Non-uniform Sampling Delta Modulation -Principles of Parameters Design

RYSZARD GOLAŃSKI, JACEK KOŁODZIEJ, STANISŁAW KUTA
Department of Electronics
AGH-UST University of Science and Technology
Al. A. Mickiewicza 30, 30-059 Kraków
POLAND
golanski@agh.edu.pl, jackolo@agh.edu.pl, kuta@agh.edu.pl

Abstract: - The parameters design of 1-bit adaptive delta modulations (ADM) systems with uniform sampling rate is well known. Analytical study of delta modulations with sampling rate adaptation have been carried out few years ago. The described ADM systems encode the input signal by changing the sampling intervals and/or step size according to its time-varying slope characteristics. A proposition for design of the parameters of delta modulators with adapting the sampling frequency have been presented. The relation between the boundary values of sampling frequency changes, step size and input power level have been described by the *LambertW* function. The method presented in this study are useful for parameters design of NS-DM and ANS-DM modulations.

Key-Words: Adaptive delta modulation, non-uniform sampling, sampling frequency adaptation, quantization step size, *LambertW* function.

1 Introduction

For the input signals (speech and images) in which, there is a great deal of redundancy the differential (predictive) methods of the a/d conversion are very effectives. For redundant signals past information should allow current information to be predicted fairly well, so that new information need only be transmitted if significant change in the input occurs.

Although the delta modulators are conceptually simple, they are difficult to analysis because of the discrete feedback implicit in their operation.

Calculation of Adaptive Delta Modulation noise is difficult because adaptation scheme changes the slope nonlinearity according to the input signal variation. Hence, rule-of-thumb or semi-empirical formulas are normally used for calculation of ADM modulation noise.

The performance of delta modulators depend on the type of input signal, the sampling rate, the step size and the form of predictor used.

Dubnowski, Crochiere [1] and next Un [2] showed an improvement of the quality of nonstationary (speech and images) input process conversion due to the application of non-uniform sampling.

In the seventies of the 20^{th} century the first ADM solutions of 1-bit delta converters with the step size adapting have been implemented. About ten years after delta modulators with sampling frequency adaptation NS-DM (Non-uniform Sampling Delta Modulation) and next, the converters with adapting of both step size and sampling frequency, named ANS-DM (Adaptive Non-uniform Sampling Delta Modulation) were suggested [3, 4]. In first solutions the number of the sampling frequency values was strongly limited. Thus whole the dynamic range (*DR*) was obtained practically due to the step size adaptation.

One of the basic problem of the correct input signal approximation using ADM techniques is an appropriate choice of the function that modify the adapted parameter. It has been found that ADM converter parameters (SNR_{max}, DR, BR_{avg}) are similar, while applying the exponential and linear changes of the step size [5]. On the other hand the number of steps when applying the exponential function of the step size changes is distinctly smaller. Then, this function is applied more frequently in practice.

Zhu [3, 4] presents the most detailed elaborations concerning the algorithm of non-uniform sampling with fixed step size.

There are some hopes of achieving the better compression results while converting, in the real time, speech signals, TV, video, by means of a delta converters with sampling frequency adaptation [5, 6].

2 Principle of the delta modulation with interval adaptation technique

At present there are some hopes of achieving the better compression results while converting, in the real time, speech signals, TV, video, by means of a delta converters with sampling frequency adaptation [5, 6]. Since first delta modulation implementation in the seventies of the 20^{th} century many authors have proposed a several different algorithms to track signal approximation by changes of sampling intervals. Two of them, NS-DM and ANS-DM, could be found most interesting due to their good speech coding performances and easy implementation into FPGA circuit.

2.1 NS-DM technique

In NS-DM conversion technique changes of the input signal level result with the sampling frequency adaptation and *SNR* is held near its maximal value (Fig.1).

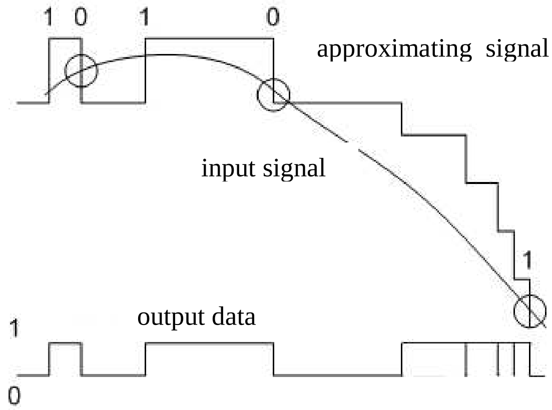


Fig. 1. Timings of the NS-DM technique.

The NS-DM schemes have been proposed and studied in [3, 7, 8]. The block diagram of its idea is presented in Fig.2.

