functions $V(t)$ and $W(t)$ during the design process for seven different switch points (from 6 to 12) for network in Fig. 6.

As discussed above, the principal element of the minimal-time design algorithm is the optimal position of the control vector switch point. Fig. 7 shows the behavior of the functions $V(t)$ and $W(t)$ for seven different positions of the switch point. The corresponding total iteration number and computer time are presented in Table 3.

Table 3. Iterations number and computer time for strategies with different switch points for network in Fig. 6.

| N | Switch point | Iterations number | Total design time (sec) |
|---|---|---|---|
| 1 | 6 | 8409 | 0,659 |
| 2 | 7 | 6408 | 0,502 |
| 3 | 8 | 3141 | 0,246 |
| 4 | 9 | 1234 | 0,096 |
| 5 | 10 | 3310 | 0,259 |
| 6 | 11 | 5918 | 0,464 |
| 7 | 12 | 7404 | 0,581 |

The integration of the system (5) was realized by the constant integration step. The analysis shows that the optimal switch point corresponds to the step 9 (graph 4 with dots in Fig. 2). The curves 1, 2, and 3 correspond to the switch point position before the optimal switch point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one. There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4

corresponds to the optimal position of the switch point.

The initial part of $W(t)$ dependencies of Fig. 7 are shown in Fig. 8 in large scale.
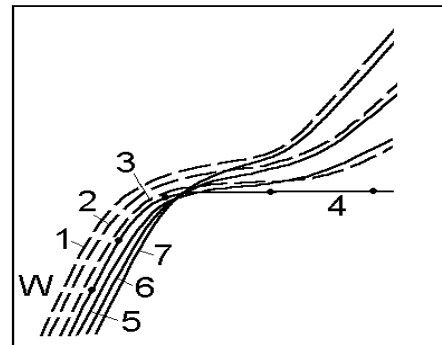


Fig. 8. Behavior of the functions $V(t)$ and $W(t)$ during the initial part of de sign process for network in Fig. 6.

We can see that the curves 1, 2, and 3, which correspond to the switch points before the optimal point (4) have not intersections. On the other hand, the curves 5, 6, and 7 that are based on the switch point after the optimal one have intersections and each this curve lies upper the curve 4 till some time point. It means that from this time moment the graph $W(t)$ for the optimal switch point lies below all of other graph. So, from one hand the optimal switch point corresponds to a minimal computer time, from the other hand, this point corresponds to the graph of $W(t)$ function that lies below all of other graphs. This property anew serves as a principal criterion for the optimal switch point selection. The function $W(t)$ that corresponds to the optimal switch point has a maximum absolute value leading off the 15th integration step. It means that from this integration step we can confidently predict the optimal switch point position that leads to the minimal computer design time.

The next example corresponds to the fourth-node passive network that is shown in Fig. 9.
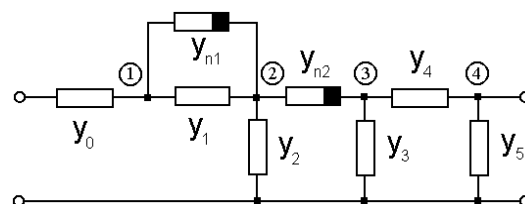


Fig. 9. Four-node nonlinear passive network.

The nonlinear elements have the same dependencies as in previous example. The vector $X$ includes nine components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$ $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$. The model of this network (2) includes four equations ($M=4$) and the optimization procedure (5) includes nine equations. This network is characterized by four dependent parameters and the control vector includes four control functions: $U=(u_1, u_2, u_3, u_4)$. Structural basis includes 16 different design strategies with corresponding control vector from (0000) to (1111). Behavior of the functions $V(t)$ and $W(t)$ help us to determine the optimal position of the switch point of the control vector.

We have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (1111) that corresponds to MTDS and the second part is defined by the control vector (0000) that corresponds to TDS. So, the switching is realized between two strategies: (1111) and (0000). The consecutive change of the switch point was realized for the integration step number from 2 to 40.

The behavior of the functions $V(t)$ and $W(t)$ during the design process after the switch point is shown in Fig. 10.
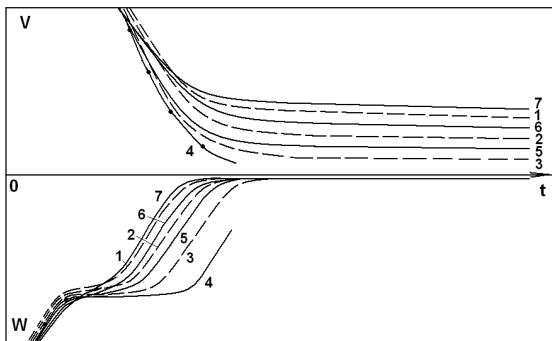


Fig. 10. Behavior of the functions $V(t)$ and $W(t)$ during the design process for seven different switch points (from 30 to 36) for network in Fig. 9.

