Nonlinear Feedback System for an Inverter-Based Ring Oscillator

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Abstract: -In integrated circuits, ring oscillator (RO) has many applications. In these applications, how to obtain the accurate oscillation frequency is an important issue for the design. In this paper, we explore the feedback problem of a N-stage ring oscillator. This paper proposes a more rigorous approach to analyze the ring oscillator. It can be approximated that the feedback system of a ring oscillator can be a nonlinear Lur'e problem. With this Lur'e problem, circle criterion can be used to determine the stability of overall feedback system. With this Lur'e problem, the describing-function method is used to determine the oscillation frequency of the ring oscillator. A new formula will be presented. It can be also observed that if N (number of inverters) is large enough, the proposed formula will approach the conventional formula. Moreover, with describing function method, a "**necessary condition"** for the existence of fundamental mode and higher order modes of oscillations are presented. Furthermore, it can be shown that as $N \geq 7$ and the voltage gain is large enough; the higher harmonic oscillation may exists. Finally, with Tsypkin Function method, a more accurate formula for the oscillation frequency of ring oscillator will be presented. Finally, Simulation examples will illustrate these results.

Key-words: -ring oscillator, describing function, Tsypkin function, Extended Nyquist Diagram

1 Introduction

For Digital Integrated Circuit designs, the ring oscillator plays an essential role [1,2]. The ring oscillator is actually a feedback circuit composed of an odd number of inverters and is one of the most fundamental circuits in large-scale integration (LSI) technology. They can be used as voltage-controlled oscillators (VCO) in applications such as clock recovery circuits for serial data communications [3], disk drive read channels [3], on-chip clock distribution [3] and integrated frequency synthesizers [3]. Despite it widespread usage, the RO still pose difficulties when it comes to analysis and modeling.

Conventionally, the ring oscillators are treated as a nonlinear negative feedback system. This means that for a nonlinear negative feedback system, the design of the system is maintained at the so-called stable "Limit Cycle Condition"[6-8].

On the other hand, the describing function method has been widely used to determine the limit cycle and the dynamical behaviors for the nonlinear systems [4-8]. The advantages of the describing function method are that it can be applied in the large signal situations. Moreover, the describing function method can be viewed as another kinds of harmonic balance method [7,8].

In this paper, we explore the feedback property of inverter-based ring oscillator. In the digital circuits, the ring oscillator contains the nonlinear feedback elements. Conventionally, the derivation of period of RO is determined by the simple concept of addition of each delay of inverter. In the meantime, several researchers have proposed some methods to determine the frequency of RO [9-11]. This paper proposes a more rigorous approach to analyze the nonlinear ring oscillator by feedback theory. It can be shown that the feedback elements of ring oscillator can be approximated by a nonlinear gain with a linear transfer function. Then, the overall system becomes a Lur'e problem. With this Lur'e problem, Circle criterion can be used to determine the stability of overall system. Also, the describing function method can be used to determine the oscillation frequency of the ring oscillator. A new formula will be presented. It can be also observed that as N (number of inverters) is large enough, the proposed formula will approach the conventional formula. On the contrary, with extended Nyquist Criterion and describing function method, we can show that if the voltage gain of the inverter in a N-stage ring oscillator is small enough, the oscillation will not exist. Furthermore, it can be shown that as $N \geq 7$ and if the voltage gain is large enough; the higher harmonic oscillation may exists. This result is the same as previous literature's points [10,12]. With Tsypkin Function method [8,13], a more accurate formula for the oscillation frequency of ring oscillator will be presented. Finally, Simulation examples will illustrate these results.

2 Ring Oscillator Circuit

In this section, we will explore the conventional derivation of the period of a N-stage ring oscillator. As seen in Fig.1, consider the cascade connection of three identical inverters, where the output node of the third inverter is connected to the input node of the first inverter. Fig.2 shows the typical output voltage waveform of the three inverters during oscillation. As the output voltage V_1 of the first inverter stage rises from V_{ol} (output low voltage) to V_{oH} (output high voltage), it trigger the second inverter output V_2 to fall, from V_{oH} to V_{ol}. Note that the difference between the $V_{50\%}$ -crossing times of V_1 and V_2 is the signal propagation delay τ_{PHL2} of the second inverter. Similarly, for V_2 and V_3 , V_3 and V_1 , τ_{PLH3} and τ_{PLH3} are the signal propagation delay of the third and first inverter respectively.

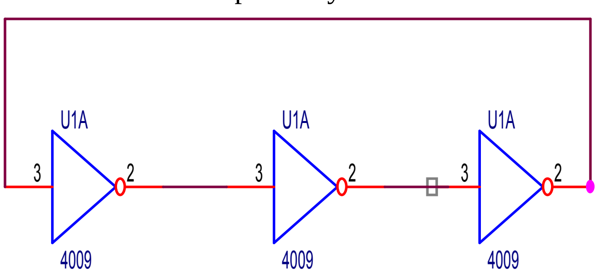

Fig. 1. Three-stage Ring Oscillator Circuit Consisting of Identical Inverters

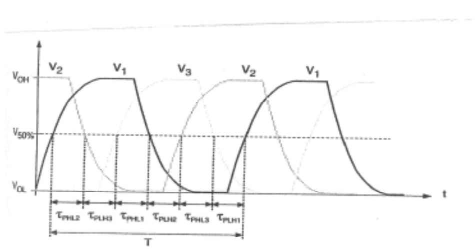

Fig. 2. Typical Voltage Waveform of the Three Inverter shown in Fig.1

In this three-stage circuit, the oscillation period T can be expressed as the sum of six propagation-delay times. Since the three inverters in the closed-loop cascade connection are assumed to be identical. We can express the oscillation period T [14][15] in terms of the average propagation delay τ as follows:

$$T = \tau_{PHL1} + \tau_{PLH1} + \tau_{PHL2} + \tau_{PLH2} + \tau_{PHL3} + \tau_{PLH3}$$
$$= 3 \cdot 2\tau = 6\tau_{av} \tag{1}$$

where τ_{av} means the average propagation delay.

