Network Optimization by Generalized Methodology

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Abstract: **-** The design process for analog network design is formulated on the basis of the optimum control theory. A special control vector is defined to redistribute the compute expensive between a network analysis and a parametric optimization. This redistribution permits the minimization of a computer time. The problem of the minimal-time network design can be formulated in this case as a classical problem of the optimal control for some functional minimization. The principal difference between the new approach and before elaborated generalized methodology is presented. This difference is based on a higher level of the problem generalization. In this case the structural basis of different design strategies is more complete and this circumstance gives possibility to obtain a great value of computer time gain. Numerical results demonstrate the efficiency and perspective of the proposed approach.

Key-Words: - Time-optimal design algorithm, control theory formulation, general methodology.

1 Introduction

The computer time reduction of a large system design is one of the sources of the total quality design improvement. This problem has a great significance because it has a lot of applications, for example on VLSI electronic circuit design. Any traditional system design strategy includes two main parts: the mathematical model of the physical system that can be defined by the algebraic equations or differential-integral equations and optimization procedure that achieves the optimum point of the design objective function. In limits of this conception it is possible to change optimization strategy and use the different models and different methods of analysis but in each step of the circuit optimization process there are a fixed number of the equations of the mathematical model and a fixed number of the independent parameters of the optimization procedure.

There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used successfully for this purpose [1]-[2]. Other approach to reduce the amount of computational required for both linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in [3], or by nodes tearing as in [4] and jointly with direct solution algorithms gives the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macromodel representation [5]. Other approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized in [6] for the iterated timing analysis and circuit simulation. Optimization technique that is used for the circuit optimization and design, exert a very strong influence on the total necessary computer time too. The numerical methods are developed both for the unconstrained and for the constrained optimization [7] and will be improved later on. The practical aspects of these methods were developed for the electronic circuits design with the different optimization criterions [8]-[9]. The fundamental problems of the development, structure elaboration, and adaptation of the automation design systems have been examine in some papers [10]-[13].

The above described system design ideas can be named as the traditional approach or the traditional strategy because the analysis method is based on the Kirchhoff laws.

The other formulation of the circuit optimization problem was developed on heuristic level some decades ago [14]. This idea was based on the Kirchhoff laws ignoring for all the circuit or for the circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization [15] and for the synthesis of high-performance analog circuits [16] in extremely case, when the total system model was eliminated. The authors of the last papers affirm that the design time was reduced significantly. This last idea can be named as the modified traditional design strategy.

Nevertheless all these ideas can be generalized to reduce the total computer design time for the system design. This generalization can be done on the basis of the control theory approach and includes the special control function to control the design process. This approach consists of the reformulation of the total design problem and generalization of it to obtain a set of different design strategies inside the same optimization procedure [17]. The number of the different design strategies, which appear in the generalized theory, is equal to 2^M for the constant value of all the control functions, where M is the number of dependent parameters. These strategies serve as the structural basis for more strategies construction with the variable control functions. The main problem of this new formulation is the unknown optimal dependency of the control function vector that satisfies to the time-optimal design algorithm.

However, the developed theory [17] is not the most general. In the limits of this approach only initially dependent system parameters can be transformed to the independent but the inverse transformation is not supposed. The next more general approach for the system design supposes that initially independent and dependent system parameters are completely equal in rights, i.e. any system parameter can be defined as independent or dependent one. In this case we have more vast set of the design strategies that compose the structural basis and more possibility to the optimal design strategy construct.

2 Problem Formulation

In accordance with the new system design methodology [17] the design process can be defined as the problem of the cost function $C(X)$ minimization for $X \in R^N$ by the optimization procedure and by the analysis of the modified electronic system model. The optimization procedure can be determined in continuous form as:

$$\frac{dx_i}{dt} = f_i(X,U), \quad i=1,2,...,N \tag{1}$$

The modified electronic system model can be expressed in the next form:

$$(1-u_j)g_j(X) = 0, \quad j=1,2,\ldots,M \tag{2}$$

where $N=K+M$, K is the number of independent system parameters, M is the number of dependent system parameters, X is the vector of all variables $X = (x_1, x_2, \ldots, x_K, x_{K+1}, x_{K+2}, \ldots, x_N)$; U is the vector of control variables $U = (u_1, u_2, \ldots, u_M)$; $u_j \in \Omega$; $\Omega = \{0;1\}$.

The functions of the right hand part of the system (1) depend on the concrete optimization algorithm and, for instance, for the gradient method are determined as:

$$f_i(X,U) = -b\frac{\delta}{\delta x_i}\left\{C(X) + \frac{1}{\varepsilon}\sum_{j=1}^{M}u_j g_j^2(X)\right\} \tag{3}$$

for $i = 1,2,\ldots,K$,

$$f_i(X,U) = -b \cdot u_{i-K}\frac{\delta}{\delta x_i}\left\{C(X) + \frac{1}{\varepsilon}\sum_{j=1}^{M}u_j g_j^2(X)\right\} + \frac{(1-u_{i-K})}{dt}\left\{-x'_i + \eta_i(X)\right\} \tag{3'}$$

for $i = K+1, K+2, \ldots, N$,

where b is the iteration parameter; the operator $\frac{\delta}{\delta x_i}$ hear and below means $\frac{\delta}{\delta x_i}\varphi(X) = \frac{\partial \varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M}\frac{\partial \varphi(X)}{\partial x_p}\frac{\partial x_p}{\partial x_i}$, x'_i is equal to $x_i(t-dt)$; $\eta_i(X)$ is the implicit function ($x_i = \eta_i(X)$) that is determined by the system (2), $C(X)$ is the cost function of the design process.