As discussed above, the principal element of the minimal-time design algorithm is the optimal position of the control vector switch point. Fig. 10 shows the behavior of the functions $V(t)$ and $W(t)$ for seven different positions of the switch point. The corresponding total iteration number and computer time are presented in Table 4.

Table 4. Iterations number and computer time for strategies with different switch points for network in Fig. 9.

| N | Switch point | Iterations number | Total design time (sec) |
|---|---|---|---|
| 1 | 30 | 12570 | 2,826 |
| 2 | 31 | 9677 | 2,174 |
| 3 | 32 | 4182 | 0,936 |
| 4 | 33 | 1020 | 0,223 |
| 5 | 34 | 8134 | 1,826 |
| 6 | 35 | 11612 | 2,609 |
| 7 | 36 | 13795 | 3,101 |

The integration of the system (5) was realized by the constant integration step. The analysis shows that the optimal switch point corresponds to the step 33 (graph 4 in Fig. 10). The curves 1, 2, and 3 correspond to the switch point position before the optimal switch point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one. There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4 corresponds to the optimal position of the switch point.

So, the optimal switch point corresponds to the minimal computer time, from the other hand, this point corresponds to the graph of $W(t)$ function that lies below of all the other graphs. This property selects the optimal switch point.

We can see that the function $W(t)$ that corresponds to the optimal switch point has a maximum absolute value leading off the 54th integration step. It means that from this integration step we can predict the optimal switch point position that minimizes the computer design time.

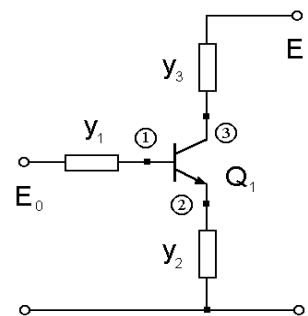Next example corresponds to the one-stage transistor amplifier in Fig. 13.



Fig. 13. One-stage transistor amplifier.

The vector $X$ includes ten components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$. The model of this network (2) includes three equations ($M$=3) and the optimization procedure (5) includes six equations. The total structural basis contains eight different design strategies. The control vector includes five control functions: $U = (u_1, u_2, u_3)$. The Ebers-Moll static model of the transistor has been used [21].

Fig. 14 shows the behavior of the functions $V(t)$ and $W(t)$ during the design process with different switch points.
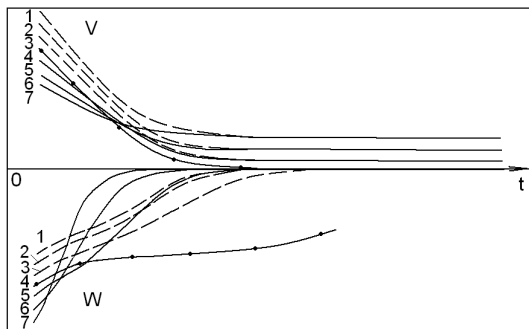


Fig. 14. Behavior of the functions $V(t)$ and $W(t)$ during the design process for seven different switch points (from 33 to 39) for network in Fig. 13.

The behavior of these functions helps us to determine the optimal position of the control vector switch point. We have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (111) that corresponds to MTDS and the second part is defined by the control vector (000) that corresponds to TDS. The optimal switch point was an aim of the analysis. The consecutive change of the switch point was realized for the integration step number from 2 to 50. The behavior of the functions $V(t)$ and $W(t)$ for the switch points from 33 to 39 are shown in this figure and the data, which correspond to these graphs, are presented in Table 6.

The analysis shows that the optimal switch point corresponds to the step 36 (graph with dots). The computer design time has a minimal value for this step. We can see that the function $W(t)$ has a maximum absolute value for the optimal switch step (number 4) leading off the 55th integration step. It means that from this integration step we can

Table 6. Iterations number and computer time for strategies with different switch points for network in Fig. 13.

| N | Switch point | Iterations number | Total design time (sec) |
|---|---|---|---|
| 1 | 33 | 2433 | 0,404 |
| 2 | 34 | 2180 | 0,361 |
| 3 | 35 | 1748 | 0,289 |
| 4 | 36 | 61 | 0,01 |
| 5 | 37 | 1705 | 0,281 |
| 6 | 38 | 2111 | 0,349 |
| 7 | 39 | 2349 | 0,389 |

confidently predict the optimal switch point position that leads to the minimal computer design time.