Generating this relationship for any arbitrary odd number (N) of cascade-connected, we obtain [14,15]

$$f_{osc} = \frac{1}{T} = \frac{1}{2N\tau_{av}} \tag{2}$$

Thus, the oscillation frequency f_{osc} is found to be a very simple function of the average propagation delay of an inverter stage; however, with the above graphical approach to derive the oscillation frequency f_{osc} of N-stage ring oscillator, even though it is simple, it is not rigorous and the accuracy is not good enough. In the sequel sections, we will explore more rigorous results with the feedback theory.

3 Preview of Describing Function

The describing function method has been extensively used to determine the limit cycle and dynamical behavior for the nonlinear systems [6-10]. According to Fig-3, a nonlinear element exists in the feedback loop described by $\phi(.)$.

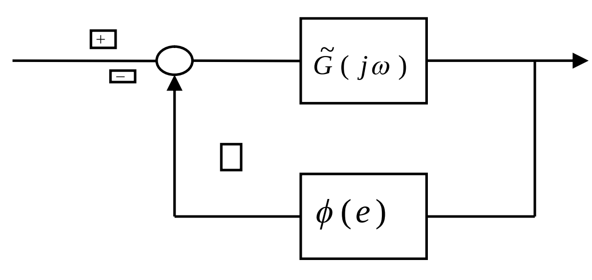

Fig. 3. A Conventional Lur'e Problem

Consider a sinusoidal input to the nonlinear element, of amplitude A and frequency ω, such as $e(t) = A\sin(\omega t)$, as displayed in Fig. 3. The output of the nonlinear element $\lambda(t)=\phi(e)$ is frequently periodic. By using Fourier series, this periodic function $\lambda(t)$ can be expanded as

$$\lambda(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}[a_n\cos(n\omega t)+b_n\sin(n\omega t)] \quad (3)$$

where the Fourier coefficients a_n 's and b_n 's are generally functions of A and ω, determined by

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}\lambda(t)d(\omega t),$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\lambda(t)\cos(n\omega t)d(\omega t), \quad (4)$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\lambda(t)\sin(n\omega t)d(\omega t),$$

If the non-linearity is an odd function, one has $a_0=0$. Furthermore, if the transfer function has the low-pass properties [4-8], i.e.,

$$|G(j\omega)|>>|G(jn\omega)| \quad \text{for } n=2,3,4.... \quad (5)$$

This assumption is called *filtering hypothesis*. In this case, the fundamental component $\lambda_1(t)$ must be considered, which can be described by

$$\lambda(t)\approx\lambda_1(t)=a_1\cos(\omega t)+b_1\sin(\omega t)=M\sin(\omega t+\theta) \quad (6)$$

where

$$M(A,\omega)=\sqrt{(a_1{}^2+b_1{}^2)} \quad \text{and} \quad \theta(A,\omega)=\tan^{-1}(a_1/b_1) \quad (7)$$

The describing function of the nonlinear element is the complex ratio of the fundamental component of the nonlinear element as defined by the input sinusoid, such as

$$N(A,\omega)=\frac{\lambda_1(t)}{e(t)}=\frac{Me^{j(\omega t+\theta)}}{Ae^{j\omega t}}=\frac{M}{A}e^{j\theta}=\frac{1}{A}(jb_1+a_1)$$

$$=\frac{1}{A\pi}\left[\int_{-\pi}^{\pi}\phi(A\sin(\omega t))\sin(\omega t)d(\omega t)+j\int_{-\pi}^{\pi}\phi(A\sin(\omega t))\cos(\omega t)d(\omega t)\right]$$

$$=\frac{j}{A\pi}\int_{-\pi}^{\pi}\phi(A\sin(\omega t))e^{-j\omega t}d(\omega t)$$

(8)

Remark 1: The describing function method is valid for the case of the feedback loop where the linear transfer function possesses low-pass filter property. According to Fig. 3, if the linear transfer function *G(s)* possesses low-pass filter property then the high-order harmonic terms in the Fourier series can be ignored

According to the definition of describing function, the characteristic equation for a feedback Lur'e problem can be expressed by

$$1+N(A)G(j\omega)=0 \quad (10)$$

4 Linear Feedback System of A N-Stage Ring Oscillator

The schematic diagram of ring oscillator is shown in Fig.1. It can be observed that in general, the number of inverters for a ring oscillator is odd [14-15]. Since the input-output DC characteristics for an inverter is shown in Fig.4. Let's assume the slope of an inverter is –k. Then for a linearized ring oscillator and latch, the feedback gain for can be described as

$$(-1)^N(k)^N \quad (11)$$

where N is the number of inverters in the feedback path.

In. Eq.(11), for a ring oscillator, the number N is odd. Therefore, the ring oscillator is a negative feedback system. As for a latch, it is a positive feedback system since n is even.

