Parameters Modelling of Transformer

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Abstract: This paper analyses transformer loss of life attending to the realistic variability of both structural and functional parameters. The article begins with the modelling of load and ambient temperature profiles, by means of chronological series theory. A simple additive model is proposed and validated with realistic data, loss of life resulting from probabilistic functional and structural parameters is analysed through a sensitivity study.

Key-Words: Transformer thermal, Estimate thermal parameters.

1 Introduction

This paper is devoted to load and ambient temperature (functional parameters) profiles modelling and to the sensitivity study of thermal and loss of life models relatively to functional and structural parameters. The functional parameters modelling will be based on realistic data and is developed on section §2. It is not objective of this section to exhaustively apply time series theory in modelling realistic load and ambient temperature profiles. Such complete analysis is out of the scope of this work. The objective is, based on data representing the load profiles of realistic distribution transformers and ambient temperature profiles, to obtain sufficiently accurate models that will give physical support to the probabilistic models that will be used on section §3. Loads modelling and forecasting play a fundamental role in power systems planning and management; due to its connection with weather characteristics, loads and weather modelling are joined subjects of some works, [1], [9] and [3-4]. Provided a transformer thermal model is chosen, deterministic hot-spot temperature can easily be computed, given the input profiles of load and ambient temperature. When analysing the time series representative of a given transformer load (or the time series of a localised ambient temperature), one can visualise a cycling (deterministic) behaviour (daily, weekly, monthly, seasonally) to which is superposed a random behaviour. Such input profile structure will be reflected on consequent hot-spot temperature profile: deterministic and random components. Apart from specific characteristics and improvements that transformer thermal model may reflect, the validity of deterministic input profiles is questionable, due to unpredictable (random) changes that realistic profiles do present. This fact determines a probabilistic analysis of the system, which is developed on section §3.

2 Functional Parameters Modelling

2.1 Time Series Descriptive Techniques

Despite the diversity of approaches, methodologies and end-use applications, when studying realistic load and weather data profiles, one can always identify trends (deterministic) components, to which are superposed irregular (random) behaviours. Deterministic data can be described by an explicit mathematical relationship (a mathematical model). Provided no unforeseen event in the future will influence the phenomenon producing the data set under consideration, for identical experience conditions, the mathematical model will reproduce the same exact data set, no matter how many times the experiment is repeated. Random data values are unpredictable in a future instant in time, and therefore must be described in terms of probability statements and statistical averages, rather than by explicit mathematical relationships. In practice problems, involving random variables, one must not expect to obtain the theoretical results, namely, a purely random variable. Main reason is that a random variable is a theoretical concept, which can not be reproduced (simulated) in practice; only samples of random variables can numerically be simulated. The statistics of samples only asymptotically (with the increasing length of the sample) tend to random variable statistics. A random variable can be viewed as a sample of infinite length.

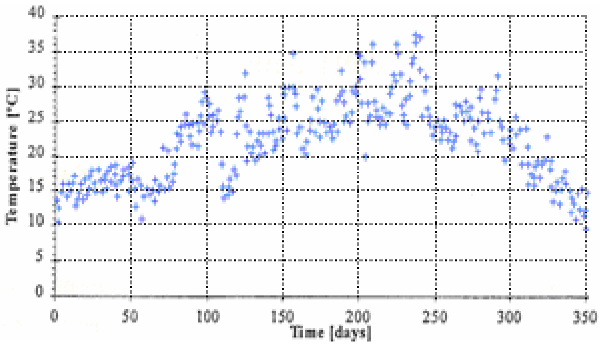


Fig. 1: Annual time series representing daily maximal ambient temperatures. Data from 2005.

Time series representative of load and ambient temperature profiles do present deterministic and random characteristics simultaneously, Fig.1. Such a data set, presenting concomitant time and random characteristics is referred as a stochastic process. Time plot will often show up the most important properties of a time series [8] and [18]. It was predictable and can be visually confirmed that ambient temperature time series do exhibit a seasonal effect that, although not representative on the sample, is expected to be cyclic. Possible long-term trends will not be considered since, although might be present, the sample length is insufficient to allow this kind of analysis. A common model to described time series as the one represented on Fig.1, is the additive model of the form [8], [10], and [14]:

$$x_t = x_{\det_t} + x_{ran_t}, \tag{1}$$

where x_{det_t} represents the deterministic cyclic component and x_{ran_t} the random component.

Most of the time series theory concerns stationary time series, which, intuitively, is a time series where no systematic temporal variations in mean and variance occur. From the analysis of series residuals, after removing the seasonal effects (and trends when existent), one may conclude that it is possible to model residuals by means of a stationary stochastic process. Several approaches, methodologies and tests can be used to detect time series characteristics such as cyclic variations, stationary, randomness, [11], [20-21] and [6]. However, a complete and powerful tool is provided by the autocorrelation function. If x_t and y_t are two samples, length N, of two stationary ergodic processes, an estimator of their correlation function $\hat{\rho}_{xy}(k)$ is, according to [8] and [20]:

$$\hat{\rho}_{xy}(k) = \frac{C\hat{O}V_{xy}(k)}{C\hat{O}V_{xy}(0)}, \tag{2}$$

where $C\hat{O}V_{xy}(k)$ denotes an estimator of the covariance function

$$C\hat{O}V_{xy}(k) = \frac{1}{N}\sum_{t=1}^{N-k}(x_t - \bar{x})(y_{t+k} - \bar{y}), \tag{3}$$

with $\bar{x}$ and $\bar{y}$ representing the samples averages given by:

$$\bar{x} = \frac{1}{N}\sum_{t=1}^{N} x_t \ and \ \bar{y} = \sum_{t=1}^{N} y_t, \tag{4}$$

when $x_t \equiv y_t$, expression (4) represents the autocorrelation function and (3) the autocovariation function. When is clear the variable it refers to, the estimator of the autocorrelation function will be denoted by $\hat{\rho}_x(k)$ or simply by $\hat{\rho}(k)$ and the estimator of the autocovariation by $C\hat{O}V_x(k)$ or simply by $C\hat{O}V(k)$. The analysis of the corresponding sample autocorrelogram (plot of the autocorrelation coefficients as a function of time lag k), often provides fully insight into the probabilistic model that describes the data. The autocorrelation function is a measuring of correlation (link) between series data values at different time distances apart. For a random variable, correlation coefficients must be null for any lag k but k=0. It should be remarked that, mathematically, the maximal time lag k in (2) is limited to N/2, although [8] states that N/4 is the usual limit. Information contained in the sample time series may not always be sufficient to completely characterise it. Fig.2(a) represents the autocorrelogram of the one-year time series of data represented on Fig.1. And although yearly cyclic variations are expected to occur, the autocorrelogram does not evidence them. However, by increasing the sample size to a two years length, the respective autocorrelogram being represented on Fig.2(b), clearly evidences an almost sinusoidal variation, which, although expected, should be confirmed with a longer size sample. If a time series could be described by a purely deterministic sinusoidal function of the form:

$$x_t = X\cos\omega t, \tag{5}$$

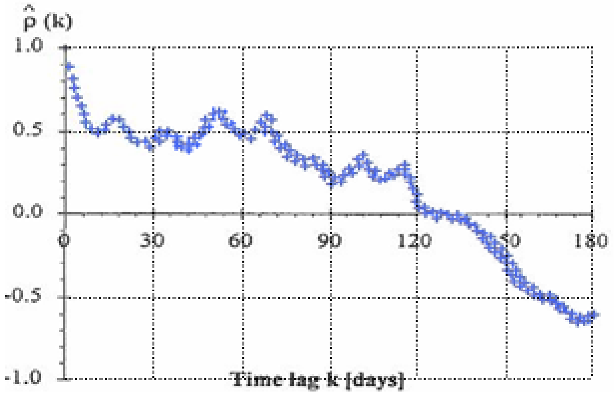
where X and (ω are constants, its autocorrelation would evidence this cyclic variation, since for large sample lengths (N $\to\infty$) it would tend to:

$$\rho(k) = \cos\omega t. \tag{6}$$

Following the evidences of Fig.2 autocorrelogram and International Standards [12] suggestion, the deterministic component, $x_{det\ t}$, of model (1) was assumed to be given by a generic deterministic sinusoidal variation represented by:

$$x_{\det_t} = \bar{x}_d + \Delta x_d \cos(\omega t + \varphi_x), \qquad (7)$$

where $\bar{x}_d$, Δx_d and φ_x are constants.