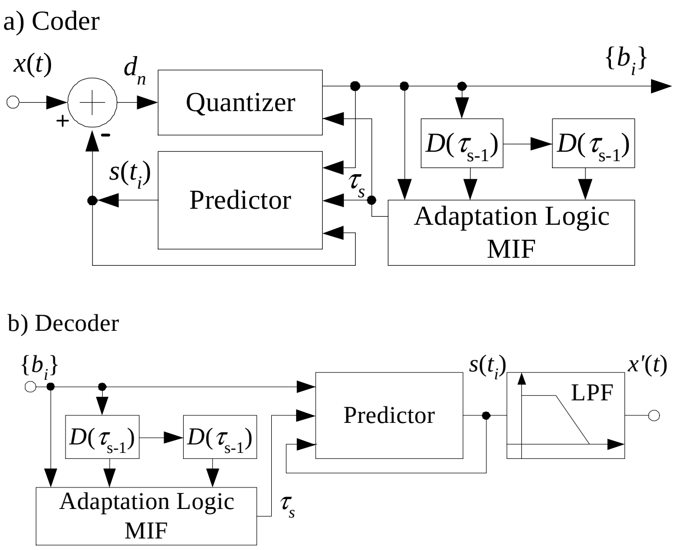


Fig. 2. Block diagram of NS-DM converter; a) coder, b) decoder.

For the input signal *x(t)* the staircase in the NS-DM modulator can be expressed as in (1):

$$s(t_i) = s(t_0) + \sum_{n=1}^{i-1} kd_n \tag{1}$$

where $d_i = \mathrm{sgn}[x(t_i) - s(t_i)]$.

An output code stream is as in the equation (2):

$$\mathrm{b}_i = \begin{cases} 1 & \text{for} \quad d_i = 1 \\ 0 & \text{for} \quad d_i = -1 \end{cases} \tag{2}$$

Let τ_s be the sampling instant and s(t_i) - the approximation signal $x(t)$ at t_i. The sampling instant $\tau_s = t_{i+1} - t_i$ vary according to the characteristics of $x(t)$ thus the next sampling time t_{i+1} is expressed as:

$$t_{i+1} = t_i + \tau_s \tag{3}$$

The sampling interval τ_s is changed according to following algorithm:

$$\tau_s = \begin{cases} Q \cdot \tau_{s\text{-}1} & \text{for} \quad b_{i\text{-}2} = b_{i\text{-}1} = b_i \\ P \cdot \tau_{s\text{-}1} & \text{for} \quad b_{i\text{-}2} = b_i \neq b_{i\text{-}1} \\ \tau_0 \text{ for other } b_{i\text{-}2}, b_{i\text{-}1}, b_i & \end{cases} \tag{4}$$

where: P, Q are constant factors of sampling instant modification and: $Q < 1 < P$.

Two other parameters establish a sampling instants border. The $\tau_{\max}$ is the upper bound of the sampling instant and $\tau_{\min}$ is the lower bound. The τ_0 is called start sampling instant and its value decides about the average output bit rate. The algorithm (4) can be also presented as a frequency modification function and then parameters assume the name f_{s_min}, f_{s_max} and f_{s_start} [7].

Formula (4) represents 3-bit adaptation algorithm of the sampling instant changing. It is simply described by the Modify Interval Function (MIF) Table 1 (<1 means increase frequency, >1 decrease frequency, 1 denotes a comeback to start frequency).

Table 1. MIF function.

B_{i-2}	b_{i-1}	b_i	MIF
0	0	0	<1
0	0	1	1
0	1	0	>1
0	1	1	1
1	0	0	1
1	0	1	>1
1	1	0	1
1	1	1	<1

One can see that the NS-DM output binary stream carries the information of not only the changing up or down of the input signal but also of the sampling instants of the modulator. So that in the demodulation process with the same algorithm the irregular (in time domain) staircase signal can be recovered [9, 10].

2.2 ANS-DM technique

The Adaptive Nonuniform Sampling Delta Modulation is a conversion technique, where both internal coder parameters: the step size and the sampling instant are adopted (Fig.3). Changes of the input signal level cause adequate a sampling frequency and a step size adaptation. This modulation has slightly better coding quality and *SNR* is closer to its maximal value then NS-DM was.

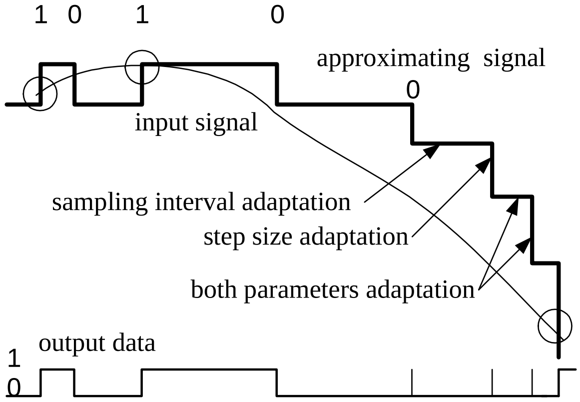
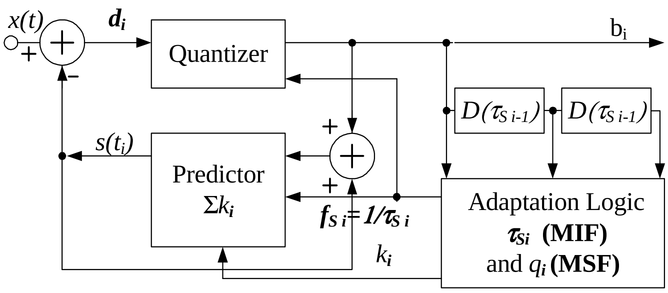


Fig. 3. Timings of the ANS-DM technique.