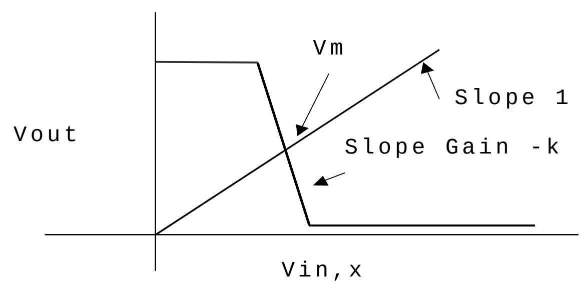

Fig. 4. Input-Output (Transfer DC) Characteristics for an Inverter

Fig.4 shows the DC transfer characteristics for an inverter, the voltage gain (i.e. the slope) of an NMOS inverter, shown in Fig.7 at the mid voltage V_M is[14]

$$k=\frac{gm1}{gm2}=\sqrt{\frac{Wn1/Ln1}{Wn2/Ln2}} \quad (12)$$

where g_{m1}, g_{m2} are the transconductance of transistors M_1, M_2 respectively and W, L are channel length and channel width for the transistors.

As to the CMOS inverter, the slope gain becomes [14]

$$k=(g_{mn}+g_{mp})(r_{on}\|r_{op}) \quad (13)$$

where $g_{mn}=\sqrt{2\frac{Wn}{Ln}K_n I_D}$, $g_{mp}=\sqrt{2\frac{Wp}{Lp}K_p I_D}$,

and r_{on}, r_{op} are output impedance of NMOS and PMOS respectively.

5 A Transformed Lur'e Problem

The overall feedback system of a ring oscillator and latch can be transformed into a Lur'e problem as shown in Fig.3 where the nonlinear element can be represented as seen in Fig.4. However, the slope is shown in Eq.(11)

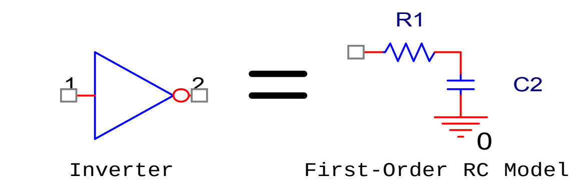

Fig. 5. Simplified first-Order RC Model for an Inverter

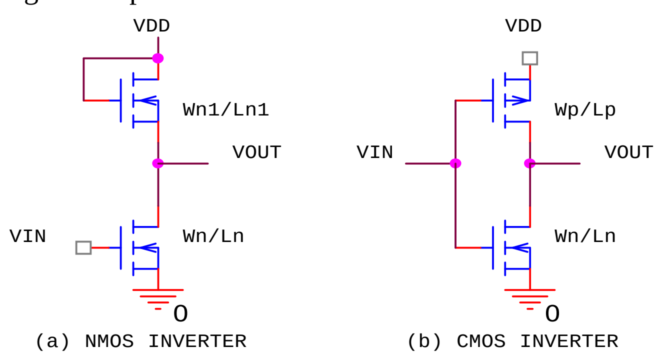

Fig. 6. The schematics of NMOS and CMOS Inverters

In this paper, we use a first-order RC delay model, shown in Fig.5, to represent the linear part of an inverter [14]. The transfer function for the first-order RC delay model can be approximated and written as

$$\frac{1}{\tau s + 1} \tag{14}$$

where $\tau = RC$ is the RC time constant.

Then, the nonlinear Input/Output transfer characteristics of an inverter can be shown as Fig.4. In Fig.4, it can be observed that the input-output characteristics can be converted to the saturation type function as Fig.7. Then the overall nonlinear model for an inverter can be approximated by

$$V_{out} = -sat\left(\frac{e}{(V_m/k)}\right) * \frac{1}{\tau s+1} \tag{14-1}$$

where * means convolution, x is the input and saturation function $Sat(.)$ can be defined as follows:

$$sat(e)=\begin{cases} e \ge 1, & 1 \\ -1 \le e \le 1, & e \\ e \prec -1, & -1 \end{cases} \tag{15}$$

and also, in Eq.(11), the time constant τ can be expressed as

$$\tau = R_{eq}C_{tot} \tag{16}$$

where $R_{eq} = R_n, or\ R_p$ and $R_n, and\ R_p$ are called the equivalent resistance for NMOS and PMOS respectively [14]. Also, C_{tot} is the effective total capacitance [14].

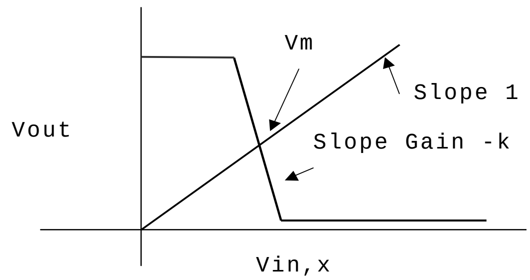

Fig.7 The Equivalent Nonlinear Function $\phi(.)$ for an inverter

Remark 2: the transfer function of an inverter can be written of Eq.(14-1). The saturation function of Eq.(14-1) is based on the input/output characteristics of an inverter.

