An Equilibrium Model for the Joint FTR/FGR Auction Market

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Abstract: - As an important instrument to hedge against the risk of congestion charges, transmission right has been successfully implemented in practice. There are two basic transmission rights, point-to-point financial transmission right (FTR) and flow-based right (FGR). The equilibrium model for the joint FTR/FGR auction market is established in this paper. Every TR (transmission right) bidder can bid for any portfolio of FTRs and FGRs in this model, and the contingency constraints are also involved in the simultaneous feasible test (SFT). Besides, the analysis for the established model is also presented. The resulting equilibrium model is formulated in terms of a mixed linear complementarity problem, which is converted into the computation of a convex quadratic programming in this paper. The standard PJM-5-bus system is used to illustrate the proposed equilibrium model.

Key-Words: FTR obligation; FTR option; FGR; simultaneous feasible test; convex quadratic programming

1 Introduction

The locational marginal price (LMP) mechanism[1], which is used to calculate the electricity price and manage the transmission congestion, has been successfully implemented in several ISOs, such as PJM, ISO-New England, California ISO and New York ISO. LMP is composed by energy cost and congestion cost while ignoring losses. If there is no transmission congestion in the electrical network, the congestion cost is zero for every node; otherwise, the LMP differences induced by the re-dispatch of generating resources appear across all system nodes. Therefore, the unpredictability of the transmission congestion creates great risk of price volatility for each market participant.

In order to hedge this uncertainty of congestion price risk and increase market liquid; transmission right market is established naturally[2]. Furthermore, the implementation of the transmission right market can also motivate the transmission investment from the view of long term[3]. Generally, there are two kinds of transmission rights (TRs), i.e. point-to-point financial transmission right (FTR) and flow-based right (FGR).

FTR is first introduced by Hogan[4], it is a kind of financial entitlement that can be purchased to hedge against congestion charges through the specified transmission path. Any FTR bid should include three components, i.e. the bid price, the maximum bid amount and two nodes separately denoted as source node and sink node. Accordingly, any FTR can be denoted as the awarded amount from source node to sink node. Based on the hedging fashion of the congestion risk, FTR can be further decomposed into two categories. One is FTR obligation and the other is FTR option. The payoff of the FTR obligation is equal to the product of the awarded amount and the LMPs difference between source node and sink node. It's clearly that the FTR obligation payoff maybe a liability when the LMPs difference is negative; while the FTR option can be exempted from this liability. As we know, the FTR scheme has been successfully implemented by some electricity markets, such as PJM, ISO-New England, and so on[5,6].

FGR is firstly proposed by Chao and Peck[7,8], it entitles the right holder to hedge against the transmission charge on the specified flowgate with positive or negative direction. Any FGR bid should also include three components, i.e. the bid price, the maximum bid amount and the flowgates with positive or negative direction. Due to the presence of a big number of contingent flowgates and the varying values of power transfer distribution factors in practice, the FGR scheme is far less popular than the FTR scheme until now. Thus far, annual, monthly and daily FGRs are explicitly auctioned by the many European TSOs[9].

In order to insure that the congestion charges collected by ISO in the energy market are not less than the payment to TR holders, ISO should also

conduct a simultaneously feasibility test (SFT) subject to the thermal capacity constraints and the contingency constraints after acquiring all the bids information from market participants.

Several research works have been done on TR auction market: detailed formulations of different financial transmission rights and the related properties are given out in [10]. A joint energy and transmission rights auction model accommodating the flowgate and point-to-point option and obligation as well as the energy trading is proposed in [11]. A new model, which incorporates the FATCS devices into the FTR optimal auction model to give the market participants more opportunities to win their bids for FTRs, is presented in [12]. The fundamental features of the FTR auction market and the implementation of the market clearing system in PJM market is introduced in [13]. A new type of transmission rights named contingent transmission right in the standard market design is studied in [14]. The risk constrained FTR bidding strategy is studied in [15-16], in which the optimization problem faced by any bidder is a bi-level optimization problem. The non-convex feasible region of the bi-level optimization problem often makes its computation become much complicated, especially for big scale system. Moreover, the existence and uniqueness of the solution for the bi-level optimization problem cannot be guaranteed[17]. Reference [18] establishes an equilibrium model for FTR auction market in terms of a mixed linear complementarity problem. However, the issuance of FGR and the contingency constraints are not taken into account in [18]; in addition, the computation approach introduced in [18] (by referring to PATH solver[19]) is not feasible if without the access to the commercial solver.

In this paper, an equilibrium model for the joint FTR/FGR market is established, in which all TR bidders can strategically bid for any combination of FTR obligations and FTR options as well as FGRs. Besides, both the thermal capacity constraints and the contingency constraints are taken into account in the SFE test. To facilitate the computation, a convex quadratic programming, which is equivalent to the resulting equilibrium model, is also derived. In the solution analysis, some analysis are given out about the essence of the conjectured transmission price function being used to model TR bidders' strategic actions and the impacts on market clearing outcome due to the contingency constraints.