The ANS-DM scheme is simillar to NS-DM, it has been proposed and studied in [3,4]. The block diagram of its idea is presented in Fig.4. Actually the ANS-DM algorithm differs from conventional linear delta modulation by the addition of the Modified Interval Function (MIF) and differs from the NSDM too, by addition the Modified Step-size Function (MSF). As was shown the MIF function (table) modifies the sampling interval while the MSF modifies the step size according to the time-varying slope characteristics of the input process. In this way a staircase waveforms fit to the source signal better by ANS-DM than others delta modulation (LDM, NS-DM).

a)



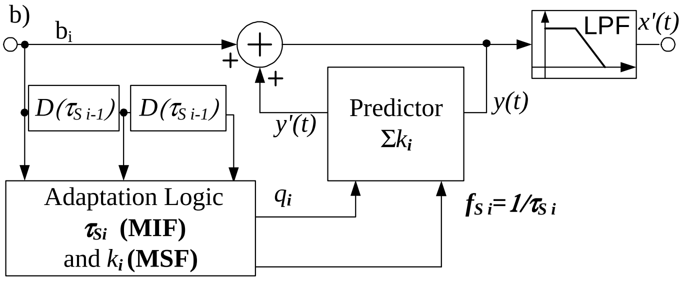


Fig. 4. Block diagram of ANS-DM converter: a) coder, b) decoder.

For the input signal $x(t)$ the predicted signal $s(t_i)$ is given by expression:

$$s(t_i) = s(t_0) + \sum_{n=1}^{i-1} k_i d_i \tag{5}$$

where $d_i = \operatorname{sgn}[x(t_i) - s(t_i)]$ and: k_i – i-th step size. The output code stream is described by equation (2).

If τ_s is the sampling instant and $s(t_i)$ approximates signal $x(t)$ at i-th sampled time t_i, the sampling instant τ_s is changed according to following algorithm:

$$\tau_s = \begin{cases} P \cdot \tau_{i\text{-}1} & \text{for} \quad MIF > 1 \\ Q \cdot \tau_{i\text{-}1} & \text{for} \quad MIF < 1 \\ \quad \tau_0 \text{ for MIF} = 1 \end{cases} \tag{6}$$

where: *MIF* function is described by Table 2; *P*, *Q* are the constant factors of sampling instant modification, and their values are: $P < 1 < Q$.

In this case τ_s is placed between the upper bound $\tau_{\max}$ and the lower one $\tau_{\min}$. The τ_0 is start sampling instant and its value decides about the average output bit rate (similar to NSDM).

A value of step size k_i is modified according formula:

$$k_i = \begin{cases} R \cdot k_{i-1} & \text{for} \quad MSF > 1 \\ \quad k_i \text{ for MSF} = 1 \end{cases} \tag{7}$$

where *MSF* function is described by Table 2; *R* is the constant factors of step size modification and $R > 1$.

Formula (6, 7) represents 3-bit Zhu adaptation algorithm - ANS-DM modulation. It could be simply described by the MIF and MSF function (Table 2). The symbols: “<1”, “>1”, “1” mean: decreasing, increasing, and coming back to start value of coder parameters.

The ANSDM output bit stream carries the information of not only the changing up or down of the input signal but also of the sampling instants and the step size value of the modulator.

Table 2. MIF and MSF Tables

b_i	b_{i-1}	b_{i-2}	S_i	T_i	S_{i-1}	T_{i-1}	MSF	MIF
0	0	0	0	1	0	0	1	<1
			1	0	0	1	>1	1
			1	1	1	0	>1	<1
			1	1	1	1	>1	<1
0	0	1	0	0	H	H	1	1
0	1	0	0	1	H	H	1	>1
0	1	1	0	0	H	H	1	1
1	0	0	0	0	H	H	1	1
1	0	1	0	1	H	H	1	>1
1	1	0	0	0	H	H	1	1
1	1	1	0	1	0	0	1	<1
			1	0	0	1	>1	1
			1	1	1	0	>1	<1
			1	1	1	1	>1	<1

3 Model of the input process and formulas describing parameters of delta modulators

Abate's [7] , Un' [2] and Taub'[11] investigations helped to determine the base dependences describing the quantization noise of delta modulators when assuming the input speech signal as a noise having an integrated power spectrum (white spectrum band-limited to ω_m and Gaussian probability density function).

Quantization distortions of delta modulations are often estimated on the basis of the so called Abate rule-of-thumb [5]. Owing to the introduction of several simplifying assumptions in the analysis of the delta modulation noise, Abate establish that the 1-bit delta modulation ensures the minimum of quantization noise (granular noise and overload noise) if the following condition is satisfied:

$$s = \frac{kB}{\chi\sqrt{S}} = \ln(2B) \qquad (8)$$

where

- s – slope loading factor,
- $B=f_s/2f_o$ – oversampling ratio,
- BR_{avg}, – average bit rate of NS-DM and ANS-DM delta modulation,
- f_s – sampling frequency,
- f_o=3,4kHz – maximum frequency for telephony speech,
- k – quantization step size,
- χ – constant determining input process,
- S – mean signal power.

Analyzing the dependence (8) it ought to be stated that the minimum value of the quantization noise power of delta modulation depends on the sampling frequency and, moreover, on the type and value of the power input signal (S). In order to obtain the noise minimum, the step size k (at a given input level and the sampling frequency f_s) ought to be chosen in such a way as to satisfy the condition (8).