Also, the relation between propagation delay, effective total capacitance and effective resistance can be expressed by [14].

$$\begin{aligned} \tau_{PLH} &= R_p C_{tot} \\ \tau_{PHL} &= R_n C_{tot} \end{aligned} \tag{17}$$

For the general case of a ring oscillator, let's consider the following cases (i) N=1 (Single-stage RO)(ii) N=odd number ($N = 3,5,7,\ldots$) (Multiple-stage RO)

Case (i) N=1 (single-stage RO)

When N=1, (i.e. there is only one inverter in the feedback system), this is the single-stage RO. Note that according to Eq.(13), the overall system is actually a negative feedback system. The overall system correspond to conventional nonlinear Lur'e problem of Fig.3 can be expressed by

$$\begin{aligned} \phi(e) &= sat\left(\frac{e}{(Vm/k)}\right) \\ G(s) &= \frac{1}{\tau s+1} \end{aligned} \tag{18}$$

In Eq.(10), the describing function N(A) of the nonlinear function of $\phi(.)$ of Eq.(18) can be written as [6-8]

$$N(A) = \frac{2k}{\pi}[\sin^{-1}(\frac{v_{sat}}{A}) + \frac{v_{sat}}{A}\sqrt{1-(\frac{v_{sat}}{A})^2}] \quad (19)$$

where $v_{sat} = \frac{V_m}{k}$ and A is the amplitude of oscillation.

According to the extended Nyquist diagram of Fig. 9(a), there is no intersection between the locus of $G(j\omega)$ and $-\frac{1}{N(A)}$. Note that the maximum value of $-\frac{1}{N(A)}$ is $-\frac{1}{k}$. Then the limit cycle not exists.

Also, on the other hand, it can be observed that the transfer function G(s) in Eq.(17) is a **Strict Positive Real (SPR)** function [6-8]; i.e. The locus $G(j\omega)$ of Nyquist plot is always in the fourth quadrant ($\mathrm{Re}(G(j\omega)) > 0, \forall \omega \in R$ .Then the overall system is globally asymptotically stable. This means there is no oscillation for single stage ring oscillator.

Case (ii) N=odd number ($N = 3,5,7,\ldots$) (Multiple-Stage RO)

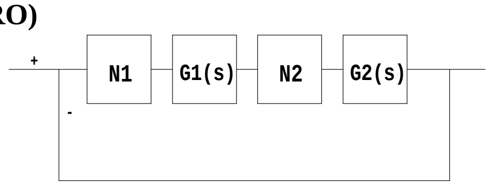

Fig. 8. General Multiple Nonlinearity Systems

Now consider the case that $N = 3,5,7,\ldots$ (multiple-stage RO). From to Eq.(11), there is a minus sign in the feedback system. This means the overall system is a negative feedback system. In this case, the overall system is actually represented a multiple nonlinear system [13] shown in Fig.8.

Then the describing function $\tilde{N}(A)$ of the overall system can be written as[14]

$$\tilde{N}(A) = N(A_{N-1})N(A_{N-2})\ldots N(A_1)N(A) \quad (20)$$

where N(A) is defined in Eq.(19) and $A_1,\ A_2, A_3,\ A_4 \ldots A_N$ satisfy

$$A_1 = AN(A)\frac{1}{|1+j\omega\tau|}$$

$$A_2 = A_1 N(A_1)\frac{1}{|1+j\omega\tau|} \quad (20\text{-}1)$$

$$\vdots$$

$$A_{N-1} = A_{N-2} N(A_{N-2})\frac{1}{|1+j\omega\tau|}$$

The derivation of Eq.(20) and Eq.(20.1), please see reference[13].

The characteristic equation of the overall system can be written as

$$1 + \tilde{N}(A)G(j\omega) = 0 \quad (21)$$

And, also the linear transfer function can be represented as

$$G(j\omega) = \frac{1}{(j\tau\omega+1)^N} \quad (21\text{-}1)$$

From the characteristic equation of Eq.(21) and In the extended Nyquist diagram of Fig.9(a), Fig.9(b)(For Describing Function), the locus of $G(j\omega)$ and $-\frac{1}{\tilde{N}(A)}$ only intersects at the frequency ω such that

$$\mathrm{Im}(\frac{1}{(j\tau\omega+1)^N}) = 0 \quad (22)$$

Then, we have

$$N \cdot Tan^{-1}(\omega\tau) = \pi + 2m \quad (23)$$

where m=0,…, $\left[\frac{N-3}{4}\right]$ and $[x]$ denotes the round the number x to the smallest integer; i.e. As $l \le x < l+1$, $[x] = l$. Moreover l is some integer.

From Eq.(23), we have the oscillation frequency for the ring oscillator

$$f_{osc} = \frac{1}{2\pi\tau}\tan(\frac{\pi(1+2m)}{N}) \quad (24)$$

Note that in Eq.(24), the unit of f is in frequency (Hz) and m=0,…, $\left[\frac{N-3}{4}\right]$.

Now, let's consider the fundamental frequency for ring oscillator, i.e. m=0, then Eq.(24) becomes

$$f_{osc} = \frac{1}{2\pi\tau}\tan(\frac{\pi}{N}) \quad (25)$$

Note that in Eq.(25), as N (the order of ring oscillator) is larger enough, then

$$\tan(\frac{\pi}{N}) \approx \frac{\pi}{N} \quad (26)$$

Therefore, with the approximation of Eq.(26), Eq.(25)

becomes

$$f_{osc} = \frac{1}{2N\tau} \qquad (27)$$

Note that Eq.(27) is the same as conventional formula for the oscillation frequency of the ring oscillator [14][15].Also, in Eq.(24) and as $m=1$, we call here the first-mode frequency of the ring oscillator. Similarly, as $m=2$, we call here the second-mode frequency of the ring oscillator and etc. The meaning of $m=2$ means that there are two intersections of for the locus of $G(j\omega)$ and negative real axis, which can be shown in Fig.9(a)