The rest of this paper is organized as follows. In Section 2, the notations used in this paper are listed; the mathematical formulation of the proposed model is formulated in Section 3; the numerical results are illustrated in Section 4; Concluding remarks are provided in Section 5.

2 Notation

In order to facilitate the presentation, the notations used in this paper are listed in this section.

v	Index for TR bidders
l	Index for TRs (including FTRs and FGRs)
x	Index for FTR obligations
y	Index for FTR options
z	Index for FGRs
c	Index for contingencies
i	Index for nodes in the system
ij	Index for transmission line from node i to node j in the systems
V	Sets of TR bidders
F_v	Sets of FTR obligations in bidder v's portfolio
O_v	Sets of FTR options in TR bidder v's portfolio
G_v	Sets of FGRs in TR bidder v's portfolio
I	Sets of nodes in the system
C	Sets of contingencies
K	Sets of transmission lines
π_v	TR bidder v's profit
π_{ISO}	The ISO's surplus coming from congestion revenue
$\beta_{v,x}$	The bid price of FTR obligation x by TR bidder v
$\beta_{v,y}$	The bid price of FTR option y by TR bidder v
$\beta_{v,z}$	The bid price of FGR z by TR bidder v
$\tau_{v,x}$	The x-th FTR obligation in TR bidder v's TR portfolio
$\tau_{v,y}$	The y-th FTR option in TR bidder v's TR portfolio
$\tau_{v,z}$	The z-th FGR in TR bidder v's TR portfolio
$p_{v,x}$	The transmission capacity allocated to TR bidder v corresponding to FTR obligation x
$p_{v,y}$	The transmission capacity allocated to TR bidder v corresponding to FTR option y
$p_{v,z}$	The transmission capacity allocated to TR bidder v corresponding to FGR z
$w_{v,x}$	The congestion price of FTR obligation x seen by TR bidder v
$w_{v,y}$	The congestion price of FTR option y seen by TR bidder v
$w_{v,z}$	The shadow price of flowgate associated with FGR z seen by TR bidder v

$A_{v,x}$ Transmission price response parameter conjectured by v corresponding to FTR obligation x

$A_{v,y}$ Transmission price response parameter conjectured by v corresponding to FTR option y

$A_{v,z}$ Transmission price response parameter conjectured by v corresponding to FGR z

F_k The power flow on flowgate k

$J_{k,x}$ The power flow through the flowgate k due to 1MW transmitted along FTR obligation x's path

$J_{k,y}$ The power flow through the flowgate k due to 1MW transmitted along FTR obligation y's path

$J_{k,z}$ The power flow through the flowgate k due to 1MW transmitted along FGR z's path

G_z^{k+} The incident element between FGR z and flowgate k in positive direction

G_z^{k-} The incident element between FGR z and flowgate k in negative direction

$\gamma_{v,x}$ The Lagrange multiplier associated with the up limit constraint for FTR obligation x bided by bidder v

$\gamma_{v,y}$ The Lagrange multiplier associated with the up limit constraint for FTR option y bided by TR bidder v

$\gamma_{v,z}$ The Lagrange multiplier associated with the up limit constraint for FGR bided by TR bidder v

λ_k^+ The Lagrange multiplier associated with flowgate k's positive transmission constraint under normal condition

λ_k^+ The Lagrange multiplier associated with flowgate k's negative transmission constraint under normal condition

$(\overline{\square})$ Maximum value for $(\square)$

$(\overline{\square})^c$ Maximum value for $(\square)$ under contingency c

$(\square)^*$ Value for $(\square)$ in equilibrium

$a \perp b$ Complementarity condition between a and b

3 Mathematical Model

There are two kinds of market participants in this model, i.e. TR bidders and ISO. Every TR bidder entering into the TR market chooses to buy or sell the relevant FTR obligations, FTR options or FGRs to maximize his own profit; and the aim of ISO is to allocate limited transmission capacity to maximize the revenue from the congestion charges.

3.1 The TR Bidder Problem

In the TR market, each TR bidder submits bids for the relevant TRs portfolio which he wants to obtain. Different from [18] where only the FTR profit component is considered, the FGR profit component is affiliated into each bidder's payoff function to make the model more practical. Therefore, the payoff π_v of any TR bidder $v \in V$ for holding a portfolio of TRs can be represented as follows:

$$\begin{aligned} max\ \pi_v = & \sum_{x \in F_v} \{\beta_{v,x}\tau_{v,x} - w_{v,x}\tau_{v,x}\} \\ & + \sum_{y \in O_v} \{\beta_{v,y}\tau_{v,y} - w_{v,y}\tau_{v,y}\} \\ & + \sum_{z \in G_v} \{\beta_{v,z}\tau_{v,z} - \mu_{v,z}\tau_{v,z}\} \end{aligned} \quad (1)$$