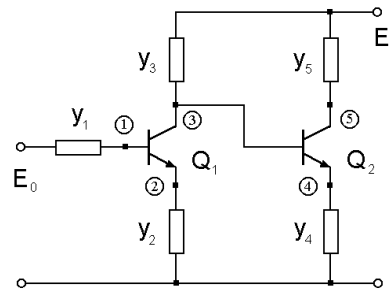The last example corresponds to the two-stage transistor amplifier in Fig. 15.



Fig. 15. Two-stage transistor amplifier.

The vector $X$ includes ten components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$, $x_{10} = V_5$. The model of this network (2) includes five equations ($M$=5) and the optimization procedure (5) includes ten equations. The total structural basis contains 32 different design strategies. The control vector includes five control functions: $U = (u_1, u_2, u_3, u_4, u_5)$. Fig. 16 shows the behavior of the functions $V(t)$ and $W(t)$ for some design strategies with different switch points including the optimal one.

The data, which correspond to these graphs, are presented in Table 7. The integration of the system (5) was realized by the optimal variable integration step.
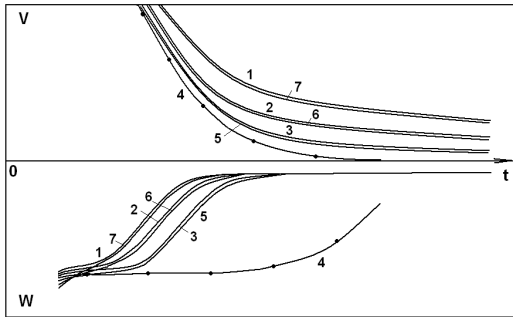
Fig. 16. Behavior of the functions *V(t)* and *W(t)* during the design process for seven different switch points (from 7 to 13) for network in Fig. 15.

Table 7. Iterations number and computer time for strategies with different switch points for network in Fig. 15.

| N | Switch point 1 | Switch point 2 | Iterations number | Total design time (sec) |
|---|---|---|---|---|
| 1 | 7 | 8 | 4900 | 9,91 |
| 2 | 8 | 9 | 4486 | 9,11 |
| 3 | 9 | 10 | 3785 | 7,69 |
| 4 | 10 | 11 | 1354 | 2,74 |
| 5 | 11 | 12 | 3618 | 7,34 |
| 6 | 12 | 13 | 4424 | 8,98 |
| 7 | 13 | 14 | 4882 | 9,89 |

As for previous example, the design of two-transistor cell amplifier has been proposed as a combination of MTDS and TDS. In this case the quasi-optimal control vector includes two switch points. We changed the control vector from (11111) to (00000) and from (00000) to (11111). The consecutive change of the switch point was realized for the integration step's number from 2 to 20. The behavior of the functions *V(t)* and *W(t)* for the optimal switch steps and some steps near the optimal confidently detect the optimal position of the switch points.

We observe a specific behavior of the function *W(t)* near the optimal switch point's position. Before the optimal switch point the function *W(t)* graphs are "parallel". Function *W(t)* has the maximum negative value for the optimal switch points. The graphs of the function *W(t)* that correspond to the optimal switch point's position (number 4) and before the optimal position (1, 2 and 3) have not intersection. After the optimal points the graphs of the function *W(t)* intersect the graphs that correspond to the optimal switch point and before the optimal one. It means that we can detect the optimal position of the switch points during the initial design interval.

So, the structure of the optimal control vector i.e. the structure of the time optimal design strategy can be defined by means of the analysis of the relative time derivative of the Lyapunov function during the initial time interval of the design process.

## 5 Conclusion

The problem of the minimal-time design algorithm construction can be solved adequately on the basis of the control theory. The design process is formulated as the controllable dynamic system. The Lyapunov function of the design process and its time derivative include the sufficient information to select more perspective design strategies from infinite set of the different design strategies that exist into the general design methodology. The special function *W(t)* was proposed to predict the structure of the time optimal design strategy. This function can be used as a main tool to construct the optimal sequence of the control vector switch points. This is the basis for the optimal design algorithm construction for the system design.

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