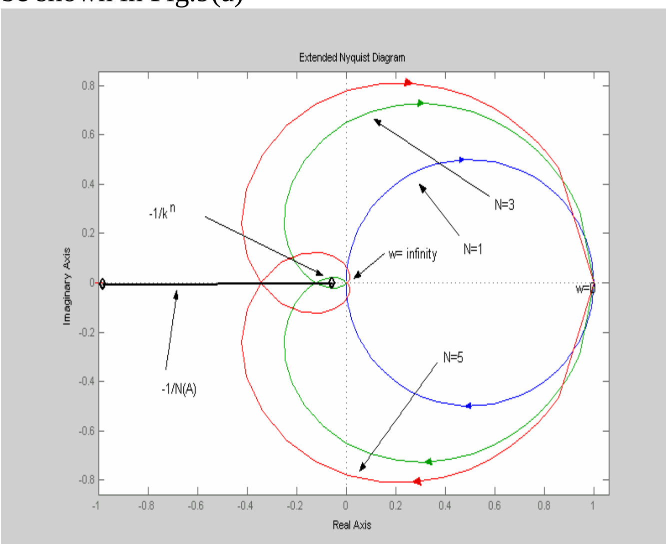

Fig.9(a) The Extended Nyquist Diagram for Ring Oscillators (N=1,3,5)

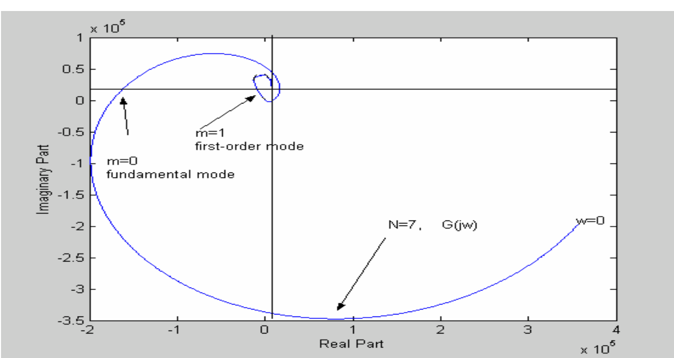

Fig. 9(b) The Extended Nyquist Diagram for Ring Oscillators (N=7)

As we know, Eq.(22) is the condition the linear transfer function $G(s)$ intersects the real axis. To guarantee the existence of limit cycle, there should be at least one intersection between the locus of $G(j\omega)$ and $-1/N(A)$.

In the sequel, we will explore the existence condition for the intersection between the locus of $G(j\omega)$ and $-1/N(A)$. From Eq.(19) and Eq.(20), it can be observed that the describing function in Eq.(19) satisfy

$$0 < N(A) \le k, \text{ As } \quad 0 < A < \infty \qquad (28)$$

Then, similarly in Eq.(20), we can have

$$0 < \tilde{N}(A) \le k^N, \text{ As } \quad 0 < A < \infty \qquad (29)$$

From Eq.(23) and substituting $\omega\tau = \tan(\frac{(2m+1)\pi}{N})$ into Eq.(21), we have

$$\mathrm{Re}\,al(\frac{1}{(j\tau\omega+1)^N}) = \cos^N(\frac{\pi(2m+1)}{N}),\ \text{m=0,1,2} \qquad (30)$$

From Eq.(29) and Eq.(30), we can conclude that for the existence of fundamental frequency of ring oscillator, it requires

$$k > Sec(\pi/N) \qquad (31)$$

where k is the voltage gain for the inverter at mid voltage V_M.

Similarly, for the existence of first-mode frequency $m=1, N \ge 7$ of ring oscillator, it requires

$$k > Sec(3\pi/N) \qquad (32)$$

For the existence of second-mode frequency $m=2, N \ge 11$ of ring oscillator, it requires

$$k > Sec(5\pi/N) \qquad (33)$$

and etc.

From Eq.(32) and Eq.(33), a "**Necessary Condition**" for the existence of fundamental and higher order frequency mode can be derived as

$$N \ge 3+4m, \quad k > k_{\min} = \sec(\frac{\pi+2m}{N}) \qquad (34)$$

where N is the number of stage for RO, m is the order of higher frequency mode (As $m=0$, it represents the fundamental mode; As $m=1$, it represents the fundamental mode.) and $k_{\min}$ is defined as the minimum required midpoint gain of the inverter.

The stability of these limit cycles for the ring oscillator can be checked by the following equations

$$N'(A)\,\mathrm{Re}(G'(j\omega_0)) > 0 \qquad (35)$$

where ω_0 (rad/s) is the oscillation frequency for RO

Note that Eq.(34) is the "**Necessary Condition**" for the existence of oscillation modes. i.e. When the voltage gain k satisfies Eq.(34), this cannot guarantee the existence of oscillation mode. However, if the voltage gain k don't satisfy Eq.(34), the oscillation modes don't exist. The multiple harmonic modes are coincide the results of some literatures [10,12]

6 Tsypkin's Method

It is known that for the nonlinear system, describing function method is only an approximate method for the derivation of frequency of RO. In this section, we will explore a more accurate method called Tsypkin method. The Tsypkin method is used the so called the Tsypkin Function to determine the exact period for a relay control system. As seen in Fig.3, a conventional Lur'e Problem. However, the non-linearity is written as

$$sign(e)=\begin{cases} e>0, & 1 \\ e<0, & -1 \end{cases} \tag{41}$$

Then the Tsypkin is defined as [8,13]

$$T(j\omega)=\sum_{k=1,3,5}^{\infty}\mathrm{Re}(G(jk\omega))+j\frac{1}{k}\sum_{k=1,3,5}^{\infty}\mathrm{Im}(G(jk\omega)) \tag{42}$$

The period of the oscillation can be determined by

$$\mathrm{Im}(T(j\omega_0))=0 \tag{43}$$

where ω_0 is the oscillation frequency.