The problem of the optimal design algorithm searching is determined now as the typical problem of the functional minimization of the control theory. The total computer design time serves as the necessary functional in this case. The optimal or quasi-optimal problem solution can be obtained on the basis of analytical [18] or numerical [19]-[22] methods. By this formulation the initially dependent parameters for $i = K+1, K+2, ..., N$ can be transformed to the independent ones when $u_j=1$ and it is dependent when $u_j=0$. On the other hand the initially independent parameters for $i = 1,2,\dots, K$, are independent ones always.

We have been developed in the present paper the new approach that permits to generalize more the above described design methodology. We suppose now that all of the system parameters can be independent or dependent ones. In this case we need to change the equation (2) for the system model definition and the equation (3) for the right parts description.

The equation (2) defines the system model and is transformed now to the next one:

$$(1-u_i)g_j(X)=0 \tag{4}$$

$i = 1,2,\dots, N$ and $j \in J$

where J is the index set for all those functions $g_j(X)$ for which $u_i = 0$, $J = \{j_1, j_2, \dots, j_z\}$, $j_s \in \Pi$ with $s = 1, 2, \dots, Z$, Π is the set of the indexes from 1 to M, $\Pi = \{1, 2, \dots, M\}$, Z is the number of the equations that will be left in the system (4), $Z \in \{0, 1\dots, M\}$.

The right hand side of the system (1) is defined now as:

$$f_i(X,U)=-b\cdot u_i\frac{\delta}{\delta x_i}F(X,U) + \frac{(1-u_i)}{dt}\{-x_i(t-dt)+\eta(X)\} \tag{5}$$

for $i = 1,2,\dots, N$,

where $F(X,U)$ is the generalized objective function and it is defined as:

$$F(X,U)=C(X)+\frac{1}{\varepsilon}\sum_{j\in\Pi\backslash J} g_j^2(X) \tag{6}$$

This new definition of the design process is more general than in [17]. It generalizes the methodology for the system design and produces more representative structural basis of different design strategies. The total number of the different design strategies, which compose the structural basis, is equal to $\sum_{i=0}^{M} C_{K+M}^{i}$. We expect the new possibilities to accelerate the design process in this case.

3 Numerical Results

New generalized methodology has been used for some non-linear electronic circuits optimization. The numerical results correspond to the integration of the system (1) with variable optimized step. The cost function $C(X)$ has been defined as a sum of squares of differences between before defined and current value of some node voltages.

3.1 Example 1

A simple two-nodes nonlinear passive circuit is presented in Fig. 1. The design procedure was realized by means of the new generalized methodology.

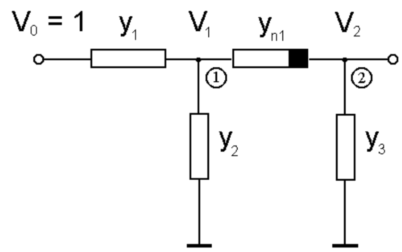

Fig. 1. Two-node circuit topology.

The nonlinear element has the following dependency: $y_{n1} = y_0 + b(V_1 - V_2)^2$. The vector X includes five components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$. The redefinition of the independent variables x_1, x_2, x_3 by squares gives us the possibility to solve the problem of feasibility. The model (4) of the circuit includes two equations (M=2). The functions $g_j(X)$ are defined by the next formulas:

$$\begin{aligned} g_1(X) &\equiv (1-x_4)x_1^2 - (x_4-x_5)\left(y_0 + a(x_4-x_5)^2\right) \\ &\quad - x_4x_2^2 = 0 \\ g_2(X) &\equiv (x_4-x_5)\left(y_0 + a(x_4-x_5)^2\right) - x_5x_3^2 = 0 \end{aligned} \tag{7}$$

The optimization procedure (1), (5) includes five equations. The structural basis includes four design strategies according to the generalized methodology of the first level. Nevertheless, in the limits of the second level of generalization, the total structural basis contains 16 different design strategies ($\sum_{i=0}^{2} C_5^i = 16$). The system (4) is solved by the Newton-Raphson method. The cost function C(X) is defined by the formula $C(X)=(x_4-k_1)^2+(x_5-k_2)^2$.

The design results for some strategies of full structural basis are presented in Table 1.

Table 1. Some strategies of the structural basis for two-node circuit.