From the condition (8) comes a simple conclusion concerning all 1-bit adaptive delta modulations. To maintain a constant maximum *SNR* value each change of the input level must be accompanied by an adequate change of step size k and/or sampling frequency f_s (B).

The LDM modulation ensures minimization of the quantization noise only for one input level since k=const and f_s=const.

The maximum *SNR* value in the LDM modulation for the white spectrum band-limited (f_o=3,4 kHz, $\chi\cong$1,08) input process according to Abate [7] and Taub [11] amounts to:

$$SNR_{max} = \left[\frac{B^3}{0{,}194(\ln B)^2 + 0{,}4\ln B + 0{,}227}\right] \qquad (9)$$

It ought to be emphasized that the SNR_{max} value does not depend on the step size. This step decides only at which input level the maximum *SNR* will be reached (Fig.5).

3.1 Companding capability of the CFDM modulation

In the ADM systems the dynamic range of constant *SNR* value is equal to a parameter called the companding gain (improvement) C [1], [2].

Hence, for all ADM modulations presented in this study

$$C \overset{def}{=} DR \qquad (10)$$

For CFDM[1] (k=var, f_s=const), in order to maximize *SNR* in some range of the input signal level, the Abate's condition (8) suggests to keep the ratio $k/\sqrt{S}$ constant.

Minimal k_{min} and maximal k_{max} step sizes needed to maximize *SNR*, in a range of input levels from S_1 to S_2 can be calculated solving the system of two equation (8):

[1] CFDM is a commonly known modulation, with constant factors of the step size adaptation [11, 13].

$$\left.\begin{aligned} \frac{k_{\min} B}{\chi\sqrt{S_1}} &= \ln(2B) \\ \frac{k_{\max} B}{\chi\sqrt{S_2}} &= \ln(2B) \end{aligned}\right\} \qquad (11', 11'')$$

We obtain:

$$DR_k = \frac{S_2}{S_1} = (K_k)^2 \qquad (12)$$

where $K_k \overset{def}{=} \frac{k_{\max}}{k_{\min}}$

and

S_2, (S_1) – maximal (minimal) input signal level,
$DR = S_2/S_1$ – dynamic range.

As seen from equation (12), *SNR* maximization within the input signal range *DR* requires the step size extended factor K_k to be equal $\sqrt{DR}$. Hence, from definition (10), the CFDM companding gain C_k is

$$C_k = (K_k)^2 \qquad (13)$$

Usually, C_k is expressed in dB

$$C_k\,[\text{dB}] = 20\log(K_k) \qquad (14)$$

As result from the last equation, the companding gain of CFDM modulation increases at a rate of 6 dB per octave, with increase of the K_k ratio. For LDM modulation (k=const, f_s=const, K_k=1) the companding gain equals zero, in decibels.

3.2. Companding capability of the NS-DM modulation

For NS-DM modulation (k=const, f_s=var), in order to maximize *SNR* in some range of the input signal level the Abate's condition (8) suggests to keep the ratio $\frac{B}{\ln(2B)}\Big/\sqrt{S}$ constant.

Minimal $B_{\min}$ and maximal $B_{\max}$ sampling frequencies needed to maximize *SNR*, in a range of input levels from S_1 to S_2 can be calculated solving the system of conditions (15', 15")

$$\left.\begin{aligned} \frac{kB_{\min}}{\chi\sqrt{S_1}} &= \ln(2B_{\min}) \\ \frac{kB_{\max}}{\chi\sqrt{S_2}} &= \ln(2B_{\max}) \end{aligned}\right\} \qquad (15', 15'')$$

In a case of given $f_{s\,max}/f_{s\,min} = B_{\max}/B_{\min} \overset{def}{=} K_f$ – sampling frequency ratio and B_{min}, from (15', 15") and (10) the following equation can be obtained

$$DR_f = C_f = \left(\frac{K_f \ln(2B_{\min})}{\ln(2B_{\min}) + \ln(K_f)}\right)^2 \qquad (16)$$

Frequency companding gain in [dB] can be found

$$C_f[\text{dB}] = 20\log\left(\frac{K_f \ln(2B_{\min})}{\ln(2B_{\min}) + \ln(K_f)}\right) \qquad (17)$$

3.3 Companding capability of the ANS-DM modulation

For ANS-DM modulation (k=var, f_s=var), in order to maximize *SNR* in some range of the input signal level the Abate's condition (8) suggests to keep the ratio $\frac{kB}{\ln(2B)}\Big/\sqrt{S}$ constant.

In similar manner as for CFDM and NS-DM modulations basing on the system of conditions (18',18'')

$$\left.\begin{aligned} \frac{k_{\min} B_{\min}}{\chi\sqrt{S_1}} &= \ln(2B_{\min}) \\ \frac{k_{\min} B_{\max}}{\chi\sqrt{S_2}} &= \ln(2B_{\max}) \end{aligned}\right\} \qquad (18', 18'')$$

the *DR* range may be written as

$$DR = \frac{S_2}{S_1} = \left(\frac{k_{\max}}{k_{\min}}\right)^2 \left(\frac{B_{\max}}{\ln(2B_{\max})}\frac{\ln(2B_{\min})}{B_{\min}}\right) = (K_k)^2 \left(\frac{[\ln(2B_{\min})]K_f}{\ln(2B_{\min}) + \ln(K_f)}\right)^2 = C_k C_f \,. \qquad (19)$$

The total companding gain of the ANS-DM modulation can be calculated as a product of the companding gain derived both from sampling frequency and step size adaptation (C_k,C_f)

As seen from (19) an infinite number of pairs of the coefficients K_f, K_k that help to provide the equation occurs. Only in practice, definite chosen applications help to determine the useful pairs.