Consider the linear transfer function G(s) of Eq.(21-1), as N=3, we have Tsypkin Function as[13]

$$\mathrm{Im}(T(j\omega))=\frac{\tau^3}{16\cosh^2(\pi/2\omega_0\tau)}(2\sinh(\pi/\omega_0\tau)-\pi^2\tanh(\pi/2\omega_0\tau)-2\,\pi/\omega_0\tau) \tag{44}$$

Substituting Eq.(44) into Eq.(43) and with the numerical solution, we have

$$f=\frac{1}{5.2602\tau} \tag{45}$$

Similarly, as N=5, we have Tsypkin Function as[13]

$$\mathrm{Im}(T(j\omega))=(4.8\cosh^4(1/2u)\sinh(1/2u)\text{-}83.21763u\cosh^3($$
$$\text{-}118.435253\cosh^2(1/2u)\sinh(1/2u)\text{-}19.87\cosh^2(1/2u)\mathrm{si}$$
$$88.82644\cosh(1/2u)u+3\cosh(1/2u)u^3+296.088133\sinh($$
$$\tag{46}$$

where $u=\pi/\omega_0\tau$

Also, from Eq.(43) and (46) and with the numerical solution, we have

$$f=\frac{1}{8.7726\tau} \tag{47}$$

In general, for the N-order Tsypkin Function, we can have

$$\mathrm{Im}(T(j\omega))=\frac{d^N}{da^N}\left(\frac{-\pi}{4a}\tanh\left(\frac{\pi a}{2\omega}\right)\right) \tag{48}$$

where $a \triangleq 1/\tau$ .The derivation of Eq.(48) can be derived from reference [13], which is neglected here.

7 Examples and Illustrations

In this section, we will illustrate the above results with two simulation examples.

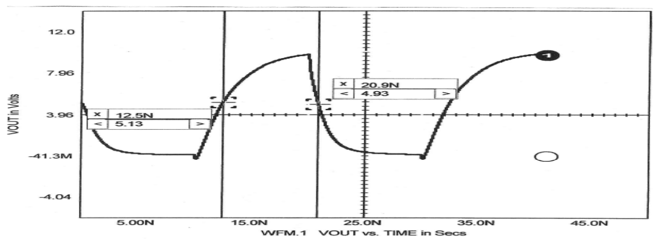

Fig.10. Simulation of Propagation Delays for Inverter

Example 1: Let's consider a N-stage ring oscillator. The 3-stage CMOS inverter is shown in Fig.3. The propagation delay is shown in Fig.10 can be obtained as (by simulation)

$$\tau_{av}=\frac{\tau_{phl}+\tau_{plh}}{2}=\frac{(12.5-10)+(20.9-20)}{2}=\frac{2.5ns+0.9ns}{2}=1.7ns \tag{47}$$

where τ_{av} means as the average propagation delay and τ_{phl},τ_{plh} are the high to low and low to high propagation delay respectively.

The simulation results of 3-stage, 5-stage, 7-stage, and 9-stage ring oscillators are shown in Fig10-Fig.14 Respectively. The simulation results can be shown in Table1.