N	Control functions vector U (u1, u2, u3, u4, u5)	Calculation results	
		Iterations number	Total design time (sec)
1	(0 1 0 1 1)	5	0.000851
2	(0 1 1 1 1)	178	0.016671
3	(1 0 0 1 1)	201	0.026235
4	(1 0 1 1 1)	3162	0.300012
5	(1 1 0 0 1)	23	0.002205
6	(1 1 0 1 0)	49	0.100011
7	(1 1 0 1 1)	49	0.002405
8	(1 1 1 0 0)	107	0.010365
9	(1 1 1 0 1)	1063	0.170011
10	(1 1 1 1 0)	143	0.013115
11	(1 1 1 1 1)	243	0.006215

Four last strategies of the table are the same that had been defined inside the previously (first level) formulated methodology. We can name these strategies as the "old" ones. It is very interesting that some new strategies have the computer time significantly lesser than all the "old" strategies.

The strategy number 1 with the control vector (01011) has the minimal computer time among all the strategies and it has the maximum time gain 12.2 with respect to the traditional design strategy (TDS) number 8 that corresponds to the control vector (11100). At the same time the modified traditional design strategy (MTDS) that corresponds to the control vector (11111) is the best among all of the "old" strategies and has the time gain 1.67 only. So, strategy 1 has an additional time gain 7.3 times.

3.2 Example 2

In Fig. 2 there is a circuit that has seven parameters, i.e. four admittances y_1, y_2, y_3, y_4 and three nodal voltages V_1, V_2, V_3. The nonlinear elements were defined by the following dependencies: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$.

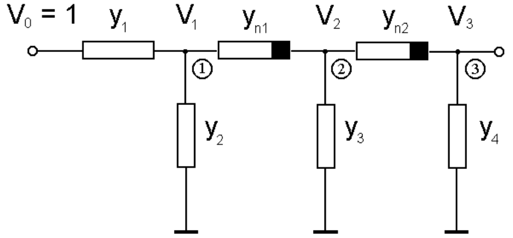

Fig. 2. Three-node circuit topology.

The vector X includes seven components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$ The mathematical model of this circuit (4) includes three equations (M=3), and the functions $g_j(X)$ are defined by the formulas:

$$g_1(X) \equiv -x_1^2 + (x_1^2 + x_2^2)x_5 + \left[a_{n1} + b_{n1}(x_5 - x_6)^2\right](x_5 - x_6) = 0$$

$$\begin{aligned} g_2(X) \equiv x_3^2 x_6 + \left[a_{n1} + b_{n1}(x_5 - x_6)^2\right](x_6 - x_5) \\ + \left[a_{n2} + b_{n2}(x_6 - x_7)^2\right](x_6 - x_7) = 0 \end{aligned} \quad (8)$$

$$g_3(X) \equiv x_4^2 x_7 + \left[a_{n2} + b_{n2}(x_6 - x_7)^2\right](x_7 - x_6) = 0$$

The optimization procedure (1), (5) includes seven equations. The cost function *C(X)* is defined by the formula: $C(X) = (V_1 - V_2 - k_1)^2 + (V_2 - V_3 - k_2)^2 + (V_3 - k_3)^2$. The total structural basis contains $\sum_{i=0}^{3} C_7^i = 64$ different strategies. For instance, the structural basis of the previous developed methodology includes only $2^3 = 8$ different strategies. The design results for all of the "old" strategies and for some of the new strategies are presented in Table 2.

Table 2. Some strategies of the structural basis for three-node circuit.

N	Control functions vector U (u1,u2,u3,u4,u5,u6,u7)	Calculation results	
		Iterations number	Total design time (sec)
1	(0 1 0 1 1 1 1)	1127	0.8414
2	(0 1 1 0 1 1 1)	63	0.0122
3	(0 1 1 1 0 1 0)	2502	1.8411
4	(0 1 1 1 1 0 1)	1390	0.9731
5	(0 1 1 1 1 1 0)	224	0.3571
6	(0 1 1 1 1 1 1)	43	0.0125
7	(1 0 1 1 1 1 0)	354	0.5205
8	(1 0 1 1 1 1 1)	2190	1.1601
9	(1 1 0 0 1 1 1)	326	0.5042
10	(1 1 1 0 0 1 1)	23	0.0161
11	(1 1 1 0 1 0 1)	14	0.0099
12	(1 1 1 0 1 1 0)	27	0.0103
13	(1 1 1 0 1 1 1)	51	0.0102
14	(1 1 1 1 0 0 0)	59	0.2291
15	(1 1 1 1 0 0 1)	167	0.2732
16	(1 1 1 1 0 1 0)	174	0.2911
17	(1 1 1 1 0 1 1)	185	0.1543
18	(1 1 1 1 1 0 0)	63	0.1228
19	(1 1 1 1 1 0 1)	198	0.2451
20	(1 1 1 1 1 1 0)	228	0.2582
21	(1 1 1 1 1 1 1)	293	0.1765

Among the "old" strategies (14-21) there are three strategies (17, 18, and 21) that have the design time lesser than the traditional strategy 14. However, the time gain is not very large. The best strategy 18 among all of the "old" strategies has the time gain 1.86 only. Nevertheless, among the new strategies we have some ones (2, 6, 10, 11, 12, 13) that have the design time significantly lesser than the TDS and they have the time gain more than 14. The optimal strategy among all of the presented is the number 11. It has the computer time gain 23.1 times with respect to the traditional design strategy.

3.3 Example 3

The four-node circuit is analyzed below (Fig. 3) by means of the new generalized methodology.