where $\beta_{v,x}\tau_{v,x}$, $w_{v,x}\tau_{v,x}$ are separately the expected revenue and the purchasing cost (congestion cost) for holding FTR obligation $\tau_{v,x}$, so $\beta_{v,x}\tau_{v,x} - w_{v,x}\tau_{v,x}$ reflects the net profit from FTR obligation $\tau_{v,x}$. In the same way, $\beta_{v,y}\tau_{v,y} - w_{v,y}\tau_{v,y}$ and $\beta_{v,z}\tau_{v,z} - w_{v,z}\tau_{v,z}$ are separately net profits from the FTR option $\tau_{v,y}$ and the FGR $\tau_{v,z}$. Note that $w_{v,x}$, $w_{v,y}$, $w_{v,z}$ are all endogenous variables faced by TR bidder v, which can be substituted by the conjectured transmission price function by which TR bidder v is modeled to have the ability to manipulate the corresponding transmission prices:

$$w_{v,x} - w_{v,x}^* - A_{v,x}(\tau_{v,x} - \tau_{v,x}^*) = 0 \quad \forall x \in F_v \quad (2)$$

$$w_{v,y} - w_{v,y}^* - A_{v,y}(\tau_{v,y} - \tau_{v,y}^*) = 0 \quad \forall y \in O_v \quad (3)$$

$$w_{v,z} - w_{v,z}^* - A_{v,z}(\tau_{v,z} - \tau_{v,z}^*) = 0 \quad \forall z \in G_v \quad (4)$$

where $A_{v,x}, A_{v,y}, A_{v,z} \geq 0$ are the conjectured price parameters separately for FTR obligation x, FTR option y and FGR z by TR bidder v. Though the conjectured parameters could be properly estimated according to historical market clearing results[20,21], it is out of the scope of this paper. For perfect competition market, $A_{v,x}, A_{v,y}, A_{v,z} = 0$. Moreover, the decision variables $\tau_{v,x}, \tau_{v,y}, \tau_{v,z} \geq 0$ should be less than their respective maximization limits:

$$\tau_{v,x} \leq \overline{\tau}_{v,x} \leftrightarrow (\gamma_{v,x}) \quad \forall x \in F_v \quad (5)$$

$$\tau_{v,y} \leq \overline{\tau}_{v,y} \leftrightarrow (\gamma_{v,y}) \quad \forall y \in O_v \quad (6)$$

$$\tau_{v,z} \leq \overline{\tau}_{v,z} \leftrightarrow (\gamma_{v,z}) \quad \forall z \in G_v \quad (7)$$

3.2 ISO's Problem

As the auctioneer of TR market, ISO first gathers all the bids from all the TR bidders; afterwards, it allocates the scarce transmission capacity to get as much congestion rent as possible. The objective function for ISO is addressed as below:

$$max\ \pi_{ISO} = \sum_{v}(\sum_{x\in F_v} w^*_{v,x}p_{v,x} + \sum_{y\in O_v} w^*_{v,y}p_{v,y} + \sum_{z\in G_v} w^*_{v,z}p_{v,z}) \quad (8)$$

where $w^*_{v,x}p_{v,x}, w^*_{v,y}p_{v,y}$ and $w^*_{v,z}p_{v,z}$ are separately the congestion charges from FTR obligation, FTR option and FGR. Unlike the TR bidder problem, the transmission prices $w^*_{v,x}, w^*_{v,y}, w^*_{v,z}$ are all exogenous variables in the ISO's optimization problem, which implies that ISO thinks he cannot influence the transmission prices.

Different from [18] where only thermal capacity constraints are considered in SFT, the contingency constraints are also taken into account in this paper to ensure that the congestion charges collected by ISO are enough to cover the FTR credits under normal or predicted contingency conditions.

$$\sum_{v}(\sum_{x\in F_v} J_{k,x}p_{v,x} + \sum_{y\in O_v} Max(0,J_{k,y})p_{v,y} + \sum_{z\in G_v} G^{k+}_z p_{v,z}) \le \overline{F}_k \ \leftrightarrow\ (\lambda^+_k) \qquad k\in K \quad (9)$$

$$\sum_{v}(-\sum_{x\in F_v} J_{k,x}p_{v,x} + \sum_{y\in O_v} Max(0,-J_{k,y})p_{v,y} + \sum_{z\in G_v} G^{k-}_z p^k_{v,z}) \le \overline{F}_k \ \leftrightarrow\ (\lambda^-_k) \qquad k\in K \quad (10)$$

$$\sum_{v}(\sum_{x\in F_v} J^c_{k,x}p_{v,x} + \sum_{y\in O_v} Max(0,J^c_{k,y})p_{v,y} + \sum_{z\in G_v} G^{k+}_z p_{v,z}) \le \overline{F}^c_k \leftrightarrow (\lambda^{c+}_k)\ \ k\in K, c\in C \quad (11)$$

$$\sum_{v}(-\sum_{x\in F_v} J^c_{k,x}p_{v,x} + \sum_{y\in O_v} Max(0,-J^c_{k,y})p_{v,y} + \sum_{z\in G_v} G^{k-}_z p_{v,z}) \le \overline{F}^c_k \leftrightarrow (\lambda^{c-}_k)\ \ k\in K, c\in C \quad (12)$$

where expressions (9-10) represent the thermal capacity constraints under normal condition; and expressions (11-12) are the contingency constraints under the contingency condition c. Note that the counter flow induced by FTR option is neglected due to the exemption of the liability.