From the equation (19) the following relation is obtained in the logarithmic measure

$$C_t\,[\text{dB}] = C_k\,[\text{dB}] + C_f\,[\text{dB}] = DR\,[\text{dB}] \qquad (20)$$

where C_t is the total companding gain of the ANS-DM modulation.

As the designing aspect is considered, in many cases the required companding gain $C=DR$ is given.

Than the equations $K_f=f_1(DR, K_k, B_{\min})$ and $K_k=f_2(DR, K_f, B_{\min})$ have to be determined from (13). Finally it have a form

$$K_f = -\frac{\sqrt{DR}}{K_k \ln(2B_{\min})} LambertW\left(-1, -\frac{K_k \ln(2B_{\min})}{2B_{\min}\sqrt{DR}}\right) \tag{21}$$

$$K_k = \frac{1}{\sqrt{DR}}\left(\frac{[\ln(2B_{\min})]K_f}{\ln(2B_{\min})+\ln(K_f)}\right). \tag{22}$$

The influence of $B_{\min}$ upon the quantity of K_f ratio is critical for small K_k ratio. In order to decrease the strong influence of $B_{\min}$ onto K_f, the big K_k values should be, if possible applied in all application of the ANS-DM modulation.

4. Design of nonouniform sampling delta modulation parameters

The analysis carried out in [9, 10, 14] comprises a derivation of all indispensible dependences describing the NS-DM parameters. It helped to establish the correct sequence of individual parameters calculations (Fig. 6).

Three proposed steps of design the NS-DM modulator parameters:

I

Minimal sampling frequency f_{s_min} ($B_{\min}$) have to be determined on the basis of formula (9) (generally we have given minimal SNR value),

II

The step size k_{opt} have to be calculated (when we already know f_{s_min} value),

III

Maximal sampling frequency f_{s_max} ($B_{\max}$) have to be determined from eq. (8)

4.1 Step size value design of LDM and NS-DM modulation

Determination of the NS-DM optimum step size helps to obtain the appropriate output parameters of coding technique. In [4, 7] the method of describe of the optimum step size for the speech signal[2] has been suggested the criterion of quality consisting in maintenance: SNR = const. As it was shown many times before [12, 13], the sampling frequency f_s has a decisive influence on the $SNR_{\max}$ value. In the case of the NS-DM modulation it is necessary to provide a sufficient conversion quality (SNR_{max}) for the lowest input S_1 by means of accepting the sufficient $B_{\min}$ value.

Step size k_{opt} that ensures obtaining minimum of quantization noise at the lowest input level (S_1) can be calculated on the basis the dependence (8). For the assumed speech signal model the constant χ equals 1,08 approximately. From (8), after transforming

$$k_{\text{opt}} \cong 1{,}08\frac{\ln 2B_{\min}}{B_{\min}}\sqrt{S_1} \cong 1{,}08\frac{\ln 2B_{\max}}{B_{\max}}\sqrt{S_2} \tag{23}$$

Using one of the expressions on the right side of the formula 23) depends on the fact which of remaining NS-DM converter parameters (B_{max}, DR, $\sqrt{S_2}$, $\sqrt{S_1}$) are given. Fig.5 illustrates an influence of the step size k value on the position of the lower and upper edge of the input signal dynamic range DR in which due to the sampling frequency adaptation, the NS-DM converter maintain the constant value of SNR. The step size value does not influence the $SNR_{\max}$ value. To determine step size value knowledge of DR value is it not enough. The maximum or minimum input signal level ($\sqrt{S_1}$ or $\sqrt{S_2}$) is necessary. Owing to it the range of the input signal levels in which the SNR remains constant is precisely determined.

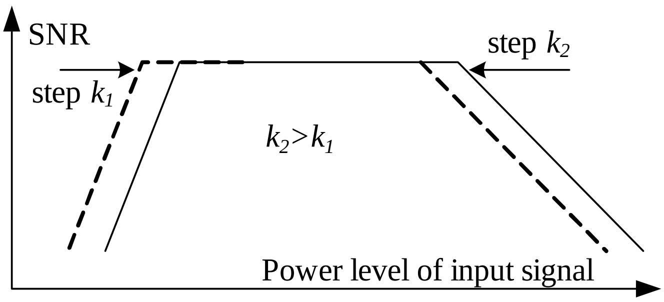


Fig.5. The influence of the step size value k on the characteristic *of SNR* vs input level for NS-DM modulation ($B_{\max}/B_{\min}$=const)

4.2 Maximal and minimal frequency oversampling ratio design of NS-DM and ANS-DM modulations

Maximal frequency oversampling ratio $B_{\max}$ have to be determined from eq. (8) or (23).