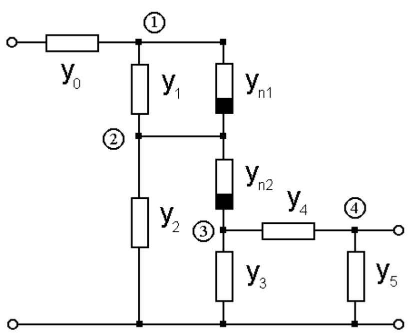

Fig. 3. Four-node circuit topology.

The design problem includes five parameters as admittances $(x_1, x_2, x_3, x_4, x_5)$, where $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, and four parameters as nodal voltages (x_6, x_7, x_8, x_9), where $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$, The nonlinear elements are defined as: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The control vector U includes nine components $(u_1, u_2, ..., u_9)$. The model of circuit (4) includes 4 equations and functions $g_j(X)$ are defined by (9):

$$g_1(X) \equiv y_0(V_0 - x_6) - \left[x_1^2 + a_{n1} + b_{n1}(x_6 - x_7)^2\right](x_6 - x_7) = 0$$

$$\begin{aligned} g_2(X) \equiv & \left[x_1^2 + a_{n1} + b_{n1}(x_6 - x_7)^2\right](x_6 - x_7) \\ & - x_2^2 x_7 - \left[a_{n2} + b_{n2}(x_7 - x_8)^2\right](x_7 - x_8) = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} g_3(X) \equiv & \left[a_{n2} + b_{n2}(x_7 - x_8)^2\right](x_7 - x_8) \\ & - \left(x_3^2 + x_4^2\right)x_8 - x_4^2 x_9 = 0 \end{aligned}$$

$$g_4(X) \equiv x_4^2 x_8 - \left(x_4^2 + x_5^2\right)x_9 = 0$$

The optimization procedure (1) includes nine equations. The cost function *C(X)* of the design process is defined by the following form: $C(X) = (x_9 - k_0)^2 + (x_6 - x_7 - k_1)^2 + (x_7 - x_8 - k_2)^2$.

The total number of the different design strategies that compose the structural basis of the generalized theory is equal $\sum_{i=0}^{4} C_9^i = 256$. At the same time the structural basis of the previous developed theory includes 16 strategies only (2^4). The results of the analysis of some strategies of structural basis that include all the "old" strategies (the last 16 strategies) and some new strategies (from 1 to 15) are shown in Table 3.

Table 3. Some strategies of the structural basis for four-node circuit.

N	Control functions vector U (u1,u2,u3,u4,u5,u6,u7,u8,u9)	Calculation results	
		Iterations number	Total design time (sec)
1	(1 1 1 0 1 0 0 0 1)	5	0.0031
2	(1 1 1 1 1 0 0 0 1)	397	0.4312
3	(1 1 1 0 1 1 0 0 1)	5	0.0029
4	(1 1 0 1 1 1 1 1 0)	119	0.0209
5	(1 1 1 1 0 0 1 0 1)	101	0.0232
6	(1 1 1 0 1 0 0 1 1)	15	0.0134
7	(1 1 1 0 1 1 1 0 1)	5	0.0009
8	(1 1 1 0 1 1 1 1 1)	101	0.0243
9	(1 1 1 1 0 0 1 1 1)	185	0.0324
10	(1 1 1 1 0 1 0 0 1)	74	0.0102
11	(1 1 1 1 0 1 0 1 1)	121	0.0254
12	(1 1 1 1 0 1 1 1 1)	159	0.0127
13	(1 1 1 1 1 0 0 0 0)	33	0.0263
14	(1 1 1 1 1 0 0 0 1)	397	0.4317
15	(1 1 1 1 1 0 0 1 0)	6548	7.1392
16	(1 1 1 1 1 0 0 1 1)	76	0.0122
17	(1 1 1 1 1 0 1 0 0)	456	0.5113
18	(1 1 1 1 1 0 1 0 1)	24	0.0052
19	(1 1 1 1 1 0 1 1 0)	3750	4.3661
20	(1 1 1 1 1 0 1 1 1)	90	0.0095
21	(1 1 1 1 1 1 0 0 0)	68	0.0354
22	(1 1 1 1 1 1 0 0 1)	596	0.6213
23	(1 1 1 1 1 1 0 1 0)	5408	6.2191
24	(1 1 1 1 1 1 0 1 1)	78	0.0255
25	(1 1 1 1 1 1 1 0 0)	238	0.2104
26	(1 1 1 1 1 1 1 0 1)	77	0.0227
27	(1 1 1 1 1 1 1 1 0)	139	0.0131
28	(1 1 1 1 1 1 1 1 1)	131	0.0103

Strategy 13 corresponds to the TDS. There are seven different strategies in the "old" group that have the design time less that the TDS. These are the strategies 16, 18, 20, 24, 26, 27 and 28. The strategy 18 is the optimal one among all of the "old" strategies and it has the time gain 5.06 with respect to the TDS. On the other hand the best strategy among all the strategies (number 7) of the Table 2 has the time gain 29.2. So, we have an additional acceleration in 5.77 times. This effect was obtained due to the utilization of more extensive structural basis and it servers as the principal result of the new generalized methodology. It is clear that further optimization of the control vector U can increase this time gain and in this case we can improve all the results as shown in [23].