3.3 Market Clearing Condition

In order to balance the required TRs by TR bidders and the transmission capacities approved by ISO, the related market clearing conditions should also be satisfied:

$$p_{v,x} = \tau_{v,x} \quad \forall v\in V, x\in F_v \quad (13)$$

$$p_{v,y} = \tau_{v,y} \quad \forall v\in V, y\in O_v \quad (14)$$

$$p_{v,z} = \tau_{v,z} \quad \forall v\in V, z\in G_v \quad (15)$$

3.4 Equilibrium Complementarity Model

It could be easily verified that all TR bidders' problems and ISO's problem belong to convex programming problem, which means any solution satisfying the first order KKT conditions is also a global optimization solution.

Therefore, the equilibrium problem can be defined through gathering all market participants' KKT conditions along with the related market clearing conditions. The corresponding equilibrium complementarity conditions can be expressed as below:

For $\tau_{v,x}$, $\forall v\in V, x\in F_v$

$$0 \ge \beta_{v,x} - (w^*_{v,x} + A_{v,x}\tau_{v,x}) - \gamma_{v,x} \perp \tau_{v,x} \ge 0 \quad (16)$$

For $\tau_{v,y}$, $\forall v\in V, y\in O_v$

$$0 \ge \beta_{v,y} - (w^*_{v,y} + A_{v,y}\tau_{v,y}) - \gamma_{v,y} \perp \tau_{v,y} \ge 0 \quad (17)$$

For $\tau_{v,z}$, $\forall v\in V, z\in G_v$

$$0 \ge \beta_{v,z} - (w^*_{v,z} + A_{v,z}\tau_{v,z}) - \gamma_{v,z} \perp \tau_{v,z} \ge 0 \quad (18)$$

For $\gamma_{v,x}$, $\forall v\in V, x\in F_v$

$$0 \ge \tau_{v,x} - \overline{\tau}_{v,x} \perp \gamma_{v,x} \ge 0 \quad (19)$$

For $\gamma_{v,y}$, $\forall v\in V, y\in O_v$

$$0 \ge \tau_{v,y} - \overline{\tau}_{v,y} \perp \gamma_{v,y} \ge 0 \quad (20)$$

For $\gamma_{v,z}$, $\forall v\in V, z\in G_v$

$$0 \ge \tau_{v,z} - \overline{\tau}_{v,z} \perp \gamma_{v,z} \ge 0 \quad (21)$$

For $p_{v,x}$, $\forall v\in V, x\in F_v$

$$0 \geq w^*_{v,x} - \sum_{k\in K} J_{k,x}(\lambda^+_k - \lambda^-_k) - \sum_{c\in C}(\sum_{k\in K} J^c_{k,x}(\lambda^{c+}_k - \lambda^{c-}_k)) \perp p_{v,x} \geq 0$$

(22)

For $p_{v,y}$, $\forall v \in V, y \in O_v$

$$0 \geq w^*_{v,y} - \sum_{k\in K}(Max(0, J_{k,y})\lambda^+_k + Max(0,-J_{k,y})\lambda^-_k) - \sum_{c\in C}(\sum_{k\in K}(Max(0, J^c_{k,y})\lambda^{c+}_k + Max(0,-J^c_{k,y})\lambda^{c-}_k)) \perp p_{v,y} \geq 0$$

(23)

For $p_{v,z}$, $\forall v \in V, z \in G_v$

$$0 \geq w^*_{v,z} - \sum_{k\in K}(G^{k+}_z\lambda^+_k + G^{k-}_z\lambda^-_k) - \sum_{c\in C}(\sum_{k\in K}(G^{k+}_z\lambda^{c+}_k + G^{k-}_z\lambda^{c-}_k)) \perp p_{v,z} \geq 0$$

(24)

For λ^+_k, $k \in K$

$$0 \geq \sum_v(\sum_{x\in F_v} J_{k,x}p_{v,x} + \sum_{y\in O_v} Max(0,J_{k,y})p_{v,y} + \sum_{z\in G_v} p_{v,z}) - \overline{F}_k \perp \lambda^+_k \geq 0$$

(25)

For λ^-_k, $k \in K$

$$0 \geq \sum_v(-\sum_{x\in F_v} J_{k,x}p_{v,x} + \sum_{y\in O_v} Max(0,-J_{k,y})p_{v,y} + \sum_{z\in G_v} p_{v,z}) - \overline{F}_k \perp \lambda^-_k \geq 0$$

(26)

For λ^{c+}_k, $k \in K$

$$0 \geq \sum_v(\sum_{x\in F_v} J^c_{k,x}p_{v,x} + \sum_{y\in O_v} Max(0,J^c_{k,y})p_{v,y} + \sum_{z\in G_v} G^{k+}_z p_{v,z}) - \overline{F}^c_k \perp \lambda^{c+}_k \geq 0$$

(27)