The problem is because these equations are transcendental in terms of $B_{\max}$:

$$\ln(2B_{\max})=cB_{\max} \tag{24}$$

where

[2] Input signal undergoing conversion, the so-called *modeled speech signal* is realized as a random process with the white spectrum band-limited to ω_m and integrated by the RC filter that satisfies the dependence $\frac{\omega_{3dB}}{\omega_m} = 0{,}23$ [5],[11].

$$c = \frac{\ln(2B_{\min})}{B_{\min}}\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{\ln(2B_{\min})}{B_{\min}}\frac{1}{\sqrt{DR}} \qquad (25)$$

Maximal frequency oversampling ratio B_{max} cannot be isolated in terms of *c*. Equation (24) has not a solutions in domain of elementary functions (in a closed form).

Positive, real values $B_{\max}$ (satisfying the Nyquist condition), which is the solution of the transcendental equation (25) can be found by using the special function: *LambertW* [15].

The *LambertW* function, known as *Omega* function is denoted also as W(*c*) or W(*n*, *c*) or $W_n(c)$.

It is special function which satisfies the condition: *W*(*z*)*exp*(*W*(*z*))=*z*. Similarly to the equation *yexp*(*y*)=*z* that has infinite number of solutions *y* for every non-zero value *z*, also the *W* has an infinite number of branches [15]. Since the function *x*exp(x)* is not injective, the *W* function is multivalued, thus it has to be divided on branches (Fig.6).

The principal branch is denoted as *W(x)* and is real valued on the interval –1/e…+∞. The –1'st branch, denoted *W(-1,x)* is real valued on the interval –1/e…0.

LambertW function has been implemented in math programs as Maple and Mathematica [16].

LambertW function solves the equation *W*exp(*W*)=*z* where *z* generally is a complex number..This equation has an infinite number of solutions, most of them are complex.

For *z*=*x* <-1/e *W(x)* has only complex solutions,

For *z*=*x* ≥ -1/e *W(x)* has real solutions.

Two branches representing *LambertW* function for real values are shown on Fig.4.

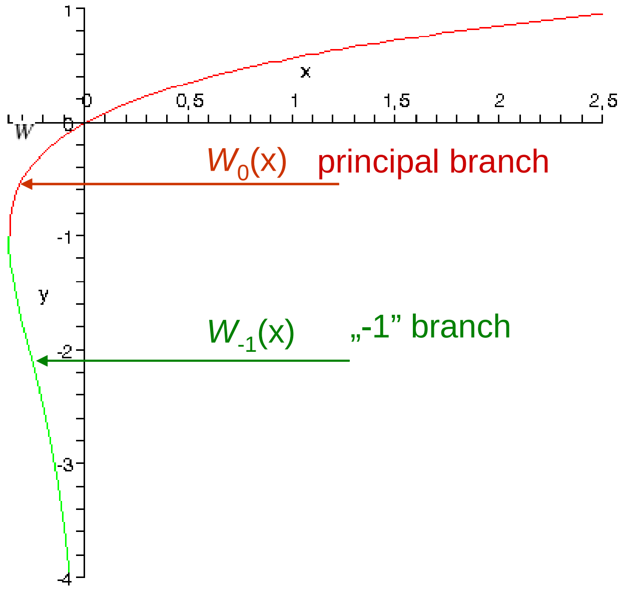


Fig.6. Two branches representing the shape of the *LambertW* function for real values of *x*.

In our considerations only the branch named "-1"satisfies the Nyquist condition.

On the basis of Maple program [16] from equation (21) is obtained:

$$B_{\max} = -\frac{W(-1,-c/2)}{c} = -\frac{W\left(-1,-\frac{\ln(2B_{\min})}{2B_{\min}\sqrt{DR}}\right)}{\frac{\ln(2B_{\min})}{B_{\min}\sqrt{DR}}} \qquad (26)$$

The run of the $B_{\max}$ function is illustrated in Fig. 7.

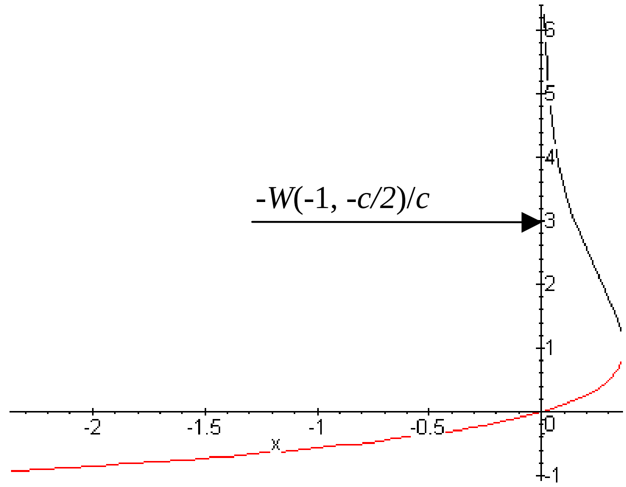


Fig. 7. The shape of the $B_{\max}$=-*W*(-1, c/2)/*c* function

Formula (25) has a real positive solutions for *c*∈[0, 1/e]. After transformations (8), (23) and (24) the following inequality is obtained

$$0 < k \le 0{,}8\sqrt{S} \qquad (27)$$

Inequality (24) has an important practical meaning pointing out of design the greatest value of the step size.