3.4 Example 4

In Fig. 4 there is a circuit that has six independent variables as admittance $y_1, y_2, y_3, y_4, y_5, y_6$ ($K=6$) and five dependent variables as nodal voltages V_1, V_2, V_3, V_4, V_5 ($M=5$) at the nodes 1, 2, 3, 4, 5.

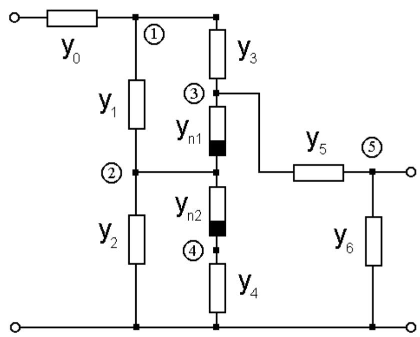

Fig. 4. Five-node circuit topology.

The nonlinear elements have next dependencies: $y_{n1} = a_{n1} + b_{n1} \cdot (V_3 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_4 - V_2)^2$. The vector X includes eleven components. The first six components are defined as: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6^2 = y_6$. The others components are defined as: $x_7 = V_1$, $x_8 = V_2$, $x_9 = V_3$, $x_{10} = V_4$, $x_{11} = V_5$. The control vector U includes eleven components too. The structural basis of the first level of generalized methodology includes 32 strategies. On the other hand the structural basis of new generalized methodology includes $\sum_{i=0}^{5} C_{11}^{i} = 1024$ different strategies.

The mathematical model (4) of this circuit is defined on the basis of nodal method and includes five equations in this case. The optimization procedure includes eleven equations and it is based on formulas (1) and (5). The cost function $C(X)$ is defined by the formula similar a previous example:

$$C(X) = (x_{11} - kk_0)^2 + \left[(x_8 - x_9)^2 - kk_1\right]^2 + \left[(x_9 - x_{10})^2 - kk_2\right]^2.$$

The results of the analysis of some design strategies that compose the “old” structural basis are shown in Table 4a.

Table 4a. Some strategies of “old” structural basis.

N	Control functions vector U(u1,u2,...,u11)	Calculation results	
		Iterations number	Total design time (sec)
1	(1 1 1 1 1 1 0 0 0 0 0)	15026	11.587
2	(1 1 1 1 1 1 0 0 0 1 1)	4387	1.522
3	(1 1 1 1 1 1 0 0 1 1 0)	1479	2.043
4	(1 1 1 1 1 1 0 0 1 1 1)	340	0.041
5	(1 1 1 1 1 1 0 1 0 1 0)	1480	1.743
6	(1 1 1 1 1 1 0 1 0 1 1)	563	0.072
7	(1 1 1 1 1 1 0 1 1 0 0)	154	0.021
8	(1 1 1 1 1 1 0 1 1 0 1)	174	0.023
9	(1 1 1 1 1 1 0 1 1 1 0)	368	0.043
10	(1 1 1 1 1 1 0 1 1 1 1)	688	0.051
11	(1 1 1 1 1 1 1 0 0 1 0)	65	0.011
12	(1 1 1 1 1 1 1 0 0 1 1)	4312	0.821
13	(1 1 1 1 1 1 1 0 1 0 0)	5601	7.112
14	(1 1 1 1 1 1 1 0 1 0 1)	854	0.081
15	(1 1 1 1 1 1 1 0 1 1 0)	483	0.052
16	(1 1 1 1 1 1 1 0 1 1 1)	367	0.031
17	(1 1 1 1 1 1 1 1 0 0 0)	354	0.352
18	(1 1 1 1 1 1 1 1 0 0 1)	548	0.063
19	(1 1 1 1 1 1 1 1 0 1 0)	98	0.012
20	(1 1 1 1 1 1 1 1 0 1 1)	1144	0.102
21	(1 1 1 1 1 1 1 1 1 0 0)	80	0.009
22	(1 1 1 1 1 1 1 1 1 0 1)	535	0.044
23	(1 1 1 1 1 1 1 1 1 1 0)	194	0.011
24	(1 1 1 1 1 1 1 1 1 1 1)	254	0.011

The strategy 1 of this table is the traditional one. All other strategies of this table have the computer time less than the TDS. The strategies 4, 7, 8, 9, 11, 16, 19, 21, 22, 23, and 24 have the time gain more than 250 with respect to the TDS. The best strategy 21 has the time gain 1287 times.

The results of the optimization process for some strategies of new structural basis are shown in Table 4b. We can see that some perspective strategies have the time gain considerably larger than the best strategy from Table 4a. The design time for the strategies 11,12, and 15 from Table 4b is lesser than the best strategy 21 from Table 4a. The best strategy from the Table 4b, the strategy 11 has the time gain 11587, i.e. almost ten times more than the strategy 21 from Table 4a. The analysis of the optimization process for nonlinear passive networks shows that the time gain, which can be obtained on the basis of new structural basis, increases when the circuit size and complexity increase. It is interesting to analyze the different active networks, which include some transistors.