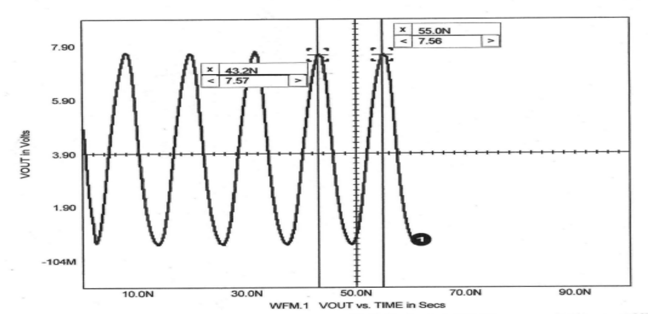

Fig.11. Simulation of Period for 3-Stage Ring Oscillator T=11.8 ns

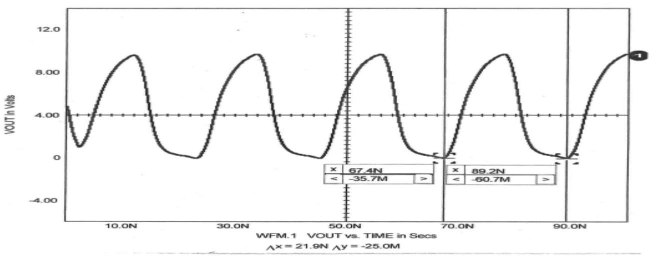

Fig.12. Simulation of Period for 5-Stage Ring Oscillator T=21.9 ns

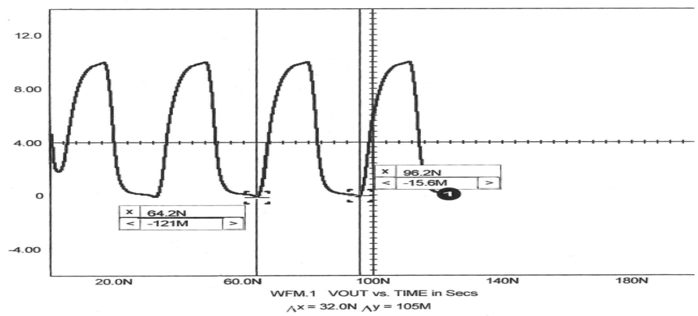

Fig.13. Simulation of Period for 7-Stage Ring Oscillator T=32 ns

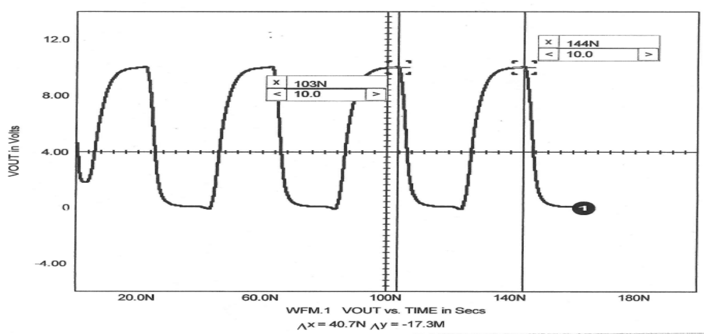

Fig.14. Simulation of Period for 9-Stage Ring Oscillator T=40.7 ns

As for the conventional formula for the N-stage ring oscillator, the period of N-stage ring oscillator can be written as

$$T = 2N\tau_{av} \qquad (48)$$

However, with the describing function method of Eq.(25), the period of N-stage ring oscillator can be written as

$$T = \frac{2\pi\tau}{\tan(\frac{\pi}{N})} \qquad (49)$$

Also, with the simplified model of transfer function of Eq.(16), we can have [9,17]

$$\tau_{av} = T_{50-50} = (\ln 0.5)\tau = 0.693\tau \qquad (50)$$

where $\tau = RC$ is the RC time constant.

The simulation is performed by P-spice Software. The NMOS and PMOS Spice parameters can be described as follows:

NMOS (LEVEL=1 VTO=0.80 KP=1.00E-02 GAMMA=1.08E-06 PHI=0.75 LAMBDA=1.40E-02 RD=3.00E+01 RS=3.60E+01 IS=1.12E-14+ CBD=5.13E-12 CBS=6.16E-12 PB=0.80 MJ=.46 +CGSO=3.60E-09+ CGDO=3.00E-09 CGBO=2.34E-08)

PMOS (LEVEL=1 VTO=-2.2 KP=2.5M GAMMA=5.43U+ PHI=.75 LAMBDA=2.14M RD=56 RS=56 IS=10.7F PB=.8 MJ=.46+ CBD=9.46P CBS=11.3P CGSO=11.7N CGDO=9.75N CGBO=16.0N)

The comparisons of simulations, convention formula of Eq.(48), theoretical describing function of Eq.(49) and Tsypkin function of Eq.(45-47) can be shown as the following Table.

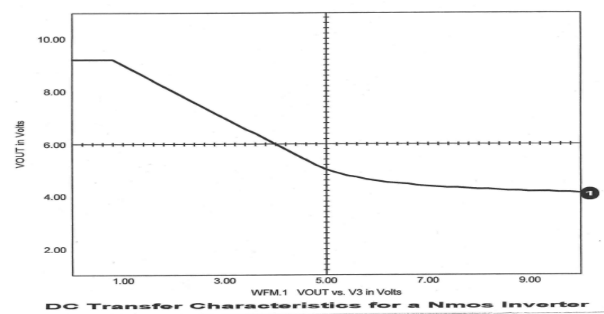

Fig.15. DC Input/Output Transfer Characteristic for A NMOS Inverter

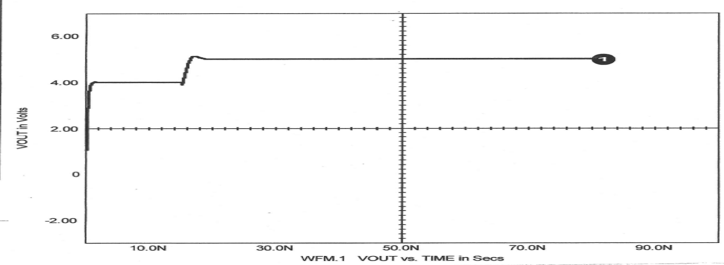

Fig.16. Simulation for a 3-stage NMOS Ring Oscillator (Example 2)

	3-stage ring oscillator	5-stage ring oscillator	7-stage ring oscillator	9-stage ring oscillator
Simulations	11.8 ns	21.9 ns	32.0 ns	40.7 ns
Conventional formula of Eq. (48) Error %	10.2 ns −13.6%	17ns −22.3%	23.8ns −25.6%	30.6ns -24.8%
Describing Function of Eq.(49) Error %	8.9ns -24.5%	21.21ns −3.1%	32ns -0.01%	42.34ns +4%
Tsypkin Function of Eq.(43)-(45) Error %	12.7ns +7.6%	21.52ns −1.7%	×	×

Table 1: The comparison of simulation and several theoretical predications of the period for N-stage ring oscillators

From Table 1, it can be observed that with the describing function method, the theoretical prediction of the period of the limit cycle has very high accuracy (especially $N \geq 5$, the error is less than 3%). However, with Tsypkin Function method, for 3-stage and 5-stage ring oscillators is more accurate than describing function method.