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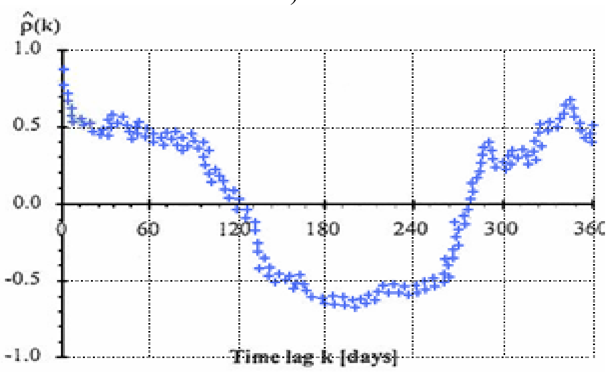
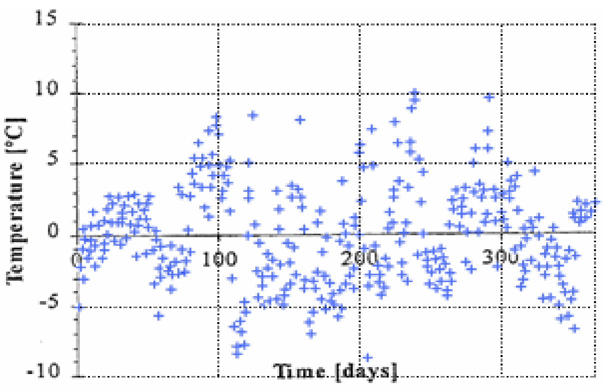


b)

Fig. 2: Autocorrelogram of daily maximal ambient temperatures in (a) 2005 and (b) 2004 and 2005.

From the analysis of time series residuals (random component x_{rant}) Fig.3(a) and respective autocorrelogram, Fig.3(b), one can extract clues towards its modelling. The interpretation of autocorrelogram does require considerably experience in time series analysis and, according to [8] and [22], this is one of the hardest aspects of time series analysis.



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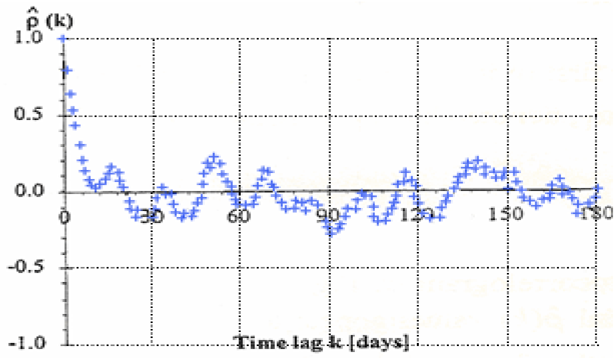


b)

Fig. 3: Random component (a) and respective autocorrelogram (b) of maximal ambient temperature in 2005.

Plot diagram of random component, Fig.3(b), shows no evident cyclic or mean variations; as a first approximation, variable could be taken as random. However a more accurate analysis, shows that autocorrelogram represented on Fig.3(b) is not typical of a random variable since it should be $\rho(0)$ =1 and $\rho(k)$=0 for $\forall k \neq 0$. Apart from small amplitude and of high frequency (probably) cyclicvariations, one can find considerably high $\hat{\rho}(k)$ values for initial time lags, k=1...6. To determine the best model that fits a given sample autocorrelation function, the methodology proposed by [7] consists in, comparing sample autocorrelation function with the theoretical autocorrelation function of several models, and choose the one which best agrees with the sample autocorrelation function. Most common models are the AutoRegressive models (AR), the moving Average Models (MA) and mixed models such as AutoRegressive Moving Average models (ARMA) and Autoregressive Integrated Moving Average models (ARIMA). As far as the objective of this work is concerned, one will limit oneself to present the AR model. The process $\{X_t\}$ is said to be AR of order m if, given a purely random process $\{Z_t\}$with null mean and variance σ_Z^2 [8] and [2]:

$$x_t = \alpha_1 x_{t-1} + \ldots + \alpha_m x_{t-m} + Z_t, \qquad (8)$$

where, $\alpha_1 \ldots \alpha_m$, are constants.

For first order AR processes (also referred as Markov process [8]) the estimator of α_1 denoted by $\hat{\alpha}_1$, is [8]:

$$\hat{\alpha}_1 = \hat{\rho}(0). \qquad (9)$$

Autocorrelogram of Fig.3 is suspicious to correspond to a first order AR model since initial $\hat{\rho}(k)$ values appear to decrease geometrically [5] and [8]. If time series x_{rant} is a sample of a first order AR process $\{X_{rant}\}$, it must be:

$$X_{rand\,t} = \alpha_1 X_{rand\,t-1} + Z_t\,. \qquad (10)$$

Using estimator (9) and sample x_{rant} the resulting z_t sample (being z_t a sample of the random variable Z_t) and corresponding autocorrelogram are represented on Fig.4. To determine whether Figure 4 corresponds to a sample autocorrelogram of a random variable ($\rho(k)$=0 for $k>$ 0) or not, confidence intervals must be determined. For large N values, being the sample autocorrelation $\hat{\rho}(k)$ normally distributed with [14] and [19]:

$$\mu_{\hat{\rho}(k)} = \rho(k) \text{ and } \sigma_{\hat{\rho}(k)} \approx \sqrt{\frac{1}{N}(1+2\hat{\rho}(0))} \text{ for } k>0 \quad (11)$$

a $(1-\alpha_S)\%$ confidence interval for $\hat{\rho}(k)$, being α_S the significance level of the test, is given by:

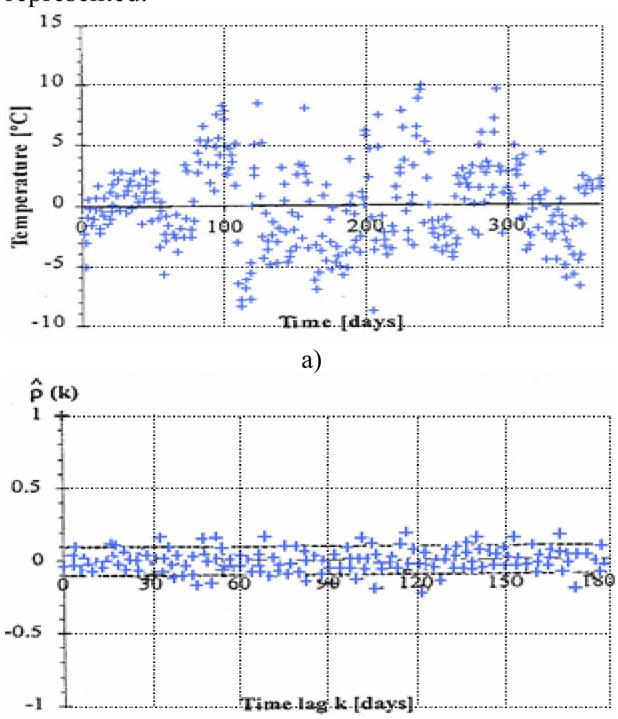
$$\mathcal{P}\left\{-\Phi^{-1}(1-\alpha_S)\% < \frac{\hat{\rho}(k)-\mu_{\hat{\rho}(k)}}{\sigma_{\hat{\rho}(k)}} < \Phi^{-1}(1-\alpha_S)\%\right\} \geq 1-\alpha_S \quad (12)$$

for k>0 being $\Phi^{-1}(1-\alpha_S)$ the inverse of the standardised normal distribution evaluated at $(1-\alpha_S)$. Attending to (11), and that for a random variable it is ($\rho(k)$=0 for k>0), probability expression (12) is traduced by the statement:

$$\hat{\rho}_k \in \left[-\sqrt{\frac{1}{N}(1+2\hat{\rho}(0))}\Phi^{-1}(1-\alpha_S); \sqrt{\frac{1}{N}(1+2\hat{\rho}(0))}\Phi^{-1}(1-\alpha_S)\right]$$

for k>0 (13)

On Figure 4, limits of (13) with α_S =5% are also represented.



a)

b)

Fig. 4: Variable z_{ran_t} (a) and respective autocorrelogram (b).

If $(1-\alpha_S)\%$ of the $\hat{\rho}(k)$ values, with k>0, are within (13) limits, $\hat{\rho}(k)$ is accepted as representative of the autocorrelation of a random variable and therefore, z_t is accepted as a random variable. A second step for the complete modelling of time series x_t, is the determination of random variable z_t distribution function. This is achieved by testing the *p*robability *d*ensity *f*unctions (*pdf*) of theoretical (expected) random variables against the realistic (observed) *pdf* one obtains for z_t. These tests are referred as goodness-of-fit tests, [2], [5], [11] and [20]. A key element associated to statistical tests is itsp-value. According to [15] test formulation, the *p-value* represents the maximal significance level at which the hypothesis should be accepted. Its value measures the closeness of the observed *pdf* relatively to the theoretical *pdf,* the *p-value* will be as close to the unity as the observed *pdf* is close to the theoretical pdf. Justification to give relevance to AR models resides on their physical base. They represent memory systems in the sense that values at instant *t* are influenced by the memory of previous values at *t*-1,..., *t-m*. Due to earth thermal inertia, ambient temperature is expected to be a function of near past ambient temperatures; due to its correlation with ambient temperature similar behaviour can be expected on the load profiles of distribution transformers.