Table 4b. Some strategies of new structural basis.

N	Control functions vector U (u1,u2,...,u11)	Calculation results	
		Iterations number	Total design time (sec)
1	(1 0 1 1 1 1 0 1 1 1 1)	95361	24.254
2	(1 0 1 1 1 1 1 1 0 1 1)	16457	14.521
3	(1 0 1 1 1 1 1 1 1 0 1)	2649	0.311
4	(1 0 1 1 1 1 1 1 1 1 0)	458	0.901
5	(1 1 1 0 0 1 1 1 1 1 1)	227	0.201
6	(1 1 1 0 1 0 1 1 1 1 1)	956	0.109
7	(1 1 1 0 1 1 0 1 1 1 1)	958	0.111
8	(1 1 1 0 1 1 1 0 1 1 1)	1369	0.162
9	(1 1 1 0 1 1 1 1 0 1 1)	1352	0.141
10	(1 1 1 0 1 1 1 1 1 1 0)	13556	1.733
11	(1 1 1 1 0 1 0 0 0 0 1)	5	0.001
12	(1 1 1 1 0 1 0 0 0 1 1)	20	0.002
13	(1 1 1 1 0 1 0 1 1 1 1)	134	0.011
14	(1 1 1 1 0 1 1 0 1 1 1)	51	0.009
15	(1 1 1 1 0 1 1 1 0 1 1)	45	0.002
16	(1 1 1 1 0 1 1 1 1 0 1)	82	0.012
17	(1 1 1 1 0 1 1 1 1 1 1)	142	0.013
18	(1 1 1 1 1 0 0 1 1 1 1)	221	0.032
19	(1 1 1 1 1 0 1 0 1 1 1)	742	0.091
20	(1 1 1 1 1 0 1 1 0 1 1)	77	0.011
21	(1 1 1 1 1 0 1 1 1 0 1)	266	0.033

3.5 Example 5

This example corresponds to the active network in Fig.5.

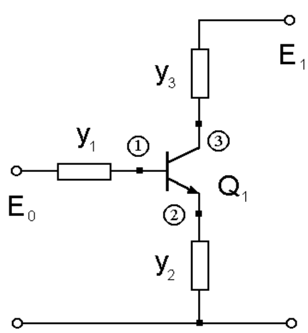

Fig. 5. One-stage transistor amplifier.

The Ebers-Moll static model of transistor has been used [24]. The vector X includes six components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$. The model (4) of this network includes three equations (M=3), the optimization procedure (1) includes six equations (K+M=6). The total "old" structural basis contains eight different design strategies. The total number of the different design strategies that compose the new structural basis of the second level of generalized theory is equal $\sum_{i=0}^{3} C_6^i = 42$. The strategy that has the control vector (111000) is the TDS in terms of the first level of generalized methodology. In this case only three first equations of the system (1) are included in optimization procedure to minimize the generalized cost function $F(X,U)$. The model of the circuit includes three equations too. The cost function $C(X)$ was defined by the formula $C(X) = [(x_4 - x_5) - m_2]^2 + [(x_6 - x_5) - m_1]^2$ where m_1, m_2 are the necessary, before defined voltages on transistor junctions.

The strategy 16 that corresponds to the control vector (111111) is the MTDS. All six equations of system (1) are involved in the optimization procedure, but the model (2) has been vanished in this case. Other strategies can be divided in two parts. The strategies that have units for three first components of the control vector define the subset of "old" strategies in limits of the first level of generalized methodology. These are the strategies from 9 to 15 of Table 5. We can see that two strategies 12 and 14 have the total computer time lesser that others. Strategy 14 corresponds to the optimal one in this case and it has time gain 198 times with respect to the TDS. Strategies numbered from 1 to 8 are the "new" strategies of the second level of generalization. Strategy 2 has the minimal design time among all strategies and has more than twice gain with respect to the best "old" strategy 14. The time gain achieves 404 times in this case. However, more impressive results were obtained analyzing more complex networks.

Table 5. Some strategies of the structural basis for one-stage transistor amplifier.

N	Control functions vector U (u1,u2,u3,u4,u5,u6)	Calculation results	
		Iterations number	Total design time (sec)
1	(0 1 1 1 0 0)	12850	10992.33
2	(0 1 1 1 0 1)	47	19.73
3	(0 1 1 1 1 0)	30015	10998.24
4	(1 0 1 1 1 0)	55992	25094.21
5	(1 0 1 1 1 1)	1195	170
6	(1 1 0 0 1 1)	174	60.01
7	(1 1 0 1 0 1)	606	220.21
8	(1 1 0 1 1 1)	778	139.11
9	(1 1 1 0 0 0)	9311	7977.01
10	(1 1 1 0 0 1)	7514	4989.11
11	(1 1 1 0 1 0)	75635	43053.12
12	(1 1 1 0 1 1)	324	60.11
13	(1 1 1 1 0 0)	25079	10970.12
14	(1 1 1 1 0 1)	243	40.11
15	(1 1 1 1 1 0)	10232	2398.53
16	(1 1 1 1 1 1)	2418	196.21

3.6 Example 6

Other example corresponds to the network in Fig.6.