Secondly, let's consider another case of a NMOS inverter of Fig.6(a). A NMOS inverter as shown in Fig.6(a). The voltage gain of this NMOS inverter can be obtained from Eq.(12) as (at the mid-voltage)

$$k = -\frac{gm1}{gm2} = -\sqrt{\frac{Wn1/Ln1}{Wn/Ln}} = -1 \quad (53)$$

It can be observed that the voltage gain at the mid-voltage is not large enough such that it not satisfies Eq.(32). From previous results, we can conclude that the limit cycle not exists. (Note that the condition of Eq.(32) is the so-called the **"Necessary Condition"**. i.e. If the condition of Eq.(32) is satisfied, it cannot guarantee the existence of limit cycle (it may exist). However, if the condition of Eq.(32) is not satisfied , the limit cycle is not existing).

The DC Input/Output transfer characteristic curve can be shown as of Fig.15. It can be observed the voltage gain at the min-voltage is –1 (slope). The simulation results for a three-stage ring oscillator with a NMOS inverter is shown in Fig.16. From Fig.16, it can be observed that this three-stage ring oscillator cannot oscillate. It coincides with the theoretical predictions.

The following Example is directly adopted from the reference [14].

Example 2: Estimate the CN20 process (Orbit Semiconductor's 2.0 μm double-poly, double-metal, n-well process).[14] (The spice parameters are listed in Appendix A). Also use hand analysis of five-stage ring oscillator with $W_n = W_p = 10um$). Also, compare it with the simulation results (with SPICE).

The effective resistance of n- and p-channel MOSFETs are

$$R_{n1} = 12k \cdot \frac{2\mu m}{10\mu m} = 2.4k\Omega \quad (54)$$
$$R_{p2} = 36k \cdot \frac{2\mu m}{10\mu m} = 7.2k\Omega$$

The total capacitance on the output of any inverter is the sum of its own output capacitance and the input capacitance of the next (identical) stage. This is given by

$$C_{tot} = C_{in} + C_{out} = \frac{5}{2} C_{ox}(W_n L_n + W_p L_p) = 80fF \quad (55)$$

Thus,

$$\tau = (R_{n1} + R_{p2}) C_{tot} / 2 = (2.4k + 7.2k) \cdot 80fF / 2 = 384ps \quad (56)$$

The oscillation frequency, from Eq.(2) is then

$$f_{osc} = \frac{1}{10 \cdot 384ps} = 260 \text{ MHz} \quad (57)$$

The SPICE simulation results are shown in Fig.17. SPICE gives a f_{osc} of approximately 300 MHz [14]. From describing function method of Eq.(25), we have

$$f_{osc} = \frac{1}{2\pi(386ps)} \tan(\frac{\pi}{3}) = 301MHz \quad (58)$$

Also, by Tsypkin function of Eq.(43)-Eq.(45), we have

$$f_{osc} = \frac{1}{8.77(386ps)} = 299MHz \quad (59)$$

It can be observed that with Describing Function and Tsypkin function methods, the predictions of oscillation frequency are very accurate such that both only have (0.3%) errors. However, with the conventional formula, it has 15.6% error.

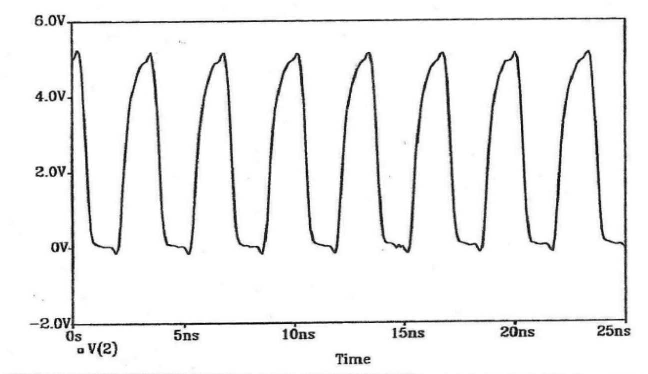

Fig. 17. Simulation for a 5-stage Ring Oscillator (CN20 Process)

8 Conclusions & Discussions

In this paper, we proposed a rigorous approach to analyze the nonlinear feedback of an inverter-based ring oscillator. It can be shown that the (RO) ring oscillator can be approximated by a nonlinear Lur'e problem. With this Lur'e problem, Circle criterion can be applied to determine the stability of overall system. A simple first-order RC delay model is used to approximated by the model of the inverter.

The describing function method is used to determine the oscillation frequency of the ring oscillator. A new formula is presented. It can be observed that if N (number of inverters) is large enough, the proposed formula will approach the conventional formula. Moreover, with extended Nyquist Criterion and describing function method, a "**Necessary Condition"** for the existence of oscillation (all modes) is shown as Eq.(34). It can be shown that as $N \geq 7$ and the voltage gain is large enough, the higher harmonic oscillation may exists. These results coincide with previous research reports [10][12]. Also, as $N \leq 5$, the higher order harmonics will not exist .

On the other hand, with Tsypkin Function method, a more accurate formula for the oscillation frequency of ring oscillator will be presented. Finally, Simulation examples already have verified these results.

It should be further stress that the above results can be applied not only MOS transistor and can be also applied to Bipolar transistor. Also, for a RO, the voltage gain k should be large enough; otherwise, the oscillation doesn't exist. In general, for the CMOS transistor the voltage gain is large enough since the output resistance (as seen in Eq.(13))for NMOS and PMOS transistors are very large. However, for NMOS inverters, the voltage gain may not be large enough [14,16]. Therefore, for a NMOS ring oscillator design, suitable channel length and width's selection should be further investigated [14,16]. Also, in this paper, we only address the ring oscillator with the number of inverter N is odd. However, in some cases, even N is even, the oscillation might be exist .

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