2.2 Case Studies

Previously described techniques were applied to four time series, representing maximal, Θ_M, minimal, Θ_m, average, Θ_{av}, and half-amplitude Θ_{am} values of daily ambient temperature in the Craiova region on the years 2002 to 2005, being:

$$\Theta_{av} \equiv (\Theta_M + \Theta_m)/2 \text{ and } \Theta_{av} \equiv (\Theta_M - \Theta_m)/2\,. \quad (14)$$

Samples length is, therefore, N=365. In order to keep exposition as clear as possible, the previous generic notations *x* and *z* of §2.1 will be used, being $x, z \equiv \Theta_M, \Theta_m, \Theta_{av}, \Theta_{am}$. By means of discrete Fourier transformer, parameters $\bar{x}_d, \Delta x_{dx}, \varphi_x$ and of deterministic model represented by (7), were determined [13] and [16]. Resulted random residuals, x_{rant}, were analysed and, although respective autocorrelograms revealed the presence of an AR model, the histogram of x_{rant} amplitudes passed a Chi-Square test regarding a Gaussian distribution. If model:

$$x_t = \bar{x}_d + \Delta x_d \cos(\omega t + \varphi_x) + \mathcal{N}(\hat{\mu}_x, \hat{\sigma}_x)\,, \quad (15)$$

where $\hat{\mu}_x = 0$ is valid, x_t can be considered as a non-stationary random variable, which mean is time dependent, $\mu_{x_t}(t)$, according to:

$$\mu_{x_t}(t) = \bar{x}_d + \Delta x_d \cos(\omega t + \varphi_x) \quad (16)$$

and which standard deviation, generically denoted by $\sigma_{x_t}(t)$, results, in fact, in a time independent (stationary) function:

$$\sigma_{x_t}(t) = \hat{\sigma}_x . \quad (17)$$

From (16) and (17) one can obtain the variation coefficient, $CV_{x_t}(t)$:

$$CV_{x_t}(t) \equiv \frac{\sigma_{x_t}(t)}{\mu_{x_t}(t)} = \frac{\hat{\sigma}_x}{\bar{x}_d + \Delta x_d \cos(\omega t + \varphi_x)}, \quad (18)$$

which mean value, $\overline{CV}_{x_t}(t)$, is:

$$\overline{CV}_{x_t}(t) = \frac{\hat{\sigma}_x}{\bar{x}_d} . \quad (19)$$

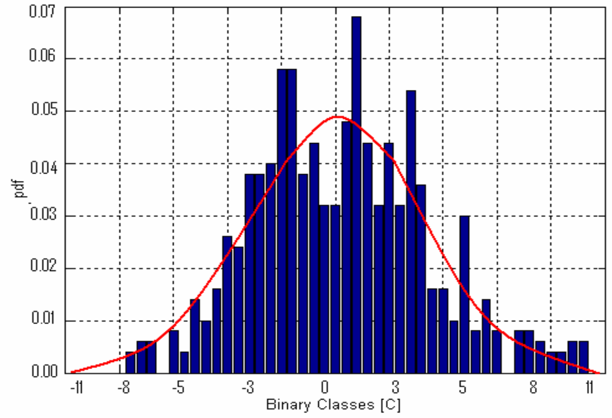
From $\overline{CV}_{x_t}(t)$ values one can realise the degree of x_t concentration around its mean $\mu_{x_t}(t)$. Deterministic model parameters, $\bar{x}_d, \Delta x_d$ and φ_x, estimators of residuals first moment, $\hat{\mu}_x$ and $\hat{\sigma}_x$, mean value of variation coefficient, $\overline{CV}_{x_t}$, and *p-value* from the Chi-Square test, are resumed in Table 1, for the 4 analysed years.

Table 1: Deterministic model parameters, random component x_{ran_t} first moment estimators and *p-value*, for ambient temperature time series.

	$\bar{x}_d[^0C]$	$\Delta x_d[^0C]$	$\varphi_x[rad]$	$\hat{\mu}_x[^0C]$	$\hat{\sigma}_x[^0C]$	$\overline{CV}_{x_t}(t)$ [p.u.]	*p-value*[%]
2002 Θ_M	20.935	7.109	2.829	0.000	3.529	0.169	53
Θ_m	12.395	5.384	2.657	0.000	2.084	0.167	40
Θ_{av}	16.233	6.237	2.753	0.000	2.435	0.145	77
Θ_{am}	4.275	1.009	-2.969	0.000	1.567	0.366	52
2003 Θ_M	20.027	6.803	2.821	0.000	3.368	0.169	22
Θ_m	12.455	5.339	2.733	0.000	2.356	0.187	90
Θ_{av}	16.247	6.067	2.783	0.000	2.485	0.154	32
Θ_{am}	3.789	0.777	3.125	0.000	1.505	0.395	97
2004 Θ_M	21.011	6.283	2.733	0.000	3.162	0.152	9
Θ_m	12.975	4.699	2.531	0.000	2.183	0.169	11
Θ_{av}	16.993	5.465	2.647	0.000	2.331	0.136	75
Θ_{am}	4.021	0.963	3.243	0.000	1.395	0.345	83
2005 Θ_M	22.111	6.797	2.810	0.000	3.527	0.158	81
Θ_m	14.011	4.639	2.606	0.000	2.654	0.188	75
Θ_{av}	18.059	5.687	2.725	0.000	2.728	0.150	47
Θ_{am}	4.051	1.223	-3.082	0.000	1.512	0.375	8

For these four analysed years, the model reproduces very well each year, although the number of considered years is insufficient to drawn generalised conclusions or forecasts for the coming years. All samples passed with relatively high p-values Chi-square tests regarding the hypotheses of being Gaussian distributed. From the low values of $\overline{CV}_{x_t}$ (one can conclude that random component $x_{ran\,t}$ is relatively concentrated around the deterministic component. The histograms and respective theoretical probabilistic density functions (*pdf*) of a Gaussian distribution with parameters $\hat{\mu}_x$ and $\hat{\sigma}_x$ are represented on Figure 5 and Fig.6. A deeper analysis of $x_{ran\,t}$ residuals with respective autocorrelation functions revealed the presence of possible first order AR models. The resulted z_t variables, once the first order AR model was removed, were studied. All passed a randomness test with a confidence level of 5%. Concerning the probabilistic distributions, in some cases z_t variable get closer to the Gaussian distribution, in others get far from the Gaussian distribution, even failing a Chi-square test at 5% level of confidence. These results indicate that more elaborate models are required to fully modelling these time series. First moment estimators, μ_z and $\hat{\sigma}_z$, of z_t variables, as well as the *p-value* of the Chi-square test concerning a

Gaussian distribution, are represented on Table 2.



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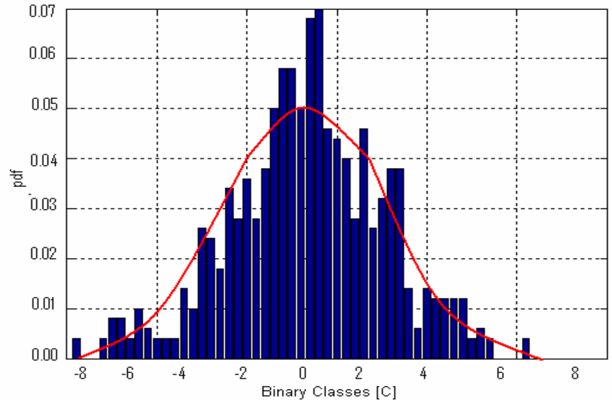

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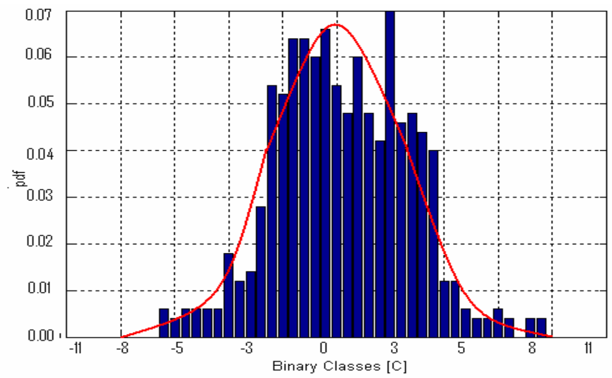
Fig. 5: Histogram and respective Gaussian *pdf* for random component of Θ_M (a) and Θ_m (b). Data from 2005.



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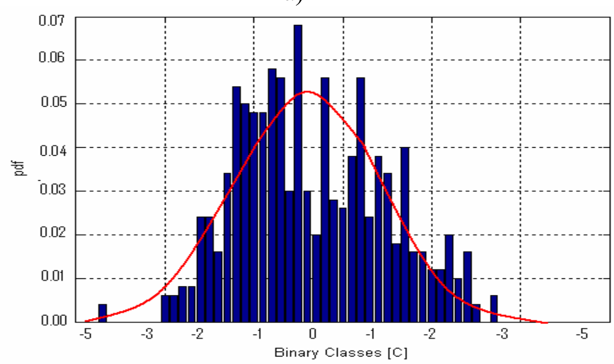


b)

Fig. 6: Histogram and respective Gaussian *pdf* for random component of Θ_{av} (a) and Θ_{am} (b). Data from 2005.