For λ^{c-}_k, $k \in K$

$$0 \geq \sum_v(-\sum_{x\in F_v} J^c_{k,x}p_{v,x} + \sum_{y\in O_v} Max(0,-J^c_{k,y})p_{v,y} + \sum_{z\in G_v} G^{k-}_z p_{v,z}) - \overline{F}^c_k \perp \lambda^{c-}_k \geq 0$$

(28)

For $w_{v,x}$, $\forall v \in V, x \in F_v$,

$$p_{v,x} = \tau_{v,x}$$

(29)

For $w_{v,y}$, $\forall v \in V, y \in O_v$

$$p_{v,y} = \tau_{v,y}$$

(30)

For $w_{v,z}$, $\forall v \in V, z \in G_v$

$$p_{v,z} = \tau_{v,z}$$

(31)

3.5 The Solution Approach

Apparently, the resulting equilibrium problem is a standard mixed linear complementarity problem (MLCP), which could be efficiently implemented by the PATH solver[19]. In addition, the existence and uniqueness of solution for the MLCP could also be guaranteed under some conditions[22].

In this paper, a convex quadratic programming problem, which is just equivalent to the proposed equilibrium model (can be verified by comparing the resulting equilibrium conditions with the KKT conditions of the quadratic programming problem (32-33)), is presented as below:

$$max\ \pi = \sum_v(\sum_{x\in F_v}(\beta_{v,x} - \frac{A_{v,x}}{2}p_{v,x})p_{v,x} + \sum_{y\in O_v}(\beta_{v,y} - \frac{A_{v,y}}{2}p_{v,y})p_{v,y} + \sum_{z\in G_v}(\beta_{v,z} - \frac{A_{v,z}}{2}p_{v,z})p_{v,z}) \quad (32)$$

$$s.t. \quad p_{v,l} \leq \overline{p}_{v,l} \leftrightarrow (\gamma_{v,l})\ \forall v, l = x, y, z$$

The thermal capacity constraints (9-10)

The contingency constraints (11-12) (33)

In this paper, we compute the above quadratic programming (32-33) by referring to the quadprog function in the Matlab environment.

3.6 The Analysis for the Equilibrium Model

Two problems are discussed in this section. The first one is why conjectured transmission price function could be used to model the bidding strategies in the TR market? And the second one is how contingency constraints impact on the market clearing results of the TR market?

Let's first look at formula (16): when $\tau_{v,x} > 0$, then $w^*_{v,x}$ can be expressed as follows:

$$w^*_{v,x} = \beta_{v,x} - A_{v,x}\tau_{v,x} - \gamma_{v,x} \quad (34)$$

According to equation (34), we can find that the auction price $w^*_{v,x}$ for FTR obligation $\tau_{v,x}$ is composed by three components: $\beta_{v,x}$ is the original expected marginal revenue; $A_{v,x}\tau_{v,x}$ is the decreased marginal revenue component incurred by the TR bidders' bidding strategies; while $\gamma_{v,x}$ is the opportunity cost incurred from the binding maximum amount constraint. Thus, $\beta_{v,x} - A_{v,x}\tau_{v,x}$ is just the actual bid price submitted to the ISO,

which embodies the pricing-depressing behaviors in the TR market. As a result of reduced auction prices, the TR bidders could capture much profit from ISO.

Moreover, the auction prices corresponding to FTR option and FGR can also be decomposed and analyzed in the same way (see (35-36)).

when $\tau_{v,y} > 0$, $w^*_{v,y} = \beta_{v,y} - A_{v,y}\tau_{v,y} - \gamma_{v,y}$ (33)

when $\tau_{v,z} > 0$, $w^*_{v,z} = \beta_{v,z} - A_{v,z}\tau_{v,z} - \gamma_{v,z}$ (34)

Let's further look at formula (22), when $p_{v,x} > 0$, then $w^*_{v,x}$ can be expressed as follows:

$$w^*_{v,x} = \sum_{k\in K} J_{k,x}(\lambda^+_k - \lambda^-_k) + \sum_{c\in C}(\sum_{k\in K} J^c_{k,x}(\lambda^{c+}_k - \lambda^{c-}_k)) \quad (35)$$

According to equation (35), we can find that the auction price $w^*_{v,x}$ for FTR obligation $\tau_{v,x}$ can be expressed by the linear combination of the shadow prices associated with the related thermal capacity constraints and the contingency constraints.