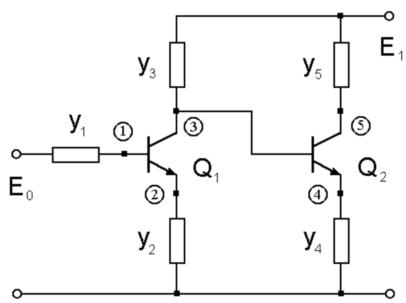

Fig. 6. Two-stage transistor amplifier.

The vector *X* includes ten components The cost function *C(X)* for the design problem was defined by the formula similar to the previous example. The presented network is characterized by five independent parameters $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$ and five dependent parameters $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$, $x_{10} = V_5$ in accordance with the traditional approach. According to the first level of generalized methodology the control vector includes five control functions, but the same control vector has 10 components following to the second level of generalized methodology. The structural basis consists of 32 design strategies according to the first level of generalization. On the other hand the total number of the different design strategies, which compose the new structural basis is equal to $\sum_{i=0}^{5} C_{10}^{i} = 638$. This structural basis can provide significantly better results for the time minimization. The results of analysis of some design strategies are presented in Table 6.

The design strategies numbered from 35 to 46 belong to subset that appears in limits of the first level of generalization. The strategy 35 that corresponds to the control vector (1111100000) is the traditional design strategy. The strategy 38 that corresponds to the control vector (1111101111) has the minimum computer time among this subset. The time gain is equal to 258 times in this case. However, there are 21 others strategies that appear among the subset of new design strategies that have the computer design time lesser that this strategy. The best strategy 19 that corresponds to the control vector (0111110111) has the time gain 4068 times with respect to the traditional design strategy and has an additional gain 15.7 times with respect to the better "old" strategy. Other strategies, for instance 1, 7, 9, 12, 13, 16, 23, 24, 25, 28 and 34 have a significant value of the time gain that is change from 1000 to 3600 times. So, we can state that the second level of the generalization of design methodology includes more perspective strategies to minimize the total computer design time. It occurs due to the broadening structural basis of different design strategies that appear in limits of the second level of design methodology generalization.

Table 6. Some strategies of the structural basis for two-stage transistor amplifier.

N	Control functions vector U (u1,u2,u3,u4,u5,u6,u7,u8,u9,u10)	Calculation results	
		Iterations number	Total design time (sec)
1	(0000011111)	55	0.159
2	(0000111110)	7912	23.985
3	(0000111111)	209	0.429
4	(0001111100)	57245	229.963
5	(0001111111)	420	0.561
6	(0011111011)	25884	52.022
7	(0011111101)	232	0.309
8	(0011111110)	138426	230.014
9	(0011111111)	381	0.319
10	(0101010111)	201	0.401
11	(0101110100)	47186	190.979
12	(0101110111)	242	0.329
13	(0101111111)	371	0.319
14	(0110110111)	338	0.441
15	(0110111111)	414	0.341
16	(0111010111)	156	0.209
17	(0111011111)	480	0.409
18	(0111110110)	8511	11.998
19	(0111110111)	68	0.082
20	(0111111011)	22381	26.012
21	(0111111100)	31525	55.061
22	(0111111110)	9264	8.961
23	(0111111111)	205	0.091
24	(1000001111)	98	0.291
25	(1000011111)	150	0.309
26	(1001101100)	40121	165.003
27	(1001101111)	286	0.379
28	(1001111101)	170	0.239
29	(1011111100)	35624	63.014
30	(1011111111)	691	0.342
31	(1100000111)	4557	22.019
32	(1110111111)	976	0.945
33	(1111000001)	79079	326.941
34	(1111011111)	542	0.271
35	(1111100000)	83402	333.601
36	(1111100011)	6695	8.991
37	(1111100111)	3395	4.007
38	(1111101111)	253	1.292
39	(1111110001)	70887	125.994
40	(1111110111)	588	2.701
41	(1111111001)	148299	158.038
42	(1111111011)	24678	15.945
43	(1111111100)	56464	57.015
44	(1111111101)	496	2.402
45	(1111111110)	5583	2.007
46	(1111111111)	614	1.699

3.7 Example 7

The last example corresponds to the three-stage transistor amplifier in Fig.7.

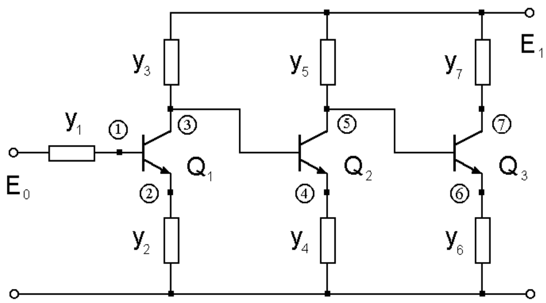

Fig. 7. Three-stage transistor amplifier.

In this case the vector *X* includes 14 components. Seven components define the independent parameters $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6^2 = y_6$, $x_7^2 = y_7$ and other seven components $x_8 = V_1$, $x_9 = V_2$, $x_{10} = V_3$, $x_{11} = V_4$, $x_{12} = V_5$, $x_{13} = V_6$, $x_{14} = V_7$ define the dependent parameters in accordance with the traditional approach. The cost function *C(X)* for the design problem was defined by the formula similar to the previous examples.