By comparing $\hat{\sigma}_x$ and $\hat{\sigma}_z$ values one concludes that the taking into consideration of the first order AR model, reduces the variance level of random component ($\hat{\sigma}_z < \hat{\sigma}_x$). However, it is $\hat{\sigma}_x / \hat{\sigma}_z \approx 1$ meaning that supplementary information carried by the AR model is quite reduced.

Table 2: Random component z_{rant} first moment estimators and *p-value*, for ambient temperature time series.

	$\hat{\sigma}_x [^0C]$	$\hat{\sigma}_x [^0C]$	*p-value* [%]	$\hat{\sigma}_x / \hat{\sigma}_z$ [p.u.]
2002 Θ_M	0.000	2.556	43	1.381
Θ_m	0.000	1.613	78	1.289
Θ_{av}	0.000	1.612	7	1.505
Θ_{am}	0.000	1.361	4	1.153
2003 Θ_M	0.000	2.447	12	1.373
Θ_m	0.000	1.963	3	1.203
Θ_{av}	0.000	1.752	64	1.416
Θ_{am}	0.000	1.330	7	1.131
2004 Θ_M	0.000	2.373	14	1.333
Θ_m	0.000	1.742	70	1.255
Θ_{av}	0.000	1.672	1	1.393
Θ_{am}	0.000	1.211	75	1.149
2005 Θ_M	0.000	2.390	75	1.473
Θ_m	0.000	1.982	19	1.336
Θ_{av}	0.000	1.753	87	1.555
Θ_{am}	0.000	1.281	99	1.185

Although model traduced by:

$$x_t = \bar{x} + \Delta x_d \cos(\omega t + \varphi_x) + \hat{\rho}_0 x_{t-1} + \mathcal{N}(\hat{\mu}_x, \hat{\sigma}_x) \quad (20)$$

where $\hat{\mu}_z = 0$ results more precise for some of the analysed time series, the model traduced by:

$$\bar{x}_d + \Delta x_d \cos(\omega t + \varphi_x) + \mathcal{N}(\hat{\mu}_x, \hat{\sigma}_x), \quad (21)$$

where $\hat{\mu}_x = 0$ can describe in a more generic way, although less accurate, the time series representative of maximal, minimal, average and half-amplitude values of daily ambient temperatures of analysed years. A similar analysis was performed for the load profiles of two distribution transformers, denoted by ES1 and BI1. Available data includes daily maximal, K_M, minimal, K_m, average, K_{av}, and half-amplitude, K_{am}, load factor for the 2003, 2004 and 2005 years, being, analogously to ambient temperature:

$$K_{av} \equiv (K_M + K_m)/2 \text{ and } K_{av} \equiv (K_M - K_m)/2 \,. \quad (22)$$

During this period no structural network changes occurred in the network. As an example, maximal and minimal load factor values of ESI transformer, relatively to 2005, are represented on Fig.7. The loads served by this transformer are, mainly, of the residential type with a small component of industry. Similar to ambient temperature modelling, the previous generic notations x and z will be used, being $x, z \equiv K_M, K_m, K_{av}, K_{am}$. Deterministic cyclic component was assumed to follow also a sinusoidal variation as represented on (7) and resulted residuals, x_{rant}, were studied. Table 3 resumes obtained values for deterministic model parameters, $\bar{x}_d$, Δx_d and φ_x, estimators of residuals first moment, $\hat{\mu}_x$ and $\hat{\sigma}_x$, and *p-value* from the Chi-Square test regarding a Gaussian distribution of the residuals.

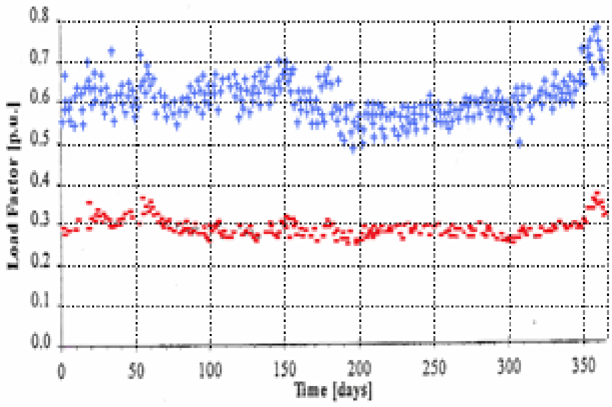


Fig. 7: Annual time series representing daily maximal (denoted with +) and minimal (denoted with -) load factor of distribution transformer ESI. Data from 2005.

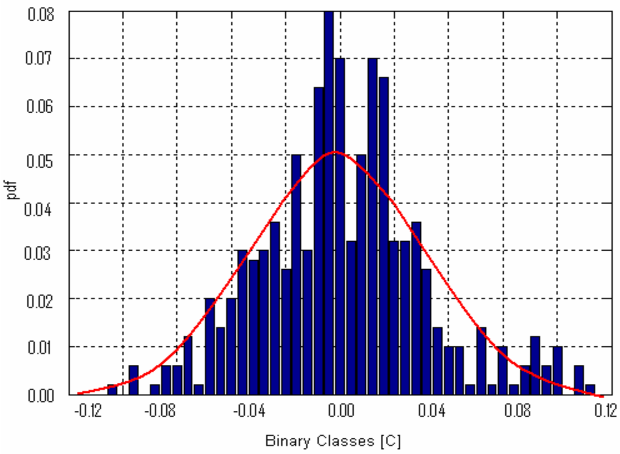
Table 3: Deterministic model parameters, random component x_{rant} first moment estimators and *p-value*, for ESI distribution transformer.

	$\bar{x}_d$ [p.u.]	Δx_d [p.u.]	φ_x [rad]
2003			
K_M	0.611	0.053	-0.029
K_m	0.277	0.016	0.292
K_{av}	0.443	0.034	0.025
K_{am}	0.165	0.023	-0.157
2004			
K_M	0.582	0.055	0.115
K_m	0.273	0.016	0.571
K_{av}	0.426	0.037	0.209
K_{am}	0.156	0.023	-0.042
2005			
K_M	0.602	0.035	-0.932
K_m	0.281	0.016	-0.773
K_{av}	0.440	0.025	-0.879
K_{am}	0.157	0.009	-1.089

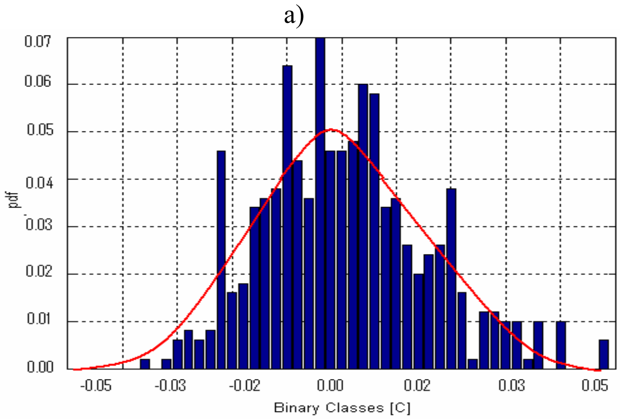
	$\hat{\mu}_x$ [p.u.]	$\hat{\sigma}_x$ [p.u.]	$\overline{CV}_{x_t}$ [p.u.]	*p-value* [%]
2003				
K_M	0.000	0.045	0.074	47
K_m	0.000	0.025	0.086	0
K_{av}	0.000	0.033	0.074	35
K_{am}	0.000	0.018	0.104	15
2004				
K_M	0.000	0.049	0.085	59
K_m	0.000	0.026	0.095	19
K_{av}	0.000	0.035	0.079	11
K_{am}	0.000	0.019	0.121	85
2005				
K_M	0.000	0.040	0.065	25
K_m	0.000	0.018	0.061	12
K_{av}	0.000	0.024	0.053	0
K_{am}	0.000	0.019	0.118	67

On Fig.7, time series representative of K_M "looks" much more disperse than the K_m time series, which is a common fact in the three analysed years. Minimal values of distribution transformer load profiles are very well defined by (usually) night loads, corresponding to "base" equipment which is almost constant if no structural changes or accidents occur in the transformer network, while maximal values traduce temporary overloads due to residential/industrial activity. From $\overline{CV}_{x_t}$ values, represented on Table 3, one realises that load profiles are much more concentrated around respective deterministic components, than ambient temperature profiles are. Globally, results resumed on Table 3 can be considered as good although residuals from K_m in 2003 and K_{av} in 2005 can not be considered to follow a Gaussian distribution. In

fact, from the study of the autocorrelation function, one realises the presence of higher frequencies than the fundamental frequency (annual) on the deterministic sinusoidal model (7). This was an expected occurrence since load profiles are constrained to a much great diversity of factors than ambient temperature profiles are. One can detect, for example, the increasing appearance of a second harmonic (bi-annual) reflecting the increase in loads due to air-conditioning equipment during summer period. However, one should recall the purpose of this work: to give a physical justification for theoretical load and ambient temperature profiles used in following simulations and not fully modelling these profiles.
The histograms and respective theoretical *pdf*'s of Gaussian distributions with parameters $\hat{\mu}_x$ and $\hat{\sigma}_x$ reproduced on Table 3 are represented on Figure 8 and Fig.9, for the 2005 data set. The great dispersion of K_M time series relatively to K_m time series can be visualised by the limits of histograms represented on Fig.8.