In the same way, the auction prices $w^*_{v,y}$ and $\mu^*_{v,z}$ for FTR option and FGR respectively could also be expressed in terms of the linear combination of the corresponding shadow prices (see (36-37)).

when $\tau_{v,y} > 0$,

$$w^*_{v,y} = \sum_{k\in K}(Max(0, J_{k,y})\lambda^+_k + Max(0, -J_{k,y})\lambda^-_k) + \sum_{c\in C}(\sum_{k\in K}(Max(0, J^c_{k,y})\lambda^{c+}_k) + Max(0, -J^c_{k,y})\lambda^{c-}_k)) \quad (36)$$

when $\tau_{v,z} > 0$,

$$w^*_{v,z} = \sum_{k\in K}(G^{k+}_z\lambda^+_k + G^{k-}_z\lambda^-_k) + \sum_{c\in C}(\sum_{k\in K}(G^{k+}_z\lambda^{c+}_k + G^{k-}_z\lambda^{c-}_k)) \quad (37)$$

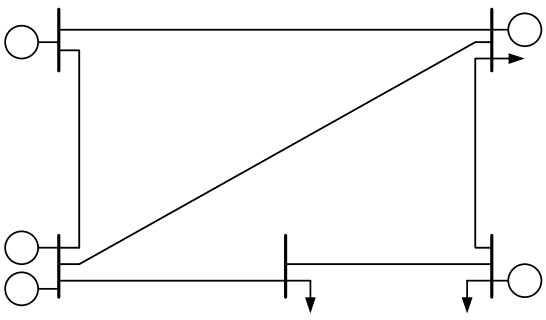

Fig.1. PJM-5-bus system configuration

4 Case study

In this section, the proposed equilibrium model is illustrated using the standard PJM-5-bus system as shown in Fig.1. The test system can be roughly divided into two areas, one is the generation center consisting of node 1 and node 3, and the other is load center consisting of node 2, node 4 and node 5. There are totally three flowgates separately denoted as line1 (node 1 to node 2), line2 (node 1 to node 3) and line3 (node 3 to node 4), and the transmission lines parameters are listed in Table 1.

Table 1 The transmission parameters of the PJM 5-bus system

k	i	j	x_{ij} (%)	$\overline{F}_{ij}$ (MW)	$\overline{F}_{ij}{}^c$ (MW)
1	1	2	0.0281	380	480
2	1	3	0.0061	400	480
3	1	4	0.0304	-	-
4	2	5	0.0108	-	-
5	3	4	0.0297	240	330
6	4	5	0.0297	-	-

4.1 Competitive Environment

In this scenario, some cases are simulated in the competitive environment. All the TR bidders are price takers under the competitive condition, which implies they believe that they can not manipulate the TR prices to earn more profit by means of their strategic bidding behaviors($A_{v,x}, A_{v,y}, A_{v,z} = 0$).

Table 2 The bid data for transmission rights

case 1					
v	l (x,y,z)	i	j	$\beta_{v,l}$(S/MW)	$\overline{\tau}_{v,l}$(MW)
1	$t^x_{1,2}$	1	2	20	300
	$t^x_{3,2}$	3	2	23	350
2	$t^x_{1,4}$	1	4	25	300
	$t^x_{3,4}$	3	4	35	120
3	$t^x_{3,5}$	3	5	25	300
	$t^x_{5,4}$	5	4	18	100
case 2					
v	l (x,y,z)	i	j	$\beta_{v,l}$(S/MW)	$\overline{\tau}_{v,l}$(MW)
1	$t^x_{1,2}$	1	2	20	300
	$t^x_{3,2}$	3	2	23	350
2	$t^x_{1,4}$	1	4	25	300
	$t^x_{3,4}$	3	4	35	120
3	$t^x_{3,5}$	3	5	25	300
	$t^y_{5,4}$	5	4	18	100
case 3					

v	l (x,y,z)	i	j	$\beta_{v,l}$(S/MW)	$\bar{\tau}_{v,l}$(MW)
1	$t^x_{1,2}$	1	2	20	300
	$t^x_{3,2}$	3	2	23	350
2	$t^x_{1,4}$	1	4	25	300
	$t^z_{3,4}$	3	4	33	80
3	$t^x_{3,5}$	3	5	25	300
	$t^y_{5,4}$	5	4	18	100

$t^x_{i,j}$: FTR obligation between node i and node j

$t^y_{i,j}$: FTR option between node i and node j

$t^z_{i,j}$: FGR associated with flowgate ij

4.1.1 Case 1:

In this case, all the TR bidders submit their bids only for FTR obligations. The detailed bid data are listed in Table 2, and the market clearing results can be found in Table 3.

As seen from Table 3, bidder 1's profit is zero because the market clearing prices (MCPs) of all his requested TRs are equal to the corresponding bid prices; bidder 2's profit is \$540.5 coming from $t^x_{3,4}$. Note that bidder 3 earns the biggest profit (\$1238.4) among all the TR bidders, being awarded all the desired quantity of $t^x_{5,4}$ at the lowest MCP in the TR market. In fact, $t^x_{5,4}$ is the only transaction inducing the counter flow on the congested flowgate1 in the TR market, which is equivalent to the augment of transmission capacity for flowgate1 in the prevailing direction of flow. Therefore, the MCP of $t^x_{5,4}$ is the lowest auction price in the TR market and the ISO issues bidder 3 the desired quantity of $t^x_{5,4}$.

4.1.2 Case 2:

In order to compare FTR obligation instance with FTR option instance, bidder 3 is assumed to bid for FTR option $t^y_{5,4}$ instead of FTR obligation $t^x_{5,4}$ in this case. The detailed bid data and the market clearing outcomes are given in Table 2 and Table 3 respectively.