Minimal sampling frequency (f_{s_min}) also can be determined while using *LambertW* function [15] but only when we have given maximal oversampling ratio $B_{\max}$ and dynamic range *DR*.

$$B_{\min} = -\frac{LambertW\left(-1,-\frac{\sqrt{DR}\ln(2B_{\max})}{2B_{\max}}\right)}{\frac{\sqrt{DR}\ln(2B_{\max})}{B_{\max}}} \qquad (28)$$

The presented principles of the NS-DM parameters design also concern ANS-DM modulation. Maximal and minimal frequency oversampling ratio design of ANS-DM modulation have to be determined from eq. (18', 18") or (19) (Fig.8).

On the basis of Maple program [16] from equation (18', 18") for ANS-DM modulation is obtained

$$B_{\max} = -\frac{\mathrm{Lambert}W\left(-1, -\dfrac{K_k \ln(2B_{\min})}{2B_{\min}\sqrt{DR}}\right)}{\dfrac{K_k \ln(2B_{\min})}{B_{\min}\sqrt{DR}}} \quad . \qquad (29)$$

Formula (26) has a real positive solutions for

$$0 < \frac{K_k \ln(2B_{\min})}{2B_{\min}\sqrt{DR}} < \frac{1}{\mathrm{e}} \qquad (30)$$

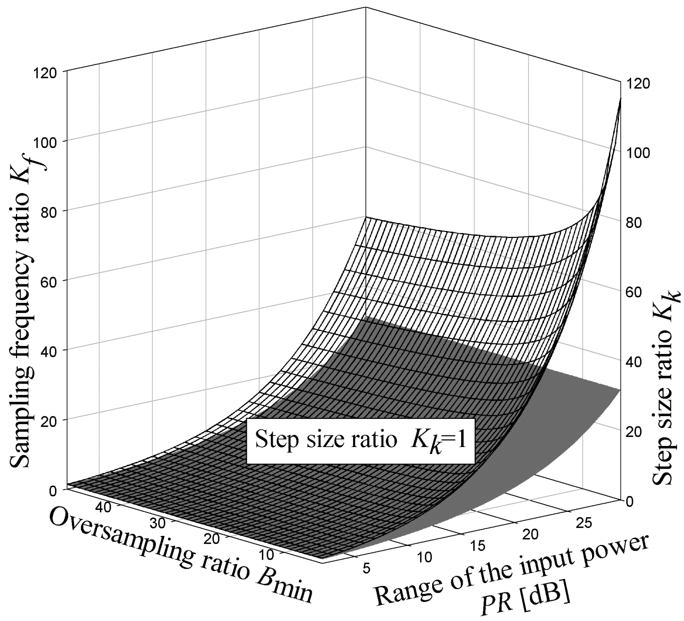


Fig.8. Require adaptation range of step size K_k (the lower surface) and frequency K_f (the upper one) versus dynamic range DR and minimum of B_{min}.

4.3 Boundary values of the step sizes for CFDM and ANS-DM modulators

Maximal $k_{\max}$ and minimal $k_{\min}$ step sizes for ANS-DM modulation can be calculated similarly as for CFDM [9] basing on equations (11', 11") and (12).

Minimal step size can be calculated on the basis of equation (11')-most often remaining multipliers are given. After it, from dependency (12) we evaluate maximal step size $k_{\max}$.

4.4 The graphs of NS-DM parameters determination

On the basis of three steps of the NS-DM modulator parameters design proposed in (4.1), two simple flowchart depending on the input data have been presented in Fig. 9.

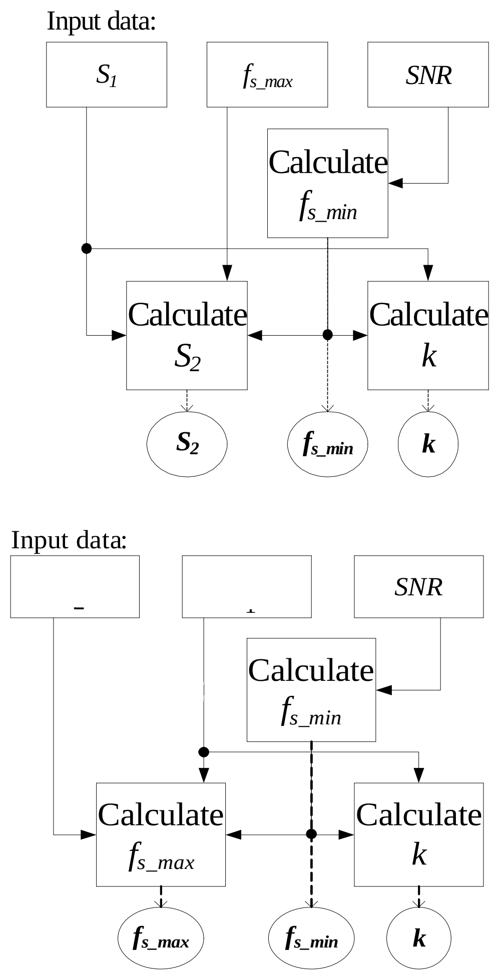


Fig.8. The graphs of the NS-DM parameters determination for two different input data

Individual output parameters are determined on the basis of their definition and from equations (8÷12, 11÷13).