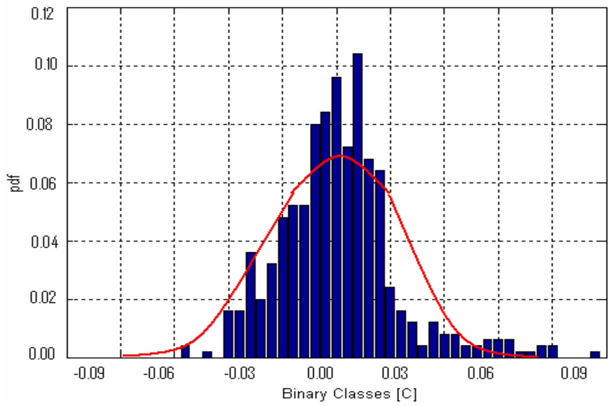
a)



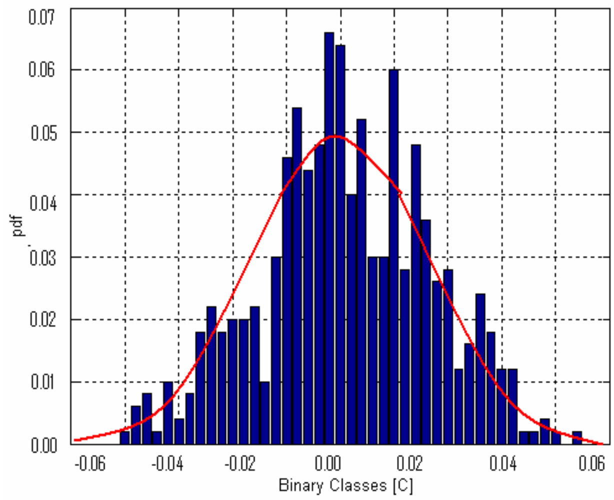
b)

Fig. 8: Histogram and respective Gaussian pdffor random component x_{ran_t} of K_M (a) and K_m (b). ESI transformer and data from 2005.

The hypothesis that random component x_{ran_t} of time series representative of load profiles could be modelled by an AR model, did not give as good results as with ambient temperature profiles. This fact is due, in part, to the already referred presence of other cyclic (bi-annual) variations in xran which were not taken into consideration on the deterministic model (7). Results are resumed on Table 4. Results obtained with the second analysed distribution transformer, referred as BI1, are resumed on Table 4, where $\hat{\mu}_z$ values were omitted since it is $\hat{\mu}_z = \hat{\mu}_x = 0$ for all samples.



a)



b)

Fig. 9: Histogram and respective Gaussian pdffor random component $x_{ran\,t}$ of K_{am} (a) and K_{av} (b). ESI transformer and data from 2005.

This transformer serves an area where loads are of residential and industrial types, in similar proportions. Residuals x_{ran_t} after removing the deterministic cyclic variation are not as normally distributed as residuals resulting from the ESI load profiles; in the 12 presented samples, 3 of them even fail the respective chi-square test. This fact does not invalidate the generic model represented by (20).

Table 4: Random component z_{ran_t} first momment estimators and *p-value*, for ESI load profiles.

	$\hat{\mu}_z[{}^0C]$	$\hat{\sigma}_x[{}^0C]$	*p-value*[%]	$\hat{\sigma}_x/\hat{\sigma}_z$ [p.u.]

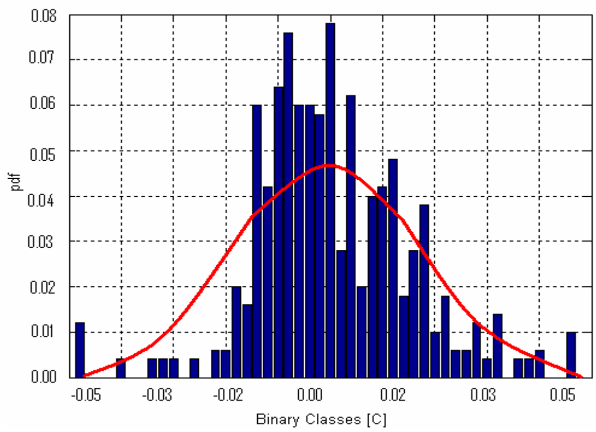
2003 K_M	0.000	0.033	5	1.417
K_m	0.000	0.011	11	1.987
K_{av}	0.000	0.017	12	1.819
K_{am}	0.000	0.015	24	1.097
2004 K_M	0.000	0.034	15	1.455
K_m	0.000	0.022	0	1.183
K_{av}	0.000	0.022	0	1.620
K_{am}	0.000	0.018	53	1.075
2005 K_M	0.000	0.032	6	1.223
K_m	0.000	0.011	84	1.427
K_{av}	0.000	0.016	56	1.337
K_{am}	0.000	0.015	11	1.155

Table 5: Deterministic model parameters, random components x_{ran_t} and z_{ran_t} first moment estimators and p-values, for BI1 distribution transformer.

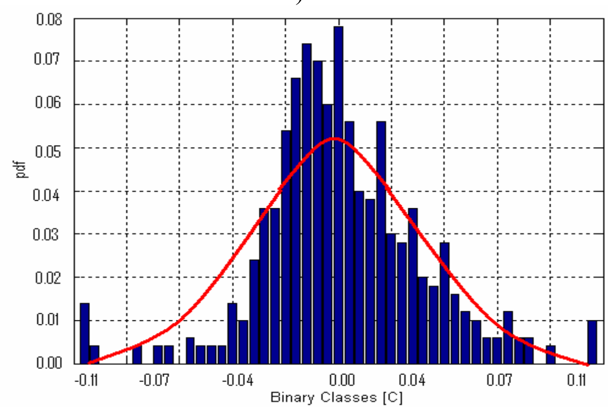
	$\bar{x}_d$ [p.u.]	Δx_d [p.u.]	φ_x [rad]	$\hat{\mu}_x$ [p.u.]	$\hat{\sigma}_x$ [p.u.]
2003 K_M	0.351	0.075	-0.196	0.000	0.035
K_m	0.166	0.014	-0.147	0.000	0.015
K_{av}	0.258	0.046	-0.189	0.000	0.023
K_{am}	0.093	0.033	-0.206	0.000	0.017
2004 K_M	0.375	0.068	-0.088	0.000	0.037
K_m	0.188	0.011	0.545	0.000	0.015
K_{av}	0.282	0.039	-0.001	0.000	0.027
K_{am}	0.095	0.031	-0.238	0.000	0.019
2005 K_M	0.391	0.071	-0.019	0.000	0.033
K_m	0.201	0.016	0.091	0.000	0.015
K_{av}	0.296	0.043	-0.001	0.000	0.022
K_{am}	0.097	0.029	-0.051	0.000	0.011

	$\overline{CV}_{x_t}$ [p.u.]	*p-value* [%]	$\hat{\sigma}_z\left[{}^0C\right]$	*p-value* [%]	$\hat{\sigma}_x / \hat{\sigma}_z$ [p.u.]
2003 K_M	0.102	63	0.033	51	1.118
K_m	0.083	31	0.012	1	1.307
K_{av}	0.089	61	0.017	21	1.236
K_{am}	0.171	5	0.016	14	1.028
2004 K_M	0.095	6	0.027	0	1.387
K_m	0.085	5	0.015	0	1.195
K_{av}	0.101	0	0.023	0	1.264
K_{am}	0.191	0	0.018	0	1.068
2005 K_M	0.081	21	0.019	0	1.726
K_m	0.079	0	0.011	1	1.546
K_{av}	0.078	9	0.014	0	1.915
K_{am}	0.116	88	0.009	4	1.263

Since 2004 is the year which data give the worst results, meaning lower *p-values* on the chi-square test for a Gaussian distribution of residuals, histograms of x_{ran_t} residuals and respective theoretical pdf's are represented on Fig.10 and Fig.11.