Because the counter flow incurred by $t^y_{5,4}$ on the congested flowgate1 is not considered in the TR market, the MCP (\$9.7/MW) of $t^y_{5,4}$ is higher than that (\$5.6/MW) of $t^x_{5,4}$ in case 1. Accordingly, TR bidder 3's profit goes down from \$1238.4 (case 1) to \$1029.5. Moreover, bidder 2 gains more profit than that in case 1 due to the decreased MCP of $t^x_{3,4}$.

4.1.3 Case 3:

When bidder 2 bids for FGR $t^z_{3,4}$ instead of FTR obligation $t^x_{3,4}$ (the bid data can be found in Table 2), the simulation results are provided in Table 3.

As illustrated in Table 3, the flowgate3 is not congested under normal condition, and the MCP of $t^z_{3,4}$ is determined by the shadow price of the related transmission constraint under contingency condition. In addition, it can be easily tested that the awarded quantity of $t^z_{3,4}$ remains constant within the bid price interval [\$32.5/MW, \$34.1/MW] while keeping other TR bids constant. Therefore, if the marginal revenue of $t^z_{3,4}$ expected by TR bidder 2 is still \$33/MW and the submitted bid price for $t^z_{3,4}$ is equal to \$32.5/MW, then TR bidder 2 can earn extra profit equal to \$35.4 (=(33-32.5)*70.8), much more than \$0/MW in this case.

Moreover, some observations could be obtained from the above cases: any bid with the MCP bigger than its bid price is rejected; any bid with the MCP lower than its bid price is totally accepted; any bid with the MCP equal to its bid price is partially accepted. As a matter of fact, these findings still hold in the imperfect competition environment.

Table 3. The market clearing results in scenario A

case 1				
v	l (x,y,z)	Awarded Quantities $P_{v,l}$(MW)	MCP $w_{v,l}$ (\$/MW)	Bidders' Profits π_v(\$)
1	$t^x_{1,2}$	74.4	20	0
	$t^x_{3,2}$	262.4	23	
2	$t^x_{1,4}$	0	25	540.5
	$t^x_{3,4}$	120	30.5	
3	$t^x_{3,5}$	252.4	25	1238.4
	$t^x_{5,4}$	100	5.6	
Shadow Prices (\$/MW)	$\lambda_1^+ = 14.8, \lambda_2^- = 1.3, \lambda_3^{c+} = 38.3,$ others=0			
ISO's Surplus π_{ISO} (\$)	18055.9			
case 2				

v	l (x,y,z)	Awarded Quantities $P_{v,l}$(MW)	MCP $w_{v,l}$ ($/MW)	Bidders' Profits π_v($)
1	$t^x_{1,2}$	45.1	20	0
	$t^x_{3,2}$	197.5	23	
2	$t^x_{1,4}$	13.2	25	840
	$t^x_{3,4}$	120	28	
3	$t^x_{3,5}$	300	24.3	1029.5
	$t^y_{5,4}$	100	9.7	
Shadow Prices ($/MW)	$\lambda_1^+ = 16.5, \lambda_2^- = 1.7, \lambda_3^{c+} = 34.2,$ others=0			
ISO's Surplus π_{ISO} ($)	17406.8			
case 3				
v	l (x,y,z)	Awarded Quantities $P_{v,l}$(MW)	MCP $w_{v,l}$ ($/MW)	Bidders' Profits π_v($)
1	$t^x_{1,2}$	0	20	0
	$t^x_{3,2}$	274.3	23	
2	$t^x_{1,4}$	16.1	25	0
	$t^z_{3,4}$	70.8	33	
3	$t^x_{3,5}$	300	24.0	1154.1
	$t^y_{5,4}$	100	9.4	
Shadow Prices ($/MW)	$\lambda_1^+ = 18.5, \lambda_2^- = 0.5, \lambda_3^{c+} = 33,$ others=0			
ISO's Surplus π_{ISO} ($)	17197.3			

4.2 Oligopolistic Environment

In the oligopolistic environment, the TR bidders are considered as price makers rather than price takers ($A_{v,x}, A_{v,y}, A_{v,z} > 0$). Due to the lack of historical data, all the conjectured parameters in this scenario are assumed to be equal to 0.03 for simplicity.

4.2.1 Case 4:

In this case, the instance with the same bid data as case 1 is tested, and the results can be found in Table 4.

Obviously, the market clearing results in case 4 are quite different from those in case 1. Most TRs' MCPs are depressed down due to TR bidders' strategic withholding behaviors, only the MCP of $t^x_{5,4}$ increases because the mitigation of flowgate1's congestion degree reduces the effects of the counter flow in TR market. At the same time, the ISO's surplus decreases from \$18055.9 to \$14162.7, while each TR bidder's profit increases greatly accordingly. This represents the profit transfer from the ISO to the TR bidders.

4.2.2 Case 5:

Let's go on to investigate the instance with the inclusion of FTR option in the oligopolistic environment. The bid data are the same as case 2, and the numerical results can be found in Table 4.