5. Further work

Basing on the proposed NS-DM parameters design method the modulator circuit has been designed in 0.35 μm technology from AMS (Fig.9, Fig. 10).

The circuit performance has been simulated with Spectre using BSIM3.3 models provided by the foundry. The circuit had been designed also at the mask level and send for manufacturing within EUROPRACTICE prototyping service.

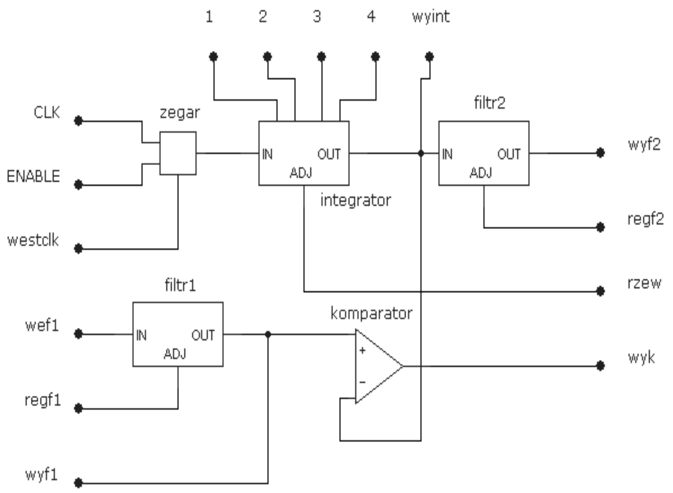


Fig. 9. Diagram of analog NS-DM codec component.

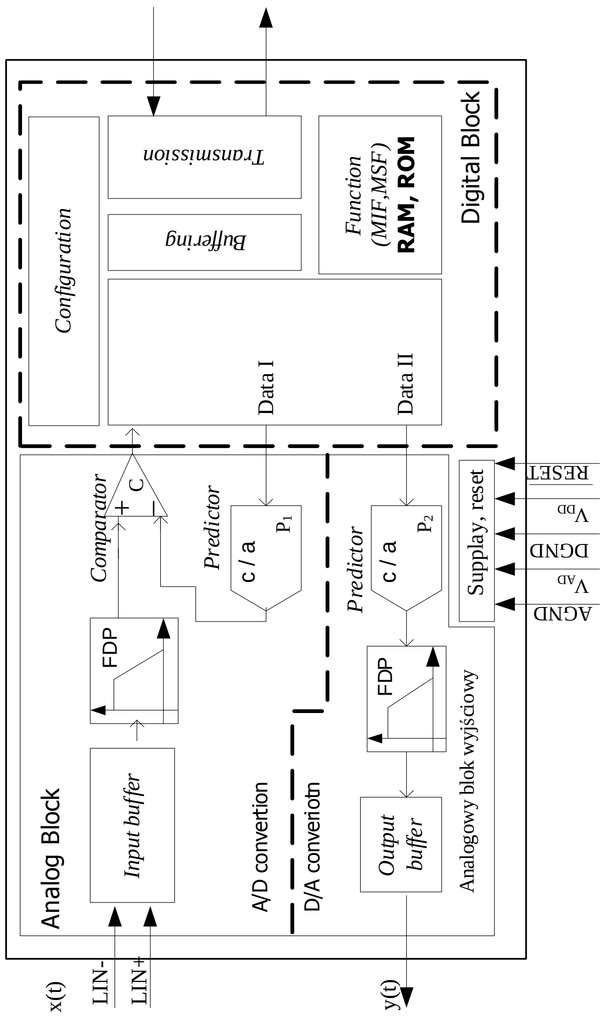


Fig. 10. Block circuit of proposed NS-DM codec implementation with digital control block.

6 Conclusion

The designed parameters values of NS-DM and ANS-DM modulators has been compared with parameters obtained by means of the simulation, and this comparison preliminarily confirms the general idea and is quite satisfied for more practical applications.

NS-DM and ANS-DM 1-bit delta modulations present better noise properties in conversion of non-stationary input processes than traditional delta modulations.

The total companding gain of the ANS-DM modulation can be calculate, in decibels, as a sum of the companding gain derived both from sampling frequency and step size adaptation (14). This relation is very advantageous because it provides a large modulator dynamic range, without considerable variation of both the companded parameters (16), (17). In practice, [4, 8] it is reflected by the increase of not only compression ratio but also SNR peak value.

The presented principles of the NS-DM parameters design base on the condition which have been proposed by Abate. It help to determine appropriate parameters indispensable for non-uniform sampling delta modulation correct functioning.

The math form of the Abate's rule-of-thumb of the quantization noise minimum, makes the special function *LambertW* extremely suitable for the determination of the boundary sampling frequencies (f_{s_min} i f_{s_max}).

Basing on the proposed parameters design method the NS-DM modulator circuit has been designed.

The theoretical evaluation of the NS-DM and ANS-DM performances agrees well with the results of computer simulation [4], [8]. Hence, the analytical results presented in this study are useful for design the dynamic range of modulations with time-varying sampling periods. Dynamic range is often most important parameter when the ADM systems employed in the speech and image coding. The authors intention is to apply the variable sampling method for the CVSD modulation in order to combine a large dynamic range of ANS-DM modulation and high tolerance to channel error of CVSD modulation.

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