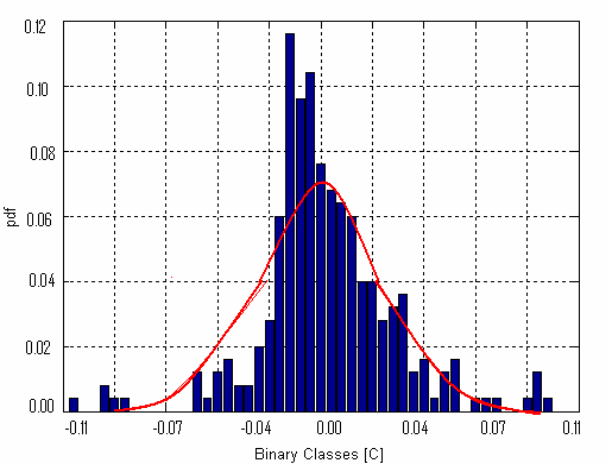
a)



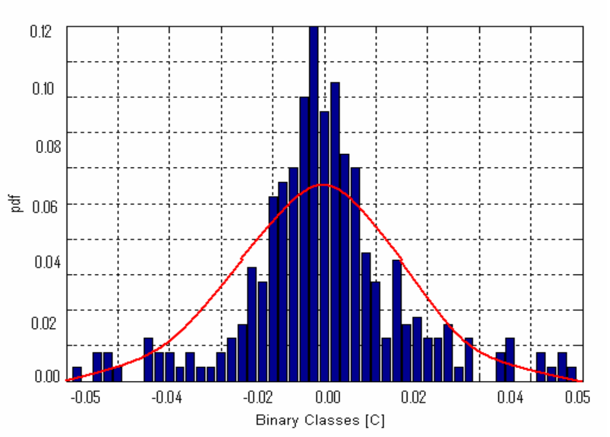
b)

Fig. 10: Histogram and respective Gaussian *pdf* for random component xran of K_M (a) and K_m (b). BI1 transformer and data from 2004.

Although not passing the Chi-square test, the statistical distribution of random component x_{ran_t} relativeley to 2004 K_M, K_m, K_{av} and K_{am}, values, is not far from a Gaussian distribution as can be visualised on Fig.10 and Fig.11. Although more elaborated models are required to fully model the load profiles of distribution transformers, it has been shown that (21) can be considered as a good generic model.



a)



b)

Fig. 11: Histogram and respective Gaussian pdf for random component X_{ran} of K(a) and K_{am} (b). BI1 transformer and data from 2004.

2.3 Global Model

On this section a global model to represent the whole set of maximal, minimal and average temperatures (or load factors) along the year, will be described. The model is based on the previously studied x_{av} and x_{am} time series where $x \equiv \Theta, k$ and is defined as a linear combination of these two:

$$x_t = x_{av} + \alpha_G x_{am}, \tag{23}$$

being α_G a real number and $\alpha_G \in [-1,1]$.

From x_{av} and x_{am} definition, (14) and (22), one can realise that the chosen α_G range, determines (23) to model variables from minimal to maximal values according to:

$$-1 \le \alpha_G \le 1 \Rightarrow x_m \le x_t \le x_M \tag{24}$$

If both x_{av} and x_{am} time series, can be assumed to follow a deterministic and random components according to (21), and attending to (23), x_t model will also result with deterministic and random components:

$$x_t = x_{\det_t} + x_{ran_t} \tag{25}$$

with:

$$x_{\det} = \bar{x}_d + \Delta x_d \cos(\omega t + \varphi_x) \tag{26}$$

$$x_{ran_t} = N(0, \sigma_x) \tag{27}$$

Each of the parameters $\bar{x}_d$, Δx_d, and φ_x can be analytically determined and result as:

$$\hat{x}_d = \hat{x}_{av} + \alpha_G \hat{x}_{amd} \tag{28}$$

$$\Delta x_d = \sqrt{(\Delta x_{av_d})^2 + (\alpha \Delta x_{am_d})^2 + 2\alpha_G \Delta x_{av_d} \Delta x_{am_d} \cos(\varphi_{xav} - \varphi_{xam})} \tag{29}$$

$$\varphi_x = arctg \frac{\Delta x_{av_d} \sin(\varphi_{xav}) + \alpha \Delta x_{am_d} \sin(\varphi_{xam})}{\Delta x_{av_d} \cos(\varphi_{xav}) + \alpha \Delta x_{am_d} \cos(\varphi_{xam})} \pm \pi \tag{30}$$

and

$$\sigma_x = \sqrt{(\sigma_{x_{av}})^2 + (\alpha \sigma_{x_{am}})^2 + 2CoV(x_{av_{rand}}, \alpha_G, x_{am_{rand}})} \tag{31}$$

where $COV(x_{av_{rand}}, \alpha_G, x_{am_{rand}})$ denotes the covariance (covariance function (3) with null time lag, k=0) between the random components $x_{av_{rand}}$ and $\alpha_G x_{am_{rand}}$. If profiles perfectly fitted model represented by (21), random components $x_{av_{rand}}$ and $x_{av_{rand}}$ would result as random variables and therefore uncorrelated from each other. Under this condition, (31) could be replaced by:

$$\sigma_x \approx \sqrt{(\sigma_{x_{av}})^2 + (\alpha \sigma_{x_{am}})^2}. \tag{32}$$

Since (21) is only an approximate model of profiles evolution, covariation between random components $x_{av_{rand}}$ and $x_{am_{rand}}$ is considerably. Since correlation is an image of covariation but normalised by variables respective variations, the strength of the link between $x_{av_{rand}}$ and $x_{am_{rand}}$ result clearer if correlation values are represented instead of covariation, Fig.12.

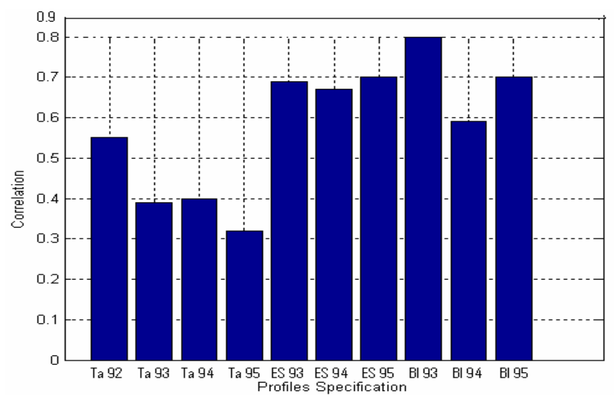


Fig. 12: Correlation between random components $x_{av_{rand}}$ and $x_{am_{rand}}$.

The usefulness of this global model resides on modelling compactness it traduces; by means of

α_G parameter $-1 \le \alpha_G \le 1$, this single model is able to reproduce ambient temperature (or load factor profiles) models previously derived, from minimal to maximal values. Numerical validation of this model is not reproduced here, since obtained values are in agreement with those reproduced on Table 1, Table 3 and Table 5.

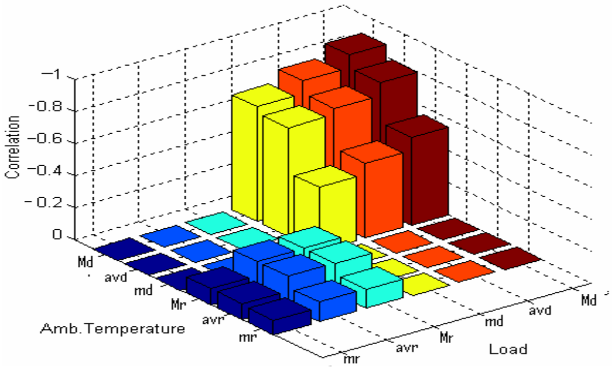
2.4 Ambient Temperature and Load Profiles Correlation

From the time evolution of load and ambient temperature profiles the distribution transformers are subjected to, one can infer a relationship between them. For the analysed cases, when ambient temperature drops, loads increase, and when ambient temperature increases, transformer loads decrease. The strength of this relationship between loads and ambient temperature is measured by the correlation between them. Since models have a deterministic and a random part (21), correlation coefficient between each of these components, will be determined, to evidence that correlation between time series is mainly due to their deterministic components; random components are practically independent (uncorrelated) from each other. Correlation coefficients between transformer ESI load profile and 2004 ambient temperature are represented on Table 6. Correlation between deterministic parts is clearly stronger than between random parts. The negative sign traduces the fact that, for the analysed data, models are inversely correlated; the ambient temperature increase implies loads decrease and vice-versa.

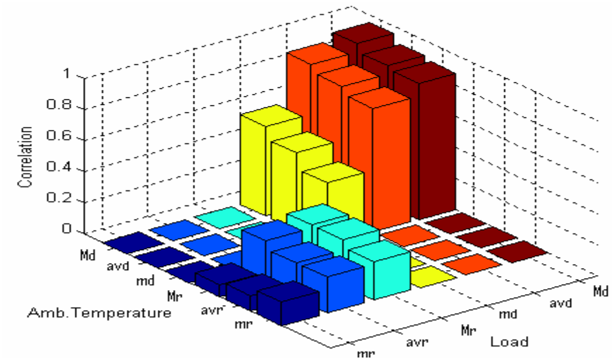
Table 6: ESI deterministic and random correlation for 2004 data set.