Similar to the scenario A, the MCP of FTR option $t^y_{5,4}$ (\$9.1/MW) is bigger than that of $t^x_{5,4}$ (\$6.7/MW) in case 4 also due to the ignorance of the counter flow. On the other hand, the TR bidders' strategic bidding behaviors depress all the issued TRs' MCPs. The ISO's surplus declines from \$17406.8 (case 2) down to \$14217.8; while the total TR bidders' profits boost from \$1869.5 (case 2) to \$4779.3.

4.2.3 Case 6:

When FGRs are taken into account in the TR market (using the same bid data as case 3), the results are given in Table 4.

As expected, the decreased TRs' MCPs increase each TR bidder's profits while cutting down the ISO's profit compared to case 3. The MCP of $t^z_{3,4}$ (\$31.1/MW) is set by the shadow price of flowgate3 in the positive direction under the predicted contingency condition.

It should be noted that the actual bid prices of the awarded TRs in the oligopolistic scenario are not the same as the corresponding original expected marginal revenue given in Table 2. Take $t^z_{3,4}$ in this case (the awarded quantity is 62.9MW, lower than its up limit 80MW) for example: the original expected marginal revenue and the decreased marginal revenue of $t^z_{3,4}$ are equal to 33.0/MW and \$1.9/MW (=0.03*62.9) respectively (according to the formula (34)); so the actual bid price of $t^z_{3,4}$ is \$31.1/MW, just equal to its MCP in the TR market.

Table 4. The market clearing results in scenario B

case 1				
V	l (x,y,z)	Awarded Quantities $P_{v,l}$ (MW)	MCP $w_{v,l}$ ($/MW)	Bidders' Profits π_v ($)
1	$t^x_{1,2}$	165.1	15.0	2085.5
	$t^x_{3,2}$	205.6	16.8	
2	$t^x_{1,4}$	31.3	24.1	1127.9
	$t^x_{3,4}$	120	25.8	
3	$t^x_{3,5}$	192.1	19.2	2239.0
	$t^x_{5,4}$	100	6.7	
Shadow Prices ($/MW)	$\lambda_1^+ = 8.8, \lambda_3^{c+} = 34.1,$ others=0			
ISO's Surplus π_{ISO} ($)	14162.7			
case 2				
V	l (x,y,z)	Awarded Quantities $P_{v,l}$ (MW)	MCP $w_{v,l}$ ($/MW)	Bidders' Profits π_v ($)
1	$t^x_{1,2}$	134.4	16.0	1530.1
	$t^x_{3,2}$	185.1	17.6	
2	$t^x_{1,4}$	61.2	23.2	1342.4
	$t^x_{3,4}$	120	24.8	
3	$t^x_{3,5}$	184.2	19.5	1906.8
	$t^y_{5,4}$	100	9.1	
Shadow Prices ($/MW)	$\lambda_1^+ = 11.0, \lambda_3^{c+} = 32.1,$ others=0			
ISO's Surplus π_{ISO} ($)	14217.8			
case 3				
v	l (x,y,z)	Awarded Quantities $P_{v,l}$ (MW)	MCP $w_{v,l}$ ($/MW)	Bidders' Profits π_v ($)
1	$t^x_{1,2}$	140.2	15.8	1667.4
	$t^x_{3,2}$	189.5	17.3	
2	$t^x_{1,4}$	82.0	22.5	320.6
	$t^z_{3,4}$	62.9	31.1	
3	$t^x_{3,5}$	196.2	19.1	2071.9
	$t^y_{5,4}$	100	8.8	
Shadow Prices ($/MW)	$\lambda_1^+ = 11.1, \lambda_3^{c+} = 31.1,$ others=0			
ISO's Surplus π_{ISO} ($)	13935.3			

5 Conclusion

An equilibrium model for the joint FTR/FGR auction market is established in this paper. In order to hedge the risk coming from the volatility of congestion charges, the TR bidders can strategically bid for any combination of FTR obligations, FTR options and FGRs in this model. Besides the thermal capacity constraints, contingency constraints are also taken into account in the SFT to make the proposed model more practical. The equilibrium model is formulated in terms of a standard mixed linear complementarity problem.

On the other hand, the corresponding convex quadratic programming problem, which is totally equivalent to the resulting equilibrium model, is also given out in this paper. In practice, the proposed equilibrium model can be efficiently applied to the evaluation of the market power for the large-scale hybrid TR market; and the existence and uniqueness of the resulting equilibria could also be guaranteed.

Some useful observations can be found from the numerical cases. In this joint FTR/FGR market, any type of TR bid will be fully or partially accepted if its bid price is greater than or equal to the relevant auction price; while any type of TR bid will be rejected if its bid price is lower than relevant auction price. Due to the disregard of the counterflow on congested flowgates, the auction price of FTR option is usually bigger than that of FTR obligation along with the same path. The auction price of any FGR is determined by the shadow price of active transmission constraint under normal or contingency condition. Under the oligopolistic environment, all TR bidders strategically cut down the bid prices for their interested combination of TRs, and this could greatly increase their respective profits while reduce ISO's surplus.

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