The structural basis consists of 128 different design strategies according to the first level of generalization. On the other hand the structural basis of the second level of generalization is equal to $\sum_{i=0}^{7} C_{14}^{i} = 9908$. Once again we have very broaden structural basis in the second case. The results of the analysis of some design strategies for this network are presented in Table 7.

The design strategies numbered from 15 to 28 belong to the subset that appears in limits of the first level of design methodology generalization. The strategy 15 that corresponds to the control vector (11111110000000) is the traditional design strategy. The strategy 22 that corresponds to the control vector (11111111011111) has the minimum computer time among all the strategies of this subset. The time gain in this case is equal to 368 times. The strategies from 1 to 14 belong to the subset of new design strategies. Six strategies of this subset have the design time lesser than the best strategy of the "old" structural basis. The best strategy among new structural basis has the time gain 11715 times with respect to the traditional design strategy and has an additional time gain 31.8 times with respect to the better "old" strategy.

Table 7. Some strategies of the structural basis for three-stage transistor amplifier.

N	Control functions vector U (u1,u2,...,u14)	Calculation results	
		Iterations number	Total design time (sec)
1	(0 0 0 0 0 0 0 1 1 1 1 1 1 1)	72	549
2	(0 0 0 0 0 0 1 1 1 1 1 1 1 1)	235	1030
3	(0 0 0 0 0 1 1 1 1 1 1 1 1 1)	506	1031
4	(0 0 0 0 1 1 1 1 1 1 1 1 1 1)	891	2980
5	(0 0 0 1 1 1 1 1 1 1 1 1 1 1)	660	1050
6	(0 0 1 1 1 1 1 1 1 1 1 1 1 1)	1262	2002
7	(0 1 1 1 1 1 1 1 1 1 1 1 1 1)	504	953
8	(1 0 1 1 1 1 1 1 1 1 1 1 1 1)	351	380
9	(1 1 0 1 1 1 1 1 1 1 1 1 1 1)	316	350
10	(1 1 1 0 1 1 1 1 1 1 1 1 1 1)	662	709
11	(1 1 1 1 0 1 1 1 1 1 1 1 1 1)	801	986
12	(1 1 1 1 1 0 1 1 1 1 1 1 1 1)	532	956
13	(1 1 1 1 1 1 0 0 0 0 0 0 0 1)	11993	129003
14	(1 1 1 1 1 1 0 1 1 1 1 1 1 1)	308	30
15	(1 1 1 1 1 1 1 0 0 0 0 0 0 0)	38775	351456
16	(1 1 1 1 1 1 1 0 0 0 0 0 0 1)	100843	742993
17	(1 1 1 1 1 1 1 0 0 0 0 1 0 0)	45407	440014
18	(1 1 1 1 1 1 1 0 0 1 0 0 0 0)	2643	29002
19	(1 1 1 1 1 1 1 0 1 0 0 0 0 0)	82811	1163987
20	(1 1 1 1 1 1 1 0 1 1 1 1 1 1)	1127	1020
21	(1 1 1 1 1 1 1 1 0 0 0 0 0 0)	10454	89019
22	(1 1 1 1 1 1 1 1 0 1 1 1 1 1)	540	955
23	(1 1 1 1 1 1 1 1 1 0 1 1 1 1)	53880	61040
24	(1 1 1 1 1 1 1 1 1 1 0 1 1 1)	1008	1007
25	(1 1 1 1 1 1 1 1 1 1 1 0 1 1)	5647	6012
26	(1 1 1 1 1 1 1 1 1 1 1 1 0 1)	226	1885
27	(1 1 1 1 1 1 1 1 1 1 1 1 1 0)	7441	7999
28	(1 1 1 1 1 1 1 1 1 1 1 1 1 1)	3979	4005

So, taking into consideration the obtained results we can state that the second level of the design methodology generalization gives the possibility to improve all characteristics of the generalized design theory. Further analysis may be focused on the problem of the optimal design strategy searching by means of the control vector manipulation. It is intuitively clear that we can obtain very great time gain by means of the new structural basis.

4 Conclusion

The traditional approach for the analog circuit design is not time-optimal. The problem of the optimum algorithm construction can be solved more adequately on the basis of the optimal control theory application. The time-optimal design algorithm is formulated as the problem of the functional minimization of the optimal control theory. In this case it is necessary to select one optimal trajectory from the quasi-infinite number of the different design strategies, which are produced. The new and more complete approach to the electronic network design methodology has been developed now. This approach generates structural basis of the different design strategies that is more broadened than for the previous developed methodology. The total number of the different design strategies, which compose the structural basis by this approach, is equal to $\sum_{i=0}^{M} C_{K+M}^{i}$. This new structural basis serves as the necessary set for searching the optimal design strategy. This approach can reduce considerably the total computer time for the system design. Analysis of the different problems of the electronic system design shows a significant potential of the new level of generalized design methodology. The potential computer time gain that can be obtain on the basis of new approach is significantly more than for the previous developed methodology.

Acknowledgment

This work was supported by the Puebla Autonomous University, under Grant VIEP103/EXC/G08.

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