Ambient Temperature		ESI Load Profile: Deterministic maximal	Deterministic average	Deterministic minimal	Random maximal	Random average	Random minimal
Deterministic	maximal	-0.471	-0.715	-0.771			
	average	-0.569	-0.793	-0.841			
	minimal	-0.636	-0.843	-0.885			
Random	maximal				-0.074	-0.089	-0.087
	average				-0.111	-0.135	-0.135
	minimal				-0.112	-0.143	-0.143

Correlation strength increases as walking towards maximal values, which means that loads, and in particular, maximal ones, are much more "sensitive" to maximal ambient temperature than minimal ambient temperature. In fact, minimal loads along the year are almost constant and they traduce, in practice, a "base" load that is almost invariant with ambient temperature changes and depends most upon load characteristics of transformers distribution network. Previous considerations about correlation result clearer on Fig.13(a), where Table 6 deterministic and random correlation values are graphically represented.



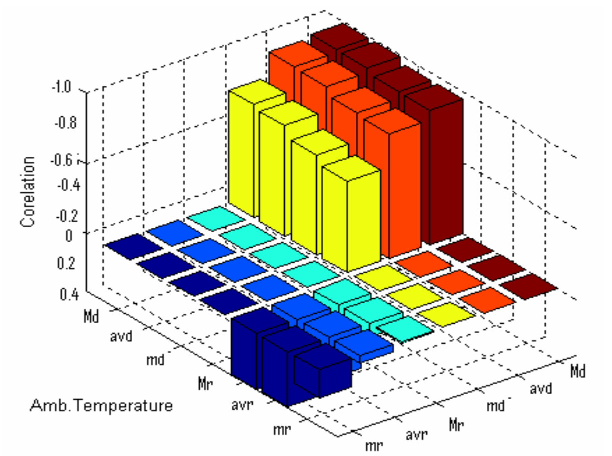
a)



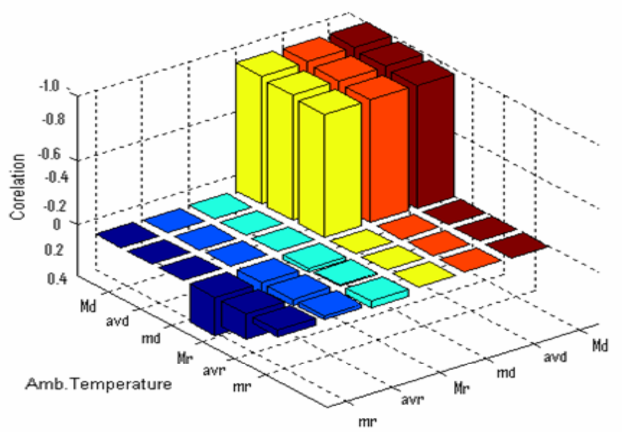
b)

Fig. 13: Deterministic and random correlation between ambient temperature and ESI (a) andBI1 (b) profiles. Data from 2004. (Table 6 for ESI transformer).

Although correlation coefficients are all negative, on Fig.13 correlation axis is in reverse order, so that graph visualisation results clearer. Similar relationship between deterministic and random correlation values can be obtained from BI1 transformer data, Fig.13(b), and from 2003 and 2005 data sets, Fig.14 and Fig.15.

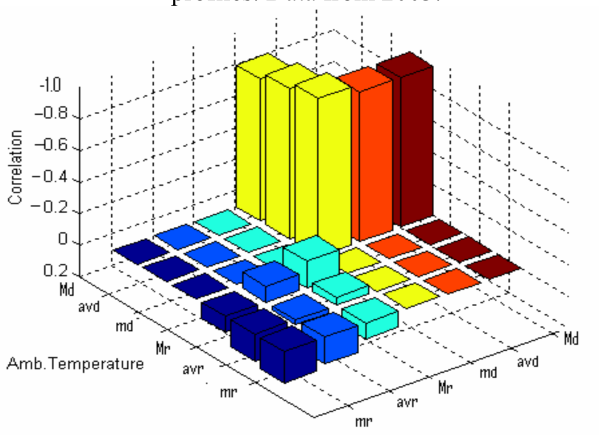


a)

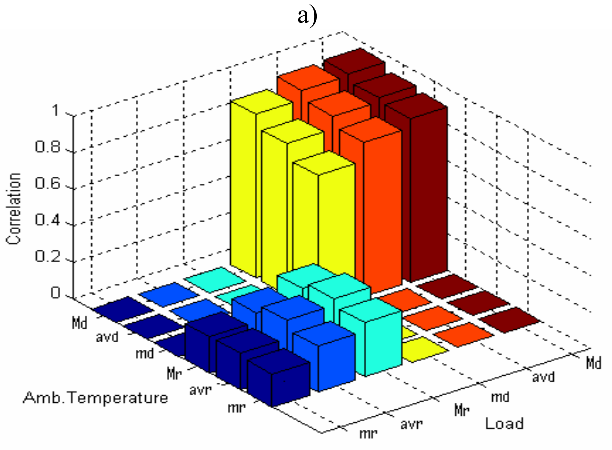


b)

Fig. 14: Deterministic and random correlation between ambient temperature and ESI (a) and Bll (b) profiles. Data from 2003.



a)



b)

Fig. 15: Deterministic and random correlation between ambient temperature and ESI (a) and Bll (b) profiles. Data from 2005.

The constancy in the sign of correlation between random parts (all negatives in 2004 or all positives in ESI 2004) is an indication that random components x_{rand} of these profiles still carry deterministic behaviours that were not removed by the assumed deterministic model (21). If profiles were perfectly modelled by (21), correlation between any random component would result as null. Attending to the magnitude of correlation between deterministic parts and random parts, and to results presented on section §2.2 one can consider that

$$x_t = \bar{x}_d + \Delta x_d \cos(\omega t + \varphi_x) + \mathcal{N}(\hat{\mu}_x, \hat{\sigma}_x), \quad (33)$$

is a generic sufficiently accurate model to traduce the load annual evolution of distribution transformers as well as ambient temperature. On next section, simulated load and ambient temperature profiles with random components will be used, to study the sensitivity of transformer thermal and loss of life models presented on [17], to such functional parameters.

3 Functional Parameters Sensitivity

3.1 Probabilistic Formulation

3.1.1 Input Profiles

System inputs, K and Θ_a, are the transformer load and ambient temperature profiles which, by assumption, can be represented by an additive model of deterministic and random components, of the form:

$$K = K_{\det} + K_{ran} \text{ and } \Theta_a = \Theta_{a_{\det}} + \Theta_{a\,ran}. \quad (34)$$

In fact, possible correlation can occur between K and T. In this general case, a non-stationary model must be considered. In this work this increase in model complexity will not be considered, since when the correlation exists, it derives, mainly, from a strong link between deterministic components (i.e. concomitant sinusoidal load and ambient temperature variations) and a weakest link between corresponding random components, as shown on section §2.4. The objective of this study is, under stationary conditions, to determine, on the output variable (LOL), its deterministic and random components, based on the previously referred additive model.

3.1.2 Methodology

The data acquisition frequency of a continuous type system must be carefully defined since it plays an important role on posterior analysis of data. Namely, the data acquisition set must represent faithfully the signal and, from this data set, one must be able to "rebuilt" the original signal in a univocal way. The sampling theorem states that a continuous signal which Fourie Transform exists and is null out of the frequency interval [-f,f], should be sampled at a frequency f_s such that:

$$f_s > 2f \quad (35)$$

Reciprocally, if the sampling frequency is f_s, no information can be inferred from the sampled data set, about signal occurrences with frequencies above the Nyquist, f_N, frequency, given by:

$$f_N = f_s / 2, \quad (35)$$

Usually, in the case of long term forecasting, the acquisition period of data for analyses is long enough and therefore it is possible to neglect variables rapid fluctuations having a period of the same order of involved thermal time constants. Typically, it is τ_0=3 h and the windings constant $\tau_w \approx 5$ to 10 minutes, [1], [12], [16] and [22]. Taking into account that input variables are approximately stationary, this simplification represents a second argument to consider a probabilistic stationary model, instead of a stochastic dynamic one. Both transformer thermal and ageing models, are strongly non-linear ones, which will determine the non-preservation of inputs statistical distribution structure [2-3], [20] and [21]. Nevertheless, provided each mathematical transformation can be defined as a one-to-one function (with inverse) of an input random variable which *pdf* is known, output variable *pdf* can be analytically determined, either directly either with recourse of characteristic functions. However, this methodology is not suitable for the system under study, since some transformations do not have an analytical exact expression for its inverse function:

$$y = \varphi(x)\,, \tag{37}$$

which must be determined numerically.

The methodology used to estimate the stochastic output variable LOL, once the random inputs K and Θ_a are defined, is based on realistic characteristics of distribution transformers load profiles and ambient temperature ones. As already shown on section §2.2, in a statistical sense, K and Θ_a can be considered as unimodal random variables concentrated around their modal values (mode) [15] and [20] which means a reduced variation coefficient CV_x. Under this condition, it will be assumed as valid the linearisation of (37) in the vicinity of its input expected value μ_x, which first three terms are:

$$y \approx \varphi(\mu_x) + \frac{\partial\varphi(x)}{\partial x}\Big|_{x=\mu_x}[x-\mu_x] + \frac{\partial^2\varphi(x)}{\partial^2 x}\Big|_{x=\mu_x}[x-\mu_x]^2 . \tag{38}$$

From (39) one can obtain estimators for y moments, denoted by $\hat{\mu}_y$ and $\hat{\sigma}_y$, as functions of x moments, denoted by μ_x and σ_x. Second order estimators will be given by:

$$\hat{\mu}_y = \varphi(\mu_x) + \frac{1}{2}\frac{\partial^2\varphi(x)}{\partial^2 x}\Big|_{x=\mu_x}\sigma_x^{\,2} \tag{39}$$

$$\hat{\sigma}_y^{\,2} = \left[\frac{\partial\varphi(x)}{\partial x}\Big|_{x=\mu_x}\right]^2 \sigma_x^{\,2} + \frac{1}{2}\left[\frac{\partial^2\varphi(x)}{\partial^2 x}\Big|_{x=\mu_x}\right]^2 \sigma_x^{\,4} \tag{40}$$

The errors one commits by considering the first order estimators, against the second order ones, can be approximately bounded by:

$$\varepsilon_\mu = \frac{1}{2}\sigma_x^{\,2}\frac{\partial^2\varphi(x)}{\partial^2 x}\Big|_{x=\mu_x}\frac{1}{\varphi(\mu_x)} \quad \text{and}$$

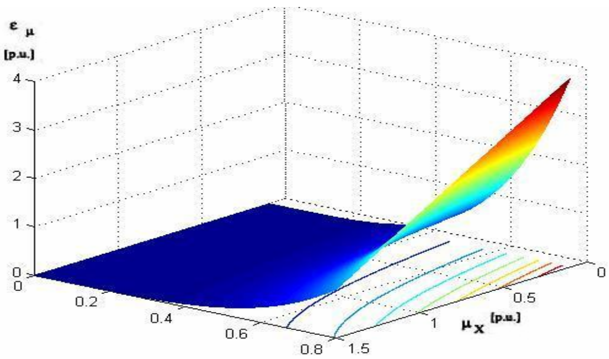
$$\varepsilon_{\sigma^2} = \frac{1}{2}\sigma_x^{\,2}\left[\frac{\partial^2\varphi(x)}{\partial^2 x}\Big|_{x=\mu_x}\frac{1}{\frac{\partial\varphi(x)}{\partial x}\Big|_{x=\mu_x}}\right]^2 . \tag{41}$$

If the linearisation of (37) is assumed to be valid, it will also lead to the preservation of input variable statistical structure. Being the x variable pdf defined, the output variable y will present a similar structure and its *pdf* can be determined, approximately, with recourse of its first moments, which estimators are given by (39) and (40).

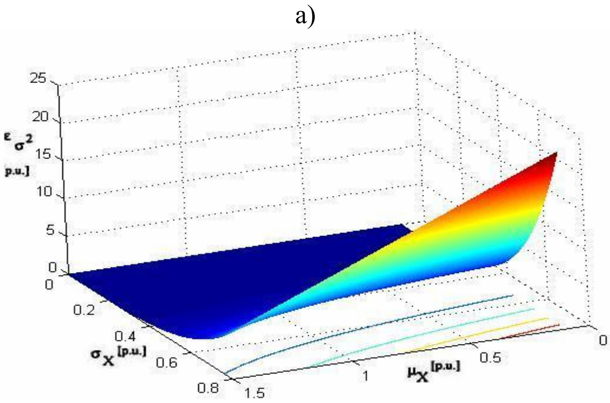
3.2 Approximate Analytical Model

3.2.1 Linearisation Error

In order to evaluate the validity of the linearisation traduced by (38), the errors ε_μ and ε_{σ^2}, (39) and (40), for a range of μ_x and σ_x corresponding to realistic values of distribution transformer load profiles, were studied: $\mu_x \in [0.1, 1.5]$ and $\sigma_x \in [0.01, 0.8]$. To μ_x=0.1 p.u. corresponds a very low load, while μ_x=1 p.u. corresponds to an overload of limited duration. The resulting variation coefficient ranges, approximately: $CV_x \in [0.007, 8]$.



a)



b)

Fig. 16: First order linearisation error (a) ε_μ and ε_{σ^2} (b).

Numerical results, presented on Fig.16, are determinant in concluding for the importance of second order estimators, as CV_x increases, traducing the limits of linearisation procedure, based on first order estimators.

3.2.2 Stationary Normal Inputs

Considering that both system input variables are normally distributed, with parameters:

$$k \sim \mathcal{N}(\mu_k, \sigma_k) \text{ and } \Theta_a \sim \mathcal{N}(\mu_k, \sigma_k), \quad (42)$$

resulting that $\hat{\mu}_{\Theta_{hs}}$ will present an approximately Normal distribution, which estimated parameters are:

$$\hat{\mu}_{\Theta_{hs}} = \hat{\mu}_{\Delta\Theta_{hs}} + \mu_{\Theta_a}, \quad (43)$$

$$\hat{\sigma}_{\Theta_{hs}}{}^2 = \hat{\sigma}_{\Delta\Theta_{hs}}{}^2 + \hat{\sigma}_{\Theta_a}{}^2, \quad (44)$$

since mutual independence between random parts was admitted. Under a probabilistic fonnulation, where time dependence does not exists and for stationary statistical distributions Vag is identical to *LOL*, and therefore [27], being Θ_{hs} approximately Normal, LOL will result strictly as a lognormal distributed random variable:

$$pdf(LOL) = \frac{1}{\hat{\sigma}_{LOL} 2\pi} \exp\left[-\frac{(\ln(LOL) - \hat{\mu}_{LOL})^2}{2\hat{\sigma}_{LOL}{}^2} \right], (45)$$

where

$$\hat{\mu}_{LOL} = \frac{\ln 2}{6}\left(\hat{\mu}_{\Theta_{hs}} - 98\right) \text{ and } \hat{\mu}_{LOL} = \frac{\ln 2}{6}\hat{\sigma}_{\Theta_{hs}}. \quad (46)$$

2.2.3 Stationary Uniform Inputs

Input variables are considered to be uniformly distributed:

$$K \sim \mathcal{U}[K_1, K_2] \text{ and } \Theta_a \sim \mathcal{U}(\mu_{\Theta_a}, \sigma_{\Theta_a}). \quad (47)$$

Their first moments are given by:

$$\mu_X = \frac{X_2 + X_1}{2} \text{ and } \sigma_X = \frac{X_2 - X_1}{2\sqrt{3}}, \quad (48)$$

and the resulting variation coefficient by:

$$CV_X = \frac{1}{\sqrt{3}} \frac{X_2 - X_1}{X_2 + X_1}, \quad (49)$$

with $X \equiv K, \Theta_a$. In this case, analytic *pdf* of output variable *LOL* is unknown because Θ_{hs} is a bounded random variable.

3 Conclusions

The modelling of the time series representative of annual evolution of ambient temperature and transformer load showed that a non-complex additive model of deterministic and random components could genetically model such time series. Good results were obtained considering the deterministic component as a time varying function represented by a constant value (mean annual value) to which a first order sinusoidal function is added (annual cyclic variation). The model can easily be extended to daily, weekly or seasonally sinusoidal variations. Resulted residuals still denoted the presence of deterministic cyclic behaviours of higher than the first order but, generally, they could be approximate to random variables closely following a Gaussian distribution.
Most detailed models, such as the autorregressive models were experienced. They proved to mostly precise model some of the analysed time series but they could not be generalised for the analysed sample of profiles. The correlation between ambient temperature and distribution transformer load was also analysed. For the studied cases, the results obtained by splitting this analysis into correlation between deterministic components and correlation between random components, showed that ambient temperature and distribution transformer load were inversely correlated and that this correlation derives mainly from a strong link between deterministic components rather than from random components. Due to their relative values, correlation between random components is practically negligible, compared to that between deterministic components. Due to the strongly non-linearity of transformer thermal and loss of life models the statistical structure of input variables (load and ambient temperature) is not preserved on the output variable (loss of life). Moreover, the analytical determination of output statistical pdf is not possible either directly either with recourse of characteristic functions, since some mathematical transformations do not have an analytical exact expression for its inverse. Since, in a statistical sense, load variable is of reduced variability, meaning concentrated around its mean, a second order linearisation of the model, valid in the vicinity of load mean, was